CBSE Board
Class IX Mathematics
Sample Paper 8

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. Simplify : $\left(6+\sqrt{27}\right)-\left(3+\sqrt{3}\right)+\left(1-2\sqrt{3}\right)$

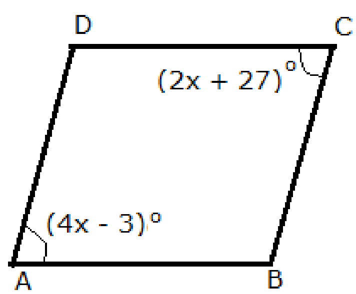
2. Find the value of the polynomial $x^2 - x - 1$ at $x = -1$.

3. In $2y - 3 = \sqrt{2}x$, what are the values of a, b and c?

OR

Find the value of k, if x = 7, y = 4 is a solution of the equation 2x + 3y = k.

4. In the following figure ABCD is a parallelogram, Find the value of x.



5. 25.7, 16.3, 2.8, 21.7, 24.3, 22.7, 24.9, what is the range of the given data?

OR

Write the class size of the given class intervals: 10-19, 20-29, 30-39.

6. What is the length of a chord which is at a distance 5 cm from the centre of a circle whose radius is 13 cm?

Section B
(Questions 7 to 12 carry 2 marks each)

7. Simplify: $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$

OR

Simplify : $\sqrt[4]{\sqrt[3]{x^2}}$

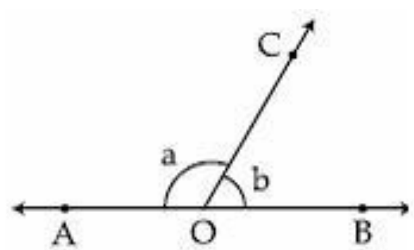
8. The perpendicular distance of a point from the x-axis is 2 units and the perpendicular distance from the y-axis is 5 units. Write the coordinates of such a point if it lies in one of the following quadrants:
(i) I Quadrant (ii) II Quadrant (iii) III Quadrant (iv) IV Quadrant

9. 10 students of Class X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys, then find the number of boys and the number of girls who took part in the quiz.

10. Find the area of an isosceles triangle with base 10 cm and perimeter 36 cm.

11. What is the area of the triangle having sides of lengths 7 cm, 8 cm and 9 cm?

12. In the figure, ∠AOC and ∠BOC form a linear pair. If a – b = 80°, then find the values of a and b.



OR

Find the measure of an angle which is complement of itself.

Section C
(Questions 13 to 22 carry 3 marks each)

13. If $\frac{3+\sqrt{8}}{3-\sqrt{8}}+\frac{3-\sqrt{8}}{3+\sqrt{8}}=a+b\sqrt{2}$, then find a and b.

OR

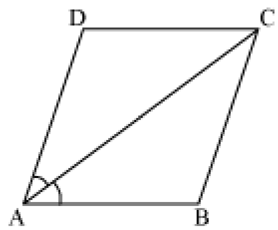
Express $0.\overline{001}$ as a fraction in the simplest form.

14. The area of a rectangle gets reduced by 80 sq. units if its length is reduced by 5 units and breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area will increase by 50 sq. units. Find the length and breadth of the rectangle.

15. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see the given figure). Show that

i. it bisects $\angle C$

ii. ABCD is a rhombus.

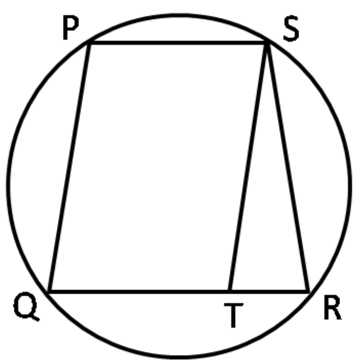


OR

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle ABD = 50^\circ$ then find $\angle DPC$.

16. The ratio of income of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save Rs. 2000 per month, find their monthly income.

17. PQST is a parallelogram. The circle through S, P and Q intersect QT produced at R. Prove that ST = SR



18. A game of chance involves spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point at

i. An odd number?

ii. A number greater than 2?

iii. A number less than 9?

19. Use a suitable identity to factorise $27p^3 + 8q^3 + 54p^2q + 36p\,q^2$.

20. The taxi fare in a city is as follows: For the first kilometre, the fare is Rs. 8 and for the remaining distance it is Rs.5 per kilometre. Taking the distance covered as x km and total fare as Rs. y, write a linear equation for this information, and draw its graph.

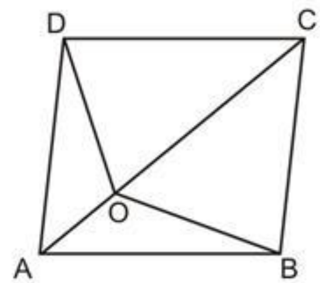
OR

If the points A(3, 5) and B(1, 4) lie on the graph of the line ax + by = 7, find the values of a and b.

21. Show that the bisectors of the angles of a parallelogram from a rectangle.

OR

A point O is taken inside an equilateral four-sided figure ABCD such that its distances from the angular points D and B are equal. Show that AO and OC are in the same straight line.



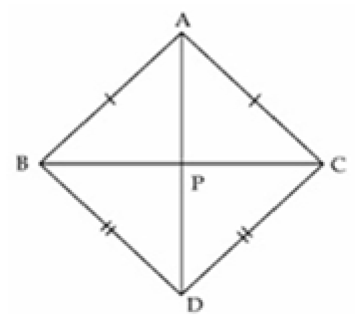
22. A survey was conducted by a group of students as a part of their Environment Awareness Programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

No of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
No of houses	1	2	1	5	6	2	3

Section D
(Questions 23 to 30 carry 4 marks each)

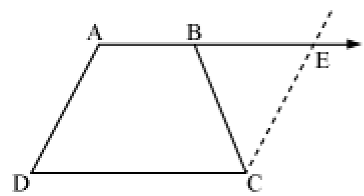
23. Simplify: $\dfrac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}$

24. In the given figure, if the two isosceles triangles have a common base, then prove that the line segment joining their vertices bisects the common base at right angles.



OR

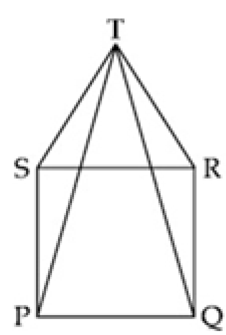
ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that



i. $\angle A = \angle B$
ii. $\angle C = \angle D$
iii. $\Delta ABC \cong \Delta BAD$
iv. Diagonal AC = Diagonal BD

25. Factorize $2x^3 - 3x^2 - 17x + 30$.

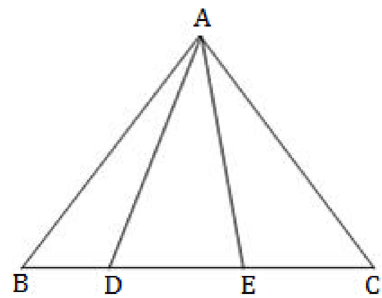
26. In the figure, PQRS is a square and SRT is an equilateral triangle. Prove that:



a) $\angle PST = \angle QRT$
b) PT = QT
c) $\Delta TSP \cong \Delta TRQ$

OR

In △ABC, points D and E are on side BC such that BD = CE and AD = AE. Prove that △ADB is congruent to △AEC. Is ∠ABC = ∠ACB? Why?



27. Sonu and Monu had adjacent triangular fields with a common boundary of 25 m. The other two sides of Sonu's field were 52 m and 63 m, while Monu's were 114 m and 101 m. If the cost of fertilization is Rs 20 per sq m, then find the total cost of fertilization for both Sonu and Monu together.

28. If AD is the median of ΔABC, then prove that AB + AC > 2AD.

29. Ajay was asked to find the sum of the four angles of a quadrilateral. He found the sum of the four angles as 270° by giving the reasoning as follows:

Sum of the three angles of a triangle [made up of three sides]

= 2 right angles = (3 – 1) right angles.

So, the sum of the four angles of quadrilateral [made up of four sides]

= (4 – 1) right angles = 3 right angles = 270°.

His classmate Anju pointed out that the sum obtained is incorrect and found the correct sum. Ajay accepted his mistake and thanked Anju for the same. Write the correct solution. What value is depicted from this action?

30. The polynomials $p(x) = ax^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + a$ leave the same remainder when divided by $x - 3$. Find the remainder when $p(x)$ is divided by $(x - 2)$.

OR

The polynomials $x^3 + 2x^2 - 5ax - 8$ and $x^3 + ax^2 - 12x - 6$ when divided by $(x - 2)$ and $(x - 3)$ leave remainders p and q, respectively. If $q - p = 10$, then find the value of a.

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Sample Paper 8 – Solution

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Section A

1.
$$\left(6+\sqrt{27}\right)-\left(3+\sqrt{3}\right)+\left(1-2\sqrt{3}\right)=6+3\sqrt{3}-3-\sqrt{3}+1-2\sqrt{3}=4$$

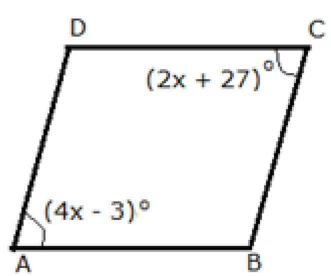
2. Substitute $x = -1$ in the polynomial $x^2 - x - 1$, we get
$x^2 - x - 1 = (-1)^2 - (-1) - 1 = 1 + 1 - 1 = 1$
$\therefore$ The value of the polynomial $x^2 - x - 1$ at $x = -1$ is 1.

3. We have, $2y - 3 = \sqrt{2}x$
$\therefore \sqrt{2}x - 2y + 3 = 0$
On comparing this equation with standard form of a linear equation,
i.e. $ax + by + c = 0$, we get
$a = \sqrt{2}$, $b = -2$ and $c = 3$

OR

If $x = 7$ and $y = 4$ is a solution of the equation $2x + 3y = k$, then
$2 \times 7 + 3 \times 4 = k$
$k = 14 + 12 = 26$

4. In a parallelogram, the opposite angles are equal.



$\therefore m\angle A = m\angle C$

$\therefore (2x+27)^\circ = (4x-3)^\circ$

$\therefore (27+3)^\circ = 4x - 2x$

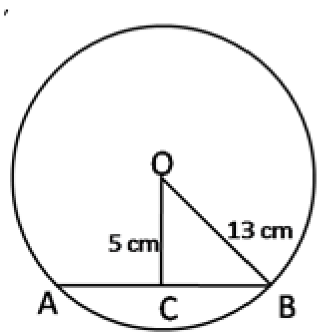
$\therefore 30^\circ = 2x \Rightarrow x = \dfrac{30^\circ}{2} = 15^\circ$

5. Range = Maximum value – Minimum value = 25.7 – 2.8 = 22.9

OR

Given intervals are discontinuous. Hence, we have to make it continuous. New intervals are 9.5 – 19.5, 19.5 – 29.5, 29.5 – 39.5
The class size of the given intervals are 19.5 – 9.5 = 10.

6.



Here, OC = 5 cm and OA = 13 cm
$\angle C = 90°$ (The line joining the centre of a circle and the mid-point of a chord is perpendicular to the chord)
$\therefore \Delta$ BOC is a right angled triangle.
Then, By Pythagoras theorem, $OB^2 = OC^2 + BC^2$
$\therefore BC = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm
Since, OC bisects the chord AB.
Hence, AB = 12 × 2 = 24 cm

Section B

7. $$\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}} = \frac{12^{\frac{1}{2}}}{27^{\frac{1}{2}}} = \frac{3^{\frac{1}{2}} \times 4^{\frac{1}{2}}}{3^{\frac{1}{2}} \times 9^{\frac{1}{2}}} = \frac{2}{3}$$

OR

$$\sqrt[4]{\sqrt[3]{x^2}} = \left[\left\{\left(x^2\right)^{\frac{1}{3}}\right\}^{\frac{1}{4}}\right] = x^{\frac{2}{3}\times\frac{1}{4}} = x^{\frac{1}{6}}$$

8. (i) I quadrant: (5, 2)
(ii) II quadrant: (–5, 2)
(iii) III quadrant: (–5, –2)
(iv) IV quadrant: (5, –2)

9. Let the number of boys be 'x' and girls be 'y'.

Number of girls is 4 more than number of boys

According to given condition,

y = x + 4 ....(1)

Total number of students = 10 (given)

Hence, x + y = 10 ....(2)

x + x + 4 = 10 [From (i)]

2x + 4 = 10

2x = 6

x = 3 ------ (3)

y = 3 + 4 Substituting (3) in (1)

y = 7

Hence, there are 3 boys and 7 girls in the class.

10. Perimeter = 36 cm = 10 cm + 2(Length of each equal side)

$\Rightarrow$ Length of each equal side = 13 cm

Here, $s = \frac{36}{2} = 18$, and the sides are 10, 13 and 13.

By Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18 \times 8 \times 5 \times 5}$

$= 60$ sq. cm

11. Let a = 7 cm, b = 8 cm and c = 9 cm.

$\therefore$ Semi-perimeter = s = $\frac{a+b+c}{2} = \frac{7\text{ cm}+8\text{ cm}+9\text{ cm}}{2} = 12\text{ cm}$

Using Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12 \times (12-7) \times (12-8) \times (12-9)}\text{ cm}^2$

$= \sqrt{12 \times 5 \times 4 \times 3}\text{ cm}^2$

$= 12\sqrt{5}\text{ cm}^2$

12. $a + b = 180°$ (Linear pair)....(i)

$a - b = 80°$ (given)....(ii)

Adding (i) and (ii),

$2a = 260°$

$\Rightarrow a = 130°$

$\Rightarrow b = 180° - 130° = 50°$

OR

Let the measure of an angle be x.

Hence, the measure of its complement is given to be x.

$x + x = 90^\circ$

$2x = 90^\circ$

$x = 45^\circ$

Section C

13.

$$\frac{(3+\sqrt{8})}{(3-\sqrt{8})} + \frac{(3-\sqrt{8})}{(3+\sqrt{8})} = \frac{(3+\sqrt{8})}{(3-\sqrt{8})} \times \frac{(3+\sqrt{8})}{(3+\sqrt{8})} + \frac{(3-\sqrt{8})}{(3+\sqrt{8})} \times \frac{(3-\sqrt{8})}{(3-\sqrt{8})}$$

$$= \frac{(3+\sqrt{8})^2}{9-8} + \frac{(3-\sqrt{8})^2}{9-8}$$

$$= \frac{(3+\sqrt{8})^2}{1} + \frac{(3-\sqrt{8})^2}{1}$$

$$= 9 + 8 + 6\sqrt{8} + 9 + 8 - 6\sqrt{8}$$

$$= 34 = a + b\sqrt{2}$$

$$\Rightarrow a = 34, \quad b = 0$$

OR

Let $x = 0.\overline{001}$

Then, x = 0.001001001……….. (i)

Therefore, 1000x = 1.001001001…………… (ii)

Subtracting (i) from (ii), we get

$999x = 1 \Rightarrow x = \frac{1}{999}$

Hence, $0.\overline{001} = \frac{1}{999}$

14. Let the present area, length & breadth of the rectangle be 'z', 'x' & 'y' respectively.

Therefore, z = xy (∵ Area = Length × Breadth) ....(1)

∵ Area is (z – 80) sq. units if length = (x – 5) and breadth = (y + 2)

Therefore, (z – 80) = (x – 5)(y + 2)

z – 80 = xy + 2x – 5y – 10

z – 80 = z + 2x – 5y – 10

–70 = –5y + 2x ....(2)

∵ Area is (z + 80) sq. units if length = (x + 10) and breadth = (y – 5)

(z + 50) = (x + 10)(y – 5)

z + 50 = xy – 5x + 10y – 50

z + 50 = z – 5x + 10y – 50

100 = 10y – 5x --- (3)

Multiply equation (2) by 2 and add it to equation (3) we get

$$\begin{array}{rl} 100 = & 10y - 5x \\ -140 = & -10y + 4x \\ \hline -40 = & -x \end{array}$$

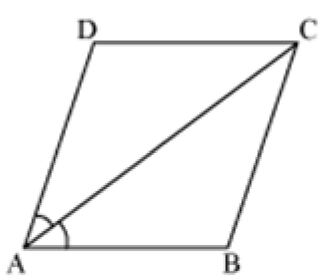
$\Rightarrow x = 40$

Substituting the value of x in equation (3) we get 100 = 10y – 5(40)

⇒ y = 30

So length of the rectangle is 40 cm and breadth of the rectangle is 30 cm.

15.



i. ABCD is a parallelogram.

∴ ∠DAC = ∠BCA (Alternate interior angles) ... (1)

And ∠BAC = ∠DCA (Alternate interior angles) ... (2)

Also, ∠DAC = ∠BAC (AC bisects ∠A) ... (3)

From equations (1), (2) and (3), we have

∠DAC = ∠BCA = ∠BAC = ∠DCA ... (4)

⇒ ∠DCA = ∠BCA

Hence, AC bisects $\angle C$.

ii. From equation (4), we have $\angle DAC = \angle DCA$

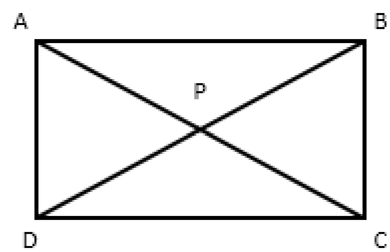
DA = DC (sides opposite to equal angles are equal)

But DA = BC and AB = CD (opposite sides of parallelogram)

$\therefore$ AB = BC = CD = DA

Hence, ABCD is rhombus.

OR



$\angle ABD = \angle ABP = 50^\circ$
$\angle PBC + \angle ABP = 90^\circ$
$\angle PBC = 40^\circ$
PB = PC
$\angle BCP = 40^\circ$
In ΔBPC,
$\angle BPC + \angle PBC + \angle BCP = 180^\circ$
$\angle BPC + 40^\circ + 40^\circ = 180^\circ$
$\angle BPC = 100^\circ$
$\angle BPC + \angle DPC = 180^\circ$
$100^\circ + \angle DPC = 180^\circ$
$\angle DPC = 80^\circ$

16. Let the common multiple for their monthly income and their expenditure be be x and y respectively.

According to the given income ratio, income of first person is 9x and that of second person is 7x.

According to the given expenditure ratio, expenditure of first person is 4y and second person is 3y.

Both of them manage to save Rs. 2000 per month

Income – Expenditure = Savings

Hence,

$9x - 4y = 2000$ ...(i)

$7x - 3y = 2000$ ...(ii)

Multiplying (i) by 3 and (ii) by 4, we get

$27x - 12y = 6000$ ...(iii)

$28x - 12y = 8000$ ...(iv)

Subtracting (iii) from (iv), we get

$\Rightarrow x = 2000$

Now, income of first person = 9x = 9(2000) = Rs. 18000

Income of second person = 7x = 7(2000) = Rs. 14000

17. Now, PQST is a cyclic quadrilateral.

$\angle SRQ + \angle SPQ = 180°$ ... (i)

(Since, opposite sides of a cyclic quadrilateral are supplementary)

Also, $\angle STQ + \angle STR = 180°$ (Linear pair of angles)

$\angle STQ = \angle SPQ$ (opposite angles of a parallelogram are equal)

Therefore, $\angle SPQ + \angle STR = 180°$ .... (ii)

From (i) and (ii),

$\angle SRQ + \angle SPQ = \angle SPQ + \angle STR$

Therefore, $\angle SRQ = \angle STR$

In Δ STR,

$\angle SRQ = \angle STR$

Δ STR is an isosceles triangle. Hence, ST = SR

18.

i. Let E denote the event 'the arrow points at an odd number'.

The favourable outcomes of the event E = 1, 3, 5, 7

The number of outcomes = n(E) = 4

So, $P(E) = \frac{4}{8} = \frac{1}{2}$

ii. Let F denote the event 'the arrow points at a number greater than 2'.

The favourable outcomes of the event F = 3, 4, 5, 6, 7, 8

The number of outcomes n(F) = 6

So, $P(F) = \frac{6}{8} = \frac{3}{4}$

iii. Let N denote the event 'the arrow points at a number less than 9'.

The favourable outcomes of the event = 1, 2, 3, 4, 5, 6, 7, 8

The number of outcomes n(N) = 8

So, $P(N)=\frac{8}{8}=1$

19. $27p^3 + 8q^3 + 54p^2q + 36pq^2$

$= (3p)^3 + (2q)^3 + 18pq(3p+2q)$

$= (3p)^3 + (2q)^3 + 3 \times 3p \times 2q\ (3p + 2q)$

$= (3p + 2q)^3$ $[(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)]$ [where a = 3p and b = 2q]

$= (3p + 2q)\ (3p + 2q)\ (3p + 2q)$

20. Total distance covered = x km.

Fare for the 1st kilometre = Rs. 8

Fare for the remaining distance per kilometre= Rs. (x – 1)5

Total fare = 8 + (x – 1)5

$y = 8 + 5x - 5$

$y = 5x + 3$

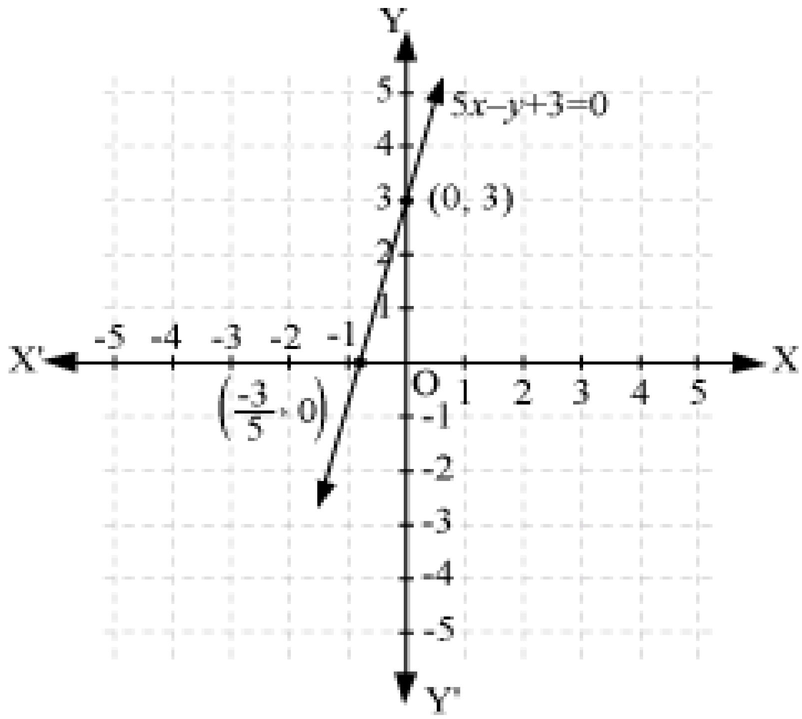
$5x - y + 3 = 0$

We observe that point (0, 3) and $\left(-\frac{3}{5}, 0\right)$ satisfy the above equation.

So these are solutions of this equation.

x	0	$\frac{-3}{5}$
y	3	0

Now join the points with a straight line as below:



Here we may find that the variables x and y are representing the distance covered and fare paid for that distance respectively and these quantities may not be negative. Hence we will consider only those values of x and y which are lying in 1^{st} quadrant.

OR

Given that points A(3, 5) and B(1, 4) lie on the graph of the line ax + by = 7. Therefore, x = 3, y = 5 and x = 1, y = 4 are solutions of the equation ax + by = 7.

3a + 5b = 7 ….(i)

a + 4b = 7…….(ii)

Multiplying by 3 to equation (ii)

3a + 12b = 21……(iii)

Subtracting (iii) from (i)

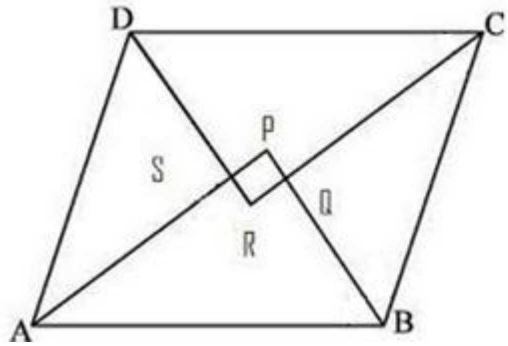
b = 2 and substituting it in equation (i)

We get a = -1

Complete the solution

a = -1 and b = 2

21. Since DS bisect ∠D and AS bisects ∠A, therefore,



$$\angle DAS + \angle ADS = \frac{1}{2}\angle A + \frac{1}{2}\angle D$$

$$\Rightarrow \angle DAS + \angle ADS = \frac{1}{2}(\angle A + \angle D) = \frac{1}{2} \times 180° = 90°$$

∠A and ∠D are interior angles on the same sides of the transversal.

Also,

∠DAS + ∠ADS + ∠DSA = 180° (Angle sum property of triangle)

⇒ 90° + ∠DSA = 180°

⇒ ∠DSA = 90°

So, ∠PSR = 90°

Similarity, it can be shown that ∠APB = 90° or ∠SPQ = 90°.

Similarly, ∠PQR = 90° and ∠SRQ = 90°.

So, PQRS is a rectangle.

OR

In ΔAOD and ΔAOB

AD = AB (given)

AO =AO (common side)

OD = OB (given)

∴ ΔAOD ≅ ΔAOB (SSS congruence rule)

∴ ∠AOD = ∠AOB (c.p.c.t)

Similarly, ΔDOC ≅ ΔBOC (SSS congruence rule)

∴ ∠DOC = ∠BOC (c.p.c.t)

∠AOD + ∠AOB + ∠DOC + ∠BOC = 360° (angles at a point)

$2\angle AOD + 2\angle DOC = 360^o$

$\angle AOD + \angle DOC = 180^o$

Hence, AO and OC are in one and the same straight line.

22. Let us find the class marks x_i of each class by taking the average of the upper class limit and lower class limit and put them in a table.

We can use the Direct Method because numerical values of x_i and f_i are small.

Class interval	No. of houses (f_i)	Class marks (x_i)	f_ix_i
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
Total	$\sum f_i = 20$		$\sum f_ix_i = 162$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Thus, the mean number of plants per house is 8.1 plants

Section D

23.

$$\frac{16\times 2^{n+1} - 4\times 2^{n}}{16\times 2^{n+2} - 2\times 2^{n+2}}$$

$$= \frac{2^4 \times 2^{n+1} - 2^2 \times 2^{n}}{2^4 \times 2^{n+2} - 2\times 2^{n+2}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n+3}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2.2^{n+5} - 2.2^{n+2}}$$

$$= \frac{\left(2^{n+5} - 2^{n+2}\right)}{2\left(2^{n+5} - 2^{n+2}\right)} = \frac{1}{2}$$

24. Since AB = AC, BD = DC, AD = AD

$\therefore$ $\Delta ABD \cong \Delta ACD$ (SSS congruence criterion)

$\Rightarrow \angle BAD = \angle CAD$ (CPCT)

In ΔABP and ΔACP

AB = AC, $\angle BAD = \angle CAD$, AP = AP

$\therefore \Delta ABP \cong \Delta ACP$ (SAS congruence criterion)

$\Rightarrow$ BP = PC and $\angle APC = \angle APB$ (CPCT)

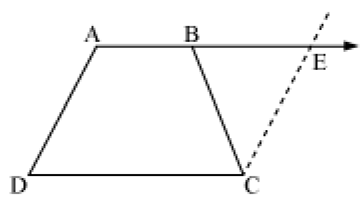
$\angle APB + \angle APC = 180^0$ (linear pair)

$\therefore \angle APB = \angle APC = 90^0$

Hence AP bisects common base at right angle.

OR

Extend AB. Draw a line through C, which is parallel to AD, intersecting AE at point E. Now, AECD is a parallelogram.



i. AD = CE (opposite sides of parallelogram AECD)
   But AD = BC (given)
   So, BC = CE
   $\angle CEB = \angle CBE$ (angle opposite to equal sides are also equal)
   Now consider parallel lines AD and CE. AE is transversal line between them
   $m\angle A + m\angle CEB = 180°$ (angles on the same side of transversal)
   $m\angle A + m\angle CBE = 180°$ (using the relation $\angle CEB = \angle CBE$) ... (1)
   But $m\angle B + m\angle CBE = 180°$ (linear pair angles) ... (2)
   From equations (1) and (2), we have
   $\angle A = \angle B$

ii. AB || CD
   $m\angle A + m\angle D = 180°$ (angles on the same side of transversal)
   Also $m\angle C + m\angle B = 180°$ (angles on the same side of transversal)
   $\therefore \angle A + \angle D = \angle C + \angle B$
   But $\angle A = \angle B$ [using the result obtained proved in (i)]
   $\therefore \angle C = \angle D$

iii. In ΔABC and ΔBAD
   AB = BA (common side)
   BC = AD (given)

$\angle B = \angle A$ (proved before)

$\therefore \Delta ABC \cong \Delta BAD$ (SAS congruence rule)

iv. $\Delta ABC \cong \Delta BAD$

$\therefore AC = BD$ (by CPCT)

25. Let $f(x) = 2x^3 - 3x^2 - 17x + 30$.

As -3 is a factor of 30.

$f(-3) = 2(-3)^3 - 3(-3)^2 - 17(-3) + 30 = -54 - 27 + 51 + 30 = -81 + 81 = 0$

Thus, $(x + 3)$ is a factor of $f(x)$.

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 10 \\
x+3 & 2x^3 - 3x^2 - 17x + 30 \\
 & 2x^3 + 6x^2 \\
 & - \quad - \\
\hline
 & -9x^2 - 17x \\
 & -9x^2 - 27x \\
 & + \quad + \\
\hline
 & 10x + 30 \\
 & 10x + 30 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

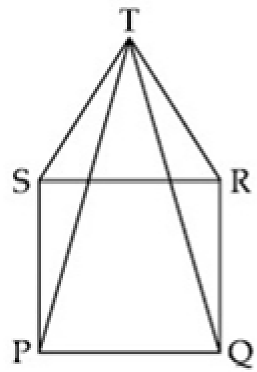
$2x^3 - 3x^2 - 17x + 30 = (x + 3)(2x^2 - 9x + 10)$

$= (x + 3)(2x^2 - 4x - 5x + 10)$

$= (x + 3)[2x(x - 2) - 5(x - 2)]$

$= (x + 3)(2x - 5)(x - 2)$

26. □PQRS is a square.



$\therefore PQ = QR = RS = SP$ ....(i)

Also $\angle RSP = \angle SRQ = \angle RQP = \angle SPQ = 90°$ ....(ii)

Also ΔTSR is equilateral.
$TS = TR = SR$......(iii)

Also $\angle STR = \angle TSR = \angle TRS = 60°$
$TR = QR$.....from (i) and (ii)

Also $\angle TSP = \angle RSP + \angle TSR$
$\angle TSP = 90° + 60° = 150°$

Similarly $\angle TRQ = 150°$

In ΔTSP and ΔTRQ,
$PS = QR$.....$(\because$ by (i))
$\angle TSP = \angle TRQ$......$(\because$ Both $150°)$
$TS = TR$......$(\because$ by (iii))
$\therefore \Delta TSP \cong \Delta TRQ$ ....(by SAS criterion)
$\therefore PT = QT$ .....(c.p.c.t)

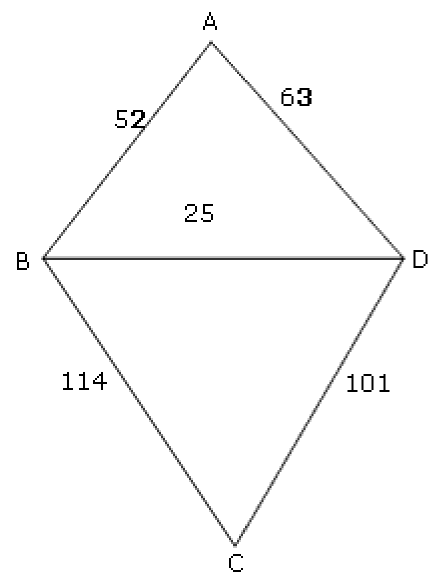
OR

Given that, AD=AE
Therefore, $\angle$ADE=$\angle$AED (angles opposite to equal sides of a triangle are equal)
So, $\angle$ADB = $\angle$AEC (remaining angles of linear pair)
In ΔADB and ΔAEC,
AD=AE (given)
$\angle$ADB= $\angle$AEC (proved above)
BD=CE (given)

Thus, ΔADB and Δ AEC are congruent.
(By SAS congruence criterion)
$\therefore \angle ABC = \angle ACB$ (Corresponding parts of congruent triangles)

27.

Sonu and Monu's field together form a quadrilateral ABCD.



Sonu's field is ΔABD,

$$s = \frac{a+b+c}{2} = \frac{52+25+63}{2} = 70$$

$s - a = 70 - 52 = 18$, $s - b = 70 - 25 = 45$ and $s - c = 70 - 63 = 7$

Area of ΔABD =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{70.18.45.7} = 630 \text{ sq m}$$

Monu's field is ΔBCD,

$$s = \frac{a+b+c}{2} = \frac{114+25+101}{2} = 120$$

$s - a = 120 - 114 = 6$, $s - b = 120 - 25 = 95$ and $s - c = 120 - 101 = 19$

Area of ΔBCD =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{120.6.95.19} = 1140 \text{ sq m}$$

Total area is = 630 + 1140 = 1770 sq m

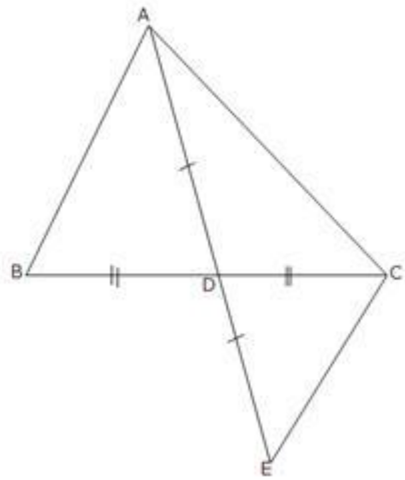
The cost of fertilization is Rs 20 per sq m.

Therefore the total cost is = 1770 x 20 = Rs 35,400.

28. Given: AD is median of triangle ABC

To Prove: AB + AC > 2AD

Proof: Produce AD so that AD = DE



Now, in triangles ADB and EDC,

AD = DE

BD = DC

∠ADB = ∠EDC

Thus, triangles ADB and EDC are congruent (By SAS congruence criterion)

Hence, AB = EC (CPCT)

Now, in triangle AEC,

AC + CE > AE

AC + CE > 2AD

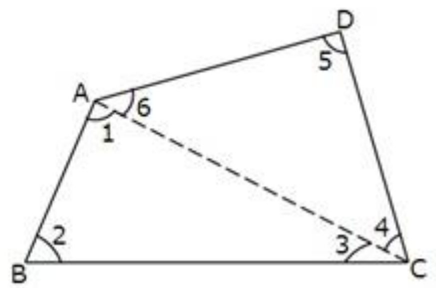
AC + AB > 2AD (since, AB = EC, proved above)

29.

Let ABCD be a quadrilateral.

We have to find $\angle A + \angle B + \angle C + \angle D$.

Join AC and mark the angles as shown in the figure.



From $\triangle ABC$, we have:

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ (Angle sum property of a triangle) ... (1)

Form $\triangle ADC$, we have:

$\angle 6 + \angle 5 + \angle 4 = 180^\circ$ (Angle sum property of a triangle) ... (2)

Adding (1) and (2),

$\angle 1 + \angle 2 + \angle 3 + \angle 6 + \angle 5 + \angle 4 = 180^\circ + 180^\circ$

$\Rightarrow (\angle 1 + \angle 6) + \angle 2 + (\angle 3 + \angle 4) + \angle 5 = 360^\circ$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$

Thus, the required sum is 360° and not 270°.

Value: Cooperative learning among students without any gender bias. Promoting secularism and self-confidence among students. Accepting own mistakes gracefully. It also provides guidance to the students not to generalize statements in haste, without any proper thinking.

30. Remainder when p(x) is divided by (x – 3) is given by

$p(3) = 27a + 36 + 9 - 4 = 27a + 41$

Remainder when q(x) is divided by (x – 3) is given by:

$q(3) = 27 - 12 + a = 15 + a$

Given, $p(3) = q(3)$

$\Rightarrow 27a + 41 = 15 + a$

$\Rightarrow a = -1$

Therefore, $p(x) = -x^3 + 4x^2 + 3x - 4$

Now, when p(x) is divided by (x – 2), the remainder is given by

$p(2) = -8 + 16 + 6 - 4 = 10$

OR

Let $f(x) = x^3 + 2x^2 - 5ax - 8$ and
$g(x) = x^3 + ax^2 - 12x - 6$
When divided by (x-2) and (x-3),f(x) and g(x) leave remainder p and q respectively
$F(x) = x^3 + 2x^2 - 5ax - 8$
$\therefore f(2) = 2^3 + 2 \times 2^2 - 5a \times 2 - 8$
$= 8 + 8 - 10a - 8$
$p = 8 - 10a$ _ _ _ _ _ (1)
$g(x) = x^3 + ax^2 - 12x - 6$
$g(3) = 3^3 + a \times 3^2 - 12 \times 3 - 6$
$= 27 + 9a - 36 - 6$
$\therefore q = -15 + 9a$ _ _ _ _ (2)
If $q - p = 10$
$\Rightarrow -15 + 9a - 8 + 10a = 10$
$\Rightarrow 19a - 23 = 10$
$\Rightarrow 19a = 33$
$\therefore a = \frac{33}{19}$