Ex 22.1

Differential Equations Ex 22.1 Q1

$$\frac{d^3x}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2=e^t$$

The highest order differential coefficient is $\frac{d^3x}{dt^3}$ and its power is 1.

So, it is a non-linear differential equation with order 3 and degree 1.

Differential Equations Ex 22.1 Q2

$$\frac{d^2y}{dx^2}+4y=0$$

It is a linear differential equation.

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q3

$$\left(\frac{dy}{dx}\right)^2+\frac{1}{\left(\frac{dy}{dx}\right)}=2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^3+1=2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^3-2\left(\frac{dy}{dx}\right)+1=0$$

This is a polynomial in $\frac{dy}{dx}$.

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 3.

So, it is a non-linear differential equation with order 1 and degree 3.

Differential Equations Ex 22.1 Q4

Consider the given differential equation, $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\left(c\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$

Squaring on both the sides, we have

$$1+\left(\frac{dy}{dx}\right)^2=\left(c\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}$$

Cubing on both the sides, we have

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left\{\left(c\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}\right\}^3$$

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^6+3\left(\frac{dy}{dx}\right)^2+3\left(\frac{dy}{dx}\right)^4=c^2\left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow c^2\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^6-3\left(\frac{dy}{dx}\right)^4-3\left(\frac{dy}{dx}\right)^2-1=0$$

The highest order differential coefficient in this equation is $\frac{d^2y}{dx^2}$ and its power is 2.

Therefore, the given differential equation is a non – linear differential equation of second order and second degree.

Differential Equations Ex 22.1 Q5

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+xy=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q6

Consider the given differential equation,

$$\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$$

Cubing on both the sides of the above equation, we have

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring on both the sides of the above equation, we have

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^{\frac{3}{2}}\right]^2$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)\right]^3$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 - \left[\left(\frac{dy}{dx}\right)\right]^3 = 0$$

The highest order differential coefficient in this equation is $\frac{d^2y}{dx^2}$ *and its power is 2.*

Therefore, the given differential equation is a non – linear differential equation of second order and second degree.

Differential Equations Ex 22.1 Q7

$$\frac{d^4y}{dx^4} = \left[c + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 = \left[c + \left(\frac{dy}{dx}\right)^2\right]^3$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 = c^3 + \left(\frac{dy}{dx}\right)^6 + 3c\left(\frac{dy}{dx}\right)^2 + 3c^2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^6 - 3c\left(\frac{dy}{dx}\right)^2 - 3c^2\left(\frac{dy}{dx}\right) - c^3 = 0$$

The highest order differential coefficient is $\left(\frac{d^4y}{dx^4}\right)$ and its power is 2.

It is a non-linear differential equation with order 4 and degree 2.

Differential Equations Ex 22.1 Q8

$$x + \left(\frac{dy}{dx}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \left(x + \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow x^2 + \left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2x\left(\frac{dy}{dx}\right) + x^2 - 1 = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{x}{2} - \frac{1}{2x} = 0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and power is 1.

So, it is a linear differential equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q9

$$y\frac{d^2x}{dy^2} = y^2 + 1$$

$$\frac{d^2x}{dy^2} - y - \frac{1}{y} = 0$$

The differential coefficient is $\frac{d^2x}{dy^2}$ and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q10

$$S^2\frac{d^2t}{ds^2}+st\frac{dt}{ds}=s$$

The differential coefficient of highest order is $\frac{d^2t}{ds^2}$ and power is 1.

So, it is a non-linear differnetial equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q11

$$x^2\left(\frac{d^2y}{dx^2}\right)^3+y\left(\frac{dy}{dx}\right)^4+y^4=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 3.

So, it is a non-linear differnetial equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q12

$$\frac{d^3y}{dx^3}+\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)+4y=\sin x$$

The highest order differential coefficient is $\frac{d^3y}{dx^3}$ and its power is 1.

So, it is a non-linear differnetial equation with order 3 and degree 1.

Differential Equations Ex 22.1 Q13

$$\left(xy^2+x\right)dx+\left(y-x^2y\right)dy=0$$
$$\left(y-x^2y\right)\frac{dy}{dx}+xy^2+x=0$$
$$y\left(1-x^2\right)\frac{dy}{dx}+x\left(y^2+1\right)=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.

So, it is a non-linear differnetial equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q14

$$\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0$$
$$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$$
$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.

So, it is a non-linear differnetial equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q15

$$\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^{\frac{2}{3}}$$
$$\left(\frac{d^2y}{dx^2}\right)^3=\left(\frac{dy}{dx}\right)^2$$
$$\left(\frac{d^2y}{dx^2}\right)^3=\left(\frac{dy}{dx}\right)^2=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 3.

So, it is a non-linear differnetial equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q16

$$2\frac{d^2y}{dx^2}+3\sqrt{1-\left(\frac{dy}{dx}\right)^2}-y=0$$

$$2\frac{d^2y}{dx^2}=-3\sqrt{1-\left(\frac{dy}{dx}\right)^2}-y$$

Squaring both the sides,

$$4\left(\frac{d^2y}{dx^2}\right)^2=9\left(1-\left(\frac{dy}{dx}\right)^2-y\right)$$

$$4\left(\frac{d^2y}{dx^2}\right)^2+9\left(\frac{dy}{dx}\right)^2+9y-9=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2.

Differential Equations Ex 22.1 Q17

$$5\ \frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$$

$$\left\{5\left(\frac{d^2y}{dx^2}\right)^2\right\}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3$$

$$25\left(\frac{d^2y}{dx^2}\right)^2=1+\left(\frac{dy}{dx}\right)^6+3\left(\frac{dy}{dx}\right)^2+3\left(\frac{dy}{dx}\right)^4$$

$$25\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^6-3\left(\frac{dy}{dx}\right)^4-3\left(\frac{dy}{dx}\right)^2-1=0$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2

Differential Equations Ex 22.1 Q18

$$y=x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\left(y-x\frac{dy}{dx}\right)^2=\left(a\sqrt{1-\left(\frac{dy}{dx}\right)^2}\right)^2$$

$$y^2+x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}=a\left(1-\left(\frac{dy}{dx}\right)^2\right)$$

$$x^2\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y+a\left(\frac{dy}{dx}\right)^2-a=0$$

$$\left(x^2+a\right)\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y-a=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and poer is 2.

So, it is a non-linear differential equation with order 1 and degree 2.

Differential Equations Ex 22.1 Q19

$$y=px+\sqrt{a^2p^2+b^2},\ p=\frac{dy}{dx}$$

$$y-px=\sqrt{a^2p^2+b^2}$$

$$(y-px)^2=\left(a^2p^2+b^2\right)$$

$$y^2+p^2x^2-2xyp=a^2p^2+b^2$$

$$x^2p^2-a^2p^2-2xyp+y^2-b^2=0$$

$$\left(x^2-a^2\right)p^2-2xyp+\left(y^2-b^2\right)=0$$

$$\left(x^2-a^2\right)\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)+\left(y^2-b^2\right)=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 2.

So, it is a non-linear differential equation of order 1 and degree 2

Differential Equations Ex 22.1 Q20

$$\frac{dy}{dx}+e^{y}=0$$

The highest order differential coefficient is $\frac{dy}{dx}$ and its power is 1.

So, it is a non-linear differential equation of order 1 and degree 1.

Differential Equations Ex 22.1 Q21

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{d^2y}{dx^2}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2-x\sin\left(\frac{d^2y}{dx^2}\right)=0$$

The highest order differential coefficient is $\left(\frac{d^2y}{dx^2}\right)$ and it is not a polynomial of derviative,

So, it is a non-linear differential equation of order 2 but degree is not defined.

Differential Equations Ex 22.1 Q22

$$(y'')^2+(y')^3+\sin y=0$$

The highest order of differential coefficient is y'' and its power is 2,

So, it is a non-linear differential equation of order 2 and degree 2.

Differential Equations Ex 22.1 Q23

$$\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$$

The highest order differential coefficient is $\frac{d^2y}{dx^2}$ and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q24

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\frac{dy}{dx}+y\sin y=0$$

The highest order differential coefficient is $\frac{d^3y}{dx^3}$ and its power is 1.

So, it is a linear differential equation of order 3 and degree 1.

Differential Equations Ex 22.1 Q25

$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2-x^2\log\left(\frac{d^2y}{dx^2}\right)=0$$

The highest order derivative is $\frac{d^2y}{dx^2}$ but it is not a polynomial in $\frac{dy}{dx}$.

So, it is a non-linear differential equation of order 2 but degree is not defined.

Differential Equations Ex 22.1 Q26

The order of a differential equation is the order of the highest order derivative appearing in the equation.
The degree of a differential equation is the degree of the highest order derivative.
Consider the given differential equation

$$\left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$$

In the above equation, the order of the highest order derivative is 1.
So the differential equation is of order 1.
In the above differential equation, the power of the highest order derivative is 3.
Hence, it is a differential equation of degree 3.
Since the degree of the above differential equation is 3, more than one, it is a non-linear differential equation.

Ex 22.2

Differential Equations Ex 22.2 Q1

$$y^2 = (x-c)^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 3(x-c)^2$$

$$(x-c)^2 = \frac{2y}{3}\frac{dy}{dx}$$

$$(x-c)^2 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Put the value of $(x-c)$ in equation (i),

$$y^2 = \left\{\left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}\right\}^3$$

$$y^2 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring both the sides,

$$y^4 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^3$$

$$y^4 = \frac{8y^3}{27}\left(\frac{dy}{dx}\right)^3$$

$$27y = 8\left(\frac{dy}{dx}\right)^3.$$

Differential Equations Ex 22.2 Q2

$$y = e^{mx} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = me^{mx} \qquad \text{—(ii)}$$

From equation (i),

$$y = e^{mx}$$

$$\log y = mx$$

$$m = \frac{\log y}{x}$$

Put the value of m and e^{mx} in equation (i),

$$\frac{dy}{dx} = \frac{\log y}{x}y$$

$$x\frac{dy}{dx} = y\log y$$

Differential Equations Ex 22.2 Q3(i)

$$y^2 = 4ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a \qquad \text{—(ii)}$$

Put the value of a from equation (i) in (ii),

$$2y\frac{dy}{dx} = 4\left(\frac{y^2}{4x}\right)$$

$$2y\frac{dy}{dx} = \frac{y^2}{x}$$

$$2x\frac{dy}{dx} = y$$

Differential Equations Ex 22.2 Q3(ii)

$$y = cx + 2c^2 + c^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = c \qquad \text{—(ii)}$$

Put the value of c from equation (ii) in (i),

$$y = \left(\frac{dy}{dx}\right)x + 2\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3$$

Differential Equations Ex 22.2 Q3(iii)

$$xy = a^2 \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$x\frac{dy}{dx} + y(1) = 0$$

$$x\frac{dy}{dx} + y = 0$$

Differential Equations Ex 22.2 Q3(iv)

$$y = ax^2 + bx + c$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ax + b$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 2a$$

Again, differentiating it with respect to x,

$$\frac{d^3y}{dx^3} = 0$$

Differential Equations Ex 22.2 Q4

$$y = Ae^{2x} + Be^{-2x} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

$$= 4\left(Ae^{2x} + Be^{-2x}\right)$$

$$\frac{d^2y}{dx^2} = 4y \qquad \left[\text{Using equation (i)}\right]$$

Differential Equations Ex 22.2 Q5

$$x = A\cos nt + B\sin nt$$

Differentiating with respect to t,

$$\frac{dx}{dt} = -An\sin nt + nB\cos nt$$

Again, differentiating with respect to t,

$$\frac{d^2x}{dt^2} = -An^2\cos nt - n^2B\sin t$$

$$= -n^2(A\cos nt + B\sin nt)$$

$$\frac{d^2x}{dt^2} = -n^2x \qquad \left[\text{Using equation (i)}\right]$$

$$\frac{d^2x}{dt^2} + n^2x = 0$$

Differential Equations Ex 22.2 Q6

$$y^2 = a\left(b - x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = a(-2x) \qquad \text{---(i)}$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx}\right] = -2a$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\left(\frac{2y}{-2x}\frac{dy}{dx}\right)$$

Using equatoin (i)

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q7

$$y^2 - 2ay + x^2 = a^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0$$

$$y\frac{dy}{dx} + x = a\frac{dy}{dx}$$

$$a = \frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}$$

Put the value of a in equation (i),

$$y^2 - 2\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]y + x^2 = \left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]^2$$

Put $\frac{dy}{dx} = y'$

$$y^2 - 2\left(\frac{yy' + x}{y'}\right)y + x^2 = \left(\frac{yy' + x}{y'}\right)^2$$

$$\frac{y'y^2 - 2y'y^2 - 2xy + y'x^2}{y'} = \frac{y'^2y^2 + x^2 + 2xyy'}{y'^2}$$

$$y'^2y^2 - 2y'^2y^2 - 2xyy' + y'^2x^2 - y'^2y^2 - x^2 - 2xyy' = 0$$

$$-4xyy' + y'^2x^2 - x^2 - 2y'^2y^2 = 0$$

$$y'^2\left(x^2 - 2y^2\right) - 4xyy' - x^2 = 0$$

Differential Equations Ex 22.2 Q8

$$(x-a)^2 + (y-b)^2 = r^2 \qquad \text{—(i)}$$

Differentiating with respect to x,

$$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$(x-a) + (y-b)\frac{dy}{dx} = 0 \qquad \text{—(ii)}$$

Differentiating with respect to x,

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = 0$$

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$(y-b) = -\left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\} \qquad \text{—(iii)}$$

Put $(y-b)$ in equation (ii),

$$(x-a) - \left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\}\frac{dy}{dx} =$$

$$(x-a)\left(\frac{d^2y}{dx^2}\right) - \left(\frac{dy}{dx}\right)^3 - \left(\frac{dy}{dx}\right) = 0$$

$$(x-a)\frac{d^2y}{dx^2} = \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$$

$$(x-a) = \frac{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}} \qquad \text{—(iv)}$$

Put the value of $(x-a)$ and $(y-b)$ from equation (iii) and (iv) in equation (i),

$$\left\{\frac{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}\right\}^2 + \left\{\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right\}^2 = r^2$$

Put $\frac{dy}{dx} = y'$ **and** $\frac{d^2y}{dx^2} = y''$

$$\left(y' + y'^3\right)^2 + \left(y'^2 + 1\right)^2 = r^2y''^2$$

$$y'^2\left(1 + y'^2\right)^2 + \left(1 + y'^2\right)^2 = r^2y''^2$$

Differential Equations Ex 22.2 Q9

We know that, equation of a circle with centre at (h,k) and radius r is given by,

$$\left(x-h^2\right)+(y-k)^2=r^2 \qquad \text{—(i)}$$

Here, centre lies, on y-axis, so $h=0$

$$\Rightarrow \quad x^2+(y-k)^2=r^2 \qquad \text{—(ii)}$$

Also, given that, circle is passing through origin, so

$$0+k^2=r^2$$
$$k^2=r^2$$

So, equation (ii) becomes,

$$x^2+(y-k)^2=k^2$$
$$x^2+y^2-2yk=0$$
$$2yk=x^2+y^2$$
$$k=\frac{x^2+y^2}{2y}$$

Differentiating with respect to x,

$$0=\frac{2y\left(2x+2y\frac{dy}{dx}\right)-\left(x^2+y^2\right)2\frac{dy}{dx}}{(2y)^2}$$
$$0=4xy+4y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}-2y^2\frac{dy}{dx}$$
$$0=2y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}+4xy$$
$$x^2\frac{dy}{dx}-y^2\frac{dy}{dx}=2xy$$
$$\left(x^2-y^2\right)\frac{dy}{dx}=2xy$$

Differential Equations Ex 22.2 Q10

Equation of circle with centre (h,k) and radius r is given by

$$(x-h)^2+(y-k)^2=r^2 \qquad \text{—(i)}$$

Here, centre lie on x-axis, so

$$k=0$$
$$\Rightarrow \quad (x-h)^2+y^2=r^2 \qquad \text{—(ii)}$$

Also, given that, circle is passing through $(0,0)$, so,

$$h^2=r^2$$

So, equation (ii) becomes,

$$(x-h)^2+y^2=h^2$$
$$x^2+h^2-2xh+y^2=h^2$$
$$x^2-2xh+y^2=0$$
$$2xh=x^2+y^2$$
$$h=\frac{x^2+y^2}{2x}$$

Differentiating it with respect to x,

$$0=\frac{\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)2}{(2x)^2}$$
$$\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)^2=0$$
$$2x^2+2yx\frac{dy}{dx}-x^2-y^2=0$$
$$\left(x^2-y^2\right)+2xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q11

Let A be the surface area of rain drain, V be its volume, and r be the radius of rain drop.
Given,

$$\frac{dV}{dt} \propto A$$

$$\frac{dV}{dt} = -kA \qquad \text{[negative because } V \text{ decreases with increase in } t\text{]}$$

where k is the constant of proportionality.

So,

$$\frac{d}{dt}\left(\frac{4\pi}{3}r^3\right) = -k\left(4\pi r^2\right)$$

$$4\pi r^2\frac{dr}{dt} = -k\left(4\pi r^2\right)$$

$$\frac{dr}{dt} = -k$$

Differential Equations Ex 22.2 Q12

Equation of parabolas with lotus rectum '$(4a)$' and whose area is parallel to x axes and vertex at (h,k) is given by,

$$(y-k)^2 = 4a(x-h)$$

Differentiating with respect to x,

$$2(y-k)y_1 = 4a(1)$$

$$(y-k)y_1 = 2a \qquad \text{---(i)}$$

Differentiating with respect to x,

$$(y-k)y_2 + (y_1)(y_1) = 0$$

$$(y-k)y_2 + (y_1)^2 = 0$$

$$\left(\frac{2a}{y_1}\right)^{y_2} + (y_1)^2 = 0$$

Using equation (i)

$$2ay_2 + (y_1)^3 = 0$$

Differential Equations Ex 22.2 Q13

$$y = 2\left(x^2-1\right) + ce^{-x^2} \qquad \text{---(i)}$$

Differentiating it in equation (i),

$$\frac{dy}{dx} = 4x - 2cxe^{-x^2} \qquad \text{---(ii)}$$

Now,

$$\frac{dy}{dx} + 2xy$$

$$= 4x - 2cxe^{-x^2} + 2x\left[2\left(x^2-1\right) + ce^{-x^2}\right]$$

$$= 4x - 2cxe^{-x^2} + 4x^3 - 4x + 2xce^{-x^2}$$

$$= 4x^3$$

So,

$$\frac{dy}{dx} + 2xy = 4x^3$$

Which is given equation, so

$y = 2\left(x^2+1\right) + ce^{-x^2}$ is the solution of the equation.

Differential Equations Ex 22.2 Q14

$$y = \left(\sin^{-1}x\right)^2 + A\cos^{-1}x + B$$

$$\frac{dy}{dx} = 2\sin^{-1}x \times \left(\frac{1}{\sqrt{1-x^2}}\right) + A \times \left(\frac{-1}{\sqrt{1-x^2}}\right) + 0$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right)(-2x) = 2 \times \left(\frac{1}{\sqrt{1-x^2}}\right) - 0$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 2 = 0$$

Note: Answer given in the book is incorrect.

Differential Equations Ex 22.2 Q15(i)

Consider the given equation.,

$(2x+a)^2+y^2=a^2 \ldots(1)$

Differentiating the above equation with respect to x, *we have,*

$$2(2x+a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (2x+a)+y\frac{dy}{dx}=0$$

$$\Rightarrow 2x+a=-y\frac{dy}{dx}$$

$$\Rightarrow a=-2x-y\frac{dy}{dx}$$

Substituting the value of a *in equation* (1), *we have*

$$\left(2x-2x-y\frac{dy}{dx}\right)^2+y^2=\left(-2x-y\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left(y\frac{dy}{dx}\right)^2+y^2=\left(4x^2+y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}\right)$$

$$\Rightarrow y^2=4x^2+4xy\frac{dy}{dx}$$

$$\Rightarrow y^2-4x^2-4xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q15(ii)

$$(2x-a)^2-y^2=a^2$$

$$4x^2+a^2-ax-y^2=a^2$$

$$4x^2-4ax-y^2=0$$

$$4ax=4x^2-y^2$$

$$a=\frac{4x^2-y^2}{4x}$$

Differentiating it with respect to x,

$$0=\left[\frac{4x\left(8x-2y\frac{dy}{dx}\right)-4\left(4x^2-y^2\right)}{(4x)^2}\right]$$

$$32x^2-8xy\frac{dy}{dx}-16x^2+4y^2=0$$

$$16x^2-8xy\frac{dy}{dx}+4y^2=0$$

$$4x^2+y^2=2xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q15(iii)

Consider the given equation,

$(x-a)^2+2y^2=a^2 \ldots(1)$

Differentiating the above equation with respect to x, *we have*

$$2(x-a)+4y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)=-2y\frac{dy}{dx}$$

$$\Rightarrow a=x+2y\frac{dy}{dx}$$

Substituting the value of a *in equation* (1), *we have*

$$\left(x-x+2y\frac{dy}{dx}\right)^2+2y^2=\left(x+2y\frac{dy}{dx}\right)^2$$

$$\Rightarrow 4y^2\left(\frac{dy}{dx}\right)^2+2y^2=x^2+4y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}$$

$$\Rightarrow 2y^2-x^2=4xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(i)

$$x^2+y^2=a^2$$

Differentiating it with respect to x,

$$2x+2y\frac{dy}{dx}=0$$

$$x+y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(ii)

$$x^2-y^2=a^2$$

Differentiating it with respect to x,

$$2x-2y\frac{dy}{dx}=0$$

$$x-y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(iii)

$$y^2=4ax$$

$$\frac{y^2}{x}=4a$$

Differentiating it with respect to x,

$$\left[\frac{x\times 2y\frac{dy}{dx}-y^2(1)}{x^2}\right]=0$$

$$2xy\frac{dy}{dx}-y^2=0$$

$$2x\frac{dy}{dx}-y=0$$

Differential Equations Ex 22.2 Q16(iv)

$$x^2+(y-b)^2=1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2x+2(y-b)\frac{dy}{dx}=0$$

$$x+(y-b)\frac{dy}{dx}=0$$

$$(y-b)\frac{dy}{dx}=-x$$

$$(y-b)=\frac{-x}{\frac{dy}{dx}}$$

Put the value of $(y-b)$ is equation (i)

$$x^2\left(\frac{-x}{\frac{dy}{dx}}\right)^2=1$$

$$x^2\left(\frac{dy}{dx}\right)^2+x^2=\left(\frac{dy}{dx}\right)^2$$

$$x^2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}=\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.2 Q16(v)

$$(x-a)^2-y^2=1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2(x-a)-2y\frac{dy}{dx}=0$$

$$(x-a)-y\frac{dy}{dx}=0$$

$$(x-a)=y\frac{dy}{dx}$$

Put the value of $(x-a)$ is equation (i)

$$\left(y\frac{dy}{dx}\right)^2-y^2=1$$

$$y^2\left(\frac{dy}{dx}\right)^2-y^2=1$$

Differential Equations Ex 22.2 Q16(vi)

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$
$$\frac{b^2x^2-a^2y^2}{a^2b^2}=1$$
$$b^2x^2-a^2y^2=a^2b^2$$

Differentiating it with respect to x,

$$2xb^2-2a^2y\frac{dy}{dx}=0$$
$$xb^2-ya^2\frac{dy}{dx}=0 \qquad \text{---(i)}$$

Again, differentiating it with respect to x,

$$b^2-a^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right)=0$$
$$b^2=a^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$$

Put the value of b^2 in equation (i)

$$xb^2-ya^2\frac{dy}{dx}=0$$
$$xa^2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)-ya^2\frac{dy}{dx}=0$$
$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$
$$x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(vii)

$$y^2=4a(x-b)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx}=4a$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right]=0$$
$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$$

Differential Equations Ex 22.2 Q16(viii)

$$y=ax^3$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=3ax^2$$
$$=3\left(\frac{y}{x^3}\right)x^2$$

Using equation (i)

$$\frac{dy}{dx}=\frac{3y}{x}$$
$$x\frac{dy}{dx}=3y$$

Differential Equations Ex 22.2 Q16(ix)

$$x^2+y^2=ax^3$$
$$\frac{x^2+y^2}{x^3}=a$$

Differentiating it with respect to x,

$$\left[\frac{(x^3)\left(2x+2y\frac{dy}{dx}\right)-(x^2+y^2)(3x^2)}{(x^3)^2}\right]=0$$
$$2x^4+2x^3y\frac{dy}{dx}-3x^4-3x^2y^2=0$$
$$2x^3y\frac{dy}{dx}-x^4-3x^2y^2=0$$
$$2x^3y\frac{dy}{dx}=x^4+3x^2y^2$$
$$2x^3y\frac{dy}{dx}=x^2(x^2+3y^2)$$
$$2xy\frac{dy}{dx}=(x^2+3y^2)$$

Differential Equations Ex 22.2 Q16(x)

$$y = e^{ax} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ae^{ax}$$
$$\frac{dy}{dx} = ay \qquad \text{—(ii)}$$

From equation (i),

$$y = e^{ax}$$
$$\log y = ax$$
$$a = \frac{\log y}{x}$$

Put the value of a in equation (ii),

$$\frac{dy}{dx} = \left(\frac{\log y}{x}\right) y$$
$$x\frac{dy}{dx} = y \log y$$

Differential Equations Ex 22.2 Q17

We know that the equation of said family of ellipses is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{--------(i)}$$

DIfferentiating (i) w.r.t. x , we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-b^2}{a^2} \qquad \text{--------(ii)}$$

DIfferentiating (ii) w.r.t. x , we get

$$\frac{y}{x}\left(\frac{d^2y}{dx^2}\right) + \left(\frac{x\frac{dy}{dx} - y}{x^2}\right)\frac{dy}{dx} = 0$$

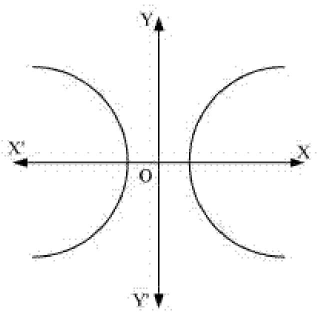
$$\Rightarrow \quad xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

which is the required differential equation.

Differential Equations Ex 22.2 Q18

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{...(1)}$$



Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$
$$\Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} = 0 \qquad \text{...(2)}$$

Again, differentiating both sides with respect to x, we get:

$$\frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} = 0$$
$$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}\left((y')^2 + yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left((y')^2 + yy''\right) - \frac{yy'}{b^2} = 0$$
$$\Rightarrow x(y')^2 + xyy'' - yy' = 0$$
$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$

This is the required differential equation.

Differential Equations Ex 22.2 Q19

Let C denote the family of circles in the second quadrant and touching the coordinate axes.

Let $(-a,a)$ be the coordinate of the centre of any member of this family.

Equation representing the family C is

$$(x+a)^2 + (y-a)^2 = a^2 \qquad --------(i)$$
$$\text{or } x^2 + y^2 + 2ax - 2ay + a^2 = 0 \qquad --------(ii)$$

Differentiating eqn (ii) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2a - 2a\frac{dy}{dx} = 0$$
$$\Rightarrow x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right)$$
$$\Rightarrow \qquad a = \frac{x + yy'}{y' - 1}$$

Substituting the value of a in (ii), we get

$$\left[x + \frac{x+yy'}{y'-1}\right]^2 + \left[y - \frac{x+yy'}{y'-1}\right]^2 = \left[\frac{x+yy'}{y'-1}\right]^2$$
$$\Rightarrow \quad \left[xy' - x + x + yy'\right]^2 + \left[yy' - y - x - yy'\right]^2 = \left[x + yy'\right]^2$$
$$\Rightarrow \quad (x+y)^2 y'' + (x+y)^2 = \left[x + yy'\right]^2$$
$$\Rightarrow \quad (x+y)^2\left[(y')^2 + 1\right] = \left[x + yy'\right]^2$$

which is the differential equation representing the given family of circles.

Ex 22.3

Differential Equations Ex 22.3 Q1

$$y = be^{x} + ce^{2x} \quad \text{---(i)}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = be^{x} + 2ce^{2x} \quad \text{---(ii)}$$

Differentiating both sides with respect to x,

$$\frac{d^2y}{dx^2} = be^{x} + 4ce^{2x} \quad \text{---(iii)}$$

Now,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$
$$= be^{x} + 4ce^{2x} - 3\left(be^{x} + 2ce^{2x}\right) + 2\left(be^{x} + ce^{2x}\right)$$
$$= be^{x} + 4ce^{2x} - 3be^{x} - 6ce^{2x} + 2be^{x} + 2ce^{2x}$$
$$= 3be^{x} - 3be^{x} + 6ce^{2x} - 6ce^{2x}$$
$$= 0$$

So,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Differential Equations Ex 22.3 Q2

$$y = 4\sin 3x \quad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 4(3)\cos 3x$$
$$\frac{dy}{dx} = 12\cos 3x \quad \text{---(ii)}$$

Differentiating it wtih respect to x,

$$\frac{d^2y}{dx^2} = -12(3)\sin 3x$$
$$\frac{d^2y}{dx^2} = -36\sin 3x \quad \text{---(iii)}$$

Now,

$$\frac{d^2y}{dx^2} + 9y$$
$$= -36\sin 3x + 9(4\sin 3x)$$
$$= -36\sin 3x + 36\sin 3x$$
$$= 0$$

So, $y = 4\sin 3x$ is a solution of

$$\frac{d^2y}{dx^2} + 9y = 0$$

Differential Equations Ex 22.3 Q3

$$y = ae^{2x} + be^{-x} \quad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ae^{2x} - be^{-x} \quad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x} \quad \text{---(ii)}$$

Now,

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$$
$$= \left(4ae^{2x} + be^{-x}\right) - \left(2ae^{2x} - be^{-x}\right) - 2\left(ae^{2x} + be^{-x}\right)$$
$$= 4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$$
$$= 4ae^{2x} - 4ae^{2x} + 2be^{-x} - 2be^{-x}$$
$$= 0$$

Differential Equations Ex 22.3 Q4

The given function is $y = A\cos x + B\sin x$ ---------(i)

Differentiating both sides of eqn (i) w.r.t x, successively, we get

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

Substituting these values of $\frac{d^2y}{dx^2}$ and y in the given differential equation,

L.H.S $= (-A\cos x - B\sin x) + (A\cos x + B\sin x) = 0 =$ R.H.S

Therefore, the given function is a solution of the given differential equation.

Differential Equations Ex 22.3 Q5

$$y = A\cos 2x - B\sin 2x \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

$$\frac{dy}{dx} = -2(A\sin 2x + B\cos 2x) \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -2[2A\cos 2x - 2B\sin 2x]$$

$$= -4[A\cos 2x - B\sin 2x]$$

$$\frac{d^2y}{dx^2} = -4y$$

$$\frac{d^2y}{dx^2} + 4y = 0$$

Differential Equations Ex 22.3 Q6

$$y = Ae^{Bx} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ABe^{Bx} \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = AB^2e^{Bx}$$

$$= \frac{(ABe^{Bx})^2}{(Ae^{Bx})}$$

$$\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.3 Q7

$$y = \frac{a}{x} + b \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{a}{x^2} \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

$$= -\frac{2}{x}\left(-\frac{a}{x^2}\right)$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x}\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$$

Differential Equations Ex 22.3 Q8

$$y^2 = 4ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$x\frac{dy}{dx} + a\frac{dy}{dx}$$

$$= 2\frac{xa}{y} + a\left(\frac{y}{2a}\right)$$

$$= \frac{4a^2x + ay^2}{2ay}$$

$$= \frac{ay^2 + ay^2}{2ay}$$

$$= y$$

So,

$$x\frac{dy}{dx} + a\frac{dx}{dy} = y$$

Differential Equations Ex 22.3 Q9

$$Ax^2 + By^2 = 1$$

Differentiating it with respect to x,

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = \frac{-2Ax}{2B}$$

$$y\frac{dy}{dx} = -\frac{Ax}{B} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

Using equation (i)

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q10

$$y = ax^3 + bx^2 + c$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 3ax^2 + 2bx$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 6ax + 2b$$

Differentiating it with respect to x

$$\frac{d^3y}{dx^3} = 6a$$

Differential Equations Ex 22.3 Q11

$$y = \frac{c - x}{1 + cx} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \left[\frac{(1+cx)(-1) - (c-x)(c)}{(1+cx)}\right]$$

$$\frac{dy}{dx} = \left[\frac{-1 - cx - c^2 + cx}{(1+cx)^2}\right]$$

$$= \frac{-1 - c^2}{(1+cx)^2}$$

$$\frac{dy}{dx} = \frac{-(1+c^2)}{(1+cx)^2} \qquad \text{---(ii)}$$

Now,

$$\left(1+x^2\right)\frac{dy}{dx} + \left(1+y^2\right)$$

$$= \left(1+x^2\right)\left[\frac{-\left(1+c^2\right)}{(1+cx)^2}\right] + \left[1 + \left(\frac{c-x}{1+cx}\right)^2\right]$$

$$= \frac{-\left(1+x^2\right)\left(1+c^2\right)}{(1+cx)^2} + \left[\frac{(1+cx)^2 + (c-x)^2}{(1+cx)^2}\right]$$

$$= \frac{-1 - x^2 - c^2 - x^2c^2 + 1 + c^2x^2 + 2cx + c^2 + x^2 - 2cx}{(1+cx)^2}$$

$$= \frac{0}{(1+cx)^2}$$

$$= 0$$

So,

$$\left(1+x^2\right)\frac{dy}{dx} + \left(1+y^2\right) = 0$$

Differential Equations Ex 22.3 Q12

$$y = e^x(A\cos x + B\sin x).....(i)$$

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

$$\frac{dy}{dx} = e^x\left[(A+B)\cos x - (A-B)\sin x\right]........(ii)$$

$$\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right] + e^x\left[-(A+B)\sin x - (A-B)\cos x\right]$$

$$\frac{d^2y}{dx^2} = 2e^x(B\cos x - A\sin x)..........(iii)$$

Adding (i) and (iii) we get

$$y + \frac{1}{2}\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right]$$

$$2y + \frac{d^2y}{dx^2} = 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence $y = e^x(A\cos x + B\sin x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Differential Equations Ex 22.3 Q13

$$y = cx + 2c^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{y}{dx} = c \qquad \text{—(ii)}$$

Now,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y$$
$$= 2c^2 + xc - cx + 2c^2 \qquad \left[\text{Using equation (i) and (ii)}\right]$$
$$= 0$$

So,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$$

Differential Equations Ex 22.3 Q14

$$y = -x - 1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -1 \qquad \text{—(ii)}$$

So,

$$(y - x)dy - \left(y^2 - x^2\right)dx$$
$$= \left[(y - 2)\frac{dy}{dx} - \left(y^2 - x^2\right)\right]dx$$
$$= \left[(-x - 1 - x)(-1) - \left\{(-x - 1)^2 - x^2\right\}\right]$$

Using equation (i) and (ii),

$$= \left[x + 1 + x - \left(x^2 + 1 + 2x - x^2\right)\right]dx$$
$$= [2x + 1 - 2x - 1]dx$$
$$= 0$$

So,

$$(y - x)dy - \left(y^2 - x^2\right)dx = 0$$

Differential Equations Ex 22.3 Q15

$$y^2 = 4a(x + a) \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a(1)$$
$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\}$$
$$= \left[y^2\left\{1 - \left(\frac{dy}{dx}\right)^2\right\}\right]\frac{1}{y}$$
$$= \left[4a(x + a) - 4a(x + a)\left(\frac{2a}{y}\right)^2\right]\frac{1}{y}$$

Using equation (i) and (ii)

$$= \left[4ax + 4a^2 - \frac{16a3x}{y^2} - \frac{16a^4}{y^2}\right]\frac{1}{y}$$
$$= \frac{4a}{y^3}\left[xy^2 + ay^2 - 4a^2x - 4a^3\right]$$
$$= \frac{4a}{y^3}\left[y^2(a + x) - 4a^2(x + a)\right]$$
$$= \frac{4a}{y^3}(a + x)\left(y^2 - 4a^2\right)$$
$$= \frac{4a}{y^3}\left(\frac{y^2}{4a}\right)\left(y^2 - 4a^2\right)$$

Using equation (i) and (ii)

$$= \frac{1}{y}\left(y^2 - 4a^2\right)$$
$$= \frac{1}{y}\left[4ax + 4a^2 - 4a\right]$$
$$= \frac{1}{y}(4ax)$$
$$= 2x\left(\frac{2a}{y}\right)$$
$$= 2x\frac{dy}{dx}$$

So,

$$y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q16

$$y = ce^{\tan^{-1} x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ce^{\tan^{-1} x} \times \left(\frac{1}{1+x^2}\right)$$

$$\left(1+x^2\right)\frac{dy}{dx} = ce^{\tan^{-1} x}$$

$$\left(1+x^2\right)\frac{dy}{dx} = y$$

Again, differentiating it with respecet to x,

$$2x\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$2x\frac{dy}{dx} - \frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

$$(2x-1)\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

Differential Equations Ex 22.3 Q17

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

$$y = e^{m\cos^{-1}x}$$

$$\frac{dy}{dx} = \frac{me^{m\cos^{-1}x}}{-\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-my}{\sqrt{1-x^2}} \text{........(i)}$$

$$\frac{d^2y}{dx^2} = \frac{\sqrt{(1-x^2)}.\left(-m\frac{dy}{dx}\right) - (-my)\frac{(-2x)}{2\sqrt{(1-x^2)}}}{(1-x^2)} \text{ [From (i)]}$$

$$\frac{d^2y}{dx^2} = \frac{(-m)(-my) - x\frac{dy}{dx}}{(1-x^2)} \text{[From (i)]}$$

$$(1-x^2)\frac{d^2y}{dx^2} = m^2y - x\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

Hence Proved

Differential Equations Ex 22.3 Q18

$$y = \log\left(x+\sqrt{x^2+a^2}\right)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx}\frac{1}{\left(x+\sqrt{a^2+x^2}\right)^2} \times 2\left(x+\sqrt{x^2+a^2}\right)\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$= \frac{2}{\left(x+\sqrt{a^2+x^2}\right)} \times \left[1+\frac{1}{2\sqrt{x^2+a^2}}(2x)\right]$$

$$= \frac{2}{\left(x+\sqrt{a^2+x^2}\right)}\left(\frac{\sqrt{x^2+a^2}+x}{2\sqrt{x^2+a^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2+x^2}}$$

$$\sqrt{a^2+x^2}\frac{dy}{dx} = 1 \qquad \text{---(i)}$$

Again, differentiating it with respecet to x,

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}(-2x)\frac{dy}{dx} = -m\frac{dy}{dx}$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} - m\left(\frac{-e^{m\cos^{-1}x}m}{\sqrt{1-x^2}}\right) = 0$$

Using equation (i),

$$\sqrt{a^2+x^2}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{a^2+x^2}}\frac{dy}{dx} = 0$$

$$\left(a^2+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Differential Equations Ex 22.3 Q19

$$y=2\left(x^2-1\right)+ce^{-x^2} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=2(2x)+ce^{-x^2}(-2x)$$

$$\frac{dy}{dx}=4x-2cxe^{-x^2} \qquad \text{—(ii)}$$

Now,

$$\frac{dy}{dx}+2xy$$

$$=4x-2cxe^{-x^2}+2x\left[2\left(x^2-1\right)+ce^{-x^2}\right]$$

Using equation (i) and (ii),

$$=4x-2cxe^{-x^2}+2x\left(2x^2-2+ce^{-x^2}\right)$$

$$=4x-2cxe^{-x^2}+4x^3-4x+2xce^{-x^2}$$

$$=4x^3$$

So,

$$\frac{dy}{dx}+2xy=4x^3$$

Differential Equations Ex 22.3 Q20

$$y=e^{-x}+ax+b$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=-e^{-x}+a$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2}=e^{-x}$$

$$\frac{1}{e^{-x}}\frac{d^2y}{dx^2}=1$$

$$e^{x}\frac{d^2y}{dx^2}=1$$

Differential Equations Ex 22.3 Q21(i)

$$y=ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=a$$

$$=\frac{ax}{x} \qquad [\because x\in R-\{0\}]$$

$$\frac{dy}{dx}=\frac{y}{x} \qquad [\text{Using equation (i)}]$$

$$x\frac{dy}{dx}=y$$

So, $y=ax$ is the solution of the given equation.

Differential Equations Ex 22.3 Q21(ii)

$$y=\pm\sqrt{a^2-x^2}$$

Squaring both the sides,

$$y^2=\left(a^2-x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx}=-2x$$

$$y\frac{dy}{dx}=-x$$

$$x+y\frac{dy}{dx}=0$$

So,

$y=\pm\sqrt{a^2-x^2}$ **is the solution of the given equation.**

Differential Equations Ex 22.3 Q21(iii)

$$y = \frac{a}{x+a}$$

$$\frac{dy}{dx} = \frac{a}{(x+a)^2} \times (-1) = -\frac{a}{(x+a)^2}$$

Consider,

$$x\frac{dy}{dx} + y = -\frac{ax}{(x+a)^2} + \frac{a}{x+a} = \frac{-ax + ax + a^2}{(x+a)^2} = \frac{a^2}{(x+a)^2} = y^2$$

$$x\frac{dy}{dx} + y = y^2$$

Hence $y = \frac{a}{x+a}$ is the solution of the differential equation $x\frac{dy}{dx} + y = y^2$.

Differential Equations Ex 22.3 Q21(iv)

$$y = ax + b + \frac{1}{2x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = a - \frac{1}{2x^2}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 0 - \frac{(-2)}{2x^3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^3}$$

$$x^3\frac{d^2y}{dx^2} = 1$$

So,

$y = ax + b + \frac{1}{2x}$ **is the solution of the given equation.**

Differential Equations Ex 22.3 Q21(v)

$$y = \frac{1}{4}(x \pm a)^2$$

Case I:

$$y = \frac{1}{4}(x + a)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4}2(x + a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x + a)$$

Squaring both sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x + a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y \qquad \text{[Using equation (i)]}$$

So,

$y = \frac{1}{4}(x + a)$ **is the solution of the given equation.**

Case II:

$$y = \frac{1}{4}(x - a)^2 \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4}2(x - a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x - a)$$

Squaring both the sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x - a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y^2 \qquad \text{[Using equation (ii)]}$$

So,

$y = \frac{1}{4}(x - a)$ **is the solution of the given equation.**

Ex 22.4

Differential Equations Ex 22.4 Q1

Here, $y = \log x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{x}$$

$$x\frac{dy}{dx} = 1$$

So, $y = \log x$ is a solutionof the equation

If $\quad x = 1, \quad y = \log 1 = 0$

So,

$$y(1) = 0$$

Differential Equations Ex 22.4 Q2

Here, $y = e^x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y$$

So, $y = e^x$ is a solution of the equation

If $x = 0, \quad y = e^0 = 1$

So,

$$y(0) = 1$$

Differential Equations Ex 22.4 Q3

Here, $y = \sin x \qquad$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x \qquad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x$$

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

So, $y = \sin x$ is a solution of the equation.

Put $\quad x = 0$ in equation (i),

$\Rightarrow \quad y = \sin 0$

$\Rightarrow \quad y = 0$

$\Rightarrow \quad y(0) = 0$

Put $x = 0$ in equation (ii),

$\quad y' = \cos 0$

$\quad y' = 1$

$\Rightarrow \quad y'(0) = 1$

Differential Equations Ex 22.4 Q4

Here, $y = e^x + 1$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y - 1 \quad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

It is given differential equation. So,

$y = e^x + 1$ is a solutionof the equation

Put $x = 0$ in equation (i),

$\Rightarrow$ $y = e^0 + 1 = 2$

$y(0) = 2$

Put $x = 0$ in equation (ii),

$y' = e^0 = 1$

$y'(0) = 1$

Differential Equations Ex 22.4 Q5

Here, $y = e^{-x} + 2$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dy}{dx} = -(y - 2) \quad \text{[Using equaton (i)]}$$

$$\frac{dy}{dx} + y = 2$$

It is given differential equation. So,

$y = e^{-x} + 2$ is a solutionof the equation

Put $x = 0$ in equation (i),

$y = e^0 + 2$

$= 1 + 2$

$y = 3$

So,

$y(0) = 3$

Differential Equations Ex 22.4 Q6

$y = \sin x + \cos x$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x - \sin x \quad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$\frac{d^2y}{dx^2} = -(\sin x + \cos x)$$

$$\frac{d^2y}{dx^2} = -y \quad \text{[Using equation (i)]}$$

$$\frac{d^2y}{dx^2} + y = 0$$

It is the given equation, so

$y = \sin x + \cos x$ is the solution of the given equation

Put $x = 0$ in equation (i),

$y = \sin 0 + \cos 0$

$y = 0 + 1$

$y = 1$

So,

$y(0) = 1$

Put $x = 0$ in equation (i),

$$\frac{dy}{dx} = \cos 0 - \sin 0$$

$y' = 1$

So,

$y'(0) = 1$

Differential Equations Ex 22.4 Q7

$$y = e^x + e^{-x} \quad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x - e^{-x} \quad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + e^{-x}$$

$$\frac{d^2y}{dx^2} = y \quad \text{[Using equation (i)]}$$

$$\frac{d^2y}{dx^2} - y = 0$$

It is the given equation, so

$y = e^x + e^{-x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$

$$y = 2$$

So,

$$y(0) = 2$$

Put $x = 0$ in equation (ii),

$$y' = e^0 - e^0$$

$$y' = 0$$

So,

$$y'(0) = 0$$

Differential Equations Ex 22.4 Q8

$$y = e^x + e^{2x} \quad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x + 2e^{2x} \quad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + 4e^{2x}$$

$$= (3-2)e^x + (6-2)e^{2x}$$

$$= 3e^x - 2e^x + 6e^{2x} - 2e^{2x}$$

$$= 3e^x + 6e^{2x} - 2e^x - 2e^{2x}$$

$$= 3\left(e^x + 2e^{2x}\right) - 2\left(e^x + e^{2x}\right)$$

$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} - 2y \quad \text{[Using equation(i) and (ii)]}$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

It is the given equation, so

$y = e^x + 2e^{2x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$

$$y = 1 + 1$$

$$y = 2$$

So,

$$y(0) = 2$$

Put $x = 0$ in equation (ii),

$$\frac{dy}{dx} = e^0 + 2e^0$$

$$y' = 1 + 2$$

$$y' = 3$$

So,

$$y'(0) = 3$$

Differential Equations Ex 22.4 Q9

$$y = xe^x + e^x \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \left[x \times \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(x)\right] + e^x$$

$$= xe^x + e^x(1) + e^x$$

$$\frac{dy}{dx} = xe^x + 2e^x \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = x\frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(x) + 2e^x$$

$$= (2-1)xe^x + (4-1)e^x$$

$$= 2xe^x - xe^x + 4e^x - e^x$$

$$= 2xe^x + 4e^x - xe^x - e^x$$

$$= 2\left(xe^x + 2e^x\right) - \left(xe^x + 1\right)$$

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y \qquad \left[\text{Using equation(i) and (ii)}\right]$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

It is the given equation, so

$y = xe^x + e^x$ **is the solution of the given equation.**

Put $y = 0$ in equation (i),

$$y = 0 + e^0$$

$$y = 1$$

So,

$$y(0) = 1$$

Put $y = 0$ in equation (ii),

$$\frac{dy}{dx} = 0 + 2e^0$$

$$y' = 2$$

So,

$$y'(0) = 2$$

Ex 22.5

Differential Equations Ex 22.5 Q1

$$\frac{dy}{dx} = x^2 + x - \frac{1}{x},\ x \neq 0$$
$$\int dy = \int\left(x^2 + x - \frac{1}{x}\right)dx$$
$$y = \frac{x^3}{3} + \frac{x^2}{2} - \log|x| + c,\ x \neq 0$$

Differential Equations Ex 22.5 Q2

$$\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x},\ x \neq 0$$
$$\int dy = \int\left(x^5 + x^2 - \frac{2}{x}\right)dx$$
$$y = \frac{x^6}{6} + \frac{x^3}{3} - 2\log|x| + c,\ x \neq 0$$

Differential Equations Ex 22.5 Q3

$$\frac{dy}{dx} + 2x = e^{3x}$$
$$\frac{dy}{dx} = e^{3x} - 2x$$
$$\int dy = \int\left(e^{3x} - 2x\right)dx$$
$$y = \frac{e^{3x}}{3} - \frac{2x^2}{2} + c$$
$$y = \frac{e^{3x}}{3} - x^2 + c$$
$$y + x^2 = \frac{1}{3}e^{3x} + c$$

Differential Equations Ex 22.5 Q4

$$\left(x^2+1\right)\frac{dy}{dx} = 1$$
$$\int dy = \int\frac{dx}{x^2+1}$$
$$y = \tan^{-1}x + c$$

Differential Equations Ex 22.5 Q6

$$(x+2)\frac{dy}{dx} = x^2 + 3x + 7$$
$$dy = \left(\frac{x^2+3x+7}{x+2}\right)dx$$
$$dy = \left(x + 1 + \frac{5}{x+2}\right)dx$$
$$\int dy = \int\left(x + 1 + \frac{5}{x+2}\right)dx$$
$$y = \frac{x^2}{2} + x + 5\log|x+2| + c$$
$$x \neq -2$$

Differential Equations Ex 22.5 Q7

$$\frac{dy}{dx} = \tan^{-1}x$$
$$dy = \tan^{-1}x\,dx$$
$$\int dy = \int \tan^{-1}x\,dx$$
$$y = \tan^{-1}x \times \int 1dx - \int\left(\frac{1}{1+x^2}\int dx\right)dx + c$$

Using integration by parts

$$y = x\tan^{-1}x - \int\frac{x}{1+x^2}dx + c$$
$$y = x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx + c$$
$$y = x\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + c$$

Differential Equations Ex 22.5 Q8

$$\frac{dy}{dx} = \log x$$
$$\Rightarrow dy = \log x \times dx$$
$$\Rightarrow \int dy = \int \log x dx$$
$$\Rightarrow y = \log x \times \int 1 dx - \int\left(\frac{1}{x}\int 1 dx\right)dx + C \quad [\textit{Using integration by parts}]$$
$$\Rightarrow y = x\log x - \int dx + C$$
$$\Rightarrow y = x\log x - x + C$$
$$\Rightarrow y = x(\log x - 1) + C, \textit{ where } x \in (0, \infty)$$

Differential Equations Ex 22.5 Q9

$$\frac{1}{x}\frac{dy}{dx} = \tan^{-1} x$$
$$dy = x\tan^{-1} x dx$$
$$\int dy = \int x\tan^{-1} x dx$$
$$y = \tan^{-1} x \int x dx - \int\left(\frac{1}{1+x^2}\int x dx\right)dx + c$$

Using integration by parts

$$y = \frac{x^2}{2}\tan^{-1} x - \int \frac{x^2}{2\left(1+x^2\right)}dx + c$$
$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx + c$$
$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(1 - \frac{1}{x^2+1}\right)dx + c$$
$$y = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$$
$$y = \frac{1}{2}\left(x^2+1\right)\tan^{-1} x - \frac{1}{2}x + c$$

Differential Equations Ex 22.5 Q10

$$\frac{dy}{dx} = \cos^3 x \sin^2 x + x\sqrt{2x+1}$$
$$dy = \left(\cos^3 x \sin^2 x + x\sqrt{2x+1}\right)dx$$
$$\int dy = \int \cos^3 x \sin^2 x dx + \int x\sqrt{2x+1}dx$$
$$y = I_1 + I_2 \qquad \text{---(i)}$$
$$I_1 = \int \cos^3 x \sin^2 x dx$$
$$= \int \cos^2 x \times \cos x \times \sin^2 x dx$$
$$I_1 = \int\left(1 - \sin^2 x\right)\sin^2 x \cos x dx$$

Put $\sin x = t$

$$\cos x dx = dt$$
$$I_1 = \int\left(1 - t^2\right)t^2 dt$$
$$= \int\left(t^2 - t^4\right)dt$$
$$= \frac{t^3}{3} - \frac{t^5}{5} + c_1$$
$$I_1 = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c_1$$

And,

$$I_2 = \int x\sqrt{2x+1}dx$$

Put $2x + 1 = v^2$

$$2dx = 2vdv$$
$$I_2 = \int\left(\frac{v^2-1}{2}\right)v \times vdv$$
$$= \frac{1}{2}\int\left(v^4 - v^2\right)dv$$
$$= \frac{1}{2}\left(\frac{v^5}{5} - \frac{v^3}{3}\right) + c_2$$
$$I_2 = \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c_2$$

Put the I_1 and I_2 in equation (i),

$$y = I_1 + I_2$$
$$y = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c$$

As $\quad c = c_1 + c_2$

Differential Equations Ex 22.5 Q11

$$(\sin x + \cos x)\,dy + (\cos x - \sin x)\,dx = 0$$
$$(\sin x + \cos x)\,dy = (\sin x - \cos x)\,dx$$
$$dy = \frac{(\sin x - \cos x)}{\sin x + \cos x}dx$$
$$\int dy = -\int\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)dx$$

Put $\sin x + \cos x = t$

$$(\cos x - \sin x)\,dx = dt$$
$$\int dy = -\int \frac{1}{t}dt$$
$$y = -\log|t| + c$$
$$y + \log|\sin x + \cos x| = c$$

Differential Equations Ex 22.5 Q12

$$\frac{dy}{dx} - x\sin^2 x = \frac{1}{x\log x}$$
$$\frac{dy}{dx} = \frac{1}{x \times \log x} + x\sin^2 x$$
$$dy = \left(\frac{1}{x\log x} + x\sin^2 x\right)dx$$
$$\int dy = \int \frac{1}{x\log x}dx + \int x\sin^2 x\,dx$$
$$y = I_1 + I_2 \qquad \text{---(i)}$$
$$I_1 = \int \frac{1}{x\log x}dx$$

Let $\log x = t$

$$\frac{1}{x}dx = dt$$
$$I_1 = \int \frac{dt}{t}$$
$$= \log|t| + c_1$$
$$I_1 = \log|\log x| + c_1$$
$$I_2 = \int x\sin^2 x\,dx$$
$$= \int x\frac{(1-\cos 2x)}{2}dx$$
$$= \frac{1}{2}\int (x - x\cos 2x)\,dx$$
$$= \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x\,dx$$
$$= \frac{1}{2}\left(\frac{x^2}{2}\right) - \frac{1}{2}\left[x\int\cos 2x\,dx - \int\left(1 \times \int\cos 2x\,dx\right)dx\right] + c_2$$
$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} - \int\frac{\sin 2x}{2}dx\right] + c_2$$
$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right] + c_2$$
$$I_2 = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c_2$$

Put the value of I_1 and I_2 in equation (i),

$$y = I_1 + I_2$$
$$y = \log|\log x| + \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c \text{ as } c_1 + c_2 = c$$

Differential Equations Ex 22.5 Q13

$$\frac{dy}{dx} = x^5\tan^{-1}\left(x^3\right)$$
$$dy = x^5\tan^{-1}\left(x^3\right)dx$$
$$\int dy = \int x^5\tan^{-1}\left(x^3\right)dx$$

Put $x^3 = t$

$\Rightarrow \quad 3x^2dx = dt$

$\Rightarrow \quad x^2dx = \frac{dt}{3}$

So,

$$\int dy = \frac{1}{3}\left[\tan^{-1}t\int t\,dt - \int\left(\frac{1}{1+t^2} \times \int t\,dt\right)\right]dt + c$$

Using integration by parts

$$y = \frac{1}{3}\left[\frac{t^2}{2} + \tan^{-1}t - \int\frac{t^2}{2\left(t^2+1\right)}dt\right] + c$$
$$= \frac{1}{6}t^2\tan^{-1}t - \frac{1}{6}\int\left(\frac{t^2}{t^2+1}\right)dt + c$$
$$y = \frac{1}{6}t^2\tan^{-1}t - \frac{1}{6}\int\left(1 - \frac{1}{t^2+1}\right)dt + c$$
$$= \frac{1}{6}t^2\tan^{-1}t - \frac{1}{6}t + \frac{1}{6}\tan^{-1}t + c$$
$$y = \frac{1}{6}\left(t^2+1\right)\tan^{-1}t - \frac{1}{6}t + c$$
$$y = \frac{1}{6}\left[\left(t^2+1\right)\tan^{-1}t - t\right] + c$$

So,

$$y = \frac{1}{6}\left[\left(x^6+1\right)\tan^{-1}\left(x^3\right) - x^3\right] + c$$

Differential Equations Ex 22.5 Q14

$$\sin^4 x \frac{dy}{dx} = \cos x$$
$$dy = \frac{\cos x}{\sin^4 x} dx$$
$$\int dy = \int \frac{\cos x}{\sin^4 x} dx$$

Put $\sin x = t$

$$\cos x dx = dt$$
$$\int dy = \int \frac{dt}{t^4}$$
$$y = \frac{1}{-3t^3} + c$$
$$y = -\frac{1}{3\sin^3 x} + c$$
$$y = -\frac{1}{3}\operatorname{cosec}^3 x + c$$

Differential Equations Ex 22.5 Q15

$$\cos x \frac{dy}{dx} - \cos 2x = \cos 3x$$
$$\cos x \frac{dy}{dx} = \cos 3x + \cos 2x$$
$$\frac{dy}{dx} = \frac{4\cos^3 x - 3\cos x + 2\cos^2 x - 1}{\cos x}$$
$$\frac{dy}{dx} = \frac{4\cos^3 x}{\cos x} - \frac{3\cos x}{\cos x} + \frac{2\cos^2 x}{\cos x} - \frac{1}{\cos x}$$
$$\frac{dy}{dx} = 4\cos^2 x - 3 + 2\cos x - \sec x$$
$$\frac{dy}{dx} = 4\left(\frac{\cos 2x + 1}{2}\right) - 3 + 2\cos x - \sec x$$
$$dy = (2\cos 2x + 2 - 3 + 2\cos x - \sec x) dx$$
$$\int dy = \int (2\cos 2x - 1 + 2\cos x - \sec x) dx$$
$$y = \sin 2x - x + 2\sin x - \log|\sec x + \tan x| + c$$

Differential Equations Ex 22.5 Q16

$$\sqrt{1 - x^4} dy = x dx$$
$$dy = \frac{x dx}{\sqrt{1 - x^4}}$$
$$\int dy = \int \frac{x dx}{\sqrt{1 - x^4}}$$

Let $x^2 = t$

$$2x dx = dt$$

$\Rightarrow$ $x dx = \frac{dt}{2}$

$$\int dy = \int \frac{dt}{2\sqrt{1 - t^2}}$$
$$y = \frac{1}{2}\sin^{-1}(t) + c$$
$$y = \frac{1}{2}\sin^{-1}(x^2) + c$$

Differential Equations Ex 22.5 Q17

$$\sqrt{a + x} dy + x dx = 0$$
$$\sqrt{a + x} dy = -x dx$$
$$dy = \frac{-x}{\sqrt{a + x}} dx$$
$$\int dy = -\int \frac{x}{\sqrt{a + x}} dx$$

Put $a + x = t^2$

$$dx = 2t dt$$
$$\int dy = -\int \left(\frac{t^2 - a}{t}\right) 2t dt$$
$$\int dy = 2\int (a - t^2) dt$$
$$y = 2\left(at - \frac{t^3}{3}\right) + c$$
$$y + \frac{2}{3}t^3 - 2at = c$$
$$y + \frac{2}{3}(a + x)^{\frac{3}{2}} - 2a\sqrt{a + x} = c$$

Differential Equations Ex 22.5 Q18

$$(1+x^2)\frac{dy}{dx}-x=2\tan^{-1}x$$
$$(1+x^2)\frac{dy}{dx}=2\tan^{-1}x+x$$
$$dy=\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$\int dy=\int\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$y=\int(2t+\tan t)dt \qquad [\tan^{-1}x=t]$$
$$=\frac{1}{2}\log\left|1+x^2\right|+(\tan^{-1}x)^2+c$$

Differential Equations Ex 22.5 Q19

$$\frac{dy}{dx}=x\log x$$
$$dy=x\log x dx$$
$$\int dy=\int x\log x dx$$
$$y=\log|x|\int x dx-\int\left(\frac{1}{x}\int x dx\right)dx+c$$

Using integration by parts

$$=\frac{x^2}{2}\log|x|-\int\frac{x^2}{2x}dx+c$$
$$=\frac{x^2}{2}\log|x|-\frac{1}{2}\int x dx+c$$
$$y=\frac{x^2}{2}\log|x|-\frac{x^2}{4}+c$$

Differential Equations Ex 22.5 Q20

$$\frac{dy}{dx}=xe^x-\frac{5}{2}+\cos^2x$$
$$dy=\left(xe^x-\frac{5}{2}+\cos^2x\right)dx$$
$$\int dy=\int xe^x dx-\frac{5}{2}\int dx+\int\cos^2x dx$$
$$\int dy=\int xe^x dx-\frac{5}{2}\int dx+\int\left(\frac{1+\cos 2x}{2}\right)dx$$
$$=\int xe^x-\frac{5}{2}\int dx+\frac{1}{2}\int dx+\frac{1}{2}\int\cos 2x dx$$
$$\int dy=\int xe^x-2\int dx+\frac{1}{2}\int\cos 2x dx$$
$$y=\left[x\times\int e^x dx-\int\left(1\times\int e^x dx\right)dx\right]-2x+\frac{1}{2}\frac{\sin 2x}{2}+c$$

Using integration by parts

$$y=xe^x-e^x-2x+\frac{1}{4}\sin 2x+c$$
$$y=xe^x-e^x-2x+\frac{1}{4}\sin 2x+c$$

Differential Equations Ex 22.5 Q21

The given differential equation is:

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x$$
$$\Rightarrow \frac{dy}{dx}=\frac{2x^2+x}{\left(x^3+x^2+x+1\right)}$$
$$\Rightarrow dy=\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides, we get:

$$\int dy=\int\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx \qquad ...(1)$$

$$\text{Let } \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}. \qquad ...(2)$$
$$\Rightarrow \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{Ax^2+A+(Bx+C)(x+1)}{(x+1)\left(x^2+1\right)}$$
$$\Rightarrow 2x^2+x=Ax^2+A+Bx^2+Bx+Cx+C$$
$$\Rightarrow 2x^2+x=(A+B)x^2+(B+C)x+(A+C)$$

Comparing the coefficients of x^2 and x, we get:

$A+B=2$

$B+C=1$

$A+C=0$

Solving these equations, we get:

$$A=\frac{1}{2},\ B=\frac{3}{2} \text{ and } C=\frac{-1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{1}{2}\cdot\frac{1}{(x+1)}+\frac{1}{2}\frac{(3x-1)}{\left(x^2+1\right)}$$

Therefore, equation (1) becomes:

$$\int dy=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{3x-1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\cdot\int\frac{2x}{x^2+1}dx-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{4}\left[2\log(x+1)+3\log\left(x^2+1\right)\right]-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{4}\left[(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+C \qquad ...(3)$$

Now, $y=1$ when $x=0$.

$$\Rightarrow 1=\frac{1}{4}\log(1)-\frac{1}{2}\tan^{-1}0+C$$
$$\Rightarrow 1=\frac{1}{4}\times0-\frac{1}{2}\times0+C$$
$$\Rightarrow C=1$$

Substituting C = 1 in equation (3), we get:

$$y=\frac{1}{4}\left[\log(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Differential Equations Ex 22.5 Q22

$$\sin\left(\frac{dy}{dx}\right) = k, \quad y(0) = 1$$

$$\frac{dy}{dx} = \sin^{-1} k$$

$$dy = \sin^{-1} k dx$$

$$\int dy = \int \sin^{-1} k dx$$

$$y = x \sin^{-1} k + c \qquad \text{---(i)}$$

Put $x = 0, y = 1$

$$1 = 0 + c$$

$$1 = c$$

Put $c = 1$ in equation (i),

$$y = x \sin^{-1} k + 1$$

$$y - 1 = x \sin^{-1} k$$

Differential Equations Ex 22.5 Q23

$$e^{\frac{dy}{dx}} = x + 1, \quad y(0) = 3$$

$$\frac{dy}{dx} = \log(x+1)$$

$$dy = \log(x+1) dx$$

$$\int dy = \int \log(x+1) dx$$

$$y = \log(x+1)\int 1 \times dx - \int\left(\frac{1}{x+1} \times \int 1 \times dx\right)dx + c$$

Using integration by parts

$$y = x \log(x+1) - \int\left(\frac{x}{x+1}\right)dx + c$$

$$= x \log(x+1) - \int\left(1 - \frac{1}{x+1}\right)dx + c$$

$$= x \log(x+1) - x + \log(x+1) + c$$

$$y = (x+1)\log(x+1) - x + c \qquad \text{---(i)}$$

Put $y = 3, x = 0$

$$3 = 0 + c$$

$\Rightarrow$ $c = 3$

Using equation (i),

$$y = (x+1)\log(x+1) - x + 3$$

Differential Equations Ex 22.5 Q24

$$c'(x) = 2 + 0.15x, \quad c(0) = 100$$

$$c'(x)dx = (2 + 0.15x)dx$$

$$\int c'(x)dx = \int 2dx + 0.15\int x dx$$

$$c(x) = 2x + 0.15\frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $x = 0, c(x) = 100$

$$100 = 2(0) + 0 + c$$

$$100 = c$$

Put $c = 100$ in equation (i),

$$c(x) = 2x + (0.15)\frac{x^2}{2} + 100$$

Differential Equations Ex 22.5 Q25

$$x\frac{dy}{dx} + 1 = 0, \quad y(-1) = 0$$

$$x\frac{dy}{dx} = -1$$

$$dy = -\frac{dx}{x}$$

$$\int dy = -\int\frac{dx}{x}$$

$$y = -\log|x| + c \qquad \text{---(i)}$$

Put $x = -1$ and $y = 0$

$$0 = 0 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$y = -\log|x|, x < 0$$

Differential Equations Ex 22.5 Q26

$$x\left(x^2-1\right)\frac{dy}{dx}=1, y(2)=0$$

$$\frac{dy}{dx}=\frac{1}{x\left(x^2-1\right)}$$

$$dy=\frac{1}{x\left(x^2-1\right)}dx$$

$$\int dy=\int\left(\frac{1}{x\left(x^2-1\right)}\right)dx$$

$$y=\frac{1}{2}\int\frac{1}{x-1}dx-\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{x+1}dx$$

$$=\frac{1}{2}\log|x-1|-\log|x|+\frac{1}{2}\log|x+1|+c$$

Putting $x=2, y=0$, we have

$$y=\frac{1}{2}\log|x-1|-1\log|x|+\frac{1}{2}\log|x+1|+c$$

$$0=\frac{1}{2}\log|2-1|-\log|2|+\frac{1}{2}\log|2+1|+c$$

$$c=\log|2|-\frac{1}{2}\log|3|$$

Putting the value of c, we have

$$y=\frac{1}{2}\log|x-1|-1\log|x|+\frac{1}{2}\log|x+1|+c$$

$$=\log\frac{4}{3}\left(\frac{x^2-1}{x^2}\right)$$

Ex 22.6

Differential Equations Ex 22.6 Q1

$$\frac{dy}{dx}+\frac{1+y^2}{y}=0, \qquad y\neq 0$$
$$\frac{dy}{dx}=-\frac{1+y^2}{y}$$
$$\int\frac{y}{1+y^2}dy=-\int dx$$
$$\int\frac{2y}{1+y^2}dy=-2\int dx$$
$$\log\left|1+y^2\right|=-2x+c_1$$
$$\frac{1}{2}\log\left|1+y^2\right|+x=c$$

Differential Equations Ex 22.6 Q2

$$\frac{dy}{dx}=\frac{1+y^2}{y^3}, \quad y\neq 0$$
$$\frac{y^3}{1+y^2}dy=dx$$
$$\int\left(y-\frac{y}{y^2+1}\right)dy=\int dx$$
$$\int y\,dy-\int\frac{y}{y^2+1}dy=\int dx$$
$$\int y\,dy-\frac{1}{2}\int\frac{2y}{y^2+1}dy=\int dx$$
$$\frac{y^2}{2}-\frac{1}{2}\log\left|y^2+1\right|=x+c$$

Differential Equations Ex 22.6 Q3

$$\frac{dy}{dx}=\sin^2 y$$
$$\frac{dy}{\sin^2 y}=dx$$
$$\int \operatorname{cosec}^2 y\,dy=\int dx$$
$$-\cot x=x+c_1$$
$$x+\cot x=c$$

Differential Equations Ex 22.6 Q4

$$\frac{dy}{dx}=\frac{1-\cos 2y}{1+\cos 2y}$$
$$=\frac{2\sin^2 y}{2\cos^2 y}$$
$$\frac{dy}{dx}=\tan^2 y$$
$$\frac{dy}{\tan^2 y}=dx$$
$$\int\cot^2 y\,dy=\int dx$$
$$\int\left(\operatorname{cosec}^2 y-1\right)dy=\int dx$$
$$-\cot y-y+c=x$$
$$c=x+y+\cot y$$

Ex 22.7

Differential Equations Ex 22.7 Q1

$$(x-1)\frac{dy}{dx} = 2xy$$

Separating the variables,

$$\int \frac{dy}{y} = \int \frac{2x}{x-1}dx$$

$$\int \frac{dy}{y} = \int \left(2 + \frac{2}{x-1}\right)dx$$

$$\log y = 2x + 2\log|x-1| + c$$

Differential Equations Ex 22.7 Q2

$$(x^2+1)dy = xydx$$

$$\int \frac{1}{y}dy = \int \frac{x}{x^2+1}dx$$

$$\int \frac{1}{y}dy = \frac{1}{2}\int \frac{2x}{x^2+1}dx$$

$$\log y = \frac{1}{2}\log|x^2+1| + \log c$$

$$y = \sqrt{x^2+1} \times c$$

Differential Equations Ex 22.7 Q3

$$\frac{dy}{dx} = (e^x+1)y$$

$$\int \frac{1}{y}dy = \int (e^x+1)dx$$

$$\log|y| = e^x + x + c$$

Differential Equations Ex 22.7 Q4

$$(x-1)\frac{dy}{dx} = 2x^3y$$

$$\frac{dy}{y} = \frac{2x^3}{x-1}dx$$

$$\int \frac{dy}{y} = 2\int \left(x^2 + x + 1 + \frac{1}{x-1}\right)dx$$

$$\log|y| = 2\left(\frac{x^3}{3} + \frac{x^2}{2} + x + \log|x-1|\right) + c$$

$$\log|y| = \frac{2}{3}x^3 + x^2 + 2x + 2\log|x-1| + c$$

Differential Equations Ex 22.7 Q5

$$xy(y+1)dy = (x^2+1)dx$$

$$y(y+1)dy = \frac{x^2+1}{x}dx$$

$$\int (y^2+y)dy = \int \left(x + \frac{1}{x}\right)dx$$

$$\frac{y^3}{3} + \frac{y^2}{2} = \frac{x^2}{2} + \log|x| + c$$

Differential Equations Ex 22.7 Q6

$$5\frac{dy}{dx} = e^xy^4$$

$$5\int \frac{dy}{y^4} = \int e^xdx$$

$$5\left(\frac{y^{-4+1}}{-4+1}\right) = e^x + c$$

$$-\frac{5}{3y^3} = e^x + c$$

Differential Equations Ex 22.7 Q7

$$x \cos y dy = \left(xe^x \log x + e^x\right) dx$$

$$\int \cos y dy = \int e^x \left(\log x + \frac{1}{x}\right) dx$$

$$\sin y = e^x \log x + c$$

Since, $\int \left(f(x) + f'(x)\right) e^x dx = e^x f(x) + c$

Differential Equations Ex 22.7 Q8

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$= e^x e^y + x^2 e^y$$

$$\frac{dy}{dx} = e^y \left(e^x + x^2\right)$$

$$\int e^{-y} dy = \int \left(e^x + x^2\right) dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + c$$

Differential Equations Ex 22.7 Q9

$$x \frac{dy}{dx} + y = y^2$$

$$x \frac{dy}{dx} = \left(y^2 - y\right)$$

$$\frac{1}{y^2 - y} dy = \frac{dx}{x}$$

$$\int \left(\frac{1}{y-1} - \frac{1}{y}\right) dy = \int \frac{dx}{x}$$

$$\log |y - 1| - \log |y| = \log |x| + \log |c|$$

$$\log \left|\frac{y-1}{y}\right| = |xc|$$

$$y - 1 = xyc$$

Differential Equations Ex 22.7 Q10

$$\left(e^y + 1\right) \cos x dx + e^y \sin x dy = 0$$

$$\left(e^y + 1\right) \cos x dx = -e^y \sin x dy$$

$$\int \frac{\cos x}{\sin x} dx = -\int \frac{e^y}{e^y + 1} dy$$

$$\int \cot x dx = -\int \frac{e^y}{e^y + 1} dy$$

$$\log |\sin x| = -\log \left|e^y + 1\right| + \log |c|$$

$$\sin x = \frac{c}{e^y + 1}$$

$$\sin x \left(e^y + 1\right) = c$$

Differential Equations Ex 22.7 Q11

$$x\cos^2 y\,dx = y\cos^2 x\,dy$$
$$\frac{x}{\cos^2 x}dx = \frac{y}{\cos^2 y}dy$$
$$\int x\sec^2 x\,dx = \int y\sec^2 y\,dy$$
$$x\times\int \sec^2 x - \int\left(1\times\int \sec^2 x\,dx\right)dx = y\int \sec^2 y\,dy - \int\left(1\times\int \sec^2 y\,dy\right)dy$$
$$x\tan x - \int \tan x\,dx = y\tan y - \int \tan y\,dy + c$$
$$x\tan x - \log|\sec x| = y\tan y - \log|\sec y| + c$$

Differential Equations Ex 22.7 Q12

$$xy\,dy = (y-1)(x+1)dx$$
$$\frac{y}{y-1}dy = \frac{x+1}{x}dx$$
$$\int\left(1+\frac{1}{y-1}\right)dy = \int\left(1+\frac{1}{x}\right)dx$$
$$y+\log|y-1| = x+\log|x|+c$$
$$y-x = \log|x| - \log|y-1| + c$$

Differential Equations Ex 22.7 Q13

$$x\frac{dy}{dx}+\cot y = 0$$
$$x\frac{dy}{dx} = -\cot y$$
$$\int \tan y\,dy = -\int\frac{dx}{x}$$
$$\log|\sec y| = -\log|x| + \log|c|$$
$$\sec y = \frac{c}{x}$$
$$x\sec y = c$$
$$x = c\cos y$$

Differential Equations Ex 22.7 Q14

$$\frac{dy}{dx} = \frac{xe^x\log x + e^x}{x\cos y}$$
$$\int \cos y\,dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$$
$$\sin y = e^x\log x + c$$

Since, $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + c$

Differential Equations Ex 22.7 Q15

$$\frac{dy}{dx} = e^{x+y} + e^y x^3$$
$$\frac{dy}{dx} = e^y\left(e^x + x^3\right)$$
$$\int e^{-y}dy = \int\left(e^x + x^3\right)dx$$
$$-e^{-y} = e^x + \frac{x^4}{4} + c_1$$
$$e^x + \frac{x^4}{4} + e^{-y} = c$$

Differential Equations Ex 22.7 Q16

$$y\sqrt{1+x^2} + x\sqrt{1+y^2}\frac{dy}{dx} = 0$$
$$x\sqrt{1+y^2}\frac{dy}{dx} = -y\sqrt{1+x^2}$$
$$\int\frac{\sqrt{1+y^2}}{y}dy = -\int\frac{\sqrt{1+x^2}}{x}dx$$
$$\int\frac{y\sqrt{1+y^2}}{y^2}dy = -\int\frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $1+y^2 = t^2$

$\Rightarrow$ $2y\,dy = 2t\,dt$

$1+x^2 = v^2$

$\Rightarrow$ $2x\,dx = 2v\,dv$

$$\int\frac{t\times t\,dt}{t^2-1} = -\int\frac{v\times v\,dv}{v^2-1}$$
$$\int\frac{t^2dt}{t^2-1} = -\int\frac{v^2dv}{v^2-1}$$
$$\int\left(1+\frac{1}{t^2-1}\right)dt = \int\left(1+\frac{1}{v^2-1}\right)dv$$
$$t+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right| = -v - \log\left|\frac{v-1}{v+1}\right| + c$$
$$\sqrt{1+y^2} + \frac{1}{2}\log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right| = -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$
$$\sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right| + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

Differential Equations Ex 22.7 Q17

$$\sqrt{1+x^2}dy + \sqrt{1+y^2}dx = 0$$
$$\sqrt{1+x^2}dy = -\sqrt{1+y^2}dx$$
$$\int \frac{dy}{\sqrt{1+y^2}} = -\int \frac{dx}{\sqrt{1+x^2}}$$
$$\log\left|y+\sqrt{1+y^2}\right| = -\log\left|x+\sqrt{1+x^2}\right| = \log|c|$$
$$\left(y+\sqrt{1+y^2}\right)\left(x+\sqrt{1+x^2}\right) = c$$

Differential Equations Ex 22.7 Q18

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$
$$\sqrt{\left(1+x^2\right)+y^2\left(1+x^2\right)} = -xy\frac{dy}{dx}$$
$$\sqrt{\left(1+x^2\right)\left(1+y^2\right)} = -xy\frac{dy}{dx}$$
$$\frac{ydy}{\sqrt{1+y^2}} = -\frac{\sqrt{1+x^2}}{x}dx$$
$$\int \frac{ydy}{\sqrt{1+y^2}} = -\int \frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $\quad 1+y^2 = t^2$

$\Rightarrow \quad 2ydy = 2tdt$

$\quad 1+x^2 = v^2$

$\Rightarrow \quad 2xdx = 2vdv$

$$\int \frac{tdt}{t} = -\int \frac{v \times vdv}{v^2-1}$$
$$\int dt = -\int \frac{v^2}{v^2-1}dv$$
$$-\int dt = \int\left(1+\frac{1}{v^2-1}\right)dv$$
$$-t = v + \frac{1}{2}\log\left|\frac{v-1}{v+1}\right| + c_1$$
$$-\sqrt{1+y^2} = \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c_1$$
$$\sqrt{1+x^2} + \sqrt{1+y^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = c$$

Differential Equations Ex 22.7 Q19

$$\frac{dy}{dx} = \frac{e^x\left(\sin^2 x + \sin x 2x\right)}{y\left(2\log y + 1\right)}$$
$$y\left(2\log y + 1\right)dy = e^x\left(\sin^2 x + \sin 2x\right)dx$$
$$\int\left(2y\log y + y\right)dy = \int e^x\left(\sin^2 x + \sin 2x\right)dx$$
$$2\left[\log y \times \int ydy - \int\left(\frac{1}{2}\int ydy\right)dy\right] + \frac{y^2}{2} = e^x\sin^2 x + c$$

Using integration by parts and

$$\int\left(f(x)+f'(x)\right)e^x dxdy + \frac{y^2}{2} = e^x\sin^2 x + c$$
$$y^2\log y - \frac{y^2}{2} + \frac{y^2}{2} = e^x\sin^2 x + c$$
$$y^2\log y = e^x\sin^2 x + c$$

Differential Equations Ex 22.7 Q20

$$\frac{dy}{dx} = \frac{x\left(2\log x + 1\right)}{\sin y + y\cos y}$$
$$\int\left(\sin y + y\cos y\right)dy = \int\left(2x\log x + x\right)dx$$
$$\int \sin ydy + \int y\cos ydy = 2\int x\log xdx + \int xdx$$
$$\int \sin ydy + \left\{y \times \left(\int\cos ydy\right) - \int\left(1\times\int\cos ydy\right)dy\right\} = 2\left\{\log x\int xdx - \int\left(\frac{1}{x}\int xdx\right)dx\right\} + \int xdx + c$$
$$\int \sin ydy + y\sin y - \int \sin ydy = x^2\log x - 2\int\frac{x}{2}dx + \int xdx + c$$
$$y\sin y = x^2\log x + c$$

Differential Equations Ex 22.7 Q21

$$\left(1-x^2\right)dy + xydx = xy^2dx$$
$$\left(1-x^2\right)dy = dx\left(xy^2 - xy\right)$$
$$\left(1-x^2\right)dy = xy\,(y-1)\,dx$$
$$\int \frac{dy}{y\,(y-1)} = \int \frac{xdx}{1-x^2}$$
$$\int\left(\frac{1}{y-1} - \frac{1}{y}\right)dy = \frac{1}{2}\int\frac{2x}{1-x^2}dx$$
$$\int\left(\frac{1}{y-1} - \frac{1}{y}\right)dy = -\frac{1}{2}\int\frac{-2x}{1-x^2}dx$$
$$\log\left|y-1\right| - \log\left|y\right| = -\frac{1}{2}\log\left|1-x^2\right| + c$$

Differential Equations Ex 22.7 Q22

$$\tan y dx + \sec^2 y \tan x dy = 0$$
$$\tan y dx = -\sec^2 y \tan x dy$$
$$-\frac{dx}{\tan x} = \frac{\sec^2 y dy}{\tan y}$$
$$-\int \cot x dx = \int \frac{\sec^2 y dy}{\tan y}$$
$$-\log\left|\sin x\right| = \log\left|\tan y\right| + \log\left|c\right|$$
$$\frac{1}{\sin x} = c\tan y$$
$$\sin x \tan y = c_1$$

Differential Equations Ex 22.7 Q23

$$(1+x)\left(1+y^2\right)dx + (1+y)\left(1+x^2\right)dy = 0$$
$$(1+x)\left(1+y^2\right)dx = -(1+y)\left(1+x^2\right)dy$$
$$\frac{(1+y)\,dy}{\left(1+y^2\right)} = -\frac{(1+x)}{\left(1+x^2\right)}dx$$
$$\int\left(\frac{1}{1+y^2} + \frac{y}{1+y^2}\right)dy = -\int\left[\frac{1}{1+x^2} + \frac{x}{1+x^2}\right]dx$$
$$\int\frac{1}{1+y^2}dy + \frac{1}{2}\int\frac{2y}{1+y^2}dy = -\int\frac{1}{1+x^2}dx - \frac{1}{2}\int\frac{2x}{1+x^2}dx$$
$$\tan^{-1}(y) + \frac{1}{2}\log\left|1+y^2\right| = -\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + c$$
$$\tan^{-1}x + \tan^{-1}y + \frac{1}{2}\log\left|\left(1+y^2\right)\left(1+x^2\right)\right| = c$$

Differential Equations Ex 22.7 Q24

$$\tan y\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$$
$$\tan y\frac{dy}{dx} = 2\sin\left\{\frac{(x+y)+(x-y)}{2}\right\}\cos\left\{\frac{(x+y)-(x-y)}{2}\right\}$$
$$= 2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)$$
$$\tan y\frac{dy}{dx} = 2\sin x\cos y$$
$$\frac{\tan y}{\cos y}dy = 2\sin x dx$$
$$\int \sec y \tan y dy = 2\int \sin x dx$$
$$\sec y = -2\cos x + c$$
$$\sec y + 2\cos x = c$$

Differential Equations Ex 22.7 Q25

$$\cos x\cos y\frac{dy}{dx} = -\sin x\sin y$$
$$\frac{\cos y}{\sin y}dy = -\frac{\sin x}{\cos x}dx$$
$$\int \cot y dy = -\int \tan x dx$$
$$\log\sin y = \log\cos x + \log c$$
$$\sin y = c\cos x$$

Differential Equations Ex 22.7 Q26

$$\frac{dy}{dx}+\frac{\cos x\sin y}{\cos y}=0$$
$$\frac{dy}{dx}=-\cos x\tan y$$
$$\frac{dy}{\tan y}=-\cos x dx$$
$$\int\cot y dy=-\int\cos x dx$$
$$\log|\sin y|=-\sin x+c$$
$$\sin x+\log|\sin y|=c$$

Differential Equations Ex 22.7 Q27

$$x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0$$
$$x\sqrt{1-y^2}dx=-y\sqrt{1-x^2}dy$$
$$\frac{ydy}{\sqrt{1-y^2}}=-\frac{xdx}{\sqrt{1-x^2}}$$
$$\frac{1}{-2}\int\frac{-2y}{\sqrt{1-y^2}}dy=\frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}dx$$
$$-\frac{1}{2}2\times\sqrt{1-y^2}=\frac{1}{2}\times2\sqrt{1-x^2}+c_1$$
$$\sqrt{1-y^2}+\sqrt{1-x^2}=c$$

Differential Equations Ex 22.7 Q28

$$y\left(1+e^x\right)dy=(y+1)e^x dx$$
$$\frac{ydy}{y+1}=\frac{e^x dx}{1+e^x}$$
$$\int\left(1-\frac{1}{y+1}\right)dy=\int\left(\frac{e^x}{1+e^x}\right)dx$$
$$y-\log|y+1|=\log\left|1+e^x\right|+c$$

Differential Equations Ex 22.7 Q29

$$(y+xy)dx+\left(x-xy^2\right)dy=0$$
$$y(1+x)dx=\left(xy^2-x\right)dy$$
$$y(1+x)dx=x\left(y^2-1\right)dy$$
$$\frac{\left(y^2-1\right)dy}{y}=\frac{1+x}{x}dx$$
$$\int\left(y-\frac{1}{y}\right)dy=\int\left(\frac{1}{x}+1\right)dx$$
$$\frac{y^2}{2}-\log|y|=\log|x|+x+c_1$$
$$\frac{y^2}{2}-x-\log|y|-\log|x|=c_1$$
$$\log|x|+x+\log|y|-\frac{y^2}{2}=c$$

Differential Equations Ex 22.7 Q30

$$\frac{dy}{dx}=1-x+y-xy$$
$$=(1-x)+y(1-x)$$
$$\frac{dy}{dx}=(1-x)(1+y)$$
$$\int\frac{dy}{1+y}=\int(1-x)dx$$
$$\log|y+1|=x-\frac{x^2}{2}+c$$

Differential Equations Ex 22.7 Q31

$$\left(y^2+1\right)dx-\left(x^2+1\right)dy=0$$
$$\left(y^2+1\right)dx=\left(x^2+1\right)dy$$
$$\int\frac{dy}{y^2+1}=\int\frac{dx}{x^2+1}$$
$$\tan^{-1}y=\tan^{-1}x+c$$

Differential Equations Ex 22.7 Q32

$$dy+(x+1)(y+1)dx=0$$
$$dy=-(x+1)(y+1)dx$$
$$\int\frac{dy}{y+1}=-\int(x+1)dx$$
$$\log|y+1|=-\frac{x^2}{2}-x+c$$
$$\log|y+1|+\frac{x^2}{2}+x=c$$

Differential Equations Ex 22.7 Q33

$$\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$
$$\int\frac{dy}{1+y^2}=\int\left(1+x^2\right)dx$$
$$\tan^{-1}y=x+\frac{x^3}{3}+c$$
$$\tan^{-1}y-x-\frac{x^3}{3}=c$$

Differential Equations Ex 22.7 Q34

$$(x-1)\frac{dy}{dx}=2x^3y$$
$$\frac{dy}{y}=\frac{2x^3dx}{x-1}$$
$$\int\frac{dy}{y}=2\int\left(x^2+x+1+\frac{1}{x-1}\right)dx$$
$$\log|y|=\log e^{\left(\frac{2}{3}x^3+x^2+2x\right)}+\log|x-1|^2+\log|c|$$
$$y=c|x-1|^2e^{\left(\frac{2}{3}x^3+x^2+2x\right)}$$

Differential Equations Ex 22.7 Q35

$$\frac{dy}{dx}=e^{x+y}+e^{-x+y}$$
$$=e^x\times e^y+e^{-x}\times e^y$$
$$\frac{dy}{dx}=e^y\left(e^x+e^{-x}\right)$$
$$\frac{dy}{e^y}=\left(e^x+e^{-x}\right)dx$$
$$\int e^{-y}dy=\int\left(e^x+e^{-x}\right)dx$$
$$-e^{-y}=e^x-e^{-x}+c$$
$$e^{-x}-e^{-y}=e^x+c$$

Differential Equations Ex 22.7 Q36

$$\frac{dy}{dx} = \left(\cos^2 x - \sin^2 x\right)\cos^2 y$$

$$\frac{dy}{\cos^2 y} = \left(\cos^2 x - \sin^2 x\right)dx$$

$$\int \sec^2 y dy = \int \cos 2x dx$$

$$\tan y = \frac{\sin 2x}{2} + c$$

Differential Equations Ex 22.7 Q37(i)

$$\left(xy^2 + 2x\right)dx + \left(x^2 y + 2y\right)dy = 0$$

$$\left(x^2 y + 2y\right)dy = -\left(xy^2 + 2x\right)dx$$

$$y\left(x^2 + 2\right)dy = -x\left(y^2 + 2\right)dx$$

$$\frac{y}{y^2 + 2}dy = -\frac{x}{x^2 + 2}dx$$

$$\int \frac{2y}{y^2 + 2}dy = -\int \frac{2x}{x^2 + 2}dx$$

$$\log\left|y^2 + 2\right| = -\log\left|x^2 + 2\right| + \log|c|$$

$$\left|y^2 + 2\right| = \left|\frac{c}{x^2 + 2}\right|$$

$$y^2 + 2 = \frac{c}{x^2 + 2}$$

Differential Equations Ex 22.7 Q37(ii)

Consider the given equation

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y^2 = 0$$

$$\Rightarrow \frac{\log y dy}{y^2} = \frac{-x^2 dx}{\operatorname{cosec} x}$$

$$\Rightarrow -\frac{\log y dy}{y^2} = x^2 \sin x dx$$

Integrating on both the sides,

$$\Rightarrow -\int \frac{\log y dy}{y^2} = \int x^2 \sin x dx$$

Using integration by parts on both sides

$$\Rightarrow \frac{\log y + 1}{y} = -x^2 \cos x + 2\left(x \sin x + \cos x\right) + C$$

$$\Rightarrow \frac{\log y + 1}{y} + x^2 \cos x - 2\left(x \sin x + \cos x\right) = C$$

Differential Equations Ex 22.7 Q38(i)

$$xy\frac{dy}{dx} = 1 + x + y + xy$$

$$= \left(1 + x\right) + y\left(1 + x\right)$$

$$xy\frac{dy}{dx} = \left(1 + x\right)\left(1 + y\right)$$

$$\int \frac{y dy}{y + 1} = \int \frac{1 + x}{x}dx$$

$$\int \left(1 - \frac{1}{y + 1}\right)dy = \int \left(\frac{1}{x} + 1\right)dx$$

$$y - \log|y + 1| = \log|x| + x + \log|c|$$

$$y = \log|cx\left(y + 1\right)| + x$$

Differential Equations Ex 22.7 Q38(ii)

$$y\left(1-x^2\right)\frac{dy}{dx}=x\left(1+y^2\right)$$
$$\frac{ydy}{\left(1+y^2\right)}=\frac{xdx}{1-x^2}$$
$$-\int\frac{2ydy}{\left(1+y^2\right)}=\int\frac{-2x}{\left(1-x^2\right)}dx$$
$$-\log\left|1+y^2\right|=\log\left|1-x^2\right|+\log\left|c_1\right|$$
$$\log|c|=\log\left|1-x^2\right|+\log\left|1+y^2\right|$$
$$c=\left(1-x^2\right)\left(1+y^2\right)$$

Differential Equations Ex 22.7 Q38(iii)

$$ye^{x/y}\,dx=(xe^{x/y}+y^2)dy$$
$$ye^{x/y}\,dx-xe^{x/y}dy=y^2dy$$
$$(ydx-xdy)e^{x/y}=y^2dy$$
$$\left(\frac{ydx-xdy}{y^2}\right)e^{x/y}=dy$$
$$e^{x/y}\,d\left(\frac{x}{y}\right)=dy$$

Integrating on both the sides we get,

$e^{x/y}=y+C$, which is the required solution.

Differential Equations Ex 22.7 Q38(iv)

$$(1+y^2)\tan^{-1}x\,dx+2y\,(1+x^2)dy=0$$
$$\left(1+y^2\right)\tan^{-1}x\,dx=-2y\left(1+x^2\right)dy$$
$$-\frac{\tan^{-1}x}{2\left(1+x^2\right)}dx=\frac{y}{\left(1+y^2\right)}dy$$

Integrating on both the sides

$$\int-\frac{\tan^{-1}x}{2\left(1+x^2\right)}dx=\int\frac{y}{\left(1+y^2\right)}dy$$
$$-\left(\tan^{-1}x\left(\frac{1}{2}\tan^{-1}x\right)-\int\frac{1}{\left(1+x^2\right)}\left(\frac{1}{2}\tan^{-1}x\right)dx\right)=\frac{1}{2}\ln\left(y^2+1\right)+C$$
$$-\frac{1}{4}\left(\tan^{-1}x\right)^2=\frac{1}{2}\ln\left(y^2+1\right)+C_1$$
$$\frac{1}{2}\left(\tan^{-1}x\right)^2+\ln\left(y^2+1\right)=C$$

Differential Equations Ex 22.7 Q39

$$\frac{dy}{dx}=y\tan2x,\ y(0)=2$$
$$\int\frac{dy}{y}=\int\tan2xdx$$
$$\log|y|=\frac{1}{2}\log|\sec2x|+\log c$$
$$y=\sqrt{\sec2x}c\qquad\text{---(i)}$$

Put $x=0, y=2$

$$2=\sqrt{\sec0}\times c$$
$$2=c$$

Put $c=2$ in equation (i),

$$y=2\sqrt{\sec2x}$$
$$y=\frac{2}{\sqrt{\cos2x}}$$

Differential Equations Ex 22.7 Q40

$$2x\frac{dy}{dx} = 3y,\ y(1) = 2$$

$$\int\frac{2dy}{y} = \int\frac{3dx}{x}$$

$$2\log|y| = 3\log|x| + \log c$$

$$y^2 = x^3 c \qquad ---(i)$$

Put $x = 1,\ y = 2$

$$4 = c$$

Put $c = 4$ in equation (i),

$$y^2 = 4x^3$$

Differential Equations Ex 22.7 Q41

$$xy\frac{dy}{dx} = y + 2,\ y(2) = 0$$

$$\frac{ydy}{y+2} = \frac{dx}{x}$$

$$\int\left(1 - \frac{2}{y+2}\right)dy = \int\frac{dx}{x}$$

$$y - 2\log|y+2| = \log|x| + \log|c| \qquad ---(i)$$

Put $y = 0,\ x = 2$

$$0 - 2\log 2 = \log 2 + \log c$$

$$-3\log 2 = \log c$$

$$\log\left(\frac{1}{8}\right) = \log c$$

$$c = \frac{1}{8}$$

Put $c = \frac{1}{8}$ in equation (i),

$$y - 2\log|y+2| = \log\left|\frac{x}{8}\right|$$

Differential Equations Ex 22.7 Q42

$$\frac{dy}{dx} = 2e^x y^3,\ y(0) = \frac{1}{2}$$

$$\int\frac{dy}{y^3} = \int 2e^x dx$$

$$-\frac{1}{2y^2} = 2e^x + c \qquad ---(i)$$

Put $x = 0,\ y = \frac{1}{2}$

$$-\frac{4}{2} = 2e^0 + c$$

$$-2 = 2 + c$$

$$c = -4$$

Put $c = -4$ in equation (i),

$$-\frac{1}{2y^2} = 2e^x - 4$$

$$-1 = 4e^x y^2 - 8y^2$$

$$-1 = -y^2\left(8 - 4e^x\right)$$

$$y^2\left(8 - 4e^x\right) = 1$$

Differential Equations Ex 22.7 Q43

$$\frac{dr}{dt} = -rt,\ r(0) = r_0$$

$$\int\frac{dr}{r} = -\int tdt$$

$$\log|r| = -\frac{t^2}{2} + c \qquad ---(i)$$

Put $t = 0,\ r = r_0$ inequation (i),

$$\log|r_0| = 0 + c$$

$$\log|r_0| = c$$

Now,

$$\log|r| = -\frac{t^2}{2} + \log|r_0|$$

$$\frac{r}{r_0} = e^{-\frac{t^2}{2}}$$

$$r = r_0 e^{-\frac{t^2}{2}}$$

Differential Equations Ex 22.7 Q44

$$\frac{dy}{dx} = y\sin 2x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \sin 2x dx$$

$$\log|y| = -\frac{\cos 2x}{2} + c \qquad ---(i)$$

Put $y = 1,\ x = 0$

$$\log|1| = -\frac{\cos 0}{2} + c$$

$$0 = -\frac{1}{2} + c$$

$$c = \frac{1}{2}$$

So,

$$\log|y| = -\frac{\cos 2x}{2} + \frac{1}{2}$$

$$= \frac{1 - \cos 2x}{2}$$

$$\log|y| = \sin^2 x$$

$$y = e^{\sin^2 x}$$

Differential Equations Ex 22.7 Q45(i)

$$\frac{dy}{dx} = y\tan x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \tan x dx$$

$$\log|y| = \log|\sec x| + c \qquad ---(i)$$

Put $y = 1,\ x = 0$

$$0 = \log(1) + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = \log|\sec x|$$

$$y = \sec x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Differential Equations Ex 22.7 Q45(ii)

$$2x\frac{dy}{dx} = 5y,\ y(1) = 1$$

$$\int \frac{2dy}{y} = \int \frac{5dx}{x}$$

$$2\log|y| = 5\log|x| + c \qquad ---(i)$$

Put $x = 1,\ y = 1$

$$2\log(1) = 5\log(1) + c$$

$$0 = c$$

Put $c = 0$ in equation (i),

$$2\log|y| = 5\log|x|$$

$$y^2 = |x|^5$$

$$y = |x|^{\frac{5}{2}}$$

Differential Equations Ex 22.7 Q45(iii)

$$\frac{dy}{dx} = 2e^{2x}y^2,\ y(0) = -1$$
$$\int \frac{dy}{y^2} = \int 2e^{2x}dx$$
$$-\frac{1}{y} = \frac{2e^{2x}}{2} + c$$
$$-\frac{1}{y} = e^{2x} + c \qquad ---(i)$$

Put $y = -1,\ x = 0$

$$1 = e^0 + c$$
$$1 = 1 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$-\frac{1}{y} = e^{2x}$$
$$y = -e^{-2x}$$

Differential Equations Ex 22.7 Q45(iv)

$$\cos y\frac{dy}{dx} = e^x,\ y(0) = \frac{\pi}{2}$$
$$\int \cos y\,dy = \int e^x dx$$
$$\sin y = e^x + c \qquad ---(i)$$

Put $x = 0,\ y = \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = e^0 + c$$
$$1 = 1 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$\sin y = e^x$$
$$y = \sin^{-1}\left(e^x\right)$$

Differential Equations Ex 22.7 Q45(v)

$$\frac{dy}{dx} = 2xy,\ y(0) = 1$$
$$\int \frac{dy}{y} = \int 2x\,dx$$
$$\log|y| = 2\frac{x^2}{2} + c$$
$$\log|y| = x^2 + c \qquad ---(i)$$

Put $x = 0,\ y = 1$

$$\log(1) = 0 + c$$
$$0 = 0 + c$$
$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = x^2$$
$$y = e^{x^2}$$

Differential Equations Ex 22.7 Q45(vi)

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2, y(0) = 1$$
$$= (1+x^2)(1+y^2)$$
$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$
$$\tan^{-1} y = x + \frac{x^3}{3} + c \qquad ---(i)$$

Put $x = 0, y = 1$

$$\tan^{-1} y = x + \frac{x^3}{3} + c$$
$$c = \frac{\pi}{4}$$

Put $c = \frac{\pi}{4}$ in equation (i)

$$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$$

Differential Equations Ex 22.7 Q45(vii)

$xy\dfrac{dy}{dx} = (x+2)(y+2),\ y(1) = -1$

$\dfrac{ydy}{(y+2)} = \dfrac{(x+2)}{x}dx$

$\int\left(1 - \dfrac{2}{y+2}\right)dy = \int\left(1 + \dfrac{2}{x}\right)dx$

$y - x - 2\log(y+2) - 2\log x = c$

Put $x = 1,\ y = -1$

$-1 - 1 - 2\log(-1+2) - 2\log 1 = c$

$\Rightarrow -2 = c$

Thus, we have

$y - x - 2\log(y+2) - 2\log x = -2$

Differential Equations Ex 22.7 Q45(viii)

$\dfrac{dy}{dx} = 1 + x + y^2 + xy^2$

$\dfrac{dy}{dx} = (1+x)(1+y^2)$

$\dfrac{1}{(1+y^2)}dy = (1+x)dx$

Integrating on both the sides we get

$\int\dfrac{1}{(1+y^2)}dy = \int(1+x)dx$

$\tan^{-1}y = x + \dfrac{x^2}{2} + C\ldots(i)$

Put $y = 0,\ x = 0$ then

$\tan^{-1}0 = 0 + 0 + C$

$C = 0$

From(i) we have

$\tan^{-1}y = x + \dfrac{x^2}{2}$

$y = \tan\left(x + \dfrac{x^2}{2}\right)$

Differential Equations Ex 22.7 Q45(ix)

$2(y+3) - xy\dfrac{dy}{dx} = 0$

$2(y+3) = xy\dfrac{dy}{dx}$

$\dfrac{2}{x}dx = \dfrac{y}{y+3}dy$

Integrating on both the sides we get

$\int\dfrac{2}{x}dx = \int\dfrac{y}{y+3}dy$

$2\ln|x| = y + 3 - 3\ln|y+3| + C\ldots.(i)$

Put $x = 1$ and $y = -2$ in eq (i)

$2\ln|1| = -2 + 3 - 3\ln|-2+3| + C$

$0 = 1 - 0 + C$

$C = -1$

From eq (i) we have

$2\ln|x| = y + 3 - 3\ln|y+3| - 1$

$\ln(|x|)^2 = y + 2 - \ln(|y+3|)^3$

$\ln(|x|)^2 + -\ln(|y+3|)^3 = y + 2$

$x^2(y+3)^3 = e^{y+2}$

Differential Equations Ex 22.7 Q46

$$x\frac{dy}{dx}+\cot y=0,\ y=\frac{\pi}{4}\ \text{at}\ x=\sqrt{2}$$

$$x\frac{dy}{dx}=-\cot y$$

$$\frac{dy}{\cot y}=-\frac{dx}{x}$$

$$\int \tan y\,dy=-\int\frac{dx}{x}+c$$

$$\log|\sec y|=-\log|x|+c \qquad \text{---(i)}$$

Put $x=\sqrt{2},\ y=\frac{\pi}{4}$

$$\log\left|\sec\frac{\pi}{4}\right|=-\log\left|\sqrt{2}\right|+c$$

$$\log\left|\sqrt{2}\right|=-\frac{1}{2}\log 2+c$$

$$\frac{1}{2}\log 2=-\frac{1}{2}\log 2+c$$

$$\log 2=c$$

Put c in equation (i),

$$\log|\sec y|=-\log|x|+\log 2$$

$$\sec y=\frac{2}{x}$$

$$x=\frac{2}{\sec y}$$

$$x=2\cos y$$

Differential Equations Ex 22.7 Q47

$$\left(1+x^2\right)\frac{dy}{dx}+\left(1+y^2\right)=0,\ y=1\ \text{at}\ x=0$$

$$\left(1+x^2\right)\frac{dy}{dx}=-\left(1+y^2\right)$$

$$\int\frac{dy}{\left(1+y^2\right)}=-\int\frac{dx}{1+x^2}$$

$$\tan^{-1}y=-\tan^{-1}x+c \qquad \text{---(i)}$$

Put $x=0,\ y=1$

$$\tan^{-1}(1)=-\tan^{-1}0+c$$

Put c in equation (1),

$$\tan^{-1}y=-\tan^{-1}x+\frac{\pi}{4}$$

$$\tan^{-1}y=\left(\frac{\pi}{4}-\tan^{-1}x\right)$$

$$y=\tan\left(\frac{\pi}{4}-\tan^{-1}x\right)$$

$$y=\frac{\tan\frac{\pi}{4}-\tan\left(\tan^{-1}x\right)}{1+\tan\frac{\pi}{4}\tan\left(\tan^{-1}x\right)}$$

$$y=\frac{1-x}{1+x}$$

$$y+xy=1-x$$

$$x+y=1-xy$$

Differential Equations Ex 22.7 Q48

$$\frac{dy}{dx}=\frac{2x(\log x+1)}{\sin y+y\cos y},\ y=0 \text{ at } x=1$$

$$\int(\sin y+y\cos y)dy=\int 2x(\log x+1)dx$$

$\Rightarrow$ $$\int\sin y\,dy+\int y\cos y\,dy=\int 2x\log x\,dx+2\int x\,dx$$

$\Rightarrow$ $$-\cos y+\left[y\times\int\cos y\,dy-\int\left(1\times\int\cos y\,dy\right)dy\right]=2\left[\log x\int x\,dx-\int\left(\frac{1}{x}\int x\,dx\right)dx\right]+x^2+c$$

$\Rightarrow$ $$-\cos y+y\sin y-\int\sin y\,dy=2\frac{x^2}{2}\log x-2\int\frac{x}{2}dx+x^2+c$$

$\Rightarrow$ $$-\cos y+y\sin y+\cos y=x^2\log x-\frac{x^2}{2}+x^2+c$$

$$y\sin y=x^2\log x+\frac{x^2}{2}+c \qquad \text{---(i)}$$

Put $y=0,\ x=1$

$$0=0+\frac{1}{2}+c$$

$$c=-\frac{1}{2}$$

Put $c=-\frac{1}{2}$ in equation (i),

$$y\sin y=x^2\log x+\frac{x^2}{2}-\frac{1}{2}$$

$$2y\sin y=2x^2\log x+x^2-1$$

Differential Equations Ex 22.7 Q49

$$e^{\frac{dy}{dx}}=x+1$$

$$\frac{dy}{dx}=\log(x+1),\ y=3 \text{ at } x=0$$

$$\int dy=\int\log(x+1)\,dx$$

$$y=\log|x+1|\times\int 1\times dx-\int\left(\frac{1}{x+1}\times\int 1dx\right)dx+c$$

Using integration by parts

$$y=x\log|x+1|-\int\frac{x}{x+1}dx+c$$

$$y=x\log|x+1|-\left(\int\left(1-\frac{1}{x+1}\right)dx\right)+c$$

$$=x\log|x+1|-(x-\log|x+1|)+c$$

$$y=x\log|x+1|-x+\log|x+1|+c$$

$$y=(x+1)\log|x+1|-x+c \qquad \text{---(i)}$$

Put $y=3$ and $x=0$

$$3=0-0+c$$

$$c=3$$

Put $c=3$ in equation (i),

$$y=(x+1)\log|x+1|-x+3$$

Differential Equations Ex 22.7 Q50

$$\cos y\,dy+\cos x\sin y\,dx=0$$

$$\cos y\,dy=-\cos x\sin y\,dx$$

$$\frac{\cos y}{\sin y}dy=-\cos x\,dx$$

$$\int\cot y\,dy=-\int\cos x\,dx$$

$$\log|\sin y|=-\sin x+c \qquad \text{---(i)}$$

Put $y=\frac{\pi}{2}$ and $x=\frac{\pi}{2}$

$$\log\left|\sin\frac{\pi}{2}\right|=-\sin\frac{\pi}{2}+c$$

$$0=-1+c$$

$$c=1$$

Put $c=1$ in equation (1),

$$\log|\sin y|=1-\sin x$$

$$\log|\sin y|+\sin x=1$$

Differential Equations Ex 22.7 Q51

$$\frac{dy}{dx} = -4xy^2,\ y = 1 \text{ when } x = 0$$

$$\int \frac{dy}{y^2} = -4\int x dx$$

$$-\frac{1}{y} = -4\frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $y = 1$ and $x = 0$

$$-1 = 0 + c$$

$$c = -1$$

Plut $c = -1$ in equation (i),

$$-\frac{1}{y} = -2x^2 - 1$$

$$\frac{1}{y} = 2x^2 + 1$$

$$y = \frac{1}{2x^2 + 1}$$

Differential Equations Ex 22.7 Q52

The differential equation of the curve is:

$$y' = e^x \sin x$$

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x\, dx \qquad ...(1)$$

Let $I = \int e^x \sin x\, dx$.

$$\Rightarrow I = \sin x \int e^x dx - \int\left(\frac{d}{dx}(\sin x)\cdot \int e^x dx\right)dx$$

$$\Rightarrow I = \sin x \cdot e^x - \int \cos x \cdot e^x dx$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot \int e^x dx - \int\left(\frac{d}{dx}(\cos x)\cdot \int e^x dx\right)dx\right]$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot e^x - \int(-\sin x)\cdot e^x dx\right]$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow 2I = e^x(\sin x - \cos x)$$

$$\Rightarrow I = \frac{e^x(\sin x - \cos x)}{2}$$

Differential Equations Ex 22.7 Q53

The differential equation of the given curve is:

$$xy\frac{dy}{dx}=(x+2)(y+2)$$
$$\Rightarrow\left(\frac{y}{y+2}\right)dy=\left(\frac{x+2}{x}\right)dx$$
$$\Rightarrow\left(1-\frac{2}{y+2}\right)dy=\left(1+\frac{2}{x}\right)dx$$

Integrating both sides, we get:

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$
$$\Rightarrow \int dy-2\int\frac{1}{y+2}dy=\int dx+2\int\frac{1}{x}dx$$
$$\Rightarrow y-2\log(y+2)=x+2\log x+C$$
$$\Rightarrow y-x-C=\log x^2+\log(y+2)^2$$
$$\Rightarrow y-x-C=\log\left[x^2(y+2)^2\right] \qquad ...(1)$$

Differential Equations Ex 22.7 Q54

Let the rate of change of the volume of the balloon be k (where k is a constant)

$$\Rightarrow \frac{dv}{dt}=k$$
$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=k \qquad \left[\text{Volume of sphere}=\frac{4}{3}\pi r^3\right]$$
$$\Rightarrow \frac{4}{3}\pi\cdot 3r^2\cdot\frac{dr}{dt}=k$$
$$\Rightarrow 4\pi r^2\,dr=k\,dt$$

Integrating both sides, we get:

$$4\pi\int r^2dr=k\int dt$$
$$\Rightarrow 4\pi\cdot\frac{r^3}{3}=kt+C$$
$$\Rightarrow 4\pi r^3=3(kt+C) \qquad ...(1)$$

Now, at $t=0$, $r=3$:

$4\pi\times 3^3=3\ (k\times 0+C)$

$108\pi=3C$

$C=36\pi$

At $t=3$, $r=6$:

$4\pi\times 6^3=3\ (k\times 3+C)$

$864\pi=3\ (3k+36\pi)$

$3k=-288\pi-36\pi=252\pi$

$k=84\pi$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3=3[84\pi t+36\pi]$$
$$\Rightarrow 4\pi r^3=4\pi(63t+27)$$
$$\Rightarrow r^3=63t+27$$
$$\Rightarrow r=(63t+27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t+27)^{\frac{1}{3}}$.

Differential Equations Ex 22.7 Q55

Let p, t, and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of $r\%$ per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$
$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100}\int dt$$
$$\Rightarrow \log p = \frac{rt}{100} + k$$
$$\Rightarrow p = e^{\frac{rt}{100}+k} \qquad \ldots(1)$$

It is given that when $t = 0$, $p = 100$.

$\Rightarrow 100 = e^{k} \ldots (2)$

Now, if $t = 10$, then $p = 2 \times 100 = 200$.

$$200 = e^{\frac{r}{10}+k}$$
$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot e^{k}$$
$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot 100 \qquad (\text{From } (2))$$
$$\Rightarrow e^{\frac{r}{10}} = 2$$
$$\Rightarrow \frac{r}{10} = \log_e 2$$
$$\Rightarrow \frac{r}{10} = 0.6931$$
$$\Rightarrow r = 6.931$$

Hence, the value of r is 6.93%.

Differential Equations Ex 22.7 Q56

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$$
$$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$$
$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20}\int dt$$
$$\Rightarrow \log p = \frac{t}{20} + C$$
$$\Rightarrow p = e^{\frac{t}{20}+C} \qquad \ldots(1)$$

Now, when $t = 0$, $p = 1000$.

$1000 = e^{C} \ldots (2)$

Differential Equations Ex 22.7 Q57

Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$
$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$
$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k \int dt$$
$$\Rightarrow \log y = kt + C \qquad ...(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$$\log y_0 = C$$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$
$$\Rightarrow \log y - \log y_0 = kt$$
$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$
$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad ...(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y_0$$
$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \qquad ...(3)$$

Substituting this value in equation (2), we get:

$$k \cdot 2 = \log\left(\frac{11}{10}\right)$$
$$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2}\log\left(\frac{11}{10}\right) \cdot t = \log\left(\frac{y}{y_0}\right)$$
$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad ...(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1.

$y = 2y_0$ at $t = t_1$

From equation (4), we get:

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Differential Equations Ex 22.7 Q58

Consider the given equation

$$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$$

$$\Rightarrow \frac{dy}{(1+y)} = \frac{-\cos x dx}{(2+\sin x)}$$

Integrating both the sides,

$$\Rightarrow \int \frac{dy}{(1+y)} = \int \frac{-\cos x dx}{(2+\sin x)}$$

$$\Rightarrow \log(1+y) = -\log(2+\sin x) + \log C$$

$$\Rightarrow \log(1+y) + \log(2+\sin x) = \log C$$

$$\Rightarrow \log(1+y)(2+\sin x) = \log C$$

$$\Rightarrow (1+y)(2+\sin x) = C \ldots(1)$$

Given that $y(0) = 1$

$$\Rightarrow (1+1)(2+\sin 0) = C$$

$$\Rightarrow C = 4$$

Substituting the value of C in equation (1), we have,

$$\Rightarrow (1+y)(2+\sin x) = 4$$

$$\Rightarrow (1+y) = \frac{4}{(2+\sin x)}$$

$$\Rightarrow y = \frac{4}{(2+\sin x)} - 1 \ldots(2)$$

We need to find the value of $y\left(\frac{\pi}{2}\right)$

Substituting the value of $x=\frac{\pi}{2}$ in equation (2), we get,

$$y = \frac{4}{\left(2+\sin\frac{\pi}{2}\right)} - 1$$

$$\Rightarrow y = \frac{4}{(2+1)} - 1$$

$$\Rightarrow y = \frac{4}{3} - 1$$

$$\Rightarrow y = \frac{1}{3}$$

Ex 22.8

Differential Equations Ex 22.8 Q1

$$\frac{dy}{dx} = (x+y+1)^2$$

Let $x+y+1 = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = v^2 + 1$$

$$\int \frac{1}{v^2+1} = \int dx$$

$$\tan^{-1}(v) = x + c$$

$$\tan^{-1}(x+y+1) = x + c$$

Differential Equations Ex 22.8 Q2

$$\frac{dy}{dx} \times \cos(x-y) = 1$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So,

$$\left(1 - \frac{dv}{dx}\right)\cos v = 1$$

$$1 - \frac{dv}{dx} = \sec v$$

$$1 - \sec v = \frac{dv}{dx}$$

$$dx = \frac{dv}{1 - \sec v}$$

$$dx = \frac{\cos v}{1 - \cos v} dv$$

$$\int dx = \int \frac{\cos^2\frac{v}{2} - \sin^2\frac{v}{2}}{2\sin^2\frac{v}{2}} dv$$

$$\int dx = \int \frac{1}{2}\cot\left(\frac{v}{2}\right) dv - \frac{1}{2} dv$$

$$2\int dx = \int \cot^2\left(\frac{v}{2}\right) dv - \int dv$$

$$2\int dx = \int\left(\operatorname{cosec}^2\frac{v}{2} - 1\right)dv - \int dv$$

$$2x = -2\cot\left(\frac{v}{2}\right)dv - v - v + c_1$$

$$2(x+v) = -2\cot\frac{v}{2} + c_1$$

$$x + x - y = -\cot\left(\frac{x-y}{2}\right) + c$$

$$c + y = \cot\left(\frac{x-y}{2}\right)$$

Differential Equations Ex 22.8 Q3

$$\frac{dy}{dx} = \frac{(x-y)+3}{2(x-y)+5}$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$1 - \frac{dv}{dx} = \frac{v+3}{2v+5}$$

$$\frac{dv}{dx} = 1 - \frac{v+3}{2v+5}$$

$$= \frac{2v+5-v-3}{2v+5}$$

$$\frac{dv}{dx} = \frac{v+2}{2v+5}$$

$$\frac{2v+5}{v+2} dv = dx$$

$$\frac{(2v+4)+1}{v+2} dv = dx$$

$$\int\left(2 + \frac{1}{v+2}\right)dv = \int dx$$

$$2v + \log|v+2| = x + c$$

$$2(x-y) + \log|x-y+2| = x + c$$

Differential Equations Ex 22.8 Q4

$$\frac{dy}{dx} = (x+y)^2$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$\frac{dv}{dx} - 1 = v^2$$
$$\frac{dv}{dx} = 1 + v^2$$
$$\int \frac{1}{1+v^2} dv = \int dx$$
$$\tan^{-1} v = x + c$$
$$\tan^{-1}(x+y) = x + c$$
$$x + y = \tan(x+c)$$

Differential Equations Ex 22.8 Q5

$$(x+y)^2 \frac{dy}{dx} = 1$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$
$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$v^2\left(\frac{dv}{dx} - 1\right) = 1$$
$$\frac{dv}{dx} = \frac{1}{v^2} + 1$$
$$\frac{dv}{dx} = \frac{v^2+1}{v^2}$$
$$\frac{v^2}{v^2+1} dv = dx$$
$$\int \frac{v^2+1-1}{v^2+1} dv = \int dx$$
$$\int \left(1 - \frac{1}{v^2+1}\right) dv = \int dx$$
$$v - \tan^{-1}(v) = x + c$$
$$x + y - \tan^{-1}(x+y) = x + c$$
$$y - \tan^{-1}(x+y) = c$$

Differential Equations Ex 22.8 Q6

$$\cos^2(x-2y) = 1 - \frac{2dy}{dx}$$

Let $x - 2y = v$

$$1 - \frac{2dy}{dx} = \frac{dv}{dx}$$

So,

$$\cos^2 v = \frac{dv}{dx}$$
$$\int dx = \int \sec^2 v dv$$
$$x = \tan v + c$$
$$x = \tan(x-2y) + c$$

Differential Equations Ex 22.8 Q7

The given differential equation can be written as

$$\frac{dy}{dx} = \frac{1}{\cos(x+y)}$$

Let $x + y = u$. Then,

$$1 + \frac{dy}{dx} = \frac{du}{dx} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$$

Putting $x + y = u$ and $\frac{dy}{dx} = \frac{du}{dx} - 1$ the given differential equation, we get

$$\Rightarrow \quad \frac{du}{dx} - 1 = \frac{1}{\cos u}$$
$$\Rightarrow \quad \frac{du}{dx} = \frac{1+\cos u}{\cos u}$$
$$\Rightarrow \quad \frac{\cos u}{1+\cos u} du = dx$$
$$\Rightarrow \quad \frac{\cos u(1-\cos u)}{1-\cos^2 u} du = dx$$
$$\Rightarrow \quad \left(\cot u \operatorname{cosec} u - \cot^2 u\right) du = dx$$
$$\Rightarrow \quad \left(\cot u \operatorname{cosec} u - \operatorname{cosec}^2 u + 1\right) du = dx$$
$$\Rightarrow \quad -\operatorname{cosec} u + \cot u + u = x + C$$
$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + x + y = x + C$$
$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + y = C$$
$$\Rightarrow \quad -\frac{1-\cos(x+y)}{\sin(x+y)} + y = C$$
$$\Rightarrow \quad -\tan\left(\frac{x+y}{2}\right) + y = C$$

We have,

$y(0) = 0$ i.e. $y = 0$ when $x = 0$

Putting $x = 0$ and $y = 0$ in (i), we get $C = 0$.

Putting $C = 0$ in (i), we get

$$-\tan\left(\frac{x+y}{2}\right) + y = 0 \Rightarrow y = \tan\left(\frac{x+y}{2}\right),$$ which is the required solution.

Differential Equations Ex 22.8 Q8

$$\frac{dy}{dx} = \tan(x+y)$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} - 1 = \tan v$$

$$\frac{dv}{dx} = 1 + \tan v$$

$$\frac{1}{1+\tan v}dv = dx$$

$$\frac{\cos v}{\cos v + \sin v}dv = dx$$

$$\left(\frac{2\cos v}{\cos v + \sin v}\right)dv = 2dx$$

$$\left(\frac{\cos v + \sin v + \cos v - \sin v}{\cos v + \sin v}\right)dv = 2dx$$

$$\int dv + \int\left(\frac{\cos v - \sin v}{\cos v + \sin v}\right)dv = 2\int dx$$

$$v + \log|\cos v + \sin v| = 2x + c$$

$$x + y + \log|\cos(x+y) + \sin(x+y)| = 2x + c$$

$$y - x + \log|\cos(x+y) + \sin(x+y)| = c$$

Differential Equations Ex 22.8 Q9

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$\Rightarrow 2v = v\frac{dv}{dx} + \frac{dv}{dx}$$

$$\Rightarrow 2v = (v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v}dv = 2dx$$

$$(x+y)(dx - dy) = dx + dy$$

$$(x+y)\left(1 - \frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$v\left(1 - \left(\frac{dv}{dx} - 1\right)\right) = \frac{dv}{dx}$$

$$v\left(2 - \frac{dv}{dx}\right) = \frac{dv}{dx}$$

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$\Rightarrow 2v = v\frac{dv}{dx} + \frac{dv}{dx}$$

$$\Rightarrow 2v = (v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v}dv = 2dx$$

$$\int\left(1 + \frac{1}{v}\right)dv = 2\int dx$$

$$v + \log|v| = 2x + c$$

$$x + y + \log|x+y| = 2x + c$$

$$y - x + \log|x+y| = c$$

Differential Equations Ex 22.8 Q10

$$(x+y+1)\frac{dy}{dx} = 1$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$(v+1)\left(\frac{dv}{dx} - 1\right) = 1$$

$$(v+1)\frac{dv}{dx} - (v+1) = 1$$

$$(1+v)\frac{dv}{dx} = 1 + 1 + v$$

$$\frac{v+1dv}{2+v} = dx$$

$$\int\left(1 - \frac{1}{v+2}\right)dv = \int dx$$

$$v - \log|v+2| = x + \log c$$

$$x + y - \log|x+y+2| = x + \log c$$

$$y = \log c|x+y+2|$$

$$e^y = c(x+y+2)$$

$$ke^y = x + y + 2 \qquad [k = 1/c]$$

$$x = ke^y - y - 2$$

Differential Equations Ex 22.8 Q11

$$\frac{dy}{dx} + 1 = e^{x+y}$$

Let $x+y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$\therefore$ Given differential equation becomes,

$$\frac{dv}{dx} = e^v$$

$$\frac{1}{e^v}dv = dx$$

Integrating on both the sides we get

$$-e^{-v} = x + C$$

$$\therefore -e^{-(x+y)} = x + C$$

Ex 22.9

Differential Equations Ex 22.9 Q1

Here, $x^2 dy + y(x+y)dx = 0$

$$\frac{dy}{dx} = -\frac{y(x+y)}{x^2}$$

It is homogeneous equation

Put $y = vx$

and,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = -\frac{vx(x+vx)}{x^2}$$

$$v + x\frac{dv}{dx} = -v - v^2$$

$$x\frac{dv}{dx} = -2v - v^2$$

$$\int \frac{1}{v^2+2v}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{v^2+2v+1-1}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{(v+1)^2-(1)^2}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\log\left|\frac{v+1-1}{v+1+1}\right| = -\log|x| + \log|c|$$

$$\log\left|\frac{v}{v+2}\right|^{\frac{1}{2}} = -\log\left|\frac{c}{x}\right|$$

$$\frac{v}{v+2} = \frac{c^2}{x^2}$$

$$\frac{\frac{y}{x}}{\frac{y}{x}+2} = \frac{c^2}{x^2}$$

$$\frac{y}{y+2x} = \frac{c^2}{x^2}$$

$$yx^2 = (y+2x)c^2$$

Differential Equations Ex 22.9 Q2

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

It is homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx-x}{vx+x}$$

$$v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$= \frac{v-1-v^2-v}{v+1}$$

$$x\frac{dv}{dx} = -\frac{(1+v^2)}{v+1}$$

$$\int \frac{v+1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\int \frac{v}{v^2+1}dv + \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v}{v^2+1}dv + \int \frac{1}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\text{og}|v^2+1| + \tan^{-1} v = -\log|x| + \log|c|$$

$$\log\left|\frac{y^2+x^2}{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log|x^2+y^2| - 2\log|x| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log(c)$$

$$\log|x^2+y^2| + 2\tan^{-1}\left(\frac{y}{x}\right) = k$$

Differential Equations Ex 22.9 Q3

Here, $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$.

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - \frac{v}{1}$$

$$= \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$$

$$\log\left|1 + v^2\right| = -\log|x| + \log|c|$$

$$1 + v^2 = \frac{c}{x}$$

$$1 + \frac{y^2}{x^2} = \frac{c}{x}$$

$$x^2 + y^2 = cx$$

Differential Equations Ex 22.9 Q4

Here, $\frac{xdy}{dx} = x + y,\ x \neq 0$

$$\frac{dy}{dx} = \frac{x + y}{x}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{x + vx}{x}$$

$$v + x\frac{dv}{dx} = 1 + v$$

$$\int dv = \int \frac{dx}{x}$$

$$v = \log|x| + c$$

$$\frac{y}{x} = \log|x| + c$$

$$y = x\log|x| + cx$$

Differential Equations Ex 22.9 Q5

Here, $\left(x^2 - y^2\right)dx - 2xydy = 0$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2}{2v}$$

$$\int \frac{2v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\frac{1}{-3}\int \frac{-6v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{-6v}{1 - 3v^2} = -3\int \frac{dx}{x}$$

$$\log\left|1 - 3v^2\right| = -3\log|x| + \log|c|$$

$$1 - 3v^2 = \frac{c}{x^3}$$

$$x^3\left(1 - \frac{3y^2}{x^2}\right) = c$$

$$\frac{x^3\left(x^2 - 3y^2\right)}{x^2} = c$$

$$x\left(x^2 - 3y^2\right) = c$$

Differential Equations Ex 22.9 Q6

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2}dv = \frac{dx}{x}$$

$$\int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\tan^{-1} v - \frac{1}{2}\log\left(1+v^2\right) = \log x + c$$

$$\tan^{-1}\frac{y}{x} = \frac{1}{2}\log\left(x^2+y^2\right) + c$$

Differential Equations Ex 22.9 Q7

Here, $2xy\frac{dy}{dx} = x^2 + y^2$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

It is a homogeneous equation

Put $y = vx$

and, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\frac{2v}{1-v^2}dv = \frac{dx}{x}$$

$$\int \frac{-2v}{1-v2}dv = -\int \frac{dx}{x}$$

$$\log\left|1-v^2\right| = -\log|x| + \log c$$

$$\left(1-v^2\right) = \frac{c}{x}$$

$$x\left(1-\frac{y^2}{x^2}\right) = c$$

$$\frac{x\left(x^2-y^2\right)}{x^2} = c$$

$$x^2 - y^2 = cx$$

Differential Equations Ex 22.9 Q8

Consider the given differential equation

$$x^2\frac{dy}{dx}=x^2-2y^2+xy$$

$$\Rightarrow\frac{dy}{dx}=\frac{x^2-2y^2+xy}{x^2}$$

This is a homogeneous differential equation.

Substituting $y=vx$ *and* $\frac{dy}{dx}=v+x\frac{dv}{dx}$, *we have*

$$v+x\frac{dv}{dx}=\frac{x^2-2v^2\times x^2+x\times v\times x}{x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=1-2v^2+v$$

$$\Rightarrow x\frac{dv}{dx}=1-2v^2$$

$$\Rightarrow\frac{dv}{1-2v^2}=\frac{dx}{x}$$

$$\Rightarrow\frac{dv}{v^2-\frac{1}{2}}=-2\frac{dx}{x}$$

$$\Rightarrow\int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow\int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow\frac{\sqrt{2}}{2}\log\left(\frac{\frac{1}{\sqrt{2}}+v}{\frac{1}{\sqrt{2}}-v}\right)=2\log x+\log C$$

$$\Rightarrow\frac{1}{\sqrt{2}}\log\left(\frac{\frac{1}{\sqrt{2}}+\frac{y}{x}}{\frac{1}{\sqrt{2}}-\frac{y}{x}}\right)=2\log x+\log C$$

$$\Rightarrow\frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=2\log x+\log C$$

$$\Rightarrow\frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\log x^2+\log C$$

$$\Rightarrow\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=\log Cx^2$$

$$\Rightarrow\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=Cx^2$$

$$\Rightarrow\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\left(Cx^2\right)^{\sqrt{2}}$$

Differential Equations Ex 22.9 Q9

Here, $xy\frac{dy}{dx} = x^2 - y^2$

$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1 - 2v^2}{v}$$

$$\frac{v}{1 - 2v^2}dv = \frac{dx}{x}$$

$$\int\frac{-4v}{1 - 2v^2}dv = -4\int\frac{dx}{x}$$

$$\log\left|1 - 2v^2\right| = -4\log|x| + \log c$$

$$\left(1 - 2\frac{y^2}{x^2}\right) = \frac{c}{x^4}$$

$$\left(\frac{x^2 - 2y^2}{x^2}\right) = \frac{c}{x^4}$$

$$x^2\left(x^2 - 2y^2\right) = c$$

Differential Equations Ex 22.9 Q10

Here, $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y\right)dy$

$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y}{ye^{\frac{x}{y}}}$

It is a homogeneous equation

Put $x = vy$

and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{vye^{\frac{vy}{y}} + y}{ye^{\frac{vy}{y}}}$$

$$v + y\frac{dv}{dy} = \frac{ve^v + 1}{e^v}$$

$$y\frac{dv}{dy} = \frac{ve^v + 1}{e^v} - v$$

$$y\frac{dv}{dy} = \frac{ve^v + 1 - ve^v}{e^v}$$

$$y\frac{dv}{dy} = \frac{1}{e^v}$$

$$\int evdv = \int\frac{dy}{y}$$

$$e^v = \log|y| + c$$

$$e^{\frac{x}{y}} = \log y + c$$

Differential Equations Ex 22.9 Q11

Here, $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

It is a homogeneous equaton

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + xvx + v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = 1 + v + v^2 - v^2$$

$$x\frac{dv}{dx} = 1 + v^2$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \log|x| + c$$

$$\tan^{-1} \frac{y}{x} = \log|x| + c$$

Differential Equations Ex 22.9 Q12

Here, $\left(y^2 - 2xy\right)dx = \left(x^2 - 2xy\right)dy$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - 2xvx}{x^2 - 2xvx}$$

$$v + x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v} - v$$

$$= \frac{v^2 - 2v - v + 2v^2}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{3v^2 - 3v}{1 - 2v}$$

$$\frac{1 - 2v}{3\left(v^2 - v\right)}dv = \frac{dx}{x}$$

$$\frac{-}{3}\frac{(2v - 1)}{v^2 - v}\ dv = \frac{dx}{x}$$

$$\int \frac{2v - 1}{v^2 - v}dv = -3\int \frac{dx}{x}$$

$$\log\left|v^2 - v\right| = -3\log|x| + \log c$$

$$v^2 - v = \frac{c}{x^3}$$

$$\frac{y^2}{x^2} - \frac{y}{x} = \frac{c}{x^3}$$

$$y^2 - xy = \frac{c}{x}$$

$$x\left(y^2 - xy\right) = c$$

Differential Equations Ex 22.9 Q13

Here, $2xydx + \left(x^2 + 2y^2\right)dy = 0$

$$\frac{dy}{dx} = \frac{2xy}{x^2 + 2y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2xvx}{x^2 + 2v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{2v}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{2v}{1 + 2v^2} - v$$

$$= \frac{2v - v - 2v^3}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{v - 2v^3}{1 + 2v^2}$$

$$\int \frac{1 + 2v^2}{v - 2v^3}dv = \int \frac{dx}{x} \qquad ---(i)$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1 + 2v^2}{v\left(1 - 2v^2\right)}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A}{v} + \frac{Bv + c}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A\left(1 - 2v^2\right) + \left(Bv + c\right)v}{v\left(1 - 2v^2\right)}$$

$$1 + 2v^2 = A - 2Av^2 + Bv^2 + cv$$

$$1 + 2v^2 = v^2\left(-2A + B\right) + cv + A$$

Comparing the coefficients of like powers of v,

$$A = 1$$

$$c = 0$$

$$-2A + B = 2$$

$$-2 + B = 0$$

$$B = 4$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} + \frac{4v}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} - \frac{\left(-4v\right)}{\left(1 - 2v^2\right)}$$

Differential Equations Ex 22.9 Q14

Here, $3x^2dy = \left(3xy + y^2\right)dx$

$$\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$$

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{3xvx + v^2x^2}{3x^2}$$

$$v + x\frac{dv}{dx} = \frac{3v + v^2}{3}$$

$$x\frac{dv}{dx} = \frac{3v + v^2}{3} - v$$

$$x\frac{dv}{dx} = \frac{3v + v^2 - 3v}{3}$$

$$x\frac{dv}{dx} = \frac{v^2}{3}$$

$$3\int \frac{1}{v^2}dv = \int \frac{dx}{x}$$

$$3\left(-\frac{1}{v}\right) = \log|x| + c$$

$$-\frac{3x}{y} = \log|x| + c$$

Differential Equations Ex 22.9 Q15

Here, $\frac{dy}{dx} = \frac{x}{2y + x}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x}{2vx + x}$$

$$v + x\frac{dv}{dx} = \frac{1}{2v + 1}$$

$$x\frac{dv}{dx} = \frac{1}{2v + 1} - v$$

$$x\frac{dv}{dx} = \frac{1 - 2v^2 - v}{2v + 1}$$

$$\int \frac{2v + 1}{1 - v - 2v^2} dv = \int \frac{dx}{x}$$

$$-\int \frac{2v + 1}{2v^2 + v - 1} dv = \int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{4v + 2}{2v^2 + v - 1} dv = -\int \frac{dx}{x}$$

$$\int \frac{4v + 1 + 1}{2v^2 + v - 1} dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1} dv + \int \frac{1}{2v^2 + v - 1} dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1} dv + \frac{1}{2}\int \frac{1}{v^2 + \frac{v}{2} - \frac{1}{2}} dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1} dv + \frac{1}{2}\int \frac{dv}{v^2 + 2v\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1} dv + \frac{1}{2}\int \frac{dv}{\left(v + \frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2} = -2\int \frac{dx}{x}$$

$$\log\left|2v^2 + v - 1\right| + \frac{1}{2} \times \frac{1}{2\left(\frac{3}{4}\right)} \log\left|\frac{v + \frac{1}{4} - \frac{3}{4}}{v + \frac{1}{4} + \frac{3}{4}}\right| = -2\log|x| + \log c$$

Differential Equations Ex 22.9 Q16

Here, $(x + 2y)dx - (2x - y)dy = 0$

$$\frac{dy}{dx} = \frac{(x + 2y)}{(2x - y)}$$

It is a homogeneous equation

Put $y = vx$

$\therefore$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x + 2vx}{2x - vx}$$

$$v + x\frac{dv}{dx} = \frac{1 + 2v}{2 - v}$$

$$x\frac{dv}{dx} = \frac{1 + 2v}{2 - v} - \frac{v}{1}$$

$$x\frac{dv}{dx} = \frac{1 + 2v - 2v + v^2}{2 - v}$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{2 - v}$$

$$\frac{2 - v}{1 + v^2} = \frac{dx}{x}$$

$$\int \frac{2 - v}{1 + v^2} dv = \int \frac{dx}{x}$$

$$\int \frac{2}{1 + v^2} dv - \int \frac{v}{1 + v^2} dv = \int \frac{dx}{x}$$

$$2\tan^{-1} v - \frac{1}{2}\log\left|1 + v^2\right| = \log|x| + \log c$$

$$2\tan^{-1} v = \log xc + \log\left|1 + v^2\right|^{\frac{1}{2}}$$

$$e^{2\tan^{-1} v} = \left(1 + v^2\right)^{\frac{1}{2}} xc$$

$$e^{2\tan^{-1}\frac{y}{x}} = \left\{\frac{\left(y^2 + x^2\right)^{\frac{1}{2}}}{x}\right\} xc$$

$$e^{2\tan^{-1}\frac{y}{x}} = \left(y^2 + x^2\right)^{\frac{1}{2}} c$$

Differential Equations Ex 22.9 Q17

Here, $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sqrt{\frac{v^2x^2}{x^2} - 1}$$

$$v + x\frac{dv}{dx} = v - \sqrt{v^2 - 1}$$

$$x\frac{dv}{dx} = v - \sqrt{v^2 - 1} - v$$

$$x\frac{dv}{dx} = -\sqrt{v^2 - 1}$$

$$\int \frac{dv}{\sqrt{v^2 - 1}} - \int \frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = -\log|x| + \log c$$

$$\left(\frac{y}{x} + \sqrt{v^2 - 1}\right) = \frac{c}{x}$$

$$y + \sqrt{y^2 - x^2} = c$$

Differential Equations Ex 22.9 Q18

$$\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x}\left\{\log\left(\frac{vx}{x}\right) + 1\right\}$$

$$v + x\frac{dv}{dx} = v\log v + v$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$$\log\log v = \log|x| + \log c$$

$$\log v = xc$$

$$\log\frac{y}{x} = xc$$

$$\frac{y}{x} = e^{xc}$$

$$y = xe^{xc}$$

Differential Equations Ex 22.9 Q19

$$\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\operatorname{cosec} v\, dv = \frac{dx}{x}$$

$$\int \operatorname{cosec} v\, dv = \int \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + \log c$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Differential Equations Ex 22.9 Q20

Here, $y^2dx+\left(x^2-xy+y^2\right)dy=0$

$$\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{-v^2x^2}{x^2-xvx+v^2x^2}$$

$$v+x\frac{dv}{dx}=\frac{-v^2}{1-v+v^2}$$

$$x\frac{dv}{dx}=\frac{-v^2}{1-v+v^2}-\frac{v}{1}$$

$$=\frac{-v^2-v+v^2-v^3}{1-v+v^2}$$

$$x\frac{dv}{dx}=\frac{-v-v^3}{v^2-v+1}$$

$$\frac{v^2-v+1}{-v\left(1+v^2\right)}dv=\frac{dx}{x}$$

$$\left(\frac{1}{1+v^2}-\frac{1}{v}\right)dv=\frac{dx}{x}$$

$$-\int\frac{1}{v}dv+\int\frac{1}{1+v^2}dv=\int\frac{dx}{x}$$

$$-\log|v|+\tan^{-1}v=\log|x|+\log c$$

$$\log\left|\frac{x}{y}\right|+\tan^{-1}\left(\frac{y}{x}\right)=\log c$$

$$\tan^{-1}\left(\frac{y}{x}\right)=\log xc-\log\frac{x}{y}$$

$$\tan^{-1}\left(\frac{y}{x}\right)=\log\left(\frac{xcy}{x}\right)$$

$$\tan^{-1}\left(\frac{y}{x}\right)=\log(cy)$$

$$e^{\tan^{-1}\left(\frac{y}{x}\right)}=cy$$

Differential Equations Ex 22.9 Q21

Here, $\left[x\sqrt{x^2+y^2}-y^2\right]dx+xydy=0$

$$\frac{dy}{dx}=\frac{\left[y^2-x\sqrt{x^2+y^2}\right]}{xy}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{\left[v^2x^2-x\sqrt{x^2+v^2x^2}\right]}{xvx}$$

$$v+x\frac{dv}{dx}=\frac{\left[v^2-\sqrt{1+v^2}\right]}{v}$$

$$x\frac{dv}{dx}=\frac{v^2-\sqrt{1+v^2}}{v}-v$$

$$=\frac{v^2-\sqrt{1+v^2}-v^2}{v}$$

$$x\frac{dv}{dx}=\frac{-\sqrt{1+v^2}}{v}$$

$$\int\frac{v}{\sqrt{1+v^2}}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v}{\sqrt{1+v^2}}dv=-\int\frac{dx}{x}$$

Let $1+v^2=t$

$$2vdv=dt$$

$$\frac{1}{2}\int\frac{1}{\sqrt{t}}dt=-\int\frac{dx}{x}$$

$$\frac{1}{2}\times2\sqrt{t}=-\log|x|+\log c$$

$$\sqrt{1+v^2}=\log\left|\frac{c}{x}\right|$$

$$\frac{\sqrt{x^2+y^2}}{x}=\log\left|\frac{c}{x}\right|$$

$$\sqrt{x^2+y^2}=x\log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q22

Here, $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v dv = -\int\frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q23

Here, $\frac{y}{x}\cos\left(\frac{y}{x}\right)dx - \left\{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{\frac{vx}{x}\cos\left(\frac{vx}{x}\right)}{\frac{x}{vx}\sin\left(\frac{vx}{x}\right) + \cos\left(\frac{vx}{x}\right)}$$

$$= \frac{v\cos v}{\frac{1}{v}\sin v + \cos v}$$

$$v + x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v} - v$$

$$x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v - v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{-v\sin v}{\sin v + v\cos v}$$

$$\frac{\sin v + v\cos v}{v\sin v}dv = -\frac{dx}{x}$$

$$\int\left(\frac{1}{v} + \cot v\right)dv = -\log|x| + \log c$$

$$\log|v| + \log|\sin v| = \log\left|\frac{c}{x}\right|$$

$$\log|v\sin v| = \log\left|\frac{c}{x}\right|$$

$$|v\sin v| = \left|\frac{c}{x}\right|$$

$$\left|x\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)\right| = |c|$$

$$\left|y\sin\frac{y}{x}\right| = c$$

Differential Equations Ex 22.9 Q24

Here, $xy\log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2\log\left(\frac{x}{y}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{x^2\log\left(\frac{x}{y}\right) - y^2}{xy\log\left(\frac{x}{y}\right)}$$

It is a homogeneous equation

Put $x = vy$

and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{v^2y^2\log\left(\frac{vy}{y}\right) - y^2}{vy\log\left(\frac{vy}{y}\right)}$$

$$v + y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v}$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v} - v$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1 - v^2\log v}{v\log v}$$

$$y\frac{dv}{dy} = \frac{-1}{v\log v}$$

$$\int v\log v dv = -\int\frac{dy}{y}$$

$$\log v \times \int v dv - \int\frac{1}{v}\times\int v dv dv = -\log|y| + \log c$$

Itegrating it by parts

$$\frac{v^2}{2}\log v\int\frac{1}{v}\times\frac{v^2}{2}dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{1}{2}\int v dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{v^2}{4} = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\left[\log v - \frac{1}{2}\right] = \log\left|\frac{c}{y}\right|$$

Differential Equations Ex 22.9 Q25

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

Here it is a homogeneous equation

Put $x = vy$

And

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

So,

$$\frac{dx}{dy} = -\frac{e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$$

$$v + y\frac{dv}{dy} = -\frac{e^{\frac{vy}{y}}\left(1 - \frac{vy}{y}\right)}{\left(1 + e^{\frac{vy}{y}}\right)}$$

$$= -\frac{e^v(1 - v)}{(1 + e^v)}$$

$$y\frac{dv}{dy} = -\frac{e^v(1 - v)}{(1 + e^v)} - v$$

$$= \frac{-e^v(1 - v) - v(1 + e^v)}{(1 + e^v)}$$

$$\frac{(1 + e^v)}{-e^v(1 - v) - v(1 + e^v)}dv = \frac{dy}{y}$$

$$x + ye^{x/y} = c$$

Differential Equations Ex 22.9 Q26

Here, $\left(x^2+y^2\right)\frac{dy}{dx}=8x^2-3xy+2y^2$

$$\frac{dy}{dx}=\frac{8x^2-3xy+2y^2}{x^2+y^2}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{8x^2-3xvx+2v^2x^2}{x^2+v^2x^2}$$

$$v+x\frac{dv}{dx}=\frac{8-3v+2v^2}{1+v^2}$$

$$x\frac{dv}{dx}=\frac{8-3v+2v^2}{1+v^2}-v$$

$$=\frac{8-3v+2v^2-v-v^3}{1+v^2}$$

$$x\frac{dv}{dx}=\frac{8-4v+2v^2-v^3}{1+v^2}$$

$$\frac{1+v^2}{8-4v+2v^2-v^3}dv=\frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv=\frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv=\frac{dx}{x}$$

$$\int\frac{1+v^2}{\left(4+v^2\right)(2-v)}dv=\int\frac{dx}{x} \quad \text{---(A)}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)}=\frac{Av+B}{4+v^2}+\frac{c}{2-v}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)}=\frac{(Av+B)(2-v)+c\left(4+v^2\right)}{\left(4+v^2\right)(2-v)}$$

$$1+v^2=2Av-Av^2+2B-Bv+4c+cv^2$$

$$1+v^2=v^2(-A+c)+v(2A-B)+2B+4c$$

Comparing the coefficients of like powers of v

$$-A+c=1 \quad \text{---(i)}$$

$$2A-B=0$$

$$\Rightarrow \quad B=2A \quad \text{---(ii)}$$

$$2B+4c=1 \quad \text{---(iii)}$$

Solving equation (i), (ii) and (iii)

$$A=-\frac{3}{8}, B=-\frac{3}{4}, C=\frac{5}{8}$$

Using equation (A)

$$\int\frac{\left(-\frac{3}{8}v-\frac{3}{4}\right)}{4+v^2}dv+\frac{5}{8}\int\frac{1}{2-v}dv=\int\frac{dx}{x}$$

$$-\frac{3}{8}\int\frac{v+2}{4+v^2}dv+\frac{5}{8}\int\frac{1}{2-v}dv=\int\frac{dx}{x}$$

$$-\frac{3}{8}\int\frac{v}{4+v^2}dv-\frac{3}{8}\int\frac{1}{4+v^2}dv+\frac{5}{8}\int\frac{1}{2-v}dv=\int\frac{dx}{x}$$

$$-\frac{3}{16}\log\left|4+v^2\right|-\frac{3}{8}\tan^{-1}\frac{v}{2}-\frac{5}{8}\log|2-v|=\log|x|+loc$$

$$-\left[\log\left|4+v^2\right|^{\frac{3}{16}}+\log e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)}+\log(2-v)^{\frac{5}{8}}\right]=\log|xc|$$

$$\left(4+v^2\right)^{\frac{3}{16}}\times e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)}\times(2-v)^{\frac{5}{8}}=\frac{c}{x}$$

$$\frac{\left(4x^2+y^2\right)^{\frac{3}{16}}}{x^{\frac{3}{8}}}\times e^{\frac{3}{8}\tan^{-1}\left(\frac{y}{2x}\right)}\frac{(2x-y)^{\frac{5}{8}}}{x^{\frac{5}{8}}}=\frac{c}{x}$$

$$\left(4x^2+y^2\right)^{\frac{3}{16}}\times(2x-y)^{\frac{5}{8}}=c\,e^{\frac{-3}{8}\tan^{-1}\left(\frac{y}{2x}\right)}$$

Differential Equations Ex 22.9 Q27

Here, $\left(x^2 - 2xy\right)dy + \left(x^2 - 3xy + 2y^2\right)dx = 0$

$$\frac{dy}{dx} = \frac{x^2 - 3xy + 2y^2}{2xy - x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - 3xvx + 2v^2x^2}{2xvx - x^2}$$

$$x\frac{dv}{dx} = \frac{1 - 3v + 2v^2}{2v - 1} - v$$

$$x\frac{dv}{dx} = \frac{1 - 3v + 2v^2 - 2v^2 + v}{2v - 1}$$

$$x\frac{dv}{dx} = \frac{1 - 2v}{2v - 1}$$

$$\frac{2v - 1}{1 - 2v}dv = \frac{dx}{x}$$

$$\frac{1 - 2v}{1 - 2v}dv = -\int\frac{dx}{x}$$

$$\int dv = -\int\frac{dx}{x}$$

$$v = -\log|x| + C$$

$$y/x + \log x = C$$

Differential Equations Ex 22.9 Q28

Here, $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int\sec^2 v\,dv = -\int\frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q29

Here, $x\frac{dy}{dx} - y = 2\sqrt{y^2 - x^2}$

$$\frac{dy}{dx} = \frac{2\sqrt{y^2 - x^2} + y}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2\sqrt{v^2x^2 - x^2} + vx}{x}$$

$$v + x\frac{dv}{dx} = 2\sqrt{v^2 - 1} + v$$

$$x\frac{dv}{dx} = 2\sqrt{v^2 - 1}$$

$$\int\frac{dv}{\sqrt{v^2 - 1}} = 2\int\frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = 2\log|x| + \log|c|$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = \log\left|cx^2\right|$$

$$v + \sqrt{v^2 - 1} = \left|cx^2\right|$$

$$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} - 1} = \left|cx^2\right|$$

$$\left(y + \sqrt{y^2 - x^2}\right) = cx^3$$

Differential Equations Ex 22.9 Q30

Here, $x\cos\left(\frac{y}{x}\right)(ydx+xdy) = y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

$$yx\cos\left(\frac{y}{x}\right)+x^2\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = xy\sin\left(\frac{y}{x}\right)-y^2\sin\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \left(\frac{-y^2\sin\left(\frac{y}{x}\right)-xy\cos\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right)-xy\sin\left(\frac{y}{x}\right)}\right)$$

$$\frac{dy}{dx} = \frac{-xy\cos\left(\frac{y}{x}\right)-y^2\sin\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right)-xy\sin\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx} = \frac{-xvx\cos\left(\frac{vx}{x}\right)-v^2x^2\sin\left(\frac{vx}{x}\right)}{x^2\cos\left(\frac{vx}{x}\right)-xvx\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{-v\cos v - v^2\sin v}{\cos v - v\sin v} - v$$

$$x\frac{dv}{dx} = \frac{-v\cos v - v^2\sin v - v\cos v + v^2\sin v}{\cos v - v\sin v}$$

$$x\frac{dv}{dx} = \frac{-2v\cos v}{\cos v - v\sin v}$$

$$\int\frac{\cos v - v\sin v}{v\cos v}dv = -2\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v}-\tan v\right)dv = -2\int\frac{dx}{x}$$

Differential Equations Ex 22.9 Q31

Here, $\left(x^2+3xy+y^2\right)dx - x^2dy = 0$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$v+x\frac{dv}{dx} = 1+3v+v^2$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$x\frac{dv}{dx} = (v+1)^2$$

$$\int\frac{1}{(v+1)^2}dv = \int\frac{dx}{x}$$

$$-\frac{1}{v+1} = \log|x| - c$$

$$\frac{x}{x+y} + \log|x| = c$$

Differential Equations Ex 22.9 Q32

Here, $(x-y)\frac{dy}{dx} = x+2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{1+2v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\frac{1-v}{v^2+v+1}dv = \frac{dx}{x}$$

$$-\frac{v-1}{v^2+v+1}dv = \frac{dx}{x}$$

$$\frac{1}{2}\times\frac{2v-2}{v^2+v+1}dv = \frac{-dx}{x}$$

$$\int\frac{(2v+1)-3}{v^2+v+1}dv = -\int\frac{2dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv - \int\frac{3}{v^2+2v\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1} = -2\int\frac{dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv - \int\frac{3}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -2\int\frac{dx}{x}$$

$$\log\left|v^2+v+1\right| - 3\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = -2\log|x| + c$$

$$\log\left|y^2+xy+x^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) + c$$

Differential Equations Ex 22.9 Q33

$$\left(2x^2y+y^3\right)dx+\left(xy^2+3x^3\right)dy=0$$
$$\frac{dy}{dx}=\frac{2x^2y+y^3}{3x^3-xy^2}$$

It is a homogeneous equation

Put $\quad y=vx$

and $\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{2x^2vx+v^3x^3}{3x^3-xv^2x^2}$$
$$x\frac{dv}{dx}=\frac{2v+v^3}{3-v^2}-v$$
$$=\frac{2v+v^3-3v+v^3}{3-v^2}$$
$$x\frac{dv}{dx}=\frac{2v^3-v}{3-v^2}$$
$$\int\frac{3-v^2}{2v^3-v}dv=\int\frac{dx}{x} \qquad \text{---(i)}$$
$$\frac{3-v^2}{v\left(2v^2-1\right)}=\frac{A}{(v)}+\frac{Bv+c}{\left(2v^2-1\right)}$$
$$3-v^2=A\left(2v^2-1\right)+(Bv+c)(v)$$
$$=2Av^2-A+Bv^2+cv$$
$$3-v^2=(2A+B)v^2cv-A$$

Comparing the coefficient of like powers of v

$A=-3$

$C=0$

and $\quad 2A+B=-1$

$\Rightarrow \quad 2(-3)+B=-1$

$\Rightarrow \quad B=5$

So,

$$\int\frac{-3}{v}dv+\int\frac{5v}{2v^2-1}dv=\int\frac{dx}{x}$$
$$-3\int\frac{1}{v}dv+\frac{5}{4}\int\frac{4v}{2v^2-1}dv=\int\frac{dx}{x}$$
$$-3\log|v|+\frac{5}{4}\log\left|2v^2-1\right|=\log|x|+\log|c|$$
$$-12\log|v|+5\log\left|2v^2-1\right|=4\log|x|+4\log|c|$$
$$\frac{\left|2v^2-1\right|^5}{v^{12}}=x^4c^4$$
$$\frac{\left|2y^2-x^2\right|^5}{x^{10}}=x^4c^4\left(\frac{y}{x}\right)^{12}$$
$$\left|2y^2-x^2\right|^5=x^{14}c^4\frac{y^{12}}{x^{12}}$$
$$x^2c^4y^{12}=\left|2y^2-x^2\right|^5$$

Differential Equations Ex 22.9 Q34

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\sin\left(\frac{vx}{x}\right)}{x}$$

$$x\frac{dv}{dx} = v - \sin v - v$$

$$\int \operatorname{cosec} v\, dv = -\int \frac{dx}{x}$$

$$\log\left|\operatorname{cosec} v + \cot v\right| = -\log\frac{c}{x}$$

$$\log\left|\operatorname{cosec} v + \cot v\right| = \log\frac{x}{c}$$

$$\operatorname{cossec}\left(\frac{y}{x}\right) + \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$\frac{\left(1 + \cos\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$x\ \sin\ \left(\frac{y}{x}\right) = c\left(1 + \cos\frac{y}{x}\right)$$

Differential Equations Ex 22.9 Q35

$$y\,dx + \left\{x\log\left(\frac{y}{x}\right)\right\}dy - 2x\,dy = 0$$

$$y + x\log\left(\frac{y}{x}\right)\frac{dy}{dx} - 2x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{2x - x\log\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v}$$

$$\int \frac{\log v - 2}{v\,(\log v - 1)}dv = -\int \frac{dx}{x}$$

Let $\log v - 1 = t$

$$\frac{1}{v}dv = dt$$

$$\int\left(\frac{t-1}{t}\right)dt = -\int\frac{dx}{x}$$

$$t - \log|t| = \log\left|\frac{c}{x}\right|$$

$$\log v - 1\log(\log v - 1) = \log\left|\frac{c}{x}\right|$$

$$\log e^{\log v - 1} - \log|\log v - 1| = \log\left|\frac{c}{x}\right|$$

$$e^{\log\left(\frac{v}{e}\right)} = \frac{c}{x}|\log v - 1|$$

$$\frac{v}{e} = \frac{c}{x}|\log v - 1|$$

$$y = c_1\left\{\log\left|\frac{y}{x}\right| - 1\right\}$$

Differential Equations Ex 22.9 Q36(i)

$$\left(x^2 + y^2\right)dx = 2xydy,\ y(1) = 0$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homogenues equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\int\frac{2v}{1 - v^2} = \int\frac{dx}{x}$$

$$\log\left|1 - v^2\right| = -\log|x| + \log|c|$$

$$\log\left|1 - v^2\right| = \log\left|\frac{c}{x}\right|$$

$$\left|\frac{x^2 - y^2}{x^2}\right| = \left|\frac{c}{x}\right|$$

$$\left|x^2 - y^2\right| = |cx| \qquad \text{---(i)}$$

Put $y = 0,\ x = 1$

$$1 - 0 = c$$

$$c = 1$$

Put the value of c in equation (i),

$$\left|x^2 - y^2\right| = |x|$$

$$\left(x^2 - y^2\right)^2 = x^2$$

Differential Equations Ex 22.9 Q36(ii)

Here, $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0,\ y(e) = 0$

$$\frac{dy}{dx} = \frac{y - xe^{\frac{y}{x}}}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - xe^{\frac{vx}{x}}}{x}$$

$$x\frac{dv}{dx} = v - e^{v} - v$$

$$x\frac{dv}{dx} = -e^{v}$$

$$\int -e^{-v} dv = \int \frac{dx}{x}$$

$$e^{v} = \log|xc|$$

$$v = \log(\log|xc|)$$

$$\frac{y}{x} = \log\log|x| + k$$

$$y = x\log(\log|x|) + k \qquad \text{---(i)}$$

Put $y = 0,\ x = e$

$$0 = e\log(\log e) + k$$

$$0 = e \times 0 + k$$

$$0 = k$$

Using equation (i),

$$y = x\log(\log|x|)$$

Differential Equations Ex 22.9 Q36(iii)

$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0, y(1) = 0$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\frac{vx}{x}$$

$$x\frac{dv}{dx} = v - \operatorname{cosec} v - v$$

$$= -\operatorname{cosec} v$$

$$\frac{dv}{\operatorname{cosec} v} = -\frac{dx}{x}$$

$$\sin v dv = -\frac{dx}{x}$$

$$-\cos v = -\log|x| + c$$

$$-\cos\frac{y}{x} = -\log|x| + c$$

Now putting $y = 0, x = 1$, we have

$$c = -1$$

Now

$$-\cos\frac{y}{x} + 1 = -\log|x|$$

$$\log|x| = \cos\frac{y}{x} - 1$$

Differential Equations Ex 22.9 Q36(iv)

$$\left(xy - y^2\right)dx - x^2dy = 0,\ y(1) = 1$$

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{xvx - v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = v - v^2 - v$$

$$x\frac{dv}{dx} = -v^2$$

$$-\int\frac{1}{v^2}dv = \int\frac{dx}{x}$$

$$-\left(-\frac{1}{v}\right) = \log|x| + c$$

$$\frac{x}{y} = \log|x| + c \qquad ---(i)$$

Put $y = 1,\ x = 1$

$1 = c$

Using equation (1),

$$x = y\left[\log|x| + 1\right]$$

$$y = \frac{x}{\left[\log|x| + 1\right]}$$

Differential Equations Ex 22.9 Q36(v)

$$\frac{dy}{dx} = \frac{y(x + 2y)}{x(2x + y)},\ y(1) = 2$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx(x + 2vx)}{x(2x + vx)}$$

$$x\frac{dv}{dx} = \frac{v(1 + 2v)}{(2 + v)} - v$$

$$x\frac{dv}{dx} = \frac{v + 2v^2 - 2v - v^2}{2 + v}$$

$$x\frac{dv}{dx} = \frac{v^2 - v}{2 + v}$$

$$\frac{2 + v}{v^2 - v}dv = \frac{dx}{x}$$

$$\int\frac{2 + v}{v^2 - v}dv = \int\frac{dx}{x} \qquad ---(i)$$

$$\frac{2 + v}{v(v - 1)} = \frac{A}{v} + \frac{B}{v - 1}$$

$$\frac{2 + v}{v(v - 1)} = \frac{A(v - 1) + Bv}{v(v - 1)}$$

$$2 + v = (A + B)v - A$$

Comparing the coefficients of like powers of v,

$A = -2$

$A + B = 1$

$\Rightarrow \quad -2 + B = 1$

$\Rightarrow \quad B = 3$

Using equation (i),

$$\int\frac{-2}{v}dv + 3\int\frac{1}{v - 1}dv = \int\frac{dx}{x}$$

$$-2\log|v| + 3\log|v - 1| = \log|cx|$$

$$|v - 1|^3 = v^2cx$$

$$\frac{|y - x|^3}{x^3} = \frac{y^2}{x^2}cx$$

Differential Equations Ex 22.9 Q36(vi)

$$\left(y^4 - 2x^3y\right)dx + \left(x^4 - 2xy^3\right)dy = 0$$

$$\frac{dy}{dx} = \frac{2x^3y - y^4}{x^4 - 2xy^3}$$

It is a homogeneous equation

Put $\quad y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{2x^3vx - x^4v^4}{x^4 - 2xv^3x^3}$$

$$x\frac{dv}{dx} = \frac{2v - v^4}{1 - 2v^3} - v$$

$$x\frac{dv}{dx} = \frac{2v - v^4 - v + 2v^4}{1 - 2v^3}$$

$$x\frac{dv}{dx} = \frac{v^4 + v}{1 - 2v^3}$$

$$\int \frac{1 - 2v^3}{v\left(v^3 + 1\right)}dv = \int \frac{dx}{x} \qquad \text{---(i)}$$

$$\frac{1 - 2v^3}{v\left(v + 1\right)\left(v^2 - v + 1\right)} = \frac{A}{v} + \frac{B}{V + 1} + \frac{cv + D}{v^2 - v + 1}$$

$$1 - 2v^3 = A\left(v^3 + 1\right) + Bv\left(v^2 - v + 1\right) + \left(cv + D\right)\left(v^2 + v\right)$$

$$= Av^3 + A + cv^3 - Bv^2 + cv + cv^3 + cv^2 + Dv^2 + Dv$$

$$1 - 2v^3 = v^3\left(A + B + C\right) + v^2\left(-B + C + D\right) + v\left(B + D\right) + A$$

Comparing the coefficients of like powers of v

$$A = 1 \qquad \text{---(ii)}$$

$$B + D = 0 \qquad \text{---(iii)}$$

$$-B + C + D = 0 \qquad \text{---(iv)}$$

$$A + B + C = -2 \qquad \text{---(v)}$$

Solution of equation (ii), (iii), (iv), (v) gives

$$A = 1,\ b = -1,\ c = -2,\ x = 1$$

Using equation (i),

$$\int \frac{1}{v}dv - \int \frac{1}{v + 1}dv - \int \frac{2v - 1}{v^2 - v + 1}dv = \int \frac{dx}{x}$$

$$\log|v| - \log|v + 1| - \log\left|v^2 - v + 1\right| = \log|xc|$$

$$\log\left|\frac{v}{v^3 + 1}\right| = \log|xc|$$

Differential Equations Ex 22.9 Q36(vii)

Here, $x\left(x^2+3y^2\right)dx + y\left(y^2+3x^2\right)dy = 0,\ y(1) = 1$

$$\frac{dy}{dx} = -\frac{x\left(x^2+3y^2\right)}{y\left(y^2+3x^2\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = -\frac{x\left(x^2+3v^2x^2\right)}{vx\left(v^2x^2+3x^2\right)}$$

$$x\frac{dv}{dx} = -\frac{\left(1+3v^2\right)}{v\left(v^2+3\right)} - v$$

$$x\frac{dv}{dx} = \frac{-1-3v^2-v^4-3v^2}{v\left(v^2+3\right)}$$

$$= \frac{-v^4-6v^2-1}{v\left(v^2+3\right)}$$

$$\frac{v\left(v^2+3\right)}{v^4+6v^2+1}dc = -\frac{dx}{x}$$

$$\int\frac{4v^3+12v}{v^4+6v^2+1}dv = -4\int\frac{dx}{x}$$

$$\log\left|v^4+6v^2+1\right| = \log\left|\frac{c}{x^4}\right|$$

$$\left|v^4+6v^2+1\right| = \left|\frac{c}{x^4}\right|$$

$$\left|y^4+6y^2x^2+x^4\right| = |c| \qquad \text{---(i)}$$

Put $y = 1,\ x = 1$

$(1+6+1) = c$

$\Rightarrow$ $c = 8$

Put $c = 8$ in equation (i),

$$\left(y^4+x^4+6x^2y^2\right) = 8$$

Differential Equations Ex 22.9 Q36(viii)

$$\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + xdy = 0$$

$$\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx = -xdy$$

$$\sin^2\left(\frac{y}{x}\right) + \frac{y}{x} = \frac{dy}{dx}\ldots\ldots..(i)$$

Let $v = \frac{y}{x}$

$$v + x\frac{dv}{dx} = \frac{dy}{dx}$$

From eq (i)

$$\sin^2 v + v = v + x\frac{dv}{dx}$$

$$\frac{1}{\sin^2 v}dv = \frac{1}{x}dx$$

Integrating on both the sides we have,

$$\int\frac{1}{\sin^2 v}dv = \int\frac{1}{x}dx$$

$$-\cot v = \log(x) + C$$

$$-\cot\left(\frac{y}{x}\right) = \log(x) + C\ldots\ldots\ldots(ii)$$

Put x= 1 y = $\frac{\pi}{4}$ in eq (ii)

$$-\cot\left(\frac{\pi}{4}\right) = \log(1) + C$$

C = -1

From eq (ii) we have

$$-\cot\left(\frac{y}{x}\right) = \log(x) - 1$$

Differential Equations Ex 22.9 Q36(ix)

$$\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + xdy = 0$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = -\sin^2\left(\frac{vx}{x}\right) + \frac{vx}{x}$$

$$x\frac{dv}{dx} = -\sin^2 v$$

$$\frac{dv}{\sin^2 v} = -\frac{dx}{x}$$

$$\cot\left(\frac{y}{x}\right) = \log|cx|$$

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0, y(2) = \pi$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right)$$

$$x\frac{dv}{dx} = -\sin v$$

$$\frac{dv}{\sin v} = -\frac{dx}{x}$$

$$\operatorname{cosec} v dv = -\frac{dx}{x}$$

$$-\log(\operatorname{cosec} v + \cot v) = -\log x + c$$

Now putting $y = \pi, x = 2$, we have

$c = 0.301$

Now

$$-\log\left(\operatorname{cosec}\left(\frac{y}{x}\right) + \cot\left(\frac{y}{x}\right)\right) = -\log x + 0.301$$

Differential Equations Ex 22.9 Q37

Consider the given equation

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$$

This is a homogeneous differential equation.

Thus, substiuting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

in the above equation, we get,

$$x\cos\left(\frac{vx}{x}\right)\left(v+x\frac{dv}{dx}\right) = vx\cos\left(\frac{vx}{x}\right)+x$$

$$\Rightarrow \cos v\left(v+x\frac{dv}{dx}\right)=v\cos\left(\frac{vx}{x}\right)+1$$

$$\Rightarrow v\cos v+x\cos v\frac{dv}{dx}=v\cos v+1$$

$$\Rightarrow x\cos v\frac{dv}{dx}=1$$

$$\Rightarrow \cos v\,dv=\frac{dx}{x}$$

Integrating both the sides,

$$\Rightarrow \int\cos v\,dv=\int\frac{dx}{x}$$

$$\Rightarrow \sin v = \log x + C$$

$$\Rightarrow \sin\left(\frac{y}{x}\right) = \log x + C\ldots(1)$$

Given that when $x = 1$, $y = \frac{\pi}{4}$

Substituting the values, $x = 1$ and $y = \frac{\pi}{4}$

in equation (1), we get,

$$\Rightarrow \sin\left(\frac{\frac{\pi}{4}}{1}\right) = \log 1 + C$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\right) = 0 + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} = C$$

Subsituting the value of C, in equation (1) we get,

$$\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{\sqrt{2}}$$

Differential Equations Ex 22.9 Q38

consider the given equation

$$(x-y)\frac{dy}{dx} = x + 2y$$

This is a homogeneous equation.

Substiuting y=vx and $\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$ in

the above equation, we have,

$$(x - vx)\left(v + x\frac{dv}{dx}\right) = x + 2vx$$

$$\Rightarrow (1-v)\left(v + x\frac{dv}{dx}\right) = 1 + 2v$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v - v(1-v)}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v - v + v^2}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \frac{(1-v)dv}{(1+v+v^2)} = \frac{dx}{x}$$

Integrating on both the sides, we have,

Integrating on both the sides, we have,

$$\Rightarrow \int \frac{(1-v)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{(1+v+v^2)} - \int \frac{1}{2}\frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{v^2 + \frac{1}{4} + v + \frac{3}{4}} - \frac{1}{2}\int \frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2}\int \frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2} \times \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{v + \frac{1}{2}}{\frac{\sqrt{3}}{2}} - \frac{1}{2}\log\left(1+v+v^2\right) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2v+1}{\sqrt{3}} - \frac{1}{2}\log\left(1+v+v^2\right) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}} - \frac{1}{2}\log\left(1 + \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\right) = \log x + C\ldots(1)$$

Given that when x = 1, y = 0

Substituting the values, in the above equation, we get,

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\times 0+1}{\sqrt{3}}-\frac{1}{2}\log\left(1+0+0^2\right)=\log 1+C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}}-\frac{1}{2}\times 0=0+C$$

$$\Rightarrow C=\sqrt{3}\times\frac{\pi}{6}$$

$$\Rightarrow C=\frac{\pi}{2\sqrt{3}}$$

Thus, equation (1) becomes,

$$\sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}}-\frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right)=\log x+\frac{\pi}{2\sqrt{3}}$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{2\sqrt{3}}=\log x+\frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{\sqrt{3}}=\log x^2+\log\left(\frac{x^2+xy+y^2}{x^2}\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}}-\frac{\pi}{\sqrt{3}}=\log\left(x^2+xy+y^2\right)$$

Differential Equations Ex 22.9 Q39

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$$

$$\frac{dy}{dx}=\left(\frac{1}{\frac{x}{y}+\frac{y}{x}}\right)\ldots\ldots(i)$$

Let $v=\frac{y}{x}$

$$x\frac{dv}{dx}+v=\frac{dy}{dx}$$

From (i) we have,

$$x\frac{dv}{dx}+v=\left(\frac{1}{\frac{1}{v}+v}\right)$$

$$\left(-\frac{1}{v^3}-\frac{1}{v}\right)dv=\frac{1}{x}dx$$

Integrating on both the sides we have

$$\frac{1}{2v^2}-\log v=\log x+C$$

$$\Rightarrow \frac{x^2}{2y^2}=\log\left(\frac{y}{x}\times x\right)+C\ldots\ldots\ldots(ii)$$

Put x =0 , y = 1

$$0=\log(1)+C$$

$$C=0$$

From eq (ii) we have

$$\frac{x^2}{2y^2}=\log(y)$$

Ex 22.10

Differential Equations Ex 22.10 Q1

Here, $\frac{dy}{dx}+2y=e^{3x}$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=2,\ Q=e^{3x}$$

I.F. $=e^{\int P dx}$

$=e^{\int 2dx}$

$=e^{2x}$

Multiplying both the sides by I.F.

$$e^{2x}\frac{dy}{dx}+e^{2x}2y=e^{2x}\times e^{3x}$$

$$e^{2x}\frac{dy}{dx}+e^{2x}2y=e^{5x}$$

Integrating it with respect to x,

$$ye^{2x}=\int e^{5x}dx+c$$

$$ye^{2x}=\frac{e^{5x}}{5}+c$$

$$y=\frac{e^{3x}}{5}+ce^{-2x}$$

Differential Equations Ex 22.10 Q2

Here, $4\frac{dy}{dx}+8y=5e^{-3x}$

$$\frac{dy}{dx}+2y=\frac{5}{4}e^{-3x}$$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=2,\ Q=\frac{5}{4}e^{-3x}$$

I.F. $=e^{\int P dx}$

$=e^{\int 2dx}$

$=e^{2x}$

Solution of the equation is given by

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$ye^{2x}=\int\frac{5}{4}e^{-3x}\times e^{2x}dx+c$$

$$ye^{2x}=\int\frac{5}{4}e^{-x}dx+c$$

$$ye^{2x}=\frac{-5}{4}e^{-x}+c$$

$$y=\frac{-5}{4}e^{-3x}+ce^{-2x}$$

Differential Equations Ex 22.10 Q3

Here, $\frac{dy}{dx}+2y=6e^{x}$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=2,\ Q=6e^{x}$$

I.F. $=e^{\int P dx}$

$=e^{\int 2dx}$

$=e^{2x}$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\times\left(e^{2x}\right)=\int 6e^{x}\times e^{2x}dx+c$$

$$=\int 6e^{3x}dx+c$$

$$ye^{2x}=\frac{6}{3}e^{3x}+c$$

$$ye^{2x}=2e^{3x}+c$$

$$y=2e^{x}+ce^{-2x}$$

Differential Equations Ex 22.10 Q4

Here, $\frac{dy}{dx}+y=e^{-2x}$

This is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=1, Q=e^{-2x}$$

I.F. $=e^{\int Pdx}$

$=e^{\int 2dx}$

$=e^{x}$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\times e^{x}=\int e^{-2x}\times e^{x}dx+c$$

$$=\int e^{-x}+c$$

$$ye^{x}=\frac{e^{-x}}{-1}+c$$

$$y=-e^{-2x}+ce^{-x}$$

Differential Equations Ex 22.10 Q6

Here, $\frac{dy}{dx}+2y=4x$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=2, Q=4x$$

I.F. $=e^{\int Pdx}$

$=e^{\int 2dx}$

$=e^{2x}$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\times e^{2x}=\int 4x\times e^{2x}dx+c$$

$$=4\left[x\times\int e^{2x}dx-\int\left(1\times\int e^{2x}dx\right)dx\right]+c$$

Using integration by parts

$$y\times e^{2x}=4\left[x\times\frac{e^{2x}}{2}-\int\frac{e^{2x}}{2}dx\right]+c$$

$$=2xe^{2x}-2\frac{e^{2x}}{2}+c$$

$$ye^{2x}=2xe^{2x}-e^{2x}+c$$

$$ye^{2x}=(2x-1)e^{2x}+c$$

$$y=(2x-1)+ce^{-2x}$$

Differential Equations Ex 22.10 Q7

Here, $x\frac{dy}{dx}+y=xe^{x}$

$$\frac{dy}{dx}+\frac{y}{x}=e^{x}$$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{x}, Q=e^{x}$$

I.F. $=e^{\int Pdx}$

$=e^{\int\frac{1}{x}dx}$

$=e^{\log x}$

$=x$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\times(x)=\int e^{x}\times xdx+c$$

$$xy=x\int e^{x}dx-\int\left(1\times\int e^{x}dx\right)dx+c$$

Using integration by parts

$$=xe^{x}-\int e^{x}dx+c$$

$$=xe^{x}-e^{x}+c$$

$$xy=(x-1)e^{x}+c$$

$$y=\left(\frac{x-1}{x}\right)e^{x}+\frac{c}{x},\ x>0$$

Differential Equations Ex 22.10 Q8

Here, $\frac{dy}{dx} + \frac{4x}{x^2+1} y = -\frac{1}{\left(x^2+1\right)^2}$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{4x}{x^2+1}, Q = -\frac{1}{\left(x^2+1\right)^2}$$

I.F. $= e^{\int P dx}$

$= e^{\int \frac{4x}{x^2+1} dx}$

$= e^{2\int \frac{2x}{x^2+1} dx}$

$= e^{2 \log\left|x^2+1\right|}$

$= \left(x^2+1\right)^2$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \left(x^2+1\right)^2 = \int -\frac{1}{\left(x^2+1\right)^2} \left(x^2+1\right)^2 x dx + c$$

$$y \left(x^2+1\right)^2 = \int -dx + c$$

$$y \left(x^2+1\right)^2 = -x + c$$

$$y = -\frac{x}{\left(x^2+1\right)^2} + \frac{c}{\left(x^2+1\right)^2}$$

Differential Equations Ex 22.10 Q9

Here, $x\frac{dy}{dx} + y = x \log x$

$$\frac{dy}{dx} + \frac{y}{x} = \log x$$

It is a linear differential equation, comparing it with

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{x}, Q = \log x$$

I.F. $= e^{\int P dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log|x|}$

$= x,\ x > 0$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times x = \int (\log x)(x) dx + c$$

$$yx = \log x \times \int x dx - \int \left(\frac{1}{x} \times \int x dx\right) dx + c$$

$$= \frac{x^2}{2} \log x - \int \frac{x^2}{2x} dx + c$$

$$= \frac{x^2}{2} \log x - \int \frac{x}{2} dx + c$$

$$yx = \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

$$y = \frac{x}{2} \log x - \frac{x}{4} + \frac{c}{x},\ x > 0$$

Differential Equations Ex 22.10 Q10

Here, $x\dfrac{dy}{dx} - y = (x-1)e^x$

$$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x-1}{x}\right)e^x$$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx} + Py = Q$$

$$P = -\frac{1}{x},\ Q = \left(\frac{x-1}{x}\right)e^x$$

I.F. $= e^{\int P dx}$

$= e^{-\int \frac{1}{x} dx}$

$= e^{-\log|x|}$

$= \frac{1}{x},\ x > 0$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y\left(\frac{1}{x}\right) = \int \left(\frac{x-1}{x}\right)e^x \left(\frac{1}{x}\right) dx + c$$

$$\frac{y}{x} = \int \left(\frac{1}{x} - \frac{1}{x^2}\right)e^x dx + c$$

$$\frac{y}{x} = \frac{1}{x}e^x + c$$

Since $\int \left[f(x) + f'(x)\right]e^x dx = f(x)e^x + c$

$$y = e^x + \text{Cx},\ x > 0$$

Differential Equations Ex 22.10 Q11

Here, $\dfrac{dy}{dx} + \dfrac{y}{x} = x^3$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{x},\ Q = x^3$$

I.F. $= e^{\int P dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log|x|}$

$= x,\ x > 0$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \times x = \int x^3 \times (x) dx + c$$

$$xy = \frac{x^5}{5} + c$$

$$y = \frac{x^4}{5} + \frac{c}{x},\ x > 0$$

Differential Equations Ex 22.10 Q12

$\frac{dy}{dx}+y=\sin x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$p=1, Q=\sin x$$

I.F.

$$=e^{\int pdx}$$

$$=e^{\int dx}$$

$$=e^{x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(e^{x}\right)=\int \sin x\times\left(e^{x}\right)dx+c$$

$$ye^{x}=\frac{e^{x}}{2}(\sin x-\cos x)+c$$

Differential Equations Ex 22.10 Q13

Here, $\frac{dy}{dx}+y=\cos x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=1, Q=\cos x$$

I.F. $=e^{\int Pdx}$

$$=e^{\int dx}$$

$$=e^{x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(e^{x}\right)=\int(\cos x)\left(e^{x}\right)+c_1 \qquad \text{---(i)}$$

Let $I=\int e^{x}\cos xdx$

$$=\cos x\times\int e^{x}dx\int\left(\sin x\int e^{x}dx\right)dx+c_2$$

Using integration by parts

$$I=e^{x}\cos x+\int \sin xe^{x}dx+c$$

$$=e^{x}\cos x+\left[\sin x\int e^{x}dx-\int\left(\cos x\int e^{x}dx\right)dx\right]+c_2$$

$$I=e^{x}\cos x+\sin e^{x}-I+c_2$$

$$2I=e^{x}\left(\cos x+\sin x\right)+c_2$$

$$I=\frac{e^{x}}{2}\left(\cos x+\sin x\right)+\frac{c_2}{2}$$

$$I=\frac{e^{x}}{2}\left(\cos x+\sin x\right)+c_3$$

Putting I in equation (i),

$$ye^{x}=\frac{e^{x}}{2}\left(\cos x+\sin x\right)+c_1+c_3$$

$$ye^{x}=\frac{e^{x}}{2}\left(\cos x+snx\right)+c$$

$$y=\frac{1}{2}\left(\cos x+\sin x\right)+ce^{-x}$$

Differential Equations Ex 22.10 Q14

$$\frac{dy}{dx}+2y=\sin x$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$p=2, Q=\sin x$$

I.F.

$$=e^{\int p dx}$$

$$=e^{\int dx}$$

$$=e^{2x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(e^{2x}\right)=\int \sin x\times\left(e^{2x}\right)dx+c$$

$$ye^{2x}=\frac{e^{2x}}{5}(2\sin x-\cos x)+c$$

Differential Equations Ex 22.10 Q15

Here, $\frac{dy}{dx}-y\tan x=-2\sin x$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\tan x, Q=-2\sin x$$

I.F. $=e^{\int P dx}$

$$=e^{-\int \tan x dx}$$

$$=e^{-\log \sec x}$$

$$=\frac{1}{\sec x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$\frac{y}{\sec x}=\int -\frac{2\sin x}{\sec x}dx+c$$

$$y\cos x=-\int 2\sin x\cos x dx+c$$

$$y\cos x=-\int \sin 2x dx+c$$

$$y\cos x=\frac{\cos 2x}{2}+c$$

$$y=\frac{\cos 2x}{2\cos x}+\frac{c}{\cos x}$$

Differential Equations Ex 22.10 Q16

Here, $\left(1+x^2\right)\frac{dy}{dx}+y=\tan^{-1}x$

$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{1+x^2}$

It is a linear differential equation. Comparing the equation by,

$\frac{dy}{dx}+Py=Q$

$P=\frac{1}{1+x^2},\ Q=\frac{\tan^{-1}x}{1+x^2}$

I.F. $=e^{\int P dx}$

$=e^{\int \frac{1}{1+x^2}dx}$

$=e^{\tan^{-1}x}$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$

$y\left(e^{\tan^{-1}x}\right)=\int \frac{\tan^{-1}x}{1+x^2}e^{\tan^{-1}x}dx+c$

Let $\tan^{-1}x=t$

$\frac{1}{1+t^2}dx=dt$

So,

$ye^t=\int t\times e^t dt+c$

$=t\times\int e^t dt-\int\left(1\times e^t dt\right)dt+c$

Using integration by parts

$ye^t=te^t-e^t+c$

$y=(t-1)ce^{-t}$

$y=\left(\tan^{-1}x-1\right)+ce^{-\tan^{-1}x}$

Differential Equations Ex 22.10 Q17

Here, $\frac{dy}{dx}+y\tan x=\cos x$

It is a linear differential equation. Comparing the equation by,

$\frac{dy}{dx}+Py=Q$

$P=\tan x,\ Q=\cos x$

I.F. $=e^{\int P dx}$

$=e^{\int \tan x dx}$

$=e^{\log|\sec x|}$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$

$y\sec x=\int \cos x\,(\sec x)dx+c$

$\frac{y}{\cos x}=\int dx+c$

$\frac{y}{\cos x}=x+c$

$y=x\cos x+c\cos x$

Differential Equations Ex 22.10 Q18

$\frac{dy}{dx}+y\cot x=x^2\cot x+2x$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}+Py=Q$

$p=\cot x, Q=x^2\cot x+2x$

I.F.

$=e^{\int pdx}$

$=e^{\int \cot dx}$

$=e^{\log \sin x}$

$=\sin x$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$

$y(\sin x)=\int(x^2\cos x+2x\sin x)dx+c$

$y\sin x=\int x^2\cos xdx+\int 2x\sin xdx+C$

$=x^2\sin x+C$

Differential Equations Ex 22.10 Q19

Here, $\frac{dy}{dx}+y\tan x=x^2\cos^2 x$

It is a linear differential equation. Comparing the equation by,

$\frac{dy}{dx}+Py=Q$

$P=\tan x, Q=x^2\cos^2 x$

I.F. $=e^{\int Pdx}$

$=e^{\int \tan xdx}$

$=e^{\log|\sec x|}$

$=\sec x$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$

$y\sec x=\int x^2\cos^2 x(\sec x)dx+c$

$=\int x^2\cos xdx+c$

$=x^2\int\cos xdx-\int(2x\int\cos xdx)dx+c$

Using integration by parts

$y(\sec x)=x^2\sin x-2\int x\sin xdx+c$

$=x^2\sin x-2\left[x\times\int\sin xdx-\int(1\times\int\sin xdx)dx\right]+c$

$y\sec x=x^2\sin x+2x\cos x-2\sin x+c$

$y=x^2\sin x\cos x+2x\cos^2 x-2\sin x\cos x+c\cos x$

Differential Equations Ex 22.10 Q20

Here, $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$$

It is a linear differential equation. Comparing the equation by,

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{1}{1+x^2},\ Q=\frac{e^{\tan^{-1}x}}{1+x^2}$$

I.F. $=e^{\int Pdx}$

$$=e^{\int\frac{1}{1+x^2}dx}$$

$$=e^{\tan^{-1}x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})\,dx+c$$

$$y\left(e^{\tan^{-1}x}\right)=\int\frac{e^{\tan^{-1}x}}{1+x^2}\times e^{\tan^{-1}x}dx+c$$

Let $e^{\tan^{-1}x}=t$

$$e^{\tan^{-1}x}*\frac{1}{1+x^2}dx=dt$$

$$y(t)=\int t\,dt+c$$

$$yt=\frac{t^2}{2}+c$$

$$y=\frac{t}{2}+\frac{c}{t}$$

$$y=\left(\frac{1}{2}e^{\tan^{-1}x}+ce^{-\tan^{-1}x}\right)$$

Differential Equations Ex 22.10 Q21

Here, $xdy=\left(2y+2x^4+x^2\right)dx$

$$x\frac{dy}{dx}=2y+2x^4+x^2$$

$$\frac{dy}{dx}-\frac{2}{x}y=2x^3+x$$

It is a linear differential equation. Comparing it with equation,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\frac{2}{x},\ Q=2x^3+x$$

I.F. $=e^{\int Pdx}$

$$=e^{-2\int\frac{1}{x}dx}$$

$$=e^{-2\log|x|}$$

$$=e^{\log\left(\frac{1}{x^2}\right)}$$

$$=\frac{1}{x^2}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})\,dx+c$$

$$y\left(\frac{1}{x^2}\right)=\int\left(2x^3+x\right)\left(\frac{1}{x^2}\right)dx+c$$

$$\frac{y}{x^2}=\int\left(2x+\frac{1}{x}\right)dx+c$$

$$\frac{y}{x^2}=2\frac{x^2}{2}+\log|x|+c$$

$$y=x^4+x^2\log|x|+cx^2$$

Differential Equations Ex 22.10 Q22

Here, $\left(1+y^2\right)+\left(x-e^{\tan^{-1} y}\right)\frac{dy}{dx}=0$

$$\left(x-e^{\tan^{-1} y}\right)\frac{dy}{dx}=-\left(1+y^2\right)$$

$$e^{\tan^{-1} y}-x=\left(1+y^2\right)\frac{dy}{dx}$$

$$\left(1+y^2\right)\frac{dx}{dy}+x=e^{\tan^{-1} y}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1} y}}{1+y^2}$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{1}{1+y^2},\ Q=\frac{e^{\tan^{-1} y}}{1+y^2}$$

I.F. $=e^{\int P dy}$

$$=e^{\int \frac{1}{1+y^2} dy}$$

$$=e^{\tan^{-1} y}$$

Solution of the equation is given by,

$$x \times (\text{I.F}) = \int Q \times (\text{I.F})\, dy + c$$

$$x\left(e^{\tan^{-1} y}\right)=\int \frac{e^{\tan^{-1} y}}{1+y^2}\left(e^{\tan^{-1} y}\right)dy+c$$

Let $e^{\tan^{-1} y}=t$

$$e^{\tan^{-1} y}\left(\frac{1}{1+y^2}\right)dy=dt$$

$$xt=\int t\,dt+c$$

$$xt=\frac{t^2}{t}+c$$

$$x=\frac{1}{2}t+\frac{c}{t}$$

$$x=\frac{1}{2}e^{\tan^{-1} y}+ce^{-\tan^{-1} y}$$

Differential Equations Ex 22.10 Q23

Here, $y^2\dfrac{dx}{dy}+x-\dfrac{1}{y}=0$

$$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{1}{y^2},\ Q=\frac{1}{y^3}$$

I.F. $=e^{\int P dy}$

$$=e^{\int \frac{1}{y^2}dy}$$

$$=e^{-\frac{1}{y}}$$

Solution of the equation is given by,

$$x\times(\text{I.F})=\int Q\times(\text{I.F})\,dy+c$$

$$x\left(e^{-\frac{1}{y}}\right)=\int\frac{1}{y^3}\left(e^{-\frac{1}{y}}\right)dy+c$$

Let $e^{-\frac{1}{y}}=t$

$\Rightarrow$ $\dfrac{1}{y}=-\log t$

$$e^{-\frac{1}{y}}\times\frac{1}{y^2}dy=dt$$

$$x(t)=\int\frac{1}{y}dt+c$$

$$=-\int \log+dt+c$$

$$=-\left[\log t\times\int 1\times dt-\int\left(\frac{1}{t}\int 1\times dt\right)dt\right]+c$$

$$=-\left[t\log t-\int\frac{t}{t}dt\right]+c$$

$$x(t)=-t\log t+t+c$$

$$x(t)=-t\left[\log t-1\right]+c$$

$$x=-\left[-\frac{1}{y}-1\right]+ce^{\frac{1}{y}}$$

$$x=\frac{1}{y}+1+ce^{\frac{1}{y}}$$

$$x=\left(\frac{1+y}{y}\right)+ce^{\frac{1}{y}}$$

Differential Equations Ex 22.10 Q24

Here, $\left(2x-10y^3\right)\dfrac{dy}{dx}+y=0$

$$y\frac{dx}{dy}+2x-10y^3=0$$

$$\frac{dx}{dy}=\frac{2}{y}x=10y^2$$

It is a linear differential equation. Comparing the equation with,

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{2}{y},\ Q=10y^2$$

I.F. $=e^{\int P dy}$

$$=e^{\int\frac{2}{y}dy}$$

$$=e^{2\log|y|}$$

$$=y^2$$

Solution of the equation is given by,

$$x\times(\text{I.F})=\int Q\times(\text{I.F})\,dy+c$$

$$x\left(y^2\right)=\int 10y^2\left(y^2\right)dy+c$$

$$xy^2=10\frac{y^5}{5}+c$$

$$xy^2=2y^5+c$$

$$x=2y^3+\frac{c}{y^2}$$

$$x=2y^3+cy^{-2}$$

Differential Equations Ex 22.10 Q25

Here, $(x+\tan y)dy = \sin 2y dx$

$$x+\tan y = \sin 2y\frac{dx}{dy}$$

$$\sin 2y\frac{dx}{dy} - x = \tan y$$

$$\frac{dx}{dy} - \operatorname{cosec}2y x = \frac{\tan y}{\sin 2y}$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = -\operatorname{cosec}2y, Q = \frac{\tan y}{\sin 2y}$$

I.F. $= e^{-\int \operatorname{cosec}2y dy}$

$= e^{-\int \operatorname{cosec}2y dy}$

$= e^{-\frac{1}{2}\log \tan y}$

$= e^{\log \sqrt{\cot y}}$

$= \sqrt{\cot y}$

Solution of the equation is given by,

$$x \times (\text{I.F}) = \int Q \times (\text{I.F}) dy + c$$

$$x\sqrt{\cot y} = \int \frac{\tan y}{\sin 2y}\sqrt{\cot y} dy + c$$

$$= \int \frac{\sqrt{\tan y}}{\left(\frac{2\tan y}{1+\tan^2 y}\right)} dy + c$$

$$x\sqrt{\cot y} = \int \frac{1+\tan^2 y}{2\sqrt{\tan y}} dy + c$$

$$\frac{x}{\sqrt{\tan y}} = \frac{1}{2}\int \frac{\sec^2 y}{\sqrt{\tan y}} dy + c$$

Put $\tan y = t$

$$\sec^2 y \times dy = dt$$

$$\frac{x}{\sqrt{\tan y}} = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + c$$

$$= \frac{1}{2} \times 2\sqrt{t} + c$$

$$\frac{x}{\sqrt{\tan y}} = \sqrt{\tan y} + c$$

$$x = \tan y + c\sqrt{\tan y}$$

Differential Equations Ex 22.10 Q26

Here, $dx + xdy = e^{-y}\sec^2 y dy$

$$\frac{dx}{dy} + x = e^{-y}\sec^2 y$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = 1, Q = e^{-y}\sec^2 y$$

I.F. $= e^{\int P dy}$

$= e^{\int dy}$

$= e^{y}$

Solution of the equation is given by,

$$x \times (\text{I.F}) = \int Q \times (\text{I.F}) dy + c$$

$$xe^{y} = \int e^{-y}\sec^2 y e^{y} dy + c$$

$$= \int \sec^2 y dy + c$$

$$xe^{y} = \int \tan y + c$$

$$x = e^{-y}(\tan y + c)$$

Differential Equations Ex 22.10 Q27

Here, $\frac{dy}{dx} = y \tan x - 2 \sin x$

$$\frac{dy}{dx} - y \tan x = -2 \sin x$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -\tan x, Q = -2 \sin x$$

I.F. $= e^{\int P dx}$

$$= e^{-\int \tan x dx}$$

$$= e^{-\log \sec x}$$

$$= \frac{1}{\sec x}$$

$$= \cos x$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \cos x = -\int 2 \sin x \cos x dx + c$$

Let $\sin x = t$

$$\cos x dx = dt$$

$$y (\cos x) = -\int 2t dt + c$$

$$= -t^2 + c$$

$$y \cos x = -\sin^2 x + c$$

$$y = \sec x \left(-\sin^2 x + c\right)$$

Differential Equations Ex 22.10 Q28

Here, $\frac{dy}{dx} + y \cos x = \sin x \cos x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = \cos x, Q = \sin x \cos x$$

I.F. $= e^{\int P dx}$

$$= e^{\int \cos x dx}$$

$$= e^{\sin x}$$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F}) dx + c$$

$$y \left(e^{\sin x}\right) = \int \sin x \cos x e^{\sin x} dx + c$$

Let $\sin x = t$

$$\cos x dx = dt$$

$$y e^t = \int t \times e^t dt + c$$

$$= t \times \int e^t dt - \int \left(1 \int e^t dt\right) dt + c$$

$$y e^t = t e^t - e^t + c$$

$$y e^t = e^t (t - 1) + c$$

$$y = t - 1 + c e^{-t}$$

$$y = \sin x - 1 + c e^{-\sin x}$$

Differential Equations Ex 22.10 Q29

Here, $\left(1+x^2\right)\frac{dy}{dx}-2xy=\left(x^2+2\right)\left(x^2+1\right)$

$$\frac{dy}{dx}-\frac{2x}{x^2+1}y=\left(x^2+2\right)$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=-\frac{2x}{x^2+1},\ Q=x^2+2$$

I.F. $=e^{\int P dx}$

$$=e^{-\int\frac{2x}{x^2+1}dx}$$

$$=e^{-\log\left|x^2+1\right|}$$

$$=\frac{1}{\left(x^2+1\right)}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y\left(\frac{1}{x^2+1}\right)=\int\left(\frac{x^2+2}{x^2+1}\right)dx+c$$

$$=\int\left(1+\frac{1}{x^2+1}\right)dx+c$$

$$\frac{y}{\left(x^2+1\right)}=x+\tan^{-1}x+c$$

$$y=\left(x^2+1\right)\left(x+\tan^{-1}x+c\right)$$

Differential Equations Ex 22.10 Q30

Here, $(\sin x)\frac{dy}{dx}+y\cos x=2\sin^2 x\cos x$

$$\frac{dy}{dx}+y\cot x=2\sin x\cos x$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=\cot x,\ Q=2\sin x\cos x$$

I.F. $=e^{\int P dx}$

$$=e^{\int \cot x dx}$$

$$=e^{\log \sin x}$$

$$=\sin x$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F})dx+c$$

$$y(\sin x)=\int 2\sin x\cos x(\sin x)dx+c$$

ysin x = (2/3)sin^3x+C

Differential Equations Ex 22.10 Q32

Here, $\frac{dy}{dx}+\frac{2y}{x}=\cos x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{2}{x},\ Q=\cos x$$

I.F. $=e^{\int P dx}$

$$=e^{2\int \frac{1}{x}dx}$$

$$=e^{2\log|x|}$$

$$=x^2$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$$

$$y\left(x^2\right)=\int \cos x\left(x^2\right)dx+c$$

$$yx^2=\int x^2\cos x\,dx+c$$

$$=x^2\int \cos x-\int\left(2x\times\int \cos x\,dx\right)dx+c$$

Using integration by parts

$$yx^2=x^2\sin x-\int 2x\sin x\,dx+c$$

$$=x^2\sin x-2\left[x\times\int \sin x\,dx-\int\left(1\times\int \sin x\,dx\right)dx\right]+c$$

$$=x^2\sin x+2x\cos x-2\int \cos x\,dx+c$$

$$yx^2=x^2\sin x+2x\cos x-2\sin x+c$$

$$y=\sin x+\frac{2}{x}\cos x-\frac{2}{x^2}\sin x+\frac{c}{x^2}$$

Differential Equations Ex 22.10 Q33

Here, $\frac{dy}{dx}-y=xe^x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx}+Py=Q$$

$$P=-1,\ Q=xe^x$$

I.F. $=e^{\int P dx}$

$$=e^{-\int dx}$$

$$=e^{-x}$$

Solution of the equation is given by,

$$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$$

$$ye^{-x}=\int xe^x\times e^{-x}dx+c$$

$$=\int x\,dx+c$$

$$ye^{-x}=\frac{x^2}{2}+c$$

$$y=e^x\left(\frac{x^2}{2}+c\right)$$

Differential Equations Ex 22.10 Q34

Here, $\frac{dy}{dx} + 2y = xe^{4x}$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = 2, Q = xe^{4x}$$

I.F. $= e^{\int P dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.})\, dx + c$$

$$y\left(e^{2x}\right) = \int xe^{4x}\left(e^{2x}\right) dx + c$$

$$= \int xe^{6x} dx + c$$

$$= x \times \int e^{6x} dx - \int \left(1 \int e^{6x} \times dx\right) + c$$

Using integration by parts

$$ye^{2x} = x \times \frac{e^{6x}}{6} - \int \frac{e^{6x}}{6} dx + c$$

$$ye^{2x} = \frac{x}{6} e^{6x} - \frac{e^{6x}}{36} + c$$

$$y = \frac{x}{6} e^{4x} - \frac{e^{4x}}{36} + ce^{-2x}$$

Differential Equations Ex 22.10 Q35

Here, $\left(x + 2y^2\right) \frac{dy}{dx} = y$

$$y \frac{dx}{dy} - x = 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

It is a linear differential equation. Comparing it with,

$$\frac{dx}{dy} + Px = Q$$

$$P = -\frac{1}{y}, Q = 2y$$

I.F. $= e^{\int P dy}$

$= e^{-\int \frac{1}{y} dy}$

$= e^{-\log|y|}$

$= \frac{1}{y}, \; y > 0$

Solution of the equation is given by,

$$x \times (\text{I.F}) = \int Q \times (\text{I.F.})\, dx + c$$

$$x\left(\frac{1}{y}\right) = \int 2y \left(\frac{1}{y}\right) dy + c$$

$$= \int 2 dy + c$$

$$x\left(\frac{1}{y}\right) = 2y + c \quad \text{---(i)}$$

Given, when $x = 2,\; y = 1$

So,

$$2 = 2 + c$$

$$c = 0$$

Put the value of c in equation (i),

$$x = 2y^2$$

Differential Equations Ex 22.10 Q36(ii)

Here, $\frac{dy}{dx} - y = \cos 2x$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -1, Q = \cos 2x$$

I.F. $= e^{\int P dx}$

$= e^{-\int dx}$

$= e^{-x}$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$y \times e^{-x} = \int \cos 2x \times e^{-x} dx + c \qquad \text{---(i)}$$

$$I = \int \cos 2x e^{-x} dx = \cos 2x \times \left(-e^{-x}\right) - \int \left(\frac{\sin 2x}{2}\right) e^{-x} dx \qquad \text{[Using integration by parts]}$$

$$I = -e^{-x}\cos 2x - \frac{1}{2}\left[\left(-\sin 2x e^{-x}\right) + \int \frac{\cos 2x}{2} e^{-x} dx\right]$$

$$I = -e^{-x}\cos 2x + \frac{1}{2}\sin 2x e^{-x} - \frac{1}{4} I$$

$$\frac{5}{4} I = \frac{e^{-x}}{2}\left(\sin 2x - 2\cos 2x\right)$$

$$I = \frac{2}{5} e^{-x}\left(\sin 2x - 2\cos 2x\right)$$

So, solution of the equation is given by

$$y = \frac{2}{5}\left(\sin 2x - 2\cos 2x\right) + ce^{x}$$

Differential Equations Ex 22.10 Q36(iii)

Here, $x\frac{dy}{dx} - y = (x+1)e^{-x}$

$$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x+1}{x}\right)e^{-x}$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = -\frac{1}{x}, Q = \left(\frac{x+1}{x}\right)e^{-x}$$

I.F. $= e^{\int P dx}$

$= e^{-\int \frac{1}{x} dx}$

$= e^{-\log|x|}$

$= e^{\log\left(\frac{1}{x}\right)}$

$= \frac{1}{x}, \quad x > 0$

Solution of the equation is given by,

$$y \times (\text{I.F}) = \int Q \times (\text{I.F.}) dx + c$$

$$y \times \left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right) e^{-x} \times \left(\frac{1}{x}\right) dx + c$$

$$\frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right) e^{-x} dx + c$$

Let $-x = t$

$$-dx = dt$$

$$y\left(-\frac{1}{x}\right) = \int \left(-\frac{1}{t} + \frac{1}{t^2}\right) e^{t} dt + c$$

$$y\left(-\frac{1}{x}\right) = -\frac{1}{t} e^{t} + c$$

$\left[\text{Since } \int \left\{f(x) + f'(x)\right\} e^{x} dx = f(x) e^{x} + c\right]$

$$-\frac{y}{x} = \frac{1}{x} e^{-x} + c$$

$$y = -\left(e^{-x} + cx\right)$$

$$y = -e^{-x} + c_1 x$$

Differential Equations Ex 22.10 Q36(iv)

Here, $x\frac{dy}{dx}+y=x^4$

$\frac{dy}{dx}+\frac{y}{x}=x^3$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}+Py=Q$

$P=\frac{1}{x}, Q=x^3$

I.F. $=e^{\int Pdx}$

$=e^{\int\frac{1}{x}dx}$

$=e^{\log|x|}$

$=x$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$

$yx=\int x^3(x)dx+c$

$xy=\frac{x^5}{5}+c$

$y=\frac{x^4}{5}+\frac{c}{x},\ x>0$

Differential Equations Ex 22.10 Q36(v)

Here, $(x\log x)\frac{dy}{dx}+y=\log x$

$\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$

It is a linear differential equation. Comparing it with,

$\frac{dy}{dx}+Py=Q$

$P=\frac{1}{x\log x}, Q=\frac{1}{x}$

I.F. $=e^{\int Pdx}$

$=e^{\int\frac{1}{x\log x}dx}$

$=e^{\log|\log x|}$

$=\log x$

Solution of the equation is given by,

$y\times(\text{I.F})=\int Q\times(\text{I.F.})\,dx+c$

$y(\log x)=\int\frac{1}{x}(\log x)dx+c$

$y(\log x)=\frac{(\log x)^2}{2}+c$

$y=\frac{1}{2}\log x+\frac{c}{\log x},\ x>0,\ x\neq 1$

Ex 22.11

Differential Equations Ex 22.11 Q1

Let A be the surface area of balloon, so

$$\frac{dA}{dt} \propto t$$

$$\Rightarrow \quad \frac{dA}{dt} = \lambda t$$

$$\Rightarrow \quad \frac{d}{dt}\left(4\pi r^2\right) = \lambda t$$

$$\Rightarrow \quad 8\pi r \frac{dr}{dt} = \lambda t$$

$$\Rightarrow \quad 8\pi r\, dr = \lambda t$$

$$\Rightarrow \quad 8\pi \int r\, dr = \lambda \int t\, dt$$

$$\Rightarrow \quad 8\pi \frac{r^2}{2} = \frac{\lambda t^2}{2} + c$$

$$\Rightarrow \quad 4\pi r^2 = \frac{\lambda t^2}{2} + c ----(1)$$

Given $r = 1$ unit when $t = 0$, so

$$4\pi (1)^2 = 0 + c$$

$$\Rightarrow \quad 4\pi = c$$

Using it is equation (i),

$$4\pi r^2 = \frac{\lambda t^2}{2} + 4\pi ----(2)$$

Also, given $r = 2$ units when $t = 3$ sec.

$$4\pi (2)^2 = \frac{\lambda (3)^2}{2} + 4\pi$$

$$\Rightarrow \quad 16\pi = \frac{9}{2}\lambda + 4\pi$$

$$\Rightarrow \quad \frac{9}{2}\lambda = 12\pi$$

$$\Rightarrow \quad \lambda = \frac{24}{9}\pi$$

$$\Rightarrow \quad \lambda = \frac{8}{3}\pi$$

Now,equation (2) becomes

$$4\pi r^2 = \frac{8\pi}{6}t^2 + 4\pi$$

$$\Rightarrow \quad 4\pi\left(r^2 - 1\right) = \frac{4}{3}\pi t^2$$

$$\Rightarrow \quad r^2 - 1 = \frac{1}{3}t^2$$

$$\Rightarrow \quad r^2 = 1 + \frac{1}{3}t^2$$

$$\therefore \quad r = \sqrt{\left(1 + \frac{1}{3}t^2\right)}$$

Differential Equations Ex 22.11 Q2

Let the population after time t be P and initial population be P_0.

So,

$$\frac{dP}{dt} = 5\% \times P$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{P}{20}$$

$$\Rightarrow \quad 20\frac{dP}{P} = dt$$

$$\Rightarrow \quad 20\int\frac{dP}{P} = \int dt$$

$$\Rightarrow \quad 20\log|P| = t + c - - - - (1)$$

Given $P = P_0$ when $t = 0$

$$20\log(P_0) = 0 + c$$

$$\Rightarrow \quad 20\log(P_0) = c$$

Now, equation (1) becomes

$$20\log(P) = t + 20\log(P_0)$$

$$\Rightarrow \quad 20\log\left(\frac{P}{P_0}\right) = t$$

Let time is t, when $P = 2P_0$, so,

$$20\log\left(\frac{2P}{P_0}\right) = t_1$$

$$\Rightarrow \quad 20\log 2 = t_1$$

Required time period $= 20\log 2$ years

Differential Equations Ex 22.11 Q3

Let P be the population at any time t and P_0 be the initial population.

So

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda\, dt$$

$$\Rightarrow \quad \int\frac{dP}{dt} = \lambda\int dt +$$

$$\Rightarrow \quad \log P = \lambda t + c - - - (1)$$

Here, $P = P_0$ t when $t = 0$,

$$\log(P_0) = 0 + c$$

$$\Rightarrow \quad c = \log(P_0)$$

Now, equation (1) becomes

$$\log(P) = \lambda t + \log(P_0)$$

$$\Rightarrow \quad \log\left(\frac{P}{P_0}\right) = \lambda t - - - (2)$$

Given $P = 2P_0$ when $t = 25$

$$\log\left(\frac{2P_0}{P_0}\right) = 25\lambda$$

$$\Rightarrow \quad \log 2 = 25\lambda$$

$$\Rightarrow \quad \lambda = \frac{\log 2}{25}$$

Now equation (2) becomes

$$\log\left(\frac{P}{P_0}\right) = \left(\frac{\log 2}{25}\right)t$$

let t_1 be the time to become population 500000 from 100000, so,

$$\log\left(\frac{500000}{100000}\right) = \frac{\log 2}{25}t_1$$

$$\Rightarrow \quad t_1 = \frac{25\log 5}{\log 2}$$

$$\Rightarrow \quad = \frac{25(1.609)}{(0.6931)} = 58$$

Required time $= 58$ years

Differential Equations Ex 22.11 Q4

Let C be the count of bacteria at any time t.

It is given that

$$\frac{dC}{dt} \propto C$$

$$\Rightarrow \frac{dC}{dt} = \lambda C, \text{ where } \lambda \text{ is a constant of proportionality}$$

$$\Rightarrow \frac{dC}{C} = \lambda dt$$

$$\Rightarrow \int \frac{dC}{C} = \lambda \int dt$$

$$\Rightarrow \log C = \lambda t + \log K \text{....(1)}$$

Initially, at $t = 0$, $C = 100000$

Thus, we have,

$$\log 100000 = \lambda \times 0 + \log K \text{....(2)}$$

$$\Rightarrow \log 100000 = \log K \text{....(3)}$$

$$\text{At } t = 2,\ C = 100000 + 100000 \times \frac{10}{100} = 110000$$

Thus, from (1), *we have,*

$$\log 110000 = \lambda \times 2 + \log K \text{....(4)}$$

Subtracting equation (2) *from* (4), *we have,*

$$\log 110000 - \log 100000 = 2\lambda$$

$$\Rightarrow \log 11 \times 10000 - \log 10 \times 10000 = 2\lambda$$

$$\Rightarrow \log \frac{11 \times 10000}{10 \times 10000} = 2\lambda$$

$$\Rightarrow \log \frac{11}{10} = 2\lambda$$

$$\Rightarrow \lambda = \frac{1}{2} \log \frac{11}{10} \text{....(5)}$$

We need to find the time 't' in which the count reaches 200000.

Substituting the values of λ *and* K *from equations* (3) *and* (5) *in equation* (1), *we have*

$$\log 200000 = \frac{1}{2} \log \frac{11}{10} t + \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 200000 - \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log \frac{200000}{100000}$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 2$$

$$\Rightarrow t = \frac{2 \log 2}{\log \frac{11}{10}} \text{ hours}$$

Differential Equations Ex 22.11 Q5

Given that, interest is compounded 6% per annum. Let P be principal

$$\frac{dP}{dt} = \frac{\Pr}{100}$$
$$\frac{dP}{dt} = \frac{r}{100}dt$$
$$\int \frac{dP}{P} = \int \frac{r}{100}dt$$
$$\log P = \frac{rt}{100} + c - - - (1)$$

Let P_0 be the initial principal at $t = 0$,

$$\log(P_0) = 0 + c$$
$$c = \log(P_0)$$

Put value of C is equation (1)

$$\log(P) = \frac{rt}{100} + \log(P_0)$$
$$\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$$

Case I:

Here, $P_0 = 1000, t = 10$ years and $r = 6$

$$\log\left(\frac{P}{1000}\right) = \frac{6 \times 10}{100}$$
$$\log P - \log 1000 = 0.6$$
$$\log P = \log e^{0.6} + \log 1000$$
$$= \log\left(e^{0.6} + 1000\right)$$
$$= \log(1.822 + 1000)$$
$$\log P = \log 1822$$

so,

$$P = \text{Rs}\, 1822$$

Rs 1000 will be Rs 1822 after 10 years

Differential Equations Ex 22.11 Q6

Let A be the amount of bacteria present at time t and A_0 be the initial amount of bacteria. Here,

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\int \frac{dA}{A} = \int \lambda dt$$

$$\log A = \lambda t + c - - - (1)$$

When $t = 0$, $A = A_0$

$$\log(A_0) = 0 + c$$

$$c = \log A_0$$

Using equation (1),

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t - - - - - (2)$$

Given, bacteria triples is 5 hours, so $A = 3A_0$, when $t = 5$

so, $$\log\left(\frac{3A_0}{A_0}\right) = 5\lambda$$

$$\log 3 = 5\lambda$$

$$\lambda = \frac{\log 3}{5}$$

Putting the value of λ in equation (2)

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} t$$

Case I: let A_1 be the number of bacteria present 10 hours, os

$$\log\left(\frac{A_1}{A_0}\right) = \frac{\log 3}{5} \times 10$$

$$\log\left(\frac{A_1}{A_0}\right) = 2\log 3$$

$$\log\left(\frac{A_1}{A_0}\right) = 2(1.0986)$$

$$\log\left(\frac{A_1}{A_0}\right) = 2.1972$$

$$A_1 = A_0 e^{2.1972}$$

$$A_1 = A_0 9$$

thus

There will be 9 times the bateria present is 10 hours.

Case II: let t_1 be the time necessary for the bacteria to be 10 times, os

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} \times t$$

$$\log\left(\frac{10A_0}{A_0}\right) = \frac{\log 3}{5} \times t_1$$

$$5 \log 10 = \log 3 t_1$$

$$5\frac{\log 10}{\log 3} = t_1$$

Required time is $\frac{5\log 10}{\log 3}$ hours

Differential Equations Ex 22.11 Q7

Let P be the population of the city at any time t.

It is given that

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = \lambda P, \text{ where } \lambda \text{ is a constant of proportionality}$$

$$\Rightarrow \frac{dP}{P} = \lambda dt$$

$$\Rightarrow \int \frac{dP}{P} = \lambda \int dt$$

$$\Rightarrow \log P = \lambda t + \log K \ldots.(1)$$

Initially, at $t = 1990,\ P = 200000$

Thus, we have,

$$\log 200000 = \lambda \times 1990 + \log K \ldots.(2)$$

At $t = 2000,\ P = 250000$

Thus, from (1), *we have,*

$$\log 250000 = \lambda \times 2000 + \log K \ldots.(3)$$

Subtracting equation (2) *from* (3), *we have,*

$$\log 250000 - \log 200000 = 10\lambda$$

$$\Rightarrow \log \frac{4}{5} = 10\lambda$$

$$\Rightarrow \lambda = \frac{1}{10}\log\frac{4}{5} \ldots.(4)$$

Substituting the value of λ *from equation* (4) *in equation* (1), *we have*

$$\log 200000 = 1990 \times \frac{1}{10}\log\frac{4}{5} + \log K$$

$$\Rightarrow \log K = \log 200000 - 199 \times \log\frac{4}{5} \ldots.(5)$$

Substituting the value of λ, $\log K$ *and* $t = 2010$ *in equation* (1), *we have*

$$\log P = \left\{\frac{1}{10}\log\frac{4}{5}\right\}2010 + \log 200000 - 199 \times \log\frac{4}{5}$$

$$\Rightarrow \log P = \log\left\{\frac{4}{5}\right\}^{201} + \log\left(200000 \times \left(\frac{5}{4}\right)^{199}\right)$$

$$\Rightarrow P = \left\{\frac{4}{5}\right\}^{201} \times 200000 \times \left(\frac{5}{4}\right)^{199}$$

$$\Rightarrow P = \left(\frac{5}{4}\right)^{2} \times 200000 = \frac{25}{16} \times 200000 = 312500$$

Differential Equations Ex 22.11 Q8

Given,

$$C'(x) = \frac{dC}{dx} = 2 + 0.15x$$

$$dC = (2 + 0.15x)\,dx$$

$$\int dC = \int (2 + 0.15x)dx$$

$$C = 2x + \frac{0.15x^2}{2} + \lambda \text{ - - - -}(1)$$

Given $C = 100$ when $x = 0$, so

$$100 = 0 + 0 + \lambda$$

$$\lambda = 100$$

Put the value of λ in equation (1) total cost function is

$$C(x) = 2x + \frac{0.15x^2}{2} + 100$$

$$C(x) = 2x + 0.075x^2 + 100$$

Differential Equations Ex 22.11 Q9

Let P be principal at any time t at the rate of $r\%$ per annum, so

$$\frac{dP}{dt} = \frac{\text{Pr}}{100}$$

$$\frac{dP}{P} = \frac{r}{100}dt$$

$$\int\frac{dP}{P} = \frac{r}{100}\int dt$$

$$\log P = \frac{rt}{100} + c - - - (1)$$

Let P_0 be the initial amount, so

$$\log(P_0) = 0 + c$$

$$c = \log(P_0)$$

Put the value of C in equation (1),

$$\log P = \frac{rt}{100} + \log P_0$$

$$\log P - \log P_0 = \frac{rt}{100}$$

$$\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$$

For $t = 1, r = 8\%$

$$\log\left(\frac{P}{P_0}\right) = \frac{8 \times 1}{100}$$

$$\log\frac{P}{P_0} = 0.08$$

$$\frac{P}{P_0} = e^{0.08}$$

$$\frac{P}{P_0} = 1.0833$$

$$\frac{P}{P_0} - 1 = 1.0833 - 1$$

$$\frac{P - P_0}{P_0} = 0.0833$$

percentage increase in amount in one year

$= 0.0833 \times 100$

$= 8.33\%$

Required percentage = 8.33%

Differential Equations Ex 22.11 Q10

Here,

$$L\frac{di}{dt}+Ri=E$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$$

It is a linear differential equation. Compound it with $\frac{dy}{dx}+Py=Q$

$$P=\frac{R}{L},\ Q=\frac{E}{L}$$

$$I.F.=e^{\int Pdt}$$

$$=e^{\int\frac{P}{L}dt}$$

$$I.F.==e^{\left(\frac{R}{L}\right)t}$$

Solution of the equation is given by

$$i\left(I.F.\right)=\int Q\left(I.F.\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\int\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\times\frac{L}{R}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i=\left(\frac{E}{L}\right)+c\left(e^{\left(\frac{R}{L}\right)t}\right)------(1)$$

Initiatially there was no current, so put $i=0, t=0$

$$0=\frac{F}{R}+ce^{0}$$

$$0=\frac{F}{R}+c$$

$$c=-\frac{F}{R}$$

Using Equation (1)

$$i=\frac{F}{R}-\frac{F}{R}e^{\left(-\frac{R}{L}\right)t}$$

$$i=\frac{F}{R}\left(1-e^{\left(-\frac{R}{L}\right)t}\right)$$

Differential Equations Ex 22.11 Q11

Let A be the quantity of mass at any time t, so

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dP} = -\lambda A$$
$$\frac{dA}{A} = -\lambda dt$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c - - - (1)$$

Let initial quantity of mass be A_0, so

$$\log A_0 = -\lambda(0) + c$$
$$\log(A_0) = c$$

Now, equation (1) becames,

$$\log A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t$$

Let t_1 be the required time to half the mass , so $A = \frac{1}{2}A_0$,

Now, $\log\left(\frac{A}{A_0}\right) = -\lambda t$

$$\log\left(\frac{A}{2A}\right) = -\lambda t$$
$$-\log 2 = -\lambda t$$
$$\frac{1}{\lambda}\log 2 = t$$

Required time is $\frac{1}{\lambda}$ log2 units where λ is constant of proportionality.

Differential Equations Ex 22.11 Q12

Let A be the quantity of radius at any time t, so

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dP} = -\lambda A$$
$$\frac{dA}{A} = -\lambda t$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c - - - (1)$$

Let A_0 be the initial amount of radius percentage , so

$$\log A_0 = -\lambda(0) + c$$
$$c = \log(A_0)$$

Using, equation (1),

$$\log A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t - - - - - (2)$$

Given,its half-life is 1590 years, so

$$\log\left(\frac{\frac{1}{2}A_0}{A_0}\right) = -\lambda(1590)$$
$$\log\left(\frac{1}{2}\right) = -\lambda(1590)$$
$$-\log 2 = -\lambda(1590)$$
$$\log 2 = \lambda(1590)$$
$$\frac{\log 2}{1590} = \lambda$$

Now, equation (1) becomes

$$\log\left(\frac{A}{A_0}\right) = -\frac{\log 2}{1590}t$$

Differential Equations Ex 22.11 Q13

Slope of tangent at point $(x, y) = -\frac{x}{y}$

$$\frac{dy}{dx} = -\frac{x}{y}$$
$$y\,dy = -x dx$$
$$\int y\,dt = -\int x\,dx$$
$$\frac{y^2}{2} + \frac{x^2}{2} = c_1$$
$$x^2 + y^2 = c - - - - - (1)$$

Given, curve is passing through $(3, -4)$, so

$$(3)^2 + (-4)^2 = c$$
$$9 + 16 = c$$
$$c = 25$$

So, using equation (1),

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 25$$

Differential Equations Ex 22.11 Q14

$$y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$$
$$\frac{dy}{dx} + x\frac{dy}{dx} = y - y^2$$
$$(1 + x)\frac{dy}{dx} = y - y^2$$
$$\frac{dy}{y - y^2} = \frac{dx}{1 + x}$$
$$\frac{dy}{y(1 - y)} = \frac{dx}{1 + x}$$
$$\int\left(\frac{1}{y} + \frac{1}{1 - y}\right)dx = \int\frac{dx}{1 + x}$$
$$\log|y| - \log|1 - y = \log|1 + x|| + \log|c|$$
$$\frac{y}{1 - y} = c(1 + x)$$
$$y = (1 - y)c(1 + x) - - - - (1)$$

It is passing through $(2,2)$ so,

$$2 = (1 - 2)c(1 + 2)$$
$$2 = -3c$$
$$c = -\frac{2}{3}$$

Now,equation (1) becomes,

$$y = -\frac{2}{3}(1 - y)(1 + x)$$
$$3y = -2(1 + x - y - xy)$$
$$3y + 2 + 2x - 2y - 2xy = 0$$
$$y + 2x - 2xy + 2 = 0$$

$$2xy - 2x - 2 - y = 0$$

Chapter 22 Differential Equations Ex 22.11 Q15

It is passing through $\left(1, \frac{\pi}{4}\right)$, so,

$$\tan\left(\frac{\pi}{4}\right) = -\log|1| + c$$
$$1 = 0 + c$$
$$c = 1$$

Now, equation (1) becomes

$$\tan\left(\frac{y}{x}\right) = -\log|x| + 1$$

Therefore,

$$\tan\left(\frac{y}{x}\right) = \log\left|\frac{e}{x}\right|$$

Differential Equations Ex 22.11 Q16

Let $P(x, y)$ be the point of contact of tangent and curve $y = f(x)$. and It cuts axes at A and B so, equatin of tangent at $P(x, y)$

$$Y - y = \frac{dy}{dx}(X - x)$$

Put $X = 0$

$$Y - y = \frac{dy}{dx}(-x)$$

$$Y = y - x\frac{dy}{dx}$$

So, coordinate of $A = \left(0, y - x\frac{dy}{dx}\right)$

Put $Y = 0$,

$$0 - y = \frac{dy}{dx}(X - x)$$

$$-y\frac{dx}{dy} = X - x$$

$$X = x - y\frac{dx}{dy}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x -axis) = 4(ordinate)

$$x - y\frac{dx}{dy} = 4y$$

$$y\frac{dx}{dy} + 4y = x$$

$$\frac{dx}{dy} + 4 = \frac{x}{y}$$

$$\frac{dx}{dy} - \frac{x}{y} = -4$$

It is a linear different equation. Comparing it with $\frac{dx}{dy} + Px = Q$

$P = -\frac{1}{y}$, $Q = -4$

$$I.F. = e^{\int p\,dy}$$

$$= e^{-\int \frac{1}{y}dy}$$

$$= e^{-\log y}$$

$$= \frac{1}{y}$$

Solution of the equation is given by,

$$x(I.F.) = \int Q(I.F.)dy + \log c$$

$$x\left(\frac{1}{y}\right) = \int(-4)\left(\frac{1}{y}\right)dy + \log c$$

$$\frac{x}{y} = -4\log y + \log c$$

$$e^{\frac{x}{y}} = \frac{c}{y^4}$$

Differential Equations Ex 22.11 Q17

Slope at any point = $y+2x$

$$\frac{dy}{dx}=y+2x$$
$$\frac{dy}{dx}-y=2x$$

It is a linear differential equation. comparing it with $\frac{dy}{dx}+Py=Q$

$P=-1, Q=2x$

$$I.F.=e^{\int P dx}$$
$$=e^{\int(-1)dx}$$
$$=e^{-x}$$

Solution of the equation is given by

$$(I.F.)=\int Q\,(I.F.)\,dx+c$$
$$y\left(e^{-x}\right)=\int(2x)\left(e^{-x}\right)dx+c$$
$$y\left(e^{-x}\right)=2\int xe^{-x}dx+c$$
$$y\left(e^{-x}\right)=2\left[x\left(-e^{-x}\right)+\int 1e^{-x}dx\right]+c$$
$$y\left(e^{-x}\right)=-2xe^{-x}-2e^{-x}+c$$
$$y=-2x-2+ce^{x}$$
$$y+2(x+1)=ce^{x}\ ---(1)$$

It is passing through origin,

$$0+2(0+1)=ce^{0}$$
$$2=c$$

Now, equation (1) becomes,

$$y+2(x+1)=2e^{x}$$

Differential Equations Ex 22.11 Q18

Given, tangent makes on angle $\tan^{-1}(2x+3y)$ with x-axis,

Slope of tangent $=\tan\theta$

$$\frac{dy}{dx}=\tan\left(\tan^{-1}(2x+3y)\right)$$
$$\frac{dy}{dx}=2x+3y$$
$$\frac{dy}{dx}-3y=2x$$

It is a linear differetial equation comparing it with $\frac{dy}{dx}+Py=Q$

$P=-3, Q=2x$

$$I.F.=e^{\int P dx}$$
$$=e^{-\int 3dx}$$
$$=e^{-3x}$$

Solution of the equation on given by

$$y\,(\text{I.F.})=\int Q\,(I.F.)\,dx+c$$
$$y\left(e^{-3x}\right)=\int 2xe^{-3x}dx+c$$
$$=2\left[x\left(\frac{-e^{-3x}}{3}\right)-\int 1.\left(\frac{-e^{-3x}}{3}\right)dx\right]+c$$
$$=-\frac{2}{3}xe^{-3x}+\frac{2}{3}\int e^{-3x}dx+c$$
$$y\left(e^{-3x}\right)=-\frac{2}{3}xe^{-3x}+\frac{2}{9}e^{-3x}+c$$
$$y=-\frac{2}{3}x-\frac{2}{9}+ce^{3x}\ -----(1)$$

It is passing through $(1,2)$,

$$2=-\frac{2}{3}-\frac{2}{9}+ce^{3}$$
$$2=-\frac{8}{9}+ce^{3}$$
$$\frac{26}{9}=ce^{3}$$
$$c=\frac{26}{9}e^{-3}$$

Now equation (1) becomes,

$$ye^{-3x}=\left(-\frac{2}{3}x-\frac{2}{9}\right)e^{-3x}+\frac{26}{9}e^{-3}$$

Differential Equations Ex 22.11 Q19

Let $P(x,y)$ be the point of contact of tangent whit curve $y = f(x)$ equatin of tangent at $P(x,y)$ is

$$Y - y = \frac{dy}{dx}(X - x)$$

Put Y = 0

$$-y = \frac{dy}{dx}(X - x)$$

$$X = X - \frac{ydx}{dx}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x - axis) = $4x$

$$x - y\frac{dx}{dy} = 2x$$

$$-y\frac{dx}{dy} = 2x - x$$

$$-y\frac{dx}{dy} = x$$

$$-\frac{dx}{x} = \frac{dy}{y}$$

$$-\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$-\log x = \log y + c \text{ - - - } (1)$$

It is passing through $(1,2)$

$$-\log 1 = \log 2 + c$$

$$c = -\log 2$$

Put c in equation (1)

$$-\log x = \log y - \log 2$$

$$\frac{1}{x} = \frac{y}{2}$$

$$xy = 2$$

Differential Equations Ex 22.11 Q20

$$x(x+1)\frac{dy}{dx} - y = x(x+1)$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = \frac{x(x+1)}{x(x+1)}$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = 1$$

It is linear differential equation coparing it with $\frac{dy}{dx} + Py = Q$

$$P = -\frac{1}{x(x+1)}, \qquad Q = 1$$

$$I.F. = e^{\int\frac{1}{x(x+1)}dx}$$

$$= e^{\int\left(\frac{1}{x} - \frac{1}{(x+1)}\right)dx}$$

$$= e^{-\log|x| + \log|x+1|}$$

$$= e^{\log\left(\frac{x+1}{x}\right)}$$

$$= \frac{x+1}{x}$$

Solution of the equation is given by

$$y(I.F.) = \int Q(I.F.)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(\frac{x+1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(1 + \frac{1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = x + \log|x| + c \text{ - - - } (1)$$

It is passing through $(1,0)$, so

$$0 = 1 + \log(1) + c$$

$$-1 = c$$

Now,equation (1) becomes,

$$y\left(\frac{x+1}{x}\right) = x + \log|x| - 1$$

$$y(x+1) = x(x + \log x - 1)$$

Differential Equations Ex 22.11 Q21

Slope of the curve $= \frac{2y}{x}$

$$\frac{dy}{dx} = \frac{2y}{x}$$
$$\frac{dy}{y} = \frac{2}{x}dx$$
$$\int \frac{dy}{y} = 2\int \frac{1}{x}dx$$
$$\log|y| = 2\log|x| + \log|c|$$
$$y = x^2 c \quad ---(1)$$

It is passing through $(3, -4)$ so,

$$-4 = (3)^2 c$$
$$-4 = 9c$$
$$c = -\frac{4}{9}$$

Now, equation (1) becomes,

$$y = -\frac{4}{9}x^2$$
$$9y = -4x^2$$

$$9y + 4x^2 = 0$$

Differential Equations Ex 22.11 Q22

Given,

Slope of the equation $= x + 3y - 1$

$$\frac{dy}{dx} = x + 3y - 1$$
$$\frac{dy}{dx} - 3y = x - 1$$

It is a linear differential equation. Camparing it with $\frac{dy}{dx} + Py = Q$

$P = -3, Q = x - 1$

$$I.F. = e^{\int P dx}$$
$$= e^{\int -3dx}$$
$$= e^{-3x}$$

Solution of the equation is given by,

$$y(I.F.) = \int Q(I.F.)dx + c$$
$$y\left(e^{-3x}\right) = \int (x-1)\left(e^{-3x}\right)dx + c$$
$$y\left(e^{-3x}\right) = (x-1)\left(-\frac{1}{3}e^{-3x}\right) - \int (1)\left(\frac{-e^{-3x}}{3}\right)dx + c$$
$$y\left(e^{-3x}\right) = -\frac{(x-1)}{3}e^{-3x} + \left(-\frac{e^{-3x}}{9}\right) + c$$
$$y = -\frac{x}{3} + \frac{1}{3} - \frac{1}{9} + ce^{3x}$$
$$y = -\frac{x}{3} + \frac{2}{9} + ce^{3x}$$

It is passing through origin, so

$$0 = 0 + \frac{2}{9} + ce^{3(0)}$$
$$0 = \frac{2}{9} + c$$
$$c = -\frac{2}{9}$$

Now, equation (1) becomes,

$$y = -\frac{x}{3} + \frac{2}{9} - \frac{2}{9}e^{3x}$$
$$9y = -3x + 2 - 2e^{3x}$$

$$3(3y + x) = 2\left(1 - e^{3x}\right)$$

Differential Equations Ex 22.11 Q23

Given,

Slope at point $(x, y) = x + xy$

$$\frac{dy}{dx} = x(y+1)$$

$$\frac{dy}{y+1} = x\,dx$$

$$\int \frac{dy}{y+1} = \int x\,dx$$

$$\log|y+1| = \frac{x^2}{2} + c - - - - - (1)$$

It is passing through $(0,1)$, so,

$$\log z = 0 + c$$

$$c = \log 2$$

Now,equation (2) becomes,

$$\log|y+1| = \frac{x^2}{2} + \log 2$$

$$y + 1 = 2e^{\frac{x^2}{2}}$$

Differential Equations Ex 22.11 Q24

$$y^2 - 2xy\frac{dy}{dx} - x^2 = 0$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$x\frac{dv}{dx} + v = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\int \frac{2v}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\log|v^2 + 1| = -\log|x| + \log|c|$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2 + x^2}{x^2} = \frac{c}{x}$$

$$y^2 + x^2 = cx$$

$$y^2 + x^2 - cx = 0$$

Differentiating it with respect to x,

$$2x + 2y\frac{dy}{dx} - c = 0$$

$$\frac{dy}{dx} = \frac{c - 2x}{2y}$$

Let (h,k) be the point where tangent passes through origin and length is equal to h. so, equation of tangent at (h,k) is

$$(y-k)=\left(\frac{dy}{dx}\right)_{(h,k)}(x-h)$$

$$(y-k)=\left(\frac{c-2h}{2k}\right)(x-h)$$

$$2ky-2k^2=xc-2hx-hc+2h^2$$

$$x(c-2h)-2ky+2k^2-hc+2h^2=0$$

$$x(c-2h)-2ky+2\left(k^2+h^2\right)-hc=0$$

$$x(c-2h)-2ky+2(ch)-hc=0$$

[Since $h^2+k^2=ch$ as (h,k) is on the curve]

$$x(c-2h)-2ky+hc=0$$

length of perpendicular as tangent from origin is

$$L=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{(0)(c-2h)+(0)(-2k)+hc}{\sqrt{(c-2h)^2+(-2k)^2}}\right|$$

$$=\frac{hc}{\sqrt{c^2+4h^2+4k^2-4ch}}$$

$$L=\frac{hc}{\sqrt{c^2+4\left(h^2+k^2-ch\right)}}$$

$$=\frac{hc}{\sqrt{c^2+4(0)}}$$

$$=\frac{hc}{c}$$

$$=c$$

Hence,

$x^2+y^2=cx$ is the required curve

Differential Equations Ex 22.11 Q25

Let $P(x,y)$ be the point of contact of tangent and curve $y=f(x)$. Equation tangent at $P(x,y)$ is

$$Y-y=\frac{dx}{dy}(X-x)$$

put $Y=0$

$$-y=\frac{dx}{dy}(X-x)$$

$$-y=\frac{dx}{dy}(X-x)$$

$$X=x-y\frac{dx}{dy}$$

coordinate of $B=\left(x-y\frac{dx}{dy},0\right)$

Given,

Distance between foot of ordinate of the point of contact and the point of intersection of tangent and x – axis $=2x$

$$BC=2x$$

$$\sqrt{\left(x-y\frac{dx}{dy}-x\right)^2+(0)^2}=2x$$

$$y\frac{dx}{dy}=2x$$

$$y\frac{dx}{x}=2\frac{dy}{y}$$

$$\int\frac{dx}{x}=2\int\frac{dy}{y}$$

$$\log x=2\log y+\log c\ ---(1)$$

It is passing through $(1,2)$,

$$\log 1=2\log 2+\log c$$

$$-2\log 2=\log c$$

$$\log\left(\frac{1}{4}\right)=\log c$$

$$c=\frac{1}{4}$$

Put value of c in equation (1),

$$\log x=2\log y+\log\left(\frac{1}{4}\right)$$

$$x=\frac{y^2}{4}$$

$$y^2=4x$$

Differential Equations Ex 22.11 Q26

Equation of normal on point (x, y) on the curve

$$Y - y = \frac{-dx}{dy}(X - x)$$

Itis passing through $(3,0)$

$$0 - y = \frac{-dx}{dy}(3 - x)$$

$$y = \frac{dx}{dy}(3 - x)$$

$$ydy = (3 - x)dx$$

$$\int y\, dy = \int (3 - x)\, dx$$

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + c - - - (1)$$

It passing through $(3,4)$, so,

$$\frac{16}{2} = 9 - \frac{9}{2} + c$$

$$\frac{16}{2} = \frac{9}{2} + c$$

$$c = 7$$

Put $c = 7$ is equation (1)

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + \frac{7}{2}$$

$$y^2 = 6x - x^2 + 7$$

Differential Equations Ex 22.11 Q27

Let A be the quantity of bacteria present in culture at any time t and initial quantity of bacteria is A_0.

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\frac{dA}{A} = \lambda dt$$

$$\int \frac{dA}{A} = \lambda \int dt$$

$$\log A = \lambda t + c - - - - (1)$$

Initially, $A = A_0, t = 0$

$$\log A_0 = 0 + c$$

$$\log A_0 = c$$

Now eqution (1) becomes,

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t - - - - - (2)$$

Given $A = 2A_0$ when $t = 6$ hours

$$\log\left(\frac{A}{A_0}\right) = 6\lambda$$

$$\frac{\log 2}{6} = \lambda$$

Now equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 2}{6} t$$

Now, $A = 8A_0$

so, $$\log\left(\frac{8A_0}{A_0}\right) = \frac{\log 2}{6} t$$

$$\log 2^3 = \frac{\log 2}{6} t$$

$$3\log 2 = \frac{\log 2}{6} t$$

$$18 = t$$

Therefore,

Bacteria becomes 8 times in 18 hours

Differential Equations Ex 22.11 Q28

Let A be the quantity of radium present at time t and A_0 be the initial quantity of radium.

$$\frac{dA}{dt} \propto A$$
$$\frac{dA}{dt} = -\lambda A$$
$$\frac{dA}{A} = -\lambda dt$$
$$\int \frac{dA}{A} = -\lambda \int dt$$
$$\log A = -\lambda t + c - - - - (2)$$

Now, $A = A_0$ when $t = 0$

$$\log A_0 = 0 + c$$
$$c = \log A_0$$

Put value of c in equation

$$\log A = -\lambda t + \log A_0$$
$$\log\left(\frac{A}{A_0}\right) = -\lambda t - - - (2)$$

Given that,

In 25 years bacteria decomposes 1.1%, so

$$A = (100 - 1.1)\% = 98.9\% = 0.989 A_0, t = 5$$
$$\log\left(\frac{0.989 A_0}{A_0}\right) = -\lambda 25$$
$$\log(0.989) = -25\lambda$$
$$\lambda = -\frac{1}{25}\log(0.989)$$

Now, equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \left\{\frac{1}{25}\log(0.989)\right\}t$$

Now $A = \frac{1}{2}A_0$

$$\log\left(\frac{A}{2A}\right) = \frac{1}{25}\log(0.989)t$$

$$\frac{-\log 2 \times 25}{\log(0.989)} = t$$
$$-\frac{0.6931 \times 25}{0.01106} = t$$

$$t = 1567 \text{ years.}$$

Required time = 1567 years

Differential Equations Ex 22.11 Q29

Given,

$$\text{Slope of tangent} = \frac{x^2 + y^2}{2xy}$$
$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$
$$x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$
$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{v}$$
$$x\frac{dv}{dx} = \frac{1 - v^2}{v}$$
$$\frac{v}{1 - v^2}dv = \frac{dx}{x}$$
$$\int \frac{v}{1 - v^2}dv = \int \frac{dx}{x}$$
$$\int \frac{-2v}{1 - v^2}dv = \int \frac{-2dx}{x}$$
$$\log\left|1 - v^2\right| = -2\log x + \log c$$
$$1 - \frac{y^2}{x^2} = \frac{c}{x^2}$$
$$\frac{x^2 - y^2}{x^2} = \frac{c}{x^2}$$

$$x^2 - y^2 = c$$

It is equation of rectangular hyperbola.

Differential Equations Ex 22.11 Q30

Given,

Slope of tangent at $(x, y) = x + y$

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} - y = x$$

It is a linear differential equation.Comparing it with $\frac{dy}{dx} + Py = Q$

$P = -1, Q = x$

$$I.F. = e^{\int P dx}$$
$$= e^{\int (-1) dx}$$
$$= e^{-x}$$

Solution of equation is given by,

$$y\left(I.F.\right) = \int Q\left(I.F.\right)dx + c$$
$$y\left(e^{-x}\right) = \int xe^{-x}dx + c$$
$$ye^{-x} = x\left(e^{-x}\right) + \int\left(1 \times e^{-x}\right)dx + c$$

[Using integration by parts]

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$
$$y = -x - 1 + ce^{x} - - - - - (1)$$

It is passing through origin

$$0 = 0 - 1 + ce^{0}$$
$$1 = c$$

Put $c = 1$ is equation

$$y = -x - 1 + e^{x}$$

$$y + x + 1 = e^{x}$$

Differential Equations Ex 22.11 Q31

We know that the slope of the tangent to the curve is $\frac{dy}{dx}$.

$\therefore \qquad \frac{dy}{dx} = x + xy$

$\Rightarrow \qquad \frac{dy}{dx} - xy = y \qquad ------(i)$

This is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$

where $P = -x$ and $Q = x$.

So, $\qquad I.F. = e^{\int -x dx} = e^{\frac{-x^2}{2}}$

$\therefore$ Solution of the given equation is given by

$$y.\, e^{\frac{-x^2}{2}} = \int x.e^{\frac{-x^2}{2}}\, dx + C \qquad ------(ii)$$

Let $\qquad I = \int x.e^{\frac{-x^2}{2}}\, dx$

Let $\frac{-x^2}{2} = t$, then $-x\, dx = dt$ or $x\, dx = -dt$

$\therefore \qquad I = \int x.e^{\frac{-x^2}{2}}\, dx = \int -e^{t}\, dt = -e^{t} = -e^{\frac{-x^2}{2}}$

Substituting the value of I in (ii), we get

$$y.\, e^{\frac{-x^2}{2}} = -e^{\frac{-x^2}{2}} + C$$

or $\qquad y = -1 + Ce^{\frac{x^2}{2}} \qquad ------(iii)$

This equation (iii) passes through (0,1)

$\therefore \qquad 1 = -1 + Ce^{0} \quad \Rightarrow \quad C = 2$

Substituting the value of C in (iii), we get

$$y = -1 + 2e^{\frac{x^2}{2}}$$

which is the equation of the required curve.

Differential Equations Ex 22.11 Q32

Given,

Slope of tangent at $(x, y) = x^2$

$$\frac{dy}{dx} = x^2$$

$$dy = x^2 dx$$

$$\int dy = \int x^2 dx$$

$$y = \frac{x^3}{3} + c - - - - (1)$$

It is passing through $(-1, 1)$

$$1 = \frac{(-1)}{3} + c$$

$$1 = -\frac{1}{3} + c$$

$$c = 1 + \frac{1}{3}$$

$$c = \frac{4}{3}$$

Put is equation

$$y = \frac{x^3}{3} + \frac{4}{3}$$

$$3y = x^3 + 4$$

Differential Equations Ex 22.11 Q33

Given,

$$y\ (\text{Slope of tangent}) = x$$

$$y\frac{dy}{dx} = x$$

$$ydy = xdx$$

$$\int ydy = \int xdx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c - - - - (1)$$

It is passing through $(0, a)$

$$\frac{a^2}{2} = 0 + c$$

$$c = \frac{a^2}{2}$$

Put $c = \frac{a^2}{2}$ is equation (1)

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{a^2}{2}$$

$$y^2 = x^2 + a^2$$

Differential Equations Ex 22.11 Q34

Let $P(x, y)$ be the point on the curve $y = f(x)$ such that tangent at P cuts the coordinate axes at A and B.

The equation of tangent is,

$$Y - y = \frac{dy}{dx}(X - x)$$

Put $Y = 0$

$$-y = \frac{dy}{dx}(X - x)$$

$$-y\frac{dy}{dx} + x = X$$

Coordinate of $B = \left(-y\frac{dy}{dx} + x, 0\right)$

Here, x intercept of tangent $= y$

$$-y\frac{dx}{dy} + x = y$$

$$\frac{dx}{dy} - \frac{x}{y} = -1$$

It is a linear differential equation on comparing it with $\frac{dx}{dy} + py = Q$

$$P = \frac{1}{y}, Q = -1$$

$$I.F. = e^{\int\left(\frac{1}{y}\right)dy}$$

$$= e^{\log y}$$

$$= \frac{1}{y}$$

Solution of the equation is given by,

$$x(I.F.) = \int Q(I.F)dy + c$$

$$x\left(\frac{1}{y}\right) = \int(-1)\left(\frac{1}{y}\right)dy + c$$

$$x\left(\frac{1}{y}\right) = -\log y + c - - - - (1)$$

It is passing through $(1, 1)$

$$\frac{1}{1} = -\log 1 + c$$

$$c = 1$$

put $c = 1$ is equation (1),

$$\frac{x}{y} = -\log y + 1$$

$$x = y - y\log y$$

$$x + y\log y = y$$