CAT 2024 Slot 3 Question Paper

Quant

47. In a group of 250 students, the percentage of girls was at least 44% and at most 60%.The rest of the students were boys. Each student opted for either swimming or running or both. If 50% of the boys and 80% of the girls opted for swimming while 70%of the boys and 60% of the girls opted for running, then the minimum and maximum possible number of students who opted for both swimming and running, are

A 72 and 88, respectively

B 75 and 96, respectively

C 72 and 80, respectively

D 75 and 90, respectively

48. If $(a+b\sqrt{3})^2 = 52 + 30\sqrt{3}$, where a and b are natural numbers, then $a+b$ equals

A 7

B 8

C 9

D 10

49. The average of three distinct real numbers is 28. If the smallest number is increased by 7 and the largest number is reduced by 10, the order of the numbers remains unchanged, and the new arithmetic mean becomes 2 more than the middle number, while the difference between the largest and the smallest numbers becomes 64.Then, the largest number in the original set of three numbers is

A 5

B 8

C 9

D 4

51. Sam can complete a job in 20 days when working alone. Mohit is twice as fast as Sam and thrice as fast as Ayna in the same job. They undertake a job with an arrangement where Sam and Mohit work together on the first day, Sam and Ayna on the second day, Mohit and Ayna on the third day, and this three-day pattern is repeated till the work gets completed. Then, the fraction of total work done by Sam is

A $\frac{1}{20}$

B $\frac{3}{10}$

C $\frac{1}{5}$

D $\frac{3}{20}$

52. A circular plot of land is divided into two regions by a chord of length $10\sqrt{3}$ meters such that the chord subtends an angle of 120° at the center. Then, the area, in square meters, of the smaller region is

A $20\left(\frac{4\pi}{3}+\sqrt{3}\right)$

B $25\left(\frac{4\pi}{3}+\sqrt{3}\right)$

C $20\left(\frac{4\pi}{3}-\sqrt{3}\right)$

D $25\left(\frac{4\pi}{3}-\sqrt{3}\right)$

53.
Consider the sequence $t_1 = 1, t_2 = -1$ and $t_n = \left(\frac{n-3}{n-1}\right) t_{n-2}$ for $n \geq 3$. Then, the value of the sum

$\frac{1}{t_2} + \frac{1}{t_4} + \frac{1}{t_6} + \ldots\ldots + \frac{1}{t_{2022}} + \frac{1}{t_{2024}}$, is

A -1024144

B -1022121

C -1023132

D -1026169

54. The number of distinct real values of x, satisfying the equation $max\ \{x, 2\} - min\ \{x, 2\} = \mid x + 2 \mid - \mid x - 2 \mid$, is

55. Aman invests Rs 4000 in a bank at a certain rate of interest, compounded annually. If the ratio of the value of the investment after 3 years to the value of the investment after 5 years is 25 : 36, then the minimum number of years required for the value of the investment to exceed Rs 20000 is

56. The sum of all distinct real values of x that satisfy the equation $10^x + \frac{4}{10^x} = \frac{81}{2}$, is

A $2 \log_{10} 2$

B $4 \log_{10} 2$

C $\log_{10} 2$

D $3 \log_{10} 2$

57. A train travelled a certain distance at a uniform speed. Had the speed been 6 km per hour more, it would have needed 4 hours less. Had the speed been 6 km per hour less, it would have needed 6 hours more. The distance, in km, travelled by the train is

A 720

B 800

C 780

D 640

58. If $3^a = 4, 4^b = 5, 5^c = 6, 6^d = 7, 7^e = 8$ and $8^f = 9$, then the value of the product abcdef is

59. Gopi marks a price on a product in order to make 20% profit. Ravi gets 10% discount on this marked price, and thus saves Rs 15. Then, the profit, in rupees, made by Gopi by selling the product to Ravi, is

A 10

B 25

C 15

D 20

60. A certain amount of water was poured into a 300 litre container and the remaining portion of the container was filled with milk. Then an amount of this solution was taken out from the container which was twice the volume of water that was earlier poured into it, and water was poured to refill the container again. If the resulting solution contains 72% milk, then the amount of water, in litres, that was initially poured into the container was

61. A regular octagon ABCDEFGH has sides of length 6 cm each. Then the area, in sq. cm, of the square ACEG is

A $72(2+\sqrt{2})$

B $36(1+\sqrt{2})$

C $72(1+\sqrt{2})$

D $36(2+\sqrt{2})$

62. The number of distinct integer solutions (x, y) of the equation $\mid x+y \mid + \mid x-y \mid = 2$, is

63.
For any non-zero real number x, let $f(x)+2f\left(\frac{1}{x}\right)=3x$. Then, the sum of all possible values of x for which $f(x)=3$, is

A 3

B -2

C -3

D 2

64. For some constant real numbers p, k and a, consider the following system of linear equations in x and y:
px - 4y = 2
3x + ky= a
A necessary condition for the system to have no solution for (x, y), is

A $ap+6=0$

B $2a+k\neq 0$

C $ap-6=0$

D $kp+12\neq 0$

65. Rajesh and Vimal own 20 hectares and 30 hectares of agricultural land, respectively, which are entirely covered by wheat and mustard crops. The cultivation area of wheat and mustard in the land owned by Vimal are in the ratio of 5 : 3. If the total cultivation area of wheat and mustard are in the ratio 11 : 9, then the ratio of cultivation area of wheat and mustard in the land owned by Rajesh is

A 4 : 3

B 7 : 9

C 3 : 7

D 1 : 1

66. The midpoints of sides AB, BC, and AC in ΔABC are M, N, and P, respectively. The medians drawn from A, B, and C intersect the line segments MP, MN and NP at X, Y, and Z, respectively. If the area of ΔABC is 1440 sq cm, then the area, in sq cm, of $\triangle XYZ$ is

67. The number of all positive integers up to 500 with non-repeating digits is

68. After two successive increments, Gopal's salary became 187.5% of his initial salary. If the percentage of salary increase in the second increment was twice of that in the first increment, then the percentage of salary increase in the first increment was

A 30

B 27.5

C 25

D 20

Answers

47.**C**	48.**B**	49.**70**	50.**C**	51.**B**	52.**D**	53.**A**	54.**2**
55.**9**	56.**A**	57.**A**	58.**2**	59.**A**	60.**30**	61.**D**	62.**8**
63.**C**	64.**B**	65.**B**	66.**90**	67.**378**	68.**C**		

Explanations

47.**C**
Total number of students is 250, and we are told that, The percentage of girls was at least 44% and at most 60%.

So the number of girls range from, $0.44\,(250) \leq Girls \leq 0.6\,(250)$
$110 \leq Girls \leq 150$

Statement 1:
If 50% of the boys and 80% of the girls opted for swimming, that means if the total number of Boys is B, Girls is G where B+G=250.
Swimming is: 0.5B+0.8G

Statement 2:
If 70%of the boys and 60% of the girls opted for running, that means
Running is 0.7B+0.6G

Total number of enrolments for swimming and running together will be
(0.7B+0.6G)+(0.5B+0.8G)=1.2B+1.4G

Using the overlapping principle, where I represents people who have enrolled only for one activity and II represents number of people who have enrolled for two activities.
We know that, $I + II = 250 = B + G$
$I + 2II = 1.2B + 1.4G$

Subtracting the two equations,
$II = 0.2B + 0.4G$
$II = 0.2\,(B + 2G)$
Using B+G=250
$II = 0.2\,(250 + G)$

G can at-most be 150 and at least 110.

So maximum value of II will be $0.2\,(250 + 150) = 80$

Minimum value of II will be $0.2\,(250 + 110) = 72$

48. **B**
Opening the square on the left-hand side, we get $a^2 + 3b^2 + 2ab\sqrt{3}$
Comparing the rational part on both side,s we get: $a^2 + 3b^2 = 52$
And comparing the irrational par,t we get: $2ab\sqrt{3} = 30\sqrt{3}$

$ab = 15$, Since we are given that a and b are natural numbers, the possible values of a and b are (1,15), (3, 5), (5, 3), or (15,1)

Putting these values in the first relation we got, we see that 15 squared would exceed the required value and would not be the case.
We need not check if a=5, b=3 or a=3, b=5 since the answer would be the same.

(a=5 and b=3 would satisfy it)

a+b = 5+3 = 8

Therefore, Option B is the correct answer.

49. **70**
We are given that average of three distinct integers is 28, that means the sum of these three integers is 28x3=84

Let us write $x + y + z = 84$
x, y, z being the three distinct integers in ascending order.

If the smallest number is increased by 7 and the largest number is reduced by 10
$(x + 7) + (y) + (z - 10) = 81$

New arithmetic mean will be $\frac{81}{3} = 27$
And this is said to be 2 more than the middle number, meaning
$27 - 2 = y = 25$

$x + z = 59$

We are given that difference between the largest and the smallest numbers becomes 64,
$(z - 10) - (x + 7) = 64$
$z - x = 81$

Adding the two equations we get, $2z = 140$
$z = 70$

50. **C**
There are multiple ways of solving such questions involving remainders; one easy way is to look for a power of numerator that leaves a remainder of 1 or 01 when divided by the denominator.

In this instance, 1000, when divided by 13, leaves a remainder of -1

We can rewrite the numerator as $\frac{10^{66} \times 100}{13}$

The remainder would be $\left[\frac{10^{66}}{13}\right]_R \times \left[\frac{100}{13}\right]_R$
$(-1)^{22} \times 9$
9

Therefore, Option C is the correct answer.

51. **B**

We are given that Sam completes a piece of work in 20 days. We are also given that Mohit is twice as fast, so he should take only 10 days. Mohit is thrice as fast as Ayna, so he would take 30 days.

Let's take the total work to be 60 units; this would give the work done per day for Mohit, Sam, and Ayna to be 6, 3 and 2, respectively.

On the first day Sam and Mohit work: doing 9 units
On the second day Sam and Ayan work: doing 5 units
On the third day, Mohit and Ayan work: doing 8 units

Essentially doing 22 units in a 3 days cycle.
After two such cycles, there will be 60-44 = 16 units of work left

On day 7, Sam and Mohit would work 9 units, leaving 7 units
On day 8, Sam and Ayan would work 5 units, leaving 2 units
And on day 9, Ayan and Mohit would complete the remaining work.

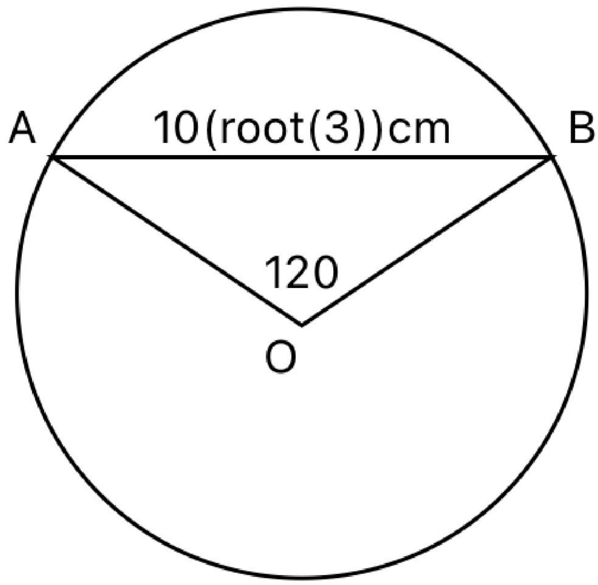
So Sam worked for a total of 2+2+2= 6 days and on each day he did 3 units of work, completing 18 units of work.

The ratio of work done by Sam would be $\frac{18}{60} = \frac{3}{10}$

Therefore, Option B is the correct answer.

52. **D**

This is the situation that is described in the question above, Angle AOB is 120 degrees and the chord AB is of length 10cm.



Using Cosine rule we can find the length of AO and AB

$\cos(AOB) = \frac{\left(r^2+r^2-300\right)}{2r^2}$

$-\frac{1}{2} = \frac{\left(2r^2-300\right)}{2r^2}$

$3r^2 = 300$

$r = 10$

Area of the Sector AOB is $\frac{120}{360} \times \pi \times r^2$
$\frac{1}{3} \times \pi \times \frac{100}{1}$ which is $\frac{100\pi}{3}$
Area of triangle AOB is $\frac{1}{2} \times r^2 \times \sin(120)$
$\frac{1}{2} \times \frac{100}{1} \times \frac{\sqrt{3}}{2}$
Area of triangle AOB is $\frac{75}{\sqrt{3}}$
The smaller region will be, $\frac{100\pi}{3} - \frac{75}{\sqrt{3}}$
Taking 25 common we will get,
$$25\left(\frac{4\pi}{3} - \sqrt{3}\right)$$

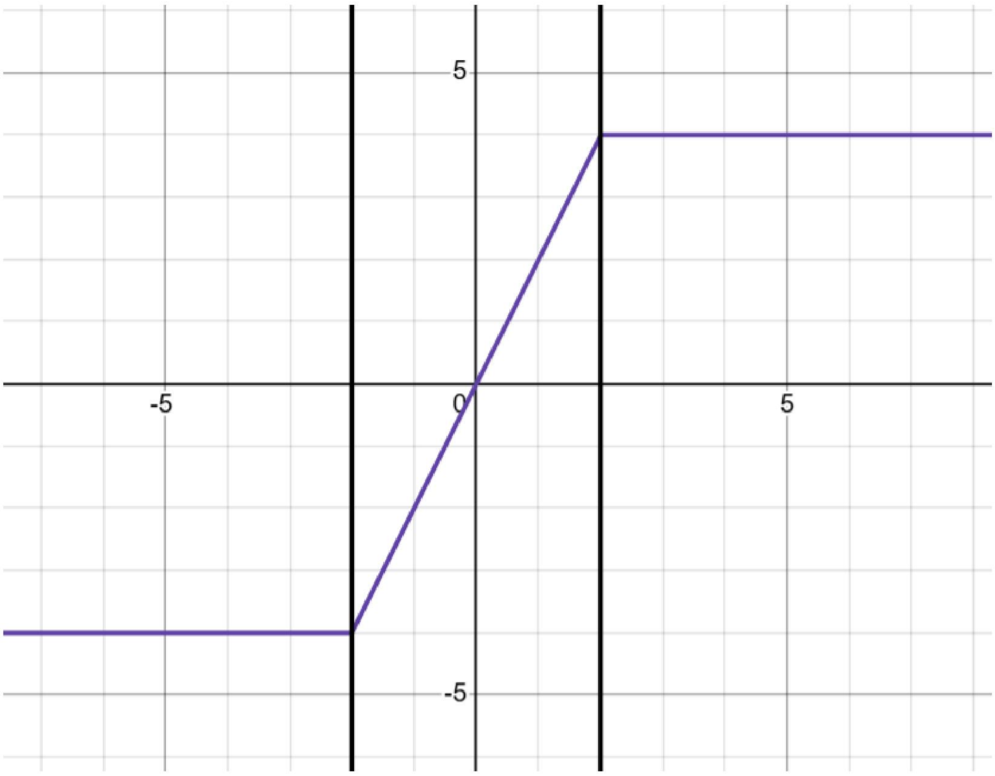
53. **A**
Finding the terms in the sequence, we see that $t_3 = 0, t_4 = -\frac{1}{3}, t_5 = 0$
We would notice that all the odd terms are 0, and we are also asked the sum of only even terms, so we do not need to consider those
$$t_6 = -\frac{1}{5}$$
We see that the even terms are in an HP: $-1,\ -\frac{1}{3},\ -\frac{1}{5},\ -\frac{1}{7},\ \ldots$
The sum we are asked is the inverse of these terms, that is: -1, -3, -5, -7, up to 1012 terms
The sum of this AP would be $\frac{[-(2\times 1) + (1012-1)(-2)]}{2} \times 1012$
Which is equal to $-1012 \times 1012 = -1024144$
Therefore, Option A is the correct answer.

54. **2**
The expression on the right-hand side will have two critical points: 2 and -2
For any value of **x greater than equal to 2**, the equation changes to x+2-(x-2) = 4
the value of min{x,2} would be 2, so we would want max{x,2} to be 4+2 = 6
Therefore, x=6 works.
For any value of **x less than equal to -2,** the equation changes to -(x+2)+(x-2) = -4
the value of max{x,2} would be 2, so we would want min{x,2} to be 6 again; this cannot be the case.
In general, subtracting the smaller (min) of the two values from the bigger (max) can not lead to a negative number. The max it can lead to is a 0
When **x lies between -2 and 2,** the equation becomes 2x
The maximum function will give back 2, and the minimum function will give back x, with the right-hand side giving 2x
Solving this we would get 2-x = 2x which is x=2/3

Therefore, there are two real values of x for which the given equation holds.

Hence, 2 is the correct answer.

The graph of the function on the right hand side can be visualised as:



Hence, 2 is the correct answer.

55. **9**

Let us assume the amount invested to be P, and the rate of interest to be r.
Value of the investment after 3 years will be, $P\left(1+r\right)^3$
Value of the investment after 5 years will be, $P\left(1+r\right)^5$

$\left(1+r\right)^2 = \frac{36}{25}$
$\left(1+r\right)^2 = 1.44$
$r = 0.2$

We need to find the value of n for which $4000\left(1+r\right)^n > 20000$
$\left(1+r\right)^n > 5$
$\left(1.2\right)^n > 5$
We see that,
$1.2^8 = 4.2999$
$1.2^9 = 5.15$

Hence it takes 9 years to grow to over 20,000.

56. **A**
Taking $10^x = a$
we get $a + \frac{4}{a} = \frac{81}{2}$
This would give the quadratic equation: $2a^2 - 81a + 8 = 0$

We want to find the sum of possible values of x, let the value of x be x1 and x2

these would correspond to log a1, and log a2

The sum of log a1 + log a2 would be log (a1 x a2)

From the quadratic equation we got above, we can see that the product of the possible values of a would-be 8/2 = 4

Threfore, the sum of values of x would be log (4) which would be $2\ \log_{10} 2$

Therefore, Option A is the correct answer.

57. **A**
Let us assume that the distance is D, speed of the train is S and time taken by the train is t.

t is nothing but $\frac{D}{S}$

Statement 1: Had the speed been 6 km per hour more, it would have needed 4 hours less
$\frac{D}{S+6} = t - 4$

$\frac{D}{S+6} = \frac{D}{S} - 4$

$4 = \frac{D}{S} - \frac{D}{S+6}$

$\frac{S+6-S}{S(S+6)} = \frac{4}{D}$

$\frac{6}{S(S+6)} = \frac{4}{D}$

$D = \frac{2S(S+6)}{3}$
Statement 2: Had the speed been 6 km per hour less, it would have needed 6 hours more
$\frac{D}{S-6} = t + 6$

$D\left[\frac{1}{S-6} - \frac{1}{S}\right] = 6$

$\frac{S-S+6}{S(S-6)} = 6$

$D = S\left(S - 6\right)$

Equating the two equations for distance,
$S\left(S - 6\right) = \frac{2S(S+6)}{3}$

$3S - 18 = 2S + 12$

$S = 30$

Hence the speed is 30 kmph

58. **2**

Taking a log for each of the expressions, we get the following:

$\log_3 4 = a,\ \log_4 5 = b,\ \log_5 6 = c,\ \log_6 7 = d,\ \log_7 8 = e,\ \log_8 9 = f$

The expression $abcef$ would then be: $\log_3 4 \times\ \log_4 5 \times\ \log_5 6 \times\ \log_6 7 \times\ \log_7 8 \times\ \log_8 9$

Next, we can use this property of log: $\frac{\log_b a}{\log_b c} = \log_c a$

Using this, we get:

$\frac{\log\ 4}{\log\ 3} \times \frac{\log\ 5}{\log\ 4} \times \frac{\log\ 6}{\log\ 5} \times \frac{\log\ 7}{\log\ 6} \times \frac{\log\ 8}{\log\ 7} \times \frac{\log\ 9}{\log\ 8}$

All the terms will cancel out except: $\frac{\log\ 9}{\log\ 3} = \log_3 9 = 2$

Therefore, 2 is the correct answer.

59. **A**

Let us say the cost price of an item is X

It is said that it is marked to make a profit of 20%.

That means it is marked at 1.2X

Ravi gets a 10% discount on the marked price,

$0.9\,(1.2X) = 1.08X$

Saves 15 rupees, so 1.2X-1.08X

0.12X=15

X=125

Profit made by Gopi is 0.08(125)=10 rupees.

60. **30**

Let us assume the amount of Milk in the container to be X and the amount of water in the container to be Y.

We are told that X+Y=300.

Now, an operation is given where "an amount of this solution was taken out from the container which was twice the volume of water that was earlier poured into it, and water was poured to refill the container again"

Volume of the water initially is Y. If twice that amount is taken out, the percentage of the solution that is taken out will be, $\frac{2Y}{X+Y}$

That means the quantity of milk that will remain in the solution will be, $X\left(1 - \frac{2Y}{X+Y}\right)$

This value is given to be 72%, 72% of 300 will be 216

$X\left(1 - \frac{2Y}{X+Y}\right) = 216$

$X\left(\frac{X+Y-2Y}{X+Y}\right) = 216$

Writing $X = 300 - Y$

$(300 - Y)\,(300 - 2Y) = 64800$

Expanding this we have,

$2Y^2 - 900Y + 25200 = 0$

Factorising this equation we have,

$2\,(Y - 30)\,(Y - 420) = 0$

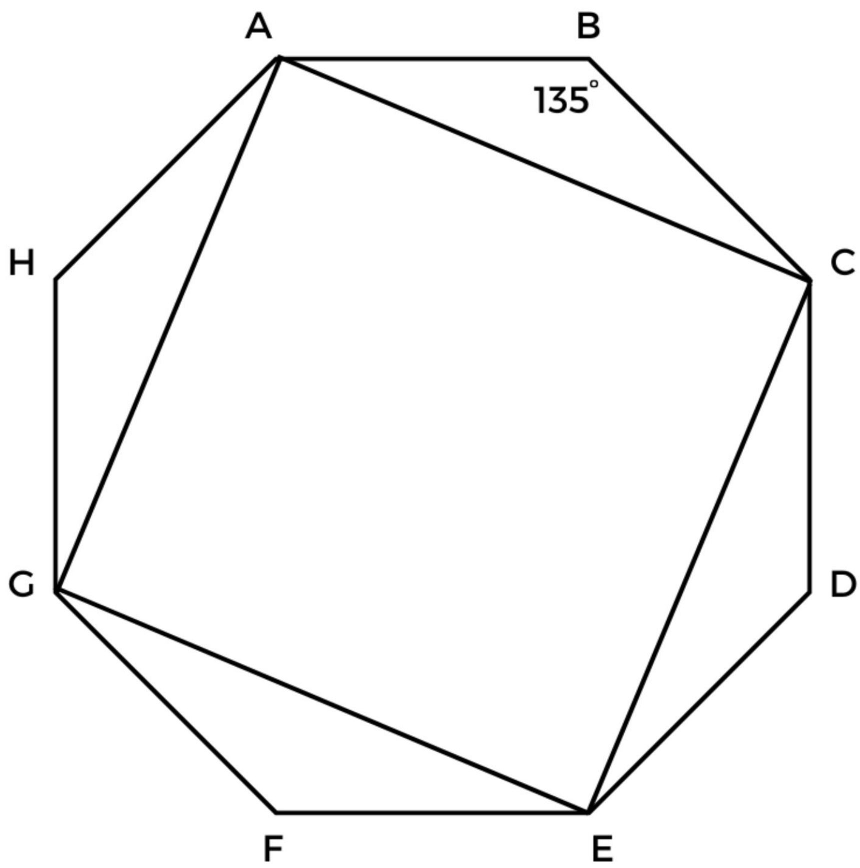
Y is either 30 or 420.

Given that the capacity of the container itself is 300, Y has to be 30.

Hence the amount of water initially is 30 Litres.

61. **D**

This is the figure in the question,



We are given that each side is 6cm long,

To find the side AC, we can use cosine rule, since we know each interior angle of the octagon(which is 135 degrees)

$$\cos\left(\angle ABC\right)=\frac{\left(AB^2+BC^2-AC^2\right)}{2\left(AB\right)\left(AC\right)}$$

$$\cos\left(135\right)=\frac{\left(36+36-AC^2\right)}{2\left(36\right)}$$

$$-\frac{1}{\sqrt{2}}=\frac{\left(72-AC^2\right)}{72}$$

$$AC^2=72+36\sqrt{2}$$

$$AC^2=36\left(2+\sqrt{2}\right)$$

Since AC is the side of the square, and the area of a square is square of the side.

Answer is $36\left(2+\sqrt{2}\right)$

62. **8**
The moduli will give out only non-negative outputs, and since we are to consider only integer values of x and y, this drastically reduces the possible cases.

We can get 2 from either 2+0 or 1+1

We get a 2+0 form when either the first term or the second term is 0
The second term is 0; this is when x=y, in this case,|2x|=2, where x can be 1 or -1; therefore, the two cases are (1,1) and (-1,-1)
The first term is 0; this is the case when x = -y, in this case, |x- (-x)|=2, giving x=1 or -1 yet again, here the two cases are (1,-1) and (-1,1)

The other way we can get 2 is through 1+1

This is possible when one of the terms is 0; if y=0, |x|+|x|=2, where x can be 1 or -1, giving two cases (1,0) and (-1,0)
Similarly,y for x=0, we get two cases, (0,1) and (0,-1)

Therefore, there are 8 pairs of (x,y) that satisfy the given equation.

63. **C**

We are given, $f(x)+2f\left(\frac{1}{x}\right)=3x$

Substituting $\frac{1}{x}$ $for\ x$

$$f\left(\frac{1}{x}\right)+2f\left(x\right)=\frac{3}{x}$$

Multiplying the second equation by 2 we will have

$$2f\left(\frac{1}{x}\right)+4f\left(x\right)=\frac{6}{x}$$

Subtracting the first equation from the second equation we have,

$3f\left(x\right)=\frac{6}{x}-3x$

$f\left(x\right)=\frac{2}{x}-x$

We want the sum of values when this function equals 3,

$\frac{2}{x}-x=3$

$x^2+3x-2=0$

Since the discriminant is greater than zero, both values of x will be real, and we can directly take the sum of values of x to be,

$-\frac{3}{1}$

Answer is -3.

64. **B**
Arranging the equation, we know that there are no solutions when the lines are parallel:
for that the condition had to $\frac{p}{3}=-\frac{4}{k}\neq\frac{2}{a}$

Checking through options:
Option A: using the first and last terms in our relation, we see that ap must not be equal to 6 for the lines to be parallel. This option puts no conditions on that and thus is not relevant.

Option C: This question is the opposite of what we want; if this is true, the lines can never be parallel.

Option D: Using the first and second terms of the relation, we see that we want kp = -12, or kp-12 = 0. Hence, this statement is not what we want.

Option B: Using the second and third terms, we see that we do not want k equals to -2a, or we do not want k+2a to be equal to 0

Therefore, B is a condition that is necessary for the lines to be parallel and have no solution.

Therefore, Option B is the correct answer.

65. **B**

We are told that Rajesh manages 20 hectares and Vimal manages 30 hectares

For Vimal, we know the distribution of the land between Wheat and Mustard, 5:3
So, wheat area will be, $\frac{5}{8}(30)$
Mustard area will be, $\frac{3}{8}(30)$

Similarly, let us assume that the distribution of crops between Wheat and Mustard to be k:1
Wheat will be, $\frac{k}{k+1}(20)$
Mustard will be, $\frac{1}{k+1}(20)$

We are told that total area of Wheat and Mustard is in the ratio 11:9

Adding them up we get,

$$\frac{\left(\frac{150}{8}+\frac{20k}{k+1}\right)}{\left(\frac{90}{8}+\frac{20}{k+1}\right)}=\frac{11}{9}$$

$$\frac{\left(\frac{15}{8}+\frac{2k}{k+1}\right)}{\left(\frac{9}{8}+\frac{2}{k+1}\right)}=\frac{11}{9}$$

$$\frac{135}{8}+\frac{18k}{k+1}=\frac{99}{8}+\frac{22}{k+1}$$

$$\frac{36}{8}=\frac{22-18k}{k+1}$$

$$44-36k=9k+9$$

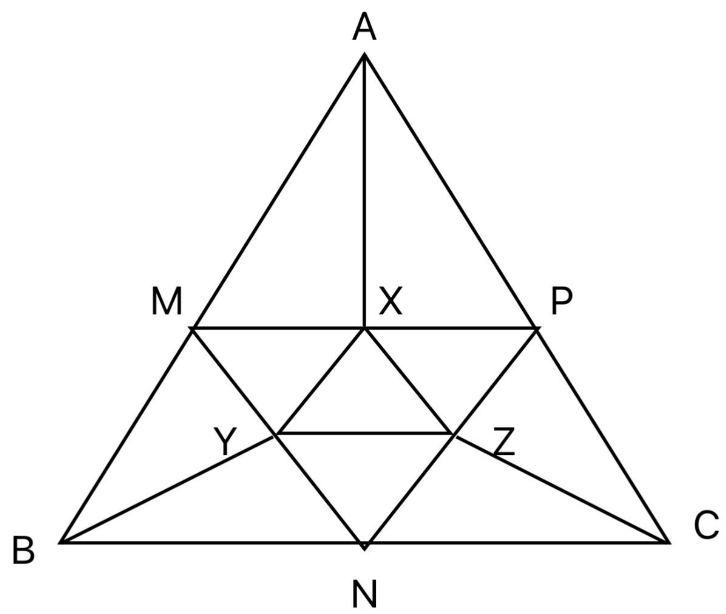
$$45k=35$$

$$k=\frac{7}{9}$$

Hence the ratio of distribution of area between Wheat and Mustard for Rajesh is $\frac{7}{9}$

66. **90**

The question describes a figure of the following nature,



We are told that the area of the triangle ABC is 1440.
By proportionality theorem, the area of MPN should be one fourth of that.
Since we know that, $\frac{(Area \triangle_1)}{Area \triangle_2} = \frac{(side\ of\ \triangle_1)^2}{(side\ of\ \triangle_2)^2}$

And since it is given that MPN are midpoints of the respective sides, area of triangle MPN is $\triangle \frac{ABC}{4} = \frac{1440}{4} = 360$

Now, the medians from each side intersect, MP MN and NP are X Y and Z respectively, since these sides are proportional to the main triangle, the point of intersection of the medians with these sides will divide the side MP, MN and NP in half.

Then, Area of Triangle XYZ will be one fourth the area MPN,
$\triangle \frac{MPN}{4} = \frac{360}{4} = 90$

The answer is 90.

67. **378**

We need to consider single-digit, two-digit and three-digit numbers

Single digit: There are only nine numbers for a single-digit number (not including zero since we are looking for positive integers only)

Double digits: The ten's digit can be chosen in 9 ways (not including 0), and the unit digit can be chosen in 9 ways (including 0 but excluding the digit used in the ten's place). Hence, a total of 81 numbers.

Three-digit: The hundred's digit can be 1, 2, 3 or 4; hence, there are four options for the hundred's digit; for the ten's digit, there will be 9 options (numbers from 0 to 9 except the one chosen in the hundred's place); for unit's digit there would be 8 options. Hence,e a total of 288 numbers

Giving the total numbers as 288+81+9 = 378

Therefore, 378 is the correct answer.

68. **C**

We are told that there was two successive increments in the salary, with the second increment percentage twice the first one. Total Increment was 187.5%.

Drawing up the equation

$(1+z)(1+2z)=1.875$

$1+3z+2z^2=1.875$

$2z^2+3z-0.875$

$z=\frac{\left(-3\pm\sqrt{9+8(0.875)}\right)}{4}$

$z=\frac{(-3\pm 4)}{4}$

$z=0.25$

Answer is 25%.