# Chapter : 30. BAYES'S THEOREM AND ITS APPLICATIONS

## Exercise : 30

**Question: 1**

In a bulb factory

**Solution:**

Let

D : Bulb is defective

We want to find P(C|D), i.e. probability that the selected defective bulb is manufactured by C

$$P(C|D) = \frac{P(C).P(D|C)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

Where, P(A) = probability that bulb is made by machine A $= \frac{60}{100}$

P(B) = probability that bulb is made by machine B $= \frac{25}{100}$

P(C) = probability that bulb is made by machine C $= \frac{15}{100}$

P(D|A) = probability of defective bulb from machine A $= \frac{1}{100}$

P(D|B) = probability of defective bulb from machine B $= \frac{2}{100}$

P(D|C) = probability of defective bulb from machine C $= \frac{1}{100}$

$$P(C|D) = \frac{15}{60 + 50 + 15}$$

$$= \frac{15}{125}$$

$$= \frac{3}{25}$$

Conclusion: Therefore, the probability of selected defective bulb is from machine C is $\frac{3}{25}$

**Question: 2**

A company manufac

**Solution:**

Let S : Standard quality

We want to find P(A|S), i.e. probability that selected standard scooter is from

plant A

$$P(A|S) = \frac{P(A).P(A|S)}{P(A).P(S|A) + P(B).P(S|B)}$$

Where, P(A) = probability that scooter is from A $= \frac{80}{100}$

P(B) = probability that scooter is from B $= \frac{20}{100}$

P(S|A) = probability that standard scooter from A $= \frac{85}{100}$

P(S|B) = probability that standard scooter from B $= \frac{65}{100}$

$$P(A|S) = \frac{(80)(85)}{(80)(85) + (20)(65)}$$

$$= \frac{6800}{6800 + 1300} = \frac{68}{81}$$

Conclusion: Therefore, the probability of selected standard scooter is from plant A is $\frac{68}{81}$

**Question: 3**

In a certain coll

**Solution:**

Let, T :students taller than 1.75

B : Boys in class

G : Girls in class

We want to find P(G|T), i.e. probability that selected taller is a girl

$$P(G|T) = \frac{P(G).P(T|G)}{P(G).P(T|G) + P(B).P(T|B)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{40}{100}\right)\left(\frac{4}{100}\right)}$$

$$= \frac{60}{220} = \frac{3}{11}$$

Conclusion : Therefore, the probability of selected taller student is a girl is $\frac{3}{11}$

**Question: 4**

In a class, 5% of

**Solution:**

Let, I : students having IQ more than 150

B : Boys in the class

G : Girls in the class

We want to find P(B|I) i.e. probability that selected student having IQ greater than 150 is a boy

$$P(B|I) = \frac{P(B).P(I|B)}{P(G).P(I|G) + P(B).P(I|B)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{5}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{5}{100}\right) + \left(\frac{40}{100}\right)\left(\frac{10}{100}\right)}$$

$$= \frac{300}{300 + 400} = \frac{3}{7}$$

Conclusion: Therefore, the probability that selected student having IQ greater than 150 is a boy is $\frac{3}{7}$

**Question: 5**

Suppose 5% of men

**Solution:**

Let MG : Men having grey hair

WG: Women having grey hair

G : Having grey hair

Given an equal number of males and females. So let's assume both the probability be $\frac{1}{2}$

We want to find P(MG|G), i.e. probability of a randomly selected grey person to be male

$$P(MG|G) = \frac{P(MG).P(G|MG)}{P(MG).P(G|MG) + P(WG).P(G|WG)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{5}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{5}{100}\right) + \left(\frac{1}{2}\right)\left(\frac{0.25}{100}\right)}$$

$$= \frac{5}{5.25}$$

$$= \frac{20}{21}$$

Conclusion: Therefore, the probability of a randomly selected grey person to be male is $\frac{20}{21}$

**Question: 6**

Two groups are co

**Solution:**

Let F : First group

S : Second group

N : Introducing a new product

We want to find P(S|N), i.e. new product introduced by the second group

$$P(S|N) = \frac{P(S).P(N|S)}{P(S).P(N|S) + P(F).P(N|F)}$$

$$= \frac{(0.4)(0.3)}{(0.6)(0.7) + (0.4)(0.3)}$$

$$= \frac{0.12}{0.54}$$

$$= \frac{2}{9}$$

Conclusion: Therefore, the probability of the second group introduced a new product is $\frac{2}{9}$

**Question: 7**

A bag A contains

**Solution:**

Let R : Red ball

W : White ball

A : Bag A

B : Bag B

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(A|W), i.e. the selected white ball is from bag A

$$P(A|W) = \frac{P(A).P(W|A)}{P(A).P(W|A) + P(B).P(W|B)}$$

$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)+\left(\frac{1}{2}\right)\left(\frac{4}{7}\right)}$

$= \frac{1}{5}$

Conclusion: Therefore, the probability of selected white ball is from

bag A is $\frac{1}{5}$

**Question: 8**

There are two I a

**Solution:**

Let W : White ball

B : Black ball

X : $1^{st}$ bag

Y : $2^{nd}$ bag

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(X|W), i.e. probability of selected white ball is from the $1^{st}$ bag

$P(X|W) = \frac{P(X).P(W|X)}{P(X).P(W|X) + P(Y).P(W|Y)}$

$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right)+\left(\frac{1}{2}\right)\left(\frac{5}{11}\right)}$

$= \frac{\frac{3}{7}}{\frac{3}{7}+\frac{5}{11}}$

$= \frac{33}{68}$

Conclusion: Therefore, the probability of selected white ball is from the $1^{st}$ bag is $\frac{33}{68}$

**Question: 9**

A box contains 2

**Solution:**

Let G : Gold coins

S : Siler coins

A : $1^{st}$ box

B : $2^{nd}$ box

Assuming, selecting bags is of equal probability i.e. $\frac{1}{2}$

We want to find P(B|G), i.e. probability of selected gold coin is from the $2^{nd}$ box

$P(B|G) = \frac{P(B).P(G|B)}{P(A).P(G|A) + P(B).P(G|B)}$

$$= \frac{\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)}{\left(\frac{2}{5}\right)\left(\frac{1}{2}\right)+\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{5}{9}$$

Conclusion: Therefore, the probability of selected gold coin is from the $2^{nd}$ box is $\frac{5}{9}$

**Question: 10**

Three urns A, B a

**Solution:**

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

R : Red ball

W : White ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(A|R), i.e. probability of selected red ball is from bag A

$$P(A|R) = \frac{P(A).P(R|A)}{P(A).P(R|A)+P(B).P(R|B)+P(C).P(R|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right)}{\left(\frac{1}{3}\right)\left(\frac{6}{10}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{8}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)}$$

$$= \frac{\left(\frac{3}{5}\right)}{\left(\frac{3}{5}\right)+\left(\frac{1}{4}\right)+\left(\frac{1}{6}\right)} = \frac{36}{61}$$

Conclusion: Therefore, the probability of selected red ball is from bag A is $\frac{36}{61}$

**Question: 11**

Three urns contai

**Solution:**

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(A|W), i.e. probability of selected White ball is from bag A

$$P(A|W) = \frac{P(A).P(W|A)}{P(A).P(W|A)+P(B).P(W|B)+P(C).P(W|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{4}{5}\right)}$$

$= \frac{2}{9}$

Conclusion: Therefore, the probability of selected white ball is from bag A is $\frac{2}{9}$

**Question: 12**

There are three b

**Solution:**

let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

RB : Red ball

Assuming, selecting bags is of equal probability i.e. $\frac{1}{3}$

We want to find P(B|WR) i.e. probability of selected White and red ball is from bag B

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)}{\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)} = \frac{6}{11}$$

Conclusion: Therefore, the probability of selected white and red ball from bag B is $\frac{6}{11}$

**Question: 13**

Urn A contains 7

**Solution:**

Let A : Ball is drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

BB : Black ball

WB : White ball

RB : Red ball

Probability of picking 2 white balls fro urn A $= \frac{7c2}{10c2} = \frac{21}{45}$

Probability of picking 2 white balls fro urn B $= \frac{4c2}{10c2} = \frac{6}{45}$

Probability of picking 2 white balls fro urn C $= \frac{2c2}{10c2} = \frac{1}{45}$

We want to find the probability of 2 white balls picked from urn C

$$= \frac{(0.2)\left(\frac{1}{45}\right)}{(0.2)\left(\frac{21}{45}\right)+(0.6)\left(\frac{6}{45}\right)+(0.2)\left(\frac{1}{45}\right)}$$

$$= \frac{1}{40}$$

Conclusion: Therefore, the probability of both selected white balls are from urn C is $\frac{1}{40}$

**Question: 14**

There are 3 bags,

**Solution:**

Let A : the set of first 3 bags

B : a set of next 2 bags

WB : White ball

BB : Black ball

Now we can change the problem to two bags, i.e. bag A containing 15 white and 9 black balls(5 white and 3 black in each bag) and bag B containing 4 white and 8 black balls(2 white and 4 black balls in each bag)

Probability of selecting bag A is $\frac{3}{5}$ (3 bags are in A) and selecting B is $\frac{2}{5}$ (2 bags are in B)

We want to find the probability of selected white ball is from bag A

$$P(A|WB) = \frac{P(A).P(WB|A)}{P(A).P(WB|A) + P(B).P(WB|B)}$$

$$= \frac{\left(\frac{3}{5}\right)\left(\frac{15}{24}\right)}{\left(\frac{3}{5}\right)\left(\frac{15}{24}\right) + \left(\frac{2}{5}\right)\left(\frac{4}{12}\right)}$$

$$= \frac{45}{61}$$

Conclusion: Therefore, the probability of selected white ball is from the first group is $\frac{45}{61}$

**Question: 15**

There are four bo

**Solution:**

Let A : Ball drawn from bag A

B : Ball is drawn from bag B

C : Ball is drawn from bag C

D : Ball is drawn from bag D

BB : Black ball

WB : White ball

RB : Red ball

Assuming all boxes have an equal probability for picking i.e. $\frac{1}{4}$

We want to find P(A|RB), i.e. probability of selected red ball is from box A

$$P(A|RB) = \frac{P(A).P(RB|A)}{P(A).P(RB|A) + P(B).P(RB|B) + P(C).P(RB|C) + P(D).P(RB|D)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{6}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{8}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{0}{10}\right)}$$

$$= \frac{1}{15}$$

Conclusion: Therefore, the probability of selected red ball is from box A is $\frac{1}{15}$

**Question: 16**

A car manufacturi

**Solution:**

Let X : Car produced from plant X

Y : Car produced from plant Y

S : Car rated as standard quality

We want to find P(X|S), i.e. selected standard quality car is from plant X

$$P(X|S) = \frac{P(X).P(S|X)}{P(X).P(S|X) + P(Y).P(S|Y)}$$

$$= \frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{90}{100}\right)}$$

$$= \frac{56}{83}$$

Conclusion: Therefore, the probability of selected standard quality car is from plant X is $\frac{56}{83}$

**Question: 17**

An insurance comp

**Solution:**

Let M : Motorcycle

S : Scooter

A : Accident vechicle

We want to find P(M|A), i.e. probability of accident vehicle was a motorcycle

$$P(M|A) = \frac{P(M).P(A|M)}{P(M).P(A|M) + P(S).P(A|S)}$$

$$= \frac{\left(\frac{3000}{5000}\right)(0.02)}{\left(\frac{3000}{5000}\right)(0.02) + \left(\frac{2000}{5000}\right)(0.01)}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Conclusion: Therefore, the probability of accident vechile was motorcycle is $\frac{3}{4}$

**Question: 18**

In a bulb factory

**Solution:**

Let A : Manufactured from machine A

B : Manufactured from machine B

C : Manufactured from machine C

D : Defective bulb

We want to find P(A|D), i.e. probability of selected defective bulb is from

machine A

$$P(A|D) = \frac{P(A).P(D|A)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)}$$

$$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{2}{100}\right) + \left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$$

$$= \frac{6}{15}$$

$$= \frac{2}{5}$$

Conclusion: Therefore, the probability of selected defective bulb is from machine A is $\frac{2}{5}$