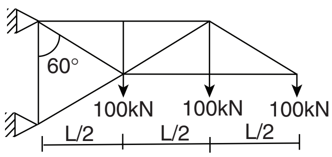
Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following statements regarding statically determinate structures are correct?
(a) Can be analyzed by equilibrium equations
(b) Stresses are caused due to temperature changes
(c) *BM* at a section does not depend on material or sectional properties of structure.
(A) a, b, c are correct (B) a, b are *c*orrect
(C) a, c are correct (D) b, c are correct

2. The pin jointed frame shown in the figure is



(A) Perfect frame (B) Redundant frame
(C) deficit frame (D) None

3. Which of the following method comes under the category of displacement method to analyze statically indeterminate structure?
(A) Elastic center method
(B) Minimum strain energy theorem
(C) Moment distribution method
(D) Column analogy method

4. The cantilever beam *AB* of length '*L*' fixed at *A* and free at *B* is subjected to a concentrated load '*W*' at its free end. The strain energy (*U*) stored in a beam is [EI: Constant)

(A) $\frac{W^2 L^2}{4EI}$ (B) $\frac{WL^3}{6EI}$

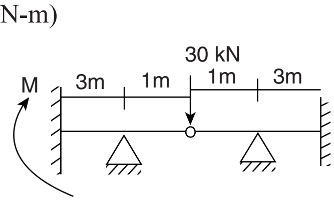
(C) $\frac{W^2 L^3}{6EI}$ (D) $\frac{WL}{EI}$

5. The Bending moment induced at fixed end of cantilever beam of span '*L*' if the free end undergoes a unit displacement without rotation is

(A) $\frac{3EI}{L^2}$ (B) $\frac{5EI}{L^2}$

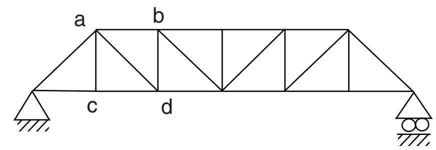
(C) $\frac{6EI}{L^2}$ (D) $\frac{4EI}{L^2}$

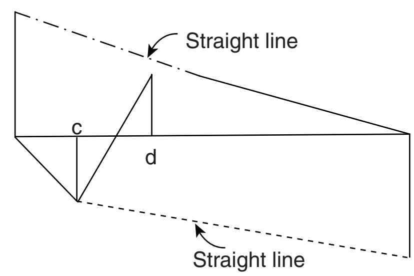
6. The value of support moment *M* for the beam shown below (in kN-m)



(A) 12.5 kN-m (B) 7.5 kN-m
(C) 15 kN-m (D) None

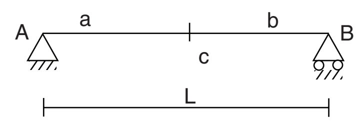
7. The Influence line diagram (I.L.D) shown is for the member ———



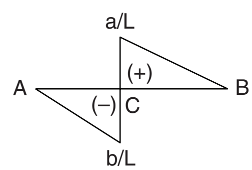


(A) *ab* (B) *ac*
(C) *cd* (D) *ad*

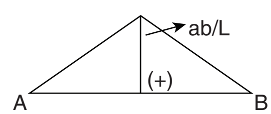
8. The ILD for shear force at a section '*c*' of simply supported beam of length '*L*', when unit load moves from one end to other is



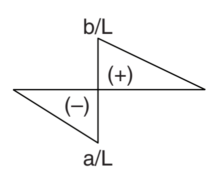
(A)



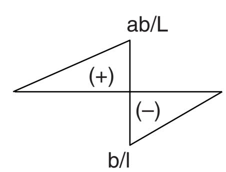
(B)



(C)



(D)



9. Which of the following statements below are correct?
(a) The stiffness coefficient k_{ji} indicates force at *j* due to a unit deformation at *i*.

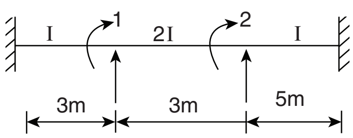
(b) Stiffness matrix is a square symmetric matrix.
(c) Stiffness matrix is possible for both stable and unstable structures also.

(A) a, b, c are correct (B) a, b are correct
(C) a, c are correct (D) b, c are correct

10. Which of the following statements are correct?
(a) The internal forces at any c/s of an arch are *SF*, *BM* only
(b) The effect of arching a beam is to reduce *BM* in the span
(c) A two hinged arch is indeterminate by one degree
(d) The internal forces at any c/s of an arch are *SF*, *BM* and normal thrust also.

(A) a, b, c, d are correct
(B) a, b, c are correct
(C) b, c, d are correct
(D) a, c, d are correct

11. Determine the stiffness matrix for a beam for the given coordinates shown in the figure

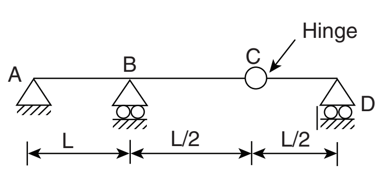


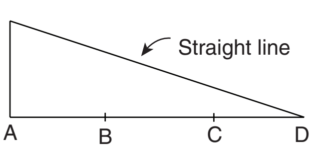
(A) $\begin{bmatrix} 4EI & \frac{4EI}{3} \\ \frac{4EI}{5} & 52EI \end{bmatrix}$ (B) $\begin{bmatrix} 4EI & \frac{4EI}{3} \\ \frac{4EI}{3} & \frac{52EI}{15} \end{bmatrix}$

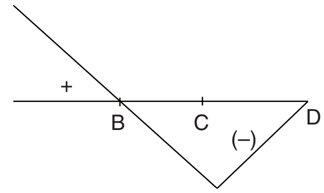
(C) $\begin{bmatrix} \frac{52EI}{15} & \frac{-4EI}{3} \\ \frac{-4EI}{3} & 4EI \end{bmatrix}$ (D) $\begin{bmatrix} 4EI & \frac{+4EI}{3} \\ \frac{4EI}{3} & 4EI \end{bmatrix}$

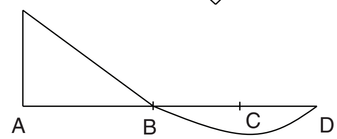
12. A cable carrying a load of 40 kN/m run of horizontal span, is stretched between supports 150m apart. The supports are at same level and the central dip is 15 m. The greatest tension and least tension in cable are
(A) 8100 kN, 7500 kN
(B) 10,000 kN, 7500 kN
(C) 9500 kN, 6000 kN
(D) None

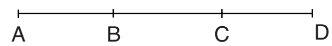
13. For the continuous beam shown below, the I.L.D for Reaction at *A* is



(A)


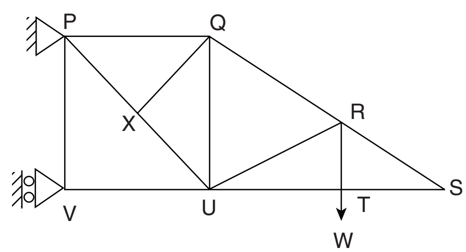
(B)


(C)


(D)


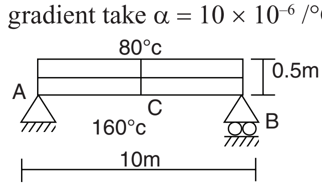
14. A beam *ABCD* is 15m long and is simply supported at *B* and *C* 8 m apart. Overhangs *BA* and *CD* are 3m and 4m respectively. A train of two point loads of 150 kN and 100 kN, 3 m apart, crosses the beam from left to right with 100 kN load leading. The maximum sagging *B.M.* under 150 kN load anywhere is
(A) 150 kN-m (B) 250 kN-m
(C) 360 kN-m (D) 400 kN-m

15. In the truss shown below, indicate how many number of members with zero forces



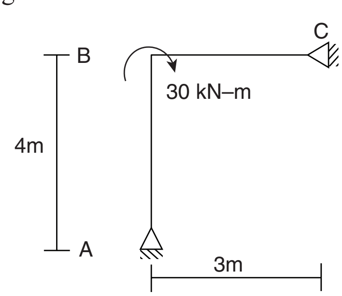
(A) 4 (B) 5
(C) 6 (D) None

16. A simply supported beam of length L = 10 m and depth = 0.5 m is subjected to a temperature differential of 80°C at top and 160°C at bottom. Determine the vertical deflection of beam at its mid point (*c*) due to temperature gradient take $\alpha = 10 \times 10^{-6}$ /°C.



(A) 25 mm (B) 30 mm
(C) 40 mm (D) None

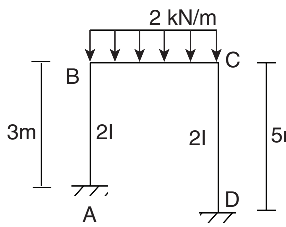
17. What is the rotation of the member at '*C*' for a frame as shown in figure below?



(A) $\frac{30}{3EI}$ (B) $\frac{60}{7EI}$

(C) $\frac{90}{EI}$ (D) $\frac{75}{EI}$

18. In the portal frame shown in the given figure, the ratio of sway moments in column AB and CD will be equal to

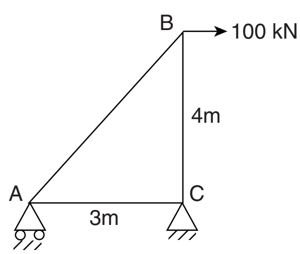


(A) 25/9 (B) 3/5
(C) 2/3 (D) 15/8

19. U_1 and U_2 are the strain energies stored in a prismatic bar due to axial tensile force w_1 and w_2 respectively. The strain energy 'U' stored in the same bar due to combined action of w_1 and w_2 be

(A) $U = U_1\, U_2$ (B) $U > U_1 + U_2$
(C) $U < U_1 + U_2$ (D) $U = U_1 + U_2$

20. The right triangular truss is made of members having equal c/s area of 1000 mm^2 and youngs modulus of 2×10^5 MPa. The horizontal deflection at B is



(A) 15 mm (B) 20 mm
(C) 12 mm (D) None

21. A two spans continuous beam having equal spans each of length $L/2$ is subjected to a uniformly distributed load $2w$ per unit m length. The beam has constant flexural rigidity. The reaction at middle support is

(A) $\frac{3wl}{4}$ (B) $\frac{3wl}{8}$

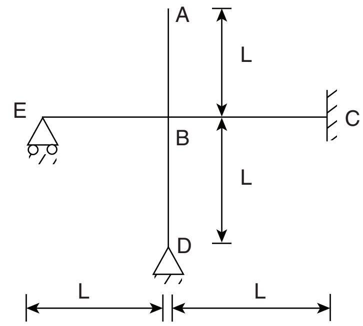
(C) $\frac{5wl}{4}$ (D) $\frac{5wl}{8}$

22. Using the data in Q No 21; find the Bending moment at the middle support

(A) $\frac{wl^2}{16}$ (B) $\frac{wl^2}{4}$

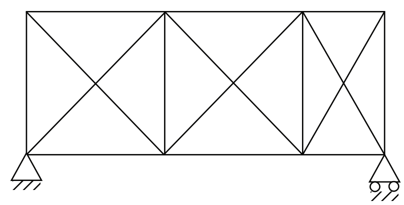
(C) $\frac{wl^2}{8}$ (D) $\frac{3wl^2}{16}$

23. In the frame shown below; what are the distribution factors for members BA, BC and BD respectively



(A) 0, 0.4, 0.6 (B) 0, 0.3, 0.7
(C) 0, 0.4, 0.3 (D) None

24. Examine the given truss below



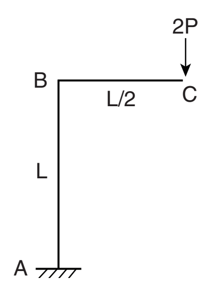
(A) statically determinate

(B) statically indeterminate but kinematically determinate

(C) statically indeterminate and kinematically indeterminate

(D) statically determinate and kinematically indeterminate

25. The horizontal deflection at C for the following frame shown below



(A) $\frac{PL^2}{2EI}$ (B) $\frac{PL^3}{3EI}$

(C) $\frac{PL^3}{2EI}$ (D) $\frac{2PL^2}{EI}$

Answer Keys

1. C	**2.** A	**3.** C	**4.** C	**5.** C	**6.** B	**7.** D	**8.** C	**9.** B	**10.** C
11. B	**12.** A	**13.** B	**14.** C	**15.** B	**16.** C	**17.** B	**18.** A	**19.** B	**20.** C
21. C	**22.** A	**23.** C	**24.** C	**25.** C					

Hints and Explanations

1. Statements (a) and (c) are correct Choice (C)

2. $m = 2j - 3$

m = number of members = 11

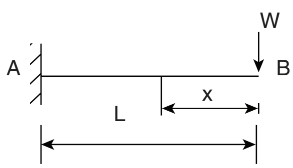
j = number of joints = 7

$m = 2(7) - 3 = 14 - 3 = 11$

Hence, a perfect frame Choice (A)

3. Moment distribution method comes under the category of displacement method. Choice (C)

4.



Strain energy (U) is given by

$$U_x = \int_0^L \frac{M_x^2\, dx}{2EI}$$

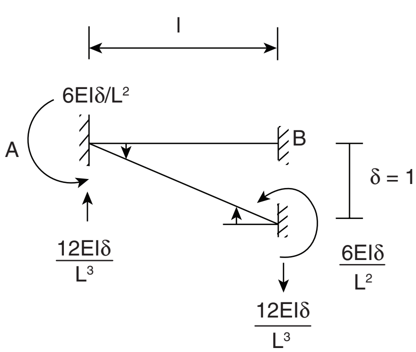
$$M_X = -wx$$

$$U_x = \int_0^L \frac{(WX)^2}{2EI} dx$$

$$= \frac{w^2}{2EI}\left[\frac{X^3}{3}\right]_o^L = \frac{W^2 L^3}{6EI}$$

Choice (C)

5. Given, free end undergoes unit displacement and without rotation, so the beam can be shown as



$$\text{Moment at } A = \frac{6EI\delta}{L^2}$$

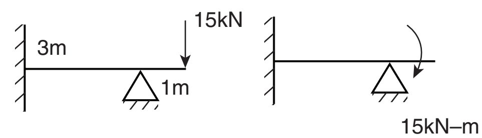
$\delta = 1$

$$M_A = \frac{6EI}{L^2}$$

[Due to sinking of support 'B'; Rotations develop at A and B; But fixed end does not allow rotations and hence fixed end moments of magnitude $\frac{6EI\delta}{L^2}$ develop]

Choice (C)

6. Because of symmetry,

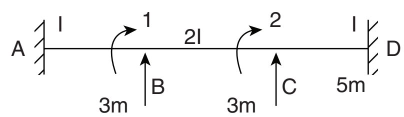


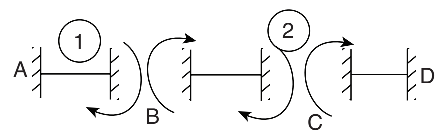
From concept of carry over moment half the moment is transferred to fixed end in some direction.

i.e., $\frac{15}{2}$ = 7.5 kN-m Clockwise

Choice (B)

11.



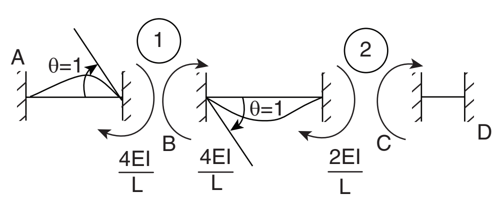


$$K = \begin{bmatrix} K_1 & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

K_{11}: moment at (1) due to unit rotation at (1)

K_{21}: moment at (2) due to unit rotation at (1)

So at first coordinate given unit rotation in C.W direction

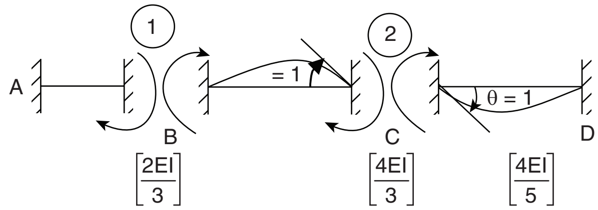


$$K_{11} = \left[\frac{4EI}{L}\right]_{BA} + \left[\frac{4EI}{L}\right]_{BC}$$

$$= \left[\frac{4EI}{3}\right] + \left[\frac{4E(2I)}{3}\right] = \frac{12EI}{3} = 4\text{EI}$$

$$K_{21}=\left[\frac{2EI}{L}\right]_{BC}=\frac{2E(2I)}{3}=\frac{4EI}{3}$$

K_{22}: moment developed at (2) due to unit rotation at (2)
K_{12}: moment developed at (1) due to unit rotation at (2)
So apply unit rotation at (2)



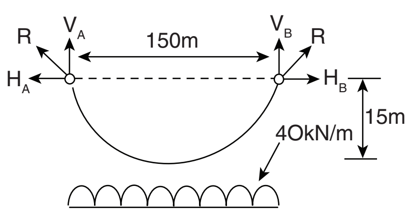
$$K_{22}=K_{22}=\left[\frac{4EI}{L}\right]_{CB}+\left[\frac{4EI}{L}\right]_{CD}=\left[\frac{4E(2I)}{3}\right]+\left[\frac{4EI}{5}\right]$$

$$=\frac{8EI}{3}+\frac{4EI}{5}=\frac{40EI+12EI}{15}=\frac{52EI}{15}$$

$$K_{12}=\left[\frac{2EI}{L}\right]_{CB}=\left[\frac{2E(2I)}{3}\right]=\frac{4EI}{3}$$

$$[K]=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}=\begin{bmatrix}4EI & \frac{4EI}{3}\\ \frac{4EI}{3} & \frac{52EI}{15}\end{bmatrix}$$ Choice (B)

12.



$$V_A=VB=\frac{wl}{2}=\frac{40\times150}{2}=3000\text{ kN}$$

$$H_A=H_B=\frac{Wl^2}{8h}=\frac{40\times150^2}{8\times15}$$

$$=\frac{5\times150\times150}{15}=7500\text{ kN}$$

Maximum tension $=R=\sqrt{V^2+H^2}$

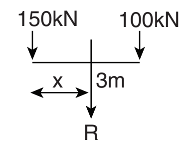
$$=\sqrt{(3000)^2+(7500)^2}=8077\text{ kN}$$

Minimum tension occurs at deepest point and is equal to Horizontal thrust

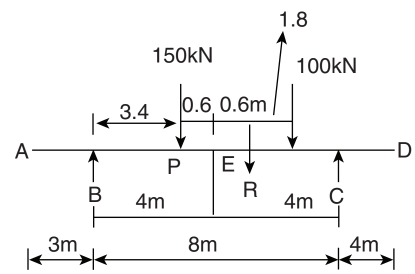
$T_{min}=7500$ kN Choice (A)

13. By Muller Breslau's principle, apply a unit vertical displacement at A. The resulting deflection profile is the I.L.D at A. Since A, B and C are one part and CD are the other part; ABC deforms as a single member and CD deforms due to downward movement of C. At 'B', there won't be any displacement because of support of B. Since the beam is statically determinate the ILD is a straight line, not a curve. Choice (B)

14. To get maximum Bending moment under a load, the Resultant and load should be kept equidistant from center of span.

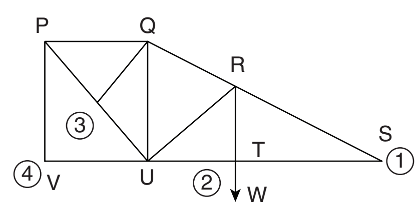


$$\bar{x}=\frac{100\times3}{150+100}=\frac{300}{250}=1.2\text{m}$$



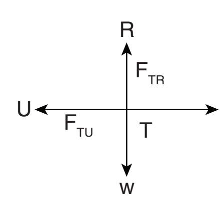
$V_b+V_C=150+100=250$ kN
$(V_C)(8)-100(6.4)-150(3.4)=0$
$V_C=143.75$ kN
$V_B=106.25$ kN
Maximum $B.M$ at $P=(V_B)$ (3.4)
$=361.25$ kN-m Choice (C)

15.



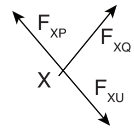
Go by Joint no's.
At joint (1); Since no load is acting at 's' force in these members to be zero

At Joint (2) i.e., at T



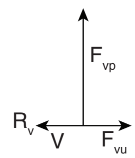
Three forces are acting at a joint; [i.e., W; F_{TR}; F_{TU}] and two of the forces are in same line i.e., W and F_{TR} the force in F_{TU} to be zero since for equilibrium of a joint

At Joint (3) i.e., at X;



The same statement as above; Hence the force in $F_{XQ} = 0$

At Joint (4) i.e., at V

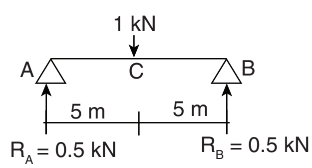


$F_{VP} = 0$

$\therefore$ no of members with zero forces = 5. Choice (B)

16. Since the vertical deflection at center of beam is to be determined

Apply a unit load at center of beam



From symmetry;

$R_A = R_B = 0.5$ kN

$$\delta_C = \int_0^L \frac{m\alpha\Delta T_m\, dX}{C}$$

m: Internal virtual moment in beam

$\alpha = 10 \times 10^{-6}/°c$

ΔT_M : temperature difference between mean temp and the temp at the top or bottom of beam

$$= \left[\frac{80+160}{2}\right] - 80 = 120 - 80 = 40°C$$

C: mid depth of beam = $\dfrac{0.5}{2} = 0.25$m

$$\delta_v = \int_0^{10m} \frac{(0.5x)(10\times10^6/°C)(40°C)}{0.25\text{m}}dx$$

$$= \left[\frac{0.5\times10\times10^{-6}\times40}{0.25}\frac{x^2}{2}\right]_0^{10}$$

$$= \frac{0.5\times10\times10^{-6}\times40\times10^2}{2\times0.25}$$

$\delta_V = 0.04$m

$\delta_V = 40$ mm Choice (C)

17. Rational stiffness at a joint B;

$$S = \frac{M}{\theta} = \frac{3EI}{L} + \frac{3EI}{L} = \frac{3EI}{3} + \frac{3EI}{4}$$

$$\frac{M}{\theta} = \frac{12EI + 9EI}{12}$$

$$\frac{M}{\theta_B} = \frac{21EI}{12}$$

$$\theta_B = \frac{12M}{21EI} \Rightarrow \theta_B = \frac{12\times30}{21EI}$$

$$\theta_B = \frac{120}{7EI}$$

From slope deflection method

$$M_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right] + M_{FCB}$$

$\delta = 0$, $M_{FCB} = 0$

$M_{CB} = 0$ [Since hinge support]

$$0 = \frac{2EI}{3}[2\theta_C + \theta_B]$$

$$\frac{2EI}{3}\left[2\theta_C + \frac{120}{7EI}\right] = 0$$

$$2\theta_c + \frac{120}{7EI} = 0$$

$$\theta_C = \frac{-120}{14EI}$$

$\theta_C = \dfrac{+60}{7EI}$ [Neglect sign] Choice (B)

18. $M_{AB}\ \alpha\ \dfrac{6EI\delta}{L^2} = \dfrac{6E(2I)\delta}{(3)^2} = \dfrac{12EI\delta}{9}$

$M_{CD}\ \alpha\ \dfrac{6EI\delta}{L^2} = \dfrac{6E(2I)\delta}{5^2} = \dfrac{12EI\delta}{25}$

$\dfrac{M_{AB}}{M_{CD}} = \dfrac{12EI\delta(25)}{9(12EI\delta)} = 25/9$ Choice (A)

19. U_1 = strain energy stored in bar due to W_1

$$U_1 = \frac{P^2L}{2AE} = \frac{W_1^2L}{2AE}$$

U_2 = strain energy stored in bar due to W_2

$$U_2 = \frac{W_2^2L}{2AE}$$

U = strain energy stored in bar due to combined $(W_1 + W_2)$

$$U = \frac{(W_1+W_2)^2L}{2AE}$$

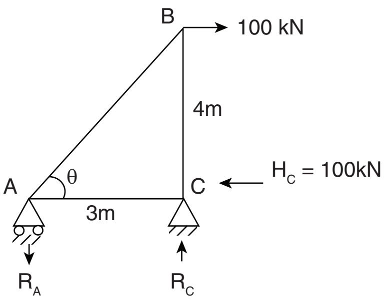
$\therefore$ $U > U_1 + U_2$ Choice (B)

20. $\delta = \Sigma\dfrac{PkL}{AE}$

P: Force in member due to applied loads

K: Force in member for a unit load in the direction in which deflection is desired

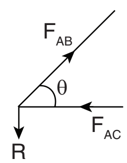
L: Length of members



$(R)(3) = 100 \times 4$

$R_A = R_C = R = \dfrac{100 \times 4}{3} = 133.33$ kN

At Joint *A*:



$\sin\theta = \dfrac{4}{5}$

$\cos\theta = 3/5$

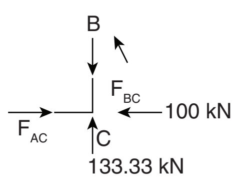
$F_{AB}\sin\theta = R$

$F_{AB} = \dfrac{R}{\sin\theta} = \dfrac{133.33}{4/5} = 166.67$ kN(Tension)

$F_{AB}\cos\theta = F_{AC}$

$\Rightarrow\ F_{AC} = 166.67 \times 3/5 = 100$ kN(compression)

At joint *C*:



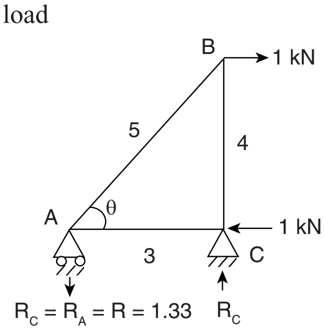
$\Sigma F_y = 0$

$F_{BC} = 133.33$ kN

$F_{BC} = 133.33$ kN (Compressive)

$F_{AC} = 100$ kN (Compressive)

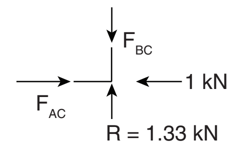
Due to unit load



$1 \times 4 = R \times 3$

$R = 4/3 = 1.33$ kN

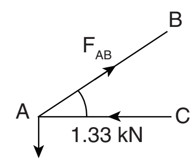
Joint C:



$F_{AC} = 1$ kN (Comp)

$F_{BC} = 1.33$ kN (Comp)

At Joint *A*:



$F_{AB}\sin\theta = 1.33$ kN

$F_{Ab} = \dfrac{1.33}{\sin\theta} = \dfrac{1.33}{415}$

$F_{AB} = 1.66$ kN (tension)

Take tension: + *ve*

compression: – *ve*

Member	Length (L)	P	K	PkL
AB	5	166.67	1.66	1383.36
BC	4	–133.33	–1.33	709.31
AC	3	–100	–1	300

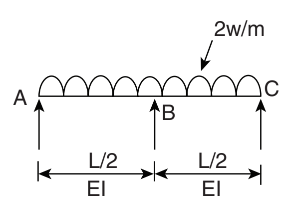
$\Sigma PKL = 2392.67$

$$\delta = \frac{\Sigma PkL}{AE}$$

$$= \frac{1}{\left(1000\ \text{mm}^2\right)\left(2\times10^5\ \text{N/mm}^2\right)}\ (2392.67)$$

$\delta = 11.96$ mm Choice (C)

21.



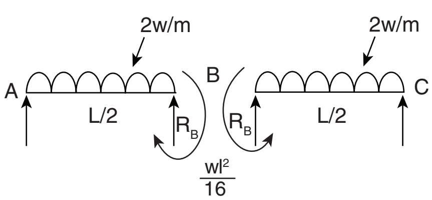
$$M_{FAB} = -\frac{Wl^2}{12} = \frac{-(2w)(L/2)^2}{12} = \frac{-(2w)(L^2/4)}{12} = \frac{-WL^2}{24}$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{(2w)(L/2)^2}{12} = \frac{+WL^2}{24}$$

$$M_{FBC} = \frac{-WL^2}{24},\ M_{FCB} = \frac{+Wl^2}{24}$$

Joint	Member	Relative stiffness k – I/L	Σk	Distribution factor = k/ Σk
B	BA	$\frac{3}{4}\frac{I}{L/2}=\frac{6I}{4L}$	$\frac{12I}{4L}$	$\frac{6I\ 4L}{4L(12I)}=\frac{1}{2}$
	BC	$\frac{3}{4}\frac{I}{L/2}=\frac{6I}{4L}$		1/2

	A	B		C
		$\frac{1}{2}$	$\frac{1}{2}$	
F EM	$\frac{-WL^2}{24}$	$\frac{WL^2}{24}$	$\frac{-WL^2}{24}$	$\frac{+WL^2}{24}$
Release moment	$\frac{+WL^2}{24}$			$\frac{-WL^2}{24}$
C.O.M		$\frac{WL^2}{48}$	$\frac{-WL^2}{48}$	
Final	O	$\frac{WL^2}{16}$	$\frac{-WL^2}{16}$	O



For SPAN *AB*:

$$(R_B)\frac{L}{2}=2w\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)=\frac{wL^2}{16}=0$$

$$R_B\ \frac{L}{2}=\frac{2\times2\times wL^2}{2\times8}+\frac{WL^2}{16}$$

$$R_B\ \frac{L}{2}=\frac{5wL^2}{16}\ w$$

$$R_B=R_B=\frac{5WL}{8}$$

For Span BC:

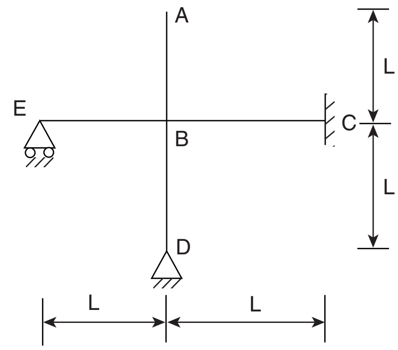
$$(R_B)\frac{L}{2}=\frac{WL^2}{16}=(2w)\frac{L}{2}\left(\frac{L}{4}\right)=0$$

$$R_B=\frac{5WL}{8}$$

$\therefore$ Prop Reaction at $B=\frac{5WL}{8}+\frac{5WL}{8}=\frac{5WL}{4}$

Choice (C)

23.

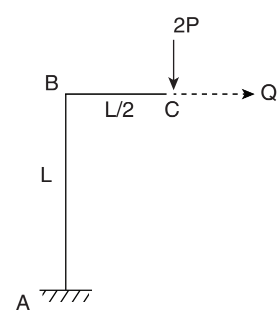


Joint	Members	K = I/L	ΣK	DF = K/ ΣK
B	BA	0	$\frac{5}{2}$I/L	0
	BC	I/L		$\frac{I2L}{L.5I}=0.4$
	BD	$\frac{3}{4}$I/L		$\frac{3I\ 2L}{4L\ 5I}=\frac{6}{20}=0.3$
	BE	$\frac{3}{4}$I/L		0.3

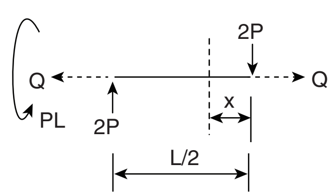
Choice (C)

24. Internal static indeterminacy, $D_{si}=m-(2j-3)$
$m=16, j=8$
$D_{si}=16-(2\times8-3)$
$D_{si}=3$
External indeterminacy $D_{se}=r-3=3-3=0$
$D_s=3+0=3$
Kinematic indeterminacy, $D_k=Nj-C$
$D_k=2\times8-3$
$D_k=13$
statically and kinematically indeterminate Choice (C)

25. Since no horizontal load acting at *c*, apply a fiction load '*Q*' at '*c*'.



Span *BC*:

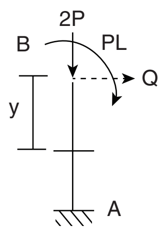


$M_X = -2PX$

$$\frac{\partial M_x}{\partial Q} = 0$$

$$(\delta_H)_{BC} = \int_0^{L/2} (M_x) \frac{\partial M_x}{\partial Q} \frac{dX}{EI} = 0$$

Span *AB*:



$M_y = PL + Qy$

$$\frac{\partial M_y}{\partial Q} = y \quad y = 0 \text{ to } L$$

$$(\delta H)_{AB} = \int_0^L (M_y)\left(\frac{\partial M_y}{\partial Q}\right)\frac{dy}{EI}$$

$$= \int_0^L (PL + Qy)(y)\frac{dy}{EI}$$

$$= \left[\frac{PLy^2}{2EI}\right]_0^L = \frac{PL^3}{2EI}$$

$$(\delta H)_c = (\delta H)_{Bc} + (\delta H)_{AB}$$

$$(\delta H)_c = \frac{PL^3}{2EI}$$

Choice (C)