Polynomial

OLYMPIAD EXCELLENCE BOOK **MATHEMATICS**

QUESTIONS

1. The zeros of the polynomials $f(x) = abx^2 + (b^2 + ac)x + bc$ is ____.

(a) $-(b^2 + ac), bc$ (b) $\left(-\frac{b}{a} \& \frac{c}{b}\right)$ (c) $\left(\frac{b^2 + ac}{a}, \frac{bc}{b}\right)$ (d) $\left(-\frac{b}{c} \& \frac{c}{a}\right)$

2. 2 and (–2) are two zeros of the polynomial $x^4 - 5x^2 + 4$. What are the other two zeros of the polynomial?

(a) $1, -1$ (b) $1, -2$ (c) $-1, 2$ (d) $1, 0$

3. Find the zeroes of the polynomials $f(y) = y^3 + y^2 - 9y - 9$, if two zeroes are equal in magnitude but opposite in sign.

$\alpha, -\alpha, \beta$

(a) $(1, -1, -3)$ (b) $(+3, -3, -1)$

(c) $(-2 + \sqrt{3}, 2 - \sqrt{3}, -1)$ (d) $\left(\frac{1}{2}, \frac{1}{2}, 9\right)$

4. Which of the following is a cubic polynomial whose zeros are (-1), (-2) and (-3)?

(a) $x^3 - 6$ (b) $x^3 - 6x^2 + 6$

(c) $x^3 - 6x^2 + 11x + 6$ (d) $x^3 - 6x^2 - 11x - 6$

5. If a and b are the roots of the equation $3m^2 + 6m - 11$, find a polynomial whose roots are $2a + 1$ and $2b + 1$.

(a) $k\left(m^2 + 3m - \frac{53}{3}\right)$ (b) $k(m^2 + 9m - 41)$

(c) $k(m^2 + 9m + 41)$ (d) $k(m^2 - 9m - 41)$

6. For what values of k is one zero of the polynomial $(k^2 - 16)x^2 + 9x + 6k$, the reciprocal of the other?

(a) $9, 0$ (b) $-3, -6$

(c) $-9, 0$ (d) $8, -2$

7. If $1 \pm \sqrt{3}$ are the two zeroes of the polynomial $f(x) = x^4 - 4x^3 + x^2 + 6x + 2$, then the other two zeroes of the polynomial f(x) are:

(a) $(-5 + \sqrt{5}, -5 - \sqrt{5})$ (b) $(1, 2)$

(c) $(3 + \sqrt{2}, 3 - \sqrt{2})$ (d) $(1 + \sqrt{2}, \text{and } 1 - \sqrt{2})$

8. Which of the following is the division algorithm of polynomials, if dividend = f(x), divisor = p(x), quotient = q(x) and remainder = r(x)?

(a) $f(x) = r(x)p(x) + q(x)$ (b) $f(x) = p(x)q(x) + r(x)$

(c) $f(x) = p(x)q(x) - r(x)$ (d) $f(x) = p(x)r(x) - q(x)$

9. Find the values of C for which the zeroes of the polynomial $f(x) = x^3 + 3x^2 + 6x + C$ are in A.P.

(a) –36 (b) 22 (c) $-\sqrt{35}$ (d) $\sqrt{21}$

10. α, β and γ are the zeros of a cubic polynomial. If $\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = (-5)$ and $\alpha\beta\gamma = (-24)$, find the polynomial.

(a) $x^3 - 3x^2 - 5x + 24$ (b) $m^3 + 3m^2 - 5m - 24$

(c) $n^3 - 2n^2 + 7n + 12$ (d) $y^3 + 6y^2 - 10y - 48$

11. Find the value of a and b such that $y^2 + 1$ is the factor of $g(z) = y^4 + y^3 + 8y^2 + ay + b$

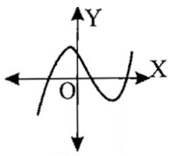
(a) $(-1, -7)$ (b) $(-1, -1)$

(c) (1, 2) (d) (1, 7)

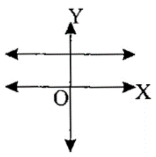
12. If a, b are the roots of the equation $ax^2 - 2bx + c = 0$ then $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2 =$

(a) $\frac{c^2(2b+c)}{a^3}$ (b) $\frac{bc^2}{a^3}$ (c) $\frac{c^3}{a^3}$ (d) $\frac{c^3(b+2c)}{a^3}$

13. Which of the following is the graph of a linear polynomial?



(a)



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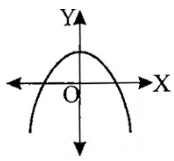

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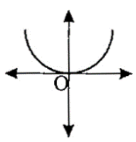


(d)

14. α and β are the zeros of a polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 4$. Identify the polynomial.

(a) $x^2 - 6x + 4$ (b) $x^2 + 8x + 6$ (c) $x^2 + 16$ (d) $x^2 - 4$

15. Find the zeros of the polynomial $7n^2 + 3\sqrt{3}n - 1$.

(a) $\frac{3}{14}(\sqrt{5} - \sqrt{3})$ and $\frac{3}{14}(-\sqrt{3} - \sqrt{5})$ (b) $3\sqrt{3}$ and $\sqrt{3}$

(c) $\frac{3\sqrt{2}}{5}, \frac{\sqrt{2}}{2}$ (d) $-3\sqrt{5}$ and $\sqrt{5}$

16. Identify the zeros of the cubic polynomial $p(m) = m^3 - 8m^2 + 19m - 12$.

(a) $-1,-3,-4$ (b) $0,-3,4$ (c) $1,3,4$ (d) $-1,1,12$

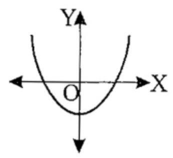
17. The zeros of a cubic polynomial $y^3 - 6y^2 + 11y + 8$ are $m, m-a, m+a$. What is the value of m?

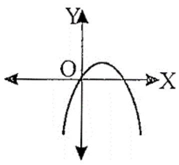
(a) 3 (b) 2 (c) 1 (d) 0

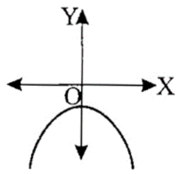
18. What is the quotient when $3m^2 - 2m + 3$ is divided by $(1-m)$ leaving a remainder 9?

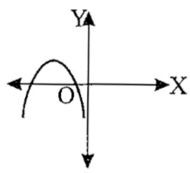
(a) $-2m+3$ (b) $-(2m-3)$ (c) $2m+3$ (d) $-(3m+1)$

19. Identify the quadratic polynomial with no zeros.

(a) 

(b) 

(c) 

(d) 

20. If a polynomial p(x) is divided by another polynomial g(x), with a quotient q(x) and remainder r(x), then p(x) = q(x) g(x) + r(x).

What is the condition that r(x) must satisfy?

(a) $r(x) = 0$

(b) $r(x) = 0$ or deg of $r(x) > \deg\ g(x)$

(c) Either $r(x) = 0$ or deg $r(x) <$ deg (g(x))

(d) $r(x) = g(x)$

21. Find the sum of zeros of the polynomial $7x^2 - 17$.

(a) 0 (b) 1 (c) -1 (d) 2

22. If α and β are the zeros of $ax^2 - a^2x + a^3$ where a < 1, and a > 0 then, which of the following is correct?

(a) $\alpha + \beta < \alpha\beta$ (b) $\alpha + \beta > \alpha\beta$ (c) $\beta - \alpha > \alpha\beta$ (d) $\alpha + \beta = \alpha\beta$

23. If α and β are two zeros of the quadratic polynomial $p(x) = 3x^2 - 6x + 5$, find the value of $\alpha^3 + \beta^3$.

(a) $\frac{30}{13}$ (b) $\frac{35}{6}$ (c) -2 (d) 2

24. What is the nature of the zeros of the quadratic polynomial $2x^2 + 63x + 63$.

(a) Both positive (b) Both negative

(c) One positive, one negative (d) Cannot be said

25. What is the relation between the zeros and the coefficients of the polynomial $4\sqrt{3}x^2+5x-2\sqrt{3}$?

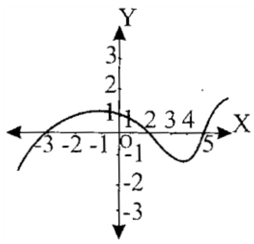
(a) Sum of zeros $= \dfrac{-(\text{Coefficient of x})}{(\text{coefficient of } x^2)}$

(b) Sum of zeros $= \dfrac{(\text{Coefficient of x})}{(\text{coefficient of } x^2)}$

(c) Sum of zeros $= \dfrac{(\text{Coefficient of } x^2)}{(\text{coefficient of x})}$

(d) Product of zeros $= \dfrac{(\text{Coefficient of } x^2)}{\text{constant}}$

26. Choose the zeros of the polynomial whose graph is given.



(a) 0, 2, 3 (b) – 3, 2, 5 (c) 0, 0, 0 (d) 0

27. Find the quadratic polynomial, one of whose zeros is $\dfrac{-\sqrt{3}}{\sqrt{2}}$ and the product of zeros is 1.

(a) $3\sqrt{2}x^2+7x+3$ (b) $\sqrt{6}x^2+5x+\sqrt{6}$ (c) $\sqrt{6}x^2+5x+\sqrt{6}$ (d) $3x^2+\dfrac{5}{2\sqrt{3}}x-\dfrac{1}{6}$

28. Find the remainder when $8p^3-10p^2+11p-24$ is divided by $(1-3p+p^2)$.

(a) $53p-24$ (b) $-10p+2$ (c) $p-10$ (d) $6p-7$

29. A quadratic polynomial $f(x)=x^2-(m+n)x+mn$ has two zeros. Find the value of $\alpha^2\beta^2$.

(a) $\frac{1}{4}(m^2+n^2)$ (b) $\frac{1}{6}(m^2+4n^3)$ (c) m^2+n^2 (d) $\frac{1}{8}(m^2+4n^2)$

30. The polynomial a^3+a^2+a+1 is divided by a polynomial g(a). The quotient and remainder obtained are $(a+1)$ and $(-2a-3)$ respectively, Find g(a).

(a) a^2+2 (b) a^2+4 (c) $2a^2-a$ (c) a^2+a+4

31. When is a real number 'a' called the zero of the polynomial f(x)?

(a) f (0) = a (b) f (a) = a (c) f (a) = 0 (d) f (a) = f (0)

32. If A and B are the zeros of the polynomial ax^2+bx+c, what is the value of A^2+B^2?

(a) $\frac{b^2+2ac}{a^2}$ (b) $\frac{a^2+2ac}{b^2}$ (c) $\frac{a^2-2ac}{b^2}$ (d) $\frac{b^2-2ac}{a^2}$

33. If a and b can take values 1, 2, 3, 4, then the number of the equations of the form $ax^2+bx+1=0$ having real roots is

(a) 10 (b) 7 (c) 6 (d) 12

34. If one zero of the quadratic polynomial $2x^2+(6m+5)x-3=0$ is negative of the other, find the value of 'm'.

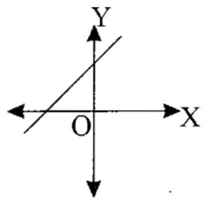
(a) $-\frac{35}{6}$ (b) 2 (c) $\frac{9}{4}$ (d) $-\frac{5}{6}$

35. The roots of $x+\frac{1}{x}=3$, when $x\neq 0$ are:

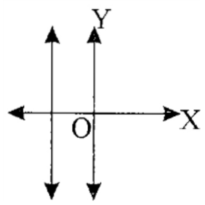
(a) $3, -3$ (b) $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

(c) $\frac{5+\sqrt{3}}{2}, \frac{5-\sqrt{3}}{2}$ (d) $\frac{5+\sqrt{6}}{2}, \frac{5-\sqrt{6}}{2}$

36. Choose the graph of a quadratic polynomial.



(a)



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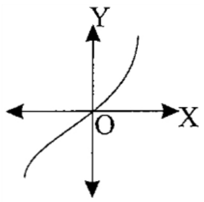

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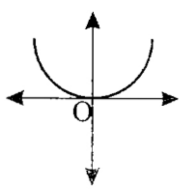

(d)

37. If α and β are the roots of the given equation $2\sqrt{3}x^2+4x-3\sqrt{3}$, then the value of $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ is ______.

(a) 0 (b) $-\frac{280\sqrt{3}}{243}$ (c) $+\frac{280\sqrt{3}}{243}$ (d) $\frac{280}{243}$

38. If the equation $(\sin\theta-1)x^2+(\sin\theta)x+\cos\theta=0$ has real roots, then θ =

(a) $[0,\pi]$ (b) $\left[0,\frac{3\pi}{2}\right]$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (d) $[0,2\pi]$

39. If α, β and γ are the zeros of p(x), which of the following is true?

(a) $p(x) = (x-\alpha)(x-\beta)(x-\gamma)$

(b) $p(x) = (x+\alpha)(x-\beta)(x-\gamma)$

(c) $p(x) = (x+\alpha)(x+\beta)(x-\gamma)$

(d) $p(x) = (x+\alpha)(x+\beta)(x+\gamma)$

40. Which of the following is not correct?

(a) The degree of a zero polynomial is zero.

(b) The polynomial $x^5 - x^3 - x^2 + 1$ has utmost five zeros (three real and two imaginary).

(c) The degree of a cubic polynomial is 3.

(d) A quadratic polynomial has a maximum of two zeros.

ANSWER - KEY				
1. D	**2.** A	**3.** B	**4.** C	**5.** A
6. D	**7.** D	**8.** B	**9.** A	**10.** A
11. D	**12.** A	**13.** B	**14.** A	**15.** A
16. C	**17.** B	**18.** D	**19.** C	**20.** C
21. A	**22.** B	**23.** C	**24.** B	**25.** A
26. B	**27.** B	**28.** A	**29.** C	**30.** B
31. C	**32.** D	**33.** B	**34.** D	**35.** B
36. D	**37.** C	**38.** C	**39.** A	**40.** A

SOLUTIONS

1. (D):

Mind of a mathematician

By observation

$(\alpha+\beta)=-\left(\frac{b}{a}+\frac{c}{b}\right)$

$\alpha\beta=\frac{b}{a}\times\frac{c}{b}$

This implies $\alpha=\left(-\frac{b}{a}\right)$ and $\beta=\left(-\frac{c}{b}\right)$

2. (A):If $f(x)=x^4-5x^2+4; g(x)=(x-2)(x+2)=x^2-4$

Then $\frac{f(x)}{g(x)}\Rightarrow$

$$
\begin{array}{r|l}
x^2-4 & x^4-5x^2+4\,(x^2-1 \\
 & x^4-4x^2 \\
 & -\quad + \\
\hline
 & -x^2+4 \\
 & -x^2+4
\end{array}
$$

$\therefore q(x)=x^2-1$ with zeros as $-1,+1$

3. (B): Let zeros be $\alpha,-\alpha,\beta$

Then, Sum of roots $=\beta=-1$:

Also, $\alpha\beta-\alpha\beta-\alpha^2=-9\Rightarrow\alpha=\pm3$

$\therefore$ Roots are $=(3,-3,-1)$ or $(-3,\ 3,-1)$.

4. (C): Polynomial

$P(x)=x^3-(-1-2-3)x^2+(2+6+3)x-(-6)$

Or $x^3+6x^2+11x+6$

5. (A): $a+b=\frac{-6}{3}=-2: ab=\frac{-11}{3}$

We need $(2a+1)(2b+1)=4ab+2(a+b)+1$

$=\frac{-44}{3}+2x-2+1$

Also, $(2a+1)+(2b+1)=2(a+b)+2=-4+2=-2$

$\Rightarrow m^2 + 2m - \frac{53}{3}$

6. (D): $\alpha, \frac{1}{\alpha} \Rightarrow \alpha + \frac{1}{\alpha} = \frac{-9}{k^2 - 16}$ and

$1 = \frac{6k}{k^2 - 16} \Rightarrow k^2 - 16 - 6k = 0:$

Solve this quadratic

7. (D): $g(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right] = x^2 + 2x - 2$: Now, $\frac{f(x)}{g(x)} : x^2 + 2x - 2)x^4 - 4x^3 + x^2 + 6x + 2$ (Find quotient q(x).

8. (B): Note: This is like Euclid's algorithm: If we choose functions as a(x) and b(x) then a(x) = b(x). q(x) + r(x).

9. (A): $x^3 + 9x^2 + 6x + c$

Let roots be (a – d), a and (a + d)

Sum of roots $\alpha + \beta + \gamma = \frac{-b}{a} = -9$

i.e. $(a - d) + a + (a + d) = -9$

$\Rightarrow 3a = -9$

$a = -3$

Sum product of roots taken two at a time:

$(a - d)a + (a + d)a + (a - d)(a + d)$

$= a^2 - ad + a^2 + ad + a^2 - d^2$

$= 3a^2 - d^2 = 3(-3)^2 - d^2 = 27 - d^2 = \frac{c}{a} = 6$

$= 27 - d^2 = \frac{c}{a} = 6$

$\Rightarrow d^2 = 21$

Product of roots = $(a - d)\ a\ (a + d)$

$= a\left(a^2 - d^2\right)$

$= a\left(9 - d^2\right) = a(9 - 21)$

$= -3(-12) = 36$

Also product of roots $= \frac{-c}{1} = 36$

$\Rightarrow c = -36$

10. (A): Note: Sign of $\alpha+\beta+\gamma=\frac{-b}{a}\Rightarrow$ a, b are of opposite signs

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}\Rightarrow$ c, a are of opposite signs

$\alpha\beta\gamma=\frac{-d}{a}=-24\Rightarrow$ d, a are of some signs: only option (A) matches.

11. (D): Not Available

12. (A): polynomial is $x^2-0x+16=x^2+16$

13. (B) Not Available

14. (A) Not Available

15. (A) Not Available

16. (C) **Mind of a mathematician**

First find out TRIVIAL SOLUTION by putting values $m=0,1,\ 2,3,-1,-2,-3p(0)=12;$

$p(1)=\ 1-8+\ 19-12=0\Rightarrow(m-1)$ is a factor

17. (B): You need not solve the whole problems just apply sum of roots = 6

18. (D): Use remainder theorem: $3m^2-2m+3$

$=(1-\mathrm{m})\times\mathrm{q(x)}+9$

19. (C): That p(x) whose graph does not intersect x - axis will have no zeros, because then $p(a)\neq 0$ for all 'a'.

20. (C) Not Available

21. (A) Not Available

22. (B): This question has been **expertly designed** by the question setter to check whether the students knows that $a>a^2$ when $0<a<1$

23. (C): This is based on the knowledge of algebraic identity: $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

24. (B): $\alpha+\beta$ is – ve and $\alpha\beta$ is +ve $\Rightarrow$ both α, β are – ve.

25. (A) Not Available

26. (B) Not Available

27. (B): Simple method $\alpha = \frac{-\sqrt{3}}{\sqrt{2}} \therefore \beta = \frac{-\sqrt{2}}{\sqrt{3}}$

$\therefore x^2 + \left(\frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}}\right)x + 1 = 0$

Elegant solution: To provide elegance to our approach, we say that the given quadratic polynomial will have α & $\frac{1}{\alpha}$ as roots. (This straightaway removes one variable as β is written as $\frac{1}{\alpha}$)

$\therefore p(\mathrm{x}) = \mathrm{x}^2 - \left(\alpha + \frac{1}{\alpha}\right)x + 1$

$= \mathrm{x}^2 - \left(\frac{\alpha^2+1}{\alpha}\right) + 1$

$\alpha = \frac{-\sqrt{3}}{\sqrt{2}} \therefore \alpha^2 = \frac{3}{2} \therefore \alpha^2 + 1 = \frac{5}{2} \therefore \frac{\alpha^2+1}{\alpha} = \frac{5\sqrt{2}}{-\sqrt{3}/\sqrt{2}} = -\frac{5}{\sqrt{6}}$

$\therefore p(\mathrm{x}) = \mathrm{x}^2 + \frac{5}{\sqrt{6}}x + 1$ or a multiple of $\sqrt{6}x^2 + 5x + \sqrt{6}$

28. (A): Again this is an application of remainder theorem $\left(-3p+p^2\right) \times q(\mathrm{p}) + \mathrm{r(p)} = 8\,\mathrm{p}^3 - 10p^2 + 11p - 24;$

Where r(p) is remainder

29. (C) Not Available

30. (B): Formulate it as: $a^3 + a^2 + a + 1$

$= g(\mathrm{a}) \times (\mathrm{a}+1) + (-2\mathrm{a}-3)$

31. (C) Not Available

32. (D) Not Available

33. (B): For real roots, $b^2 - 4ac \geq 0$

$\Rightarrow b^2 - 4ac \geq 0 \Rightarrow b^2 \geq 4a$

For a = 1; can take values 2, 3, 4

∴ Admissible pairs are (1, 2) (1, 3) (1, 4)

Similarly other ADMISSIBLE PAIRS are: (2, 3), (2, 4), (3, 4), (4, 4)

Hence total seven pairs $\Rightarrow$ seven eqns.

34. (D): This is very easy as

$$\alpha + \beta = 0 \Rightarrow -\left(\frac{6m+5}{2}\right) = 0$$

35. (B): $\text{Roots} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Cross - Check: See that product of roots = 1; if you multiply roots in option (B), product = 1

36. (D): Graph is always a parabola (for quadratic polynomial).

The sum of the roots is $\alpha + \beta = -\dfrac{b}{a} = -\dfrac{2}{\sqrt{3}}$

37. (C)

Product of the roots $\alpha + \beta = -\dfrac{c}{a} = -\dfrac{-3}{2}$;

$$\text{Now} = \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3}$$

$$= \frac{(\alpha+\beta)\left[\{\alpha+\beta\}^2 - 3\alpha\beta\right]}{(\alpha\beta)^3}$$

$$= \frac{\frac{-2}{\sqrt{3}} \times \left[\frac{4}{3} + \frac{9}{2}\right]}{\left(\frac{-27}{8}\right)} = \frac{280\sqrt{3}}{243}$$

38. (C): $b^2 - 4ac \geq 0 \Rightarrow \sin^2\theta - 4(\sin\theta - 1)\cos\theta \geq 0$

39. (A) Not Available

40. (A) Not Available