Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. If a is rational and $\sqrt{b}$ is irrational, then $a + \sqrt{b}$ is: **[1]**

a) an irrational number b) an integer
c) a natural number d) a rational number

2. 120 can be expressed as a product of its prime factors as **[1]**

a) 15×2^3 b) $5 \times 2^3 \times 3$
c) $5 \times 8 \times 3$ d) $10 \times 2^2 \times 3$

3. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? **[1]**

a) -2 or 0 b) 0 only
c) 2 or 0 d) 2 or -2

4. The value of k for which the system of linear equations x + 2y = 3, 5x + ky + 7 = 0 is inconsistent is: **[1]**

a) $-\frac{14}{3}$ b) 5
c) $\frac{2}{5}$ d) 10

5. A quadratic equation whose one root is 3 is **[1]**

a) $x^2 - 5x + 6 = 0$ b) $x^2 - 6x - 6 = 0$
c) $x^2 - 5x - 6 = 0$ d) $x^2 + 6x - 5 = 0$

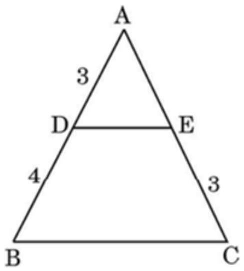
6. If (a, 0), (0, b) and (x, y) are collinear, then **[1]**

a) ay - bx = 1
b) ax + by = 1
c) ay + bx = ab
d) ax - by = ab

7. $\triangle$ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. If $\triangle DEF \sim \triangle ABC$ and EF = 4 cm, then perimeter of $\triangle$DEF is **[1]**

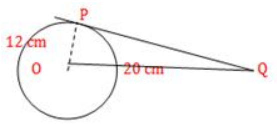
a) 30 cm
b) 15 cm
c) 22.5 cm
d) 7.5 cm

8. In the given figure, DE || BC and all measurements are given in centimetres. The length of AE is: **[1]**



a) 2.75 cm
b) 2.5 cm
c) 2 cm
d) 2.25 cm

9. A tangent PQ at point of contact P to a circle of radius 12 cm meets the line through centre O to a point Q such that OQ = 20 cm, length of tangent PQ is: **[1]**



a) 15 cm
b) 12 cm
c) 13 cm
d) 16 cm

10. If $\sqrt{3}\tan 2\theta - 3 = 0$ then θ = ? **[1]**

a) 30^o
b) 60^o
c) 15^o
d) 45^o

11. There is a small island in the middle of a 50 m wide river. A tall tree stands on the island. P and Q are points directly opposite to each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 60^o and 30^o, then find the height of the tree. **[1]**

a) 22.65 m
b) 23.56 m
c) 24.69 m
d) 21.65 m

12. If cos θ = $\frac{2}{3}$, then 2 $\sec^2\theta$ + 2 $\tan^2\theta$ - 7 is equal to **[1]**

a) 1
b) 4
c) 0
d) 3

13. The area of a quadrant of a circle whose circumference is 616 cm will be **[1]**

a) 7546 cm^2
b) 7500 cm^2
c) 7564 cm^2
d) 7456 cm^2

14. Find the area of the sector if the radius is 5 cm and with an angle of 50^o. **[1]**

a) 10.90 cm
b) 12.90 cm
c) 13.90 cm
d) 11.90 cm

15. One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a black face card? **[1]**

a) $\frac{3}{13}$
b) $\frac{3}{14}$
c) $\frac{3}{26}$
d) $\frac{1}{26}$

16. In a data, if l = 60, h = 15, f_1 = 16, f_0 = 6, f_2 = 6, then the mode is **[1]**

a) 67.5
b) 72
c) 60
d) 62

17. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. The volume of the solid is **[1]**

a) $\pi\ cm^3$
b) $4\pi\ cm^3$
c) $2\pi\ cm^3$
d) $3\pi\ cm^3$

18. The median class for the data given below is: **[1]**

Class	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	12	14	13	17

a) 80 - 100
b) 60 - 80
c) 20 - 40
d) 40 - 60

19. **Assertion (A):** Distance of point (a, b) from origin is $\sqrt{b^2 - a^2}$ **[1]**
Reason (R): Distance of point (x, y) from origin is $\sqrt{(x-0)^2 + (y-0)^2}$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** L.C.M. and H.C.F. of a and 20 are 100 and 10 respectively, then a = 50. **[1]**
Reason (R): L.C.M $\times$ H.C.F. = First number $\times$ Second number

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

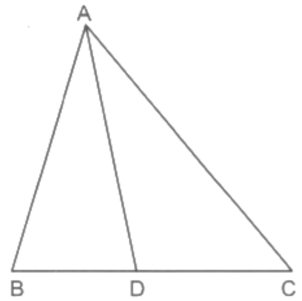
Section B

21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincide: $6x - 3y + 10 = 0$; $2x - y + 9 = 0$. **[2]**

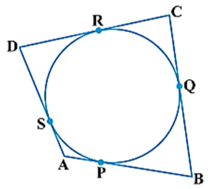
22. In $\triangle$ABC, D and E are the points on the sides AB and AC respectively such that DE||BC. If AD = 6x - 7, DB = 4x - 3, AE = 3x - 3 and EC = 2x - 1, find the value of x. **[2]**

OR

In Fig. check whether AD is the bisector of $\angle A$ of ΔABC if AB =6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm

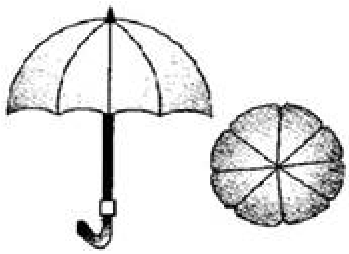


23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC **[2]**



24. If sin $\alpha = \frac{1}{\sqrt{2}}$ and cot $\beta = \sqrt{3}$, then find the value of cosec α + cosec β. **[2]**

25. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, Find the area between the two consecutive ribs of the umbrella. **[2]**



OR

Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm.

Section C

26. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers. **[3]**

27. Find the zeroes of the given quadratic polynomials and verify the relationship between the zeroes and the coefficients.$6x^2 - 3 - 7x$ **[3]**

28. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the number. Find the number. Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.

29. ABCD is a quadrilateral such that ∠D = 90°. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, Find r. **[3]**

30. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **[3]**

OR

Prove: $\frac{1}{(\cot A)(\sec A) - \cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A) + \cot A}$

31. Two different dice are rolled together. Find the probability of getting (i) the sum of numbers on two dice to be 5, (ii) even number on both dice, (iii) a doublet. **[3]**

Section D

32. A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it, having an area of 111 sq m. Find the width of the path. **[5]**

OR

If the price of a book is reduced by ₹5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

33. If BD and QM are medians of triangles ABC and PQR, respectively, where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{BD}{QM}$. **[5]**

34. A solid is in the shape of a cone surmounted on a hemisphere with both their diameters being equal to 7 cm and the height of the cone is equal to its radius. Find the volume of the solid. **[5]**

OR

A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm, the radius of the hemisphere is 60 cm and height of the cone is 120 cm, assuming that the hemisphere and the cone have common base.

35. The following table gives the distribution of the life time of 400 neon lamps: **[5]**

Lite time (in hours)	**Number of lamps**
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life time of a lamp.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Saving money is a good habit and it should be inculcated in children right from the beginning. Rehan's mother brought a piggy bank for Rehan and puts one ₹ 5 coin of her savings in the piggy bank on the first day. She increases his savings by one ₹ 5 coin daily.

Based on the above information, answer the following questions:

i. How many coins were added to the piggy bank on 8^{th} day?

ii. How much money will be there in the piggy bank after 8 days?

iii. a. If the piggy bank can hold one hundred twenty ₹ 5 coins in all find the number of days she can contribute to put ₹ 5 coins into it.

OR

b. Find the total money saved, when the piggy bank is full.

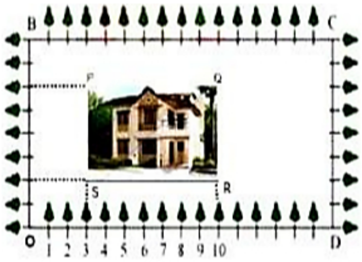
37. **Read the following text carefully and answer the questions that follow:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

In Green Park, New Delhi Suresh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. In the plot, Suresh builds his house in the rectangular area PQRS. In the remaining part of plot, Suresh wants to plant grass.



i. Find the coordinates of the midpoints of the diagonal QS. (1)
ii. Find the length and breadth of rectangle PQRS? (1)
iii. Find Area of rectangle PQRS. (2)

OR

Find the diagonal of rectangle. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Two trees are standing on flat ground. The angle of elevation of the top of Both the trees from a point X on the ground is 60^{o}. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m.

i. Calculate the distance between the point X and the top of the smaller tree. (1)
ii. Calculate the horizontal distance between the two trees. (1)
iii. Find the height of big tree. (2)

OR

Find the height of small tree. (2)

Solution

Section A

1. **(a)** an irrational number

   Explanation: Let a be rational and $\sqrt{b}$ is irrational.

   If possible let $a+\sqrt{b}$ be rational.

   Then $a+\sqrt{b}$ is rational and a is rational.

   $\Rightarrow \left[\left(a+\sqrt{b}\right)-a\right]$ is rational [Difference of two rationals is rational]

   $\Rightarrow \sqrt{b}$ is rational.

   This contradicts the fact that $\sqrt{b}$ is irrational.

   The contradiction arises by assuming that $a+\sqrt{b}$ is rational.

   Therefore, $a+\sqrt{b}$ is irrational.

2. **(b)** $5 \times 2^3 \times 3$

   Explanation: We have,

   $120 = 5 \times 2^3 \times 3$

3. **(d)** 2 or -2

   Explanation: Since the roots are equal, we have D = 0.

   $\therefore 36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

4. **(d)** 10

   Explanation: For a system of equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ to have no solution, the condition to be satisfied is

   $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

   $\Rightarrow \frac{1}{5}=\frac{2}{k}\neq\frac{-3}{7}$

   $\therefore$ For k = 10, the given system of equation is inconsistent.

5. **(a)** $x^2 - 5x + 6 = 0$

   Explanation: since 3 is the root of the equation, x = 3 must satisfy the equation.

   Applying x = 3 in the equation $x^2 - 5x + 6 = 0$

   gives, $(3)^2 - 5(3) + 6 = 0$

   $\Rightarrow 9 - 15 + 6 = 0$

   $\Rightarrow 15 - 15 = 0$

   $\Rightarrow 0 = 0$

   $\Rightarrow$ L.H.S. = R.H.S.

   Hence, $x^2 - 5x + 6 = 0$ is a required equation which has 3 as root.

6. **(c)** ay + bx = ab

   Explanation: If given points are collinear, then the area of the triangle formed by these three points is 0.

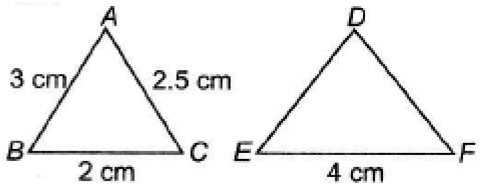
   $\therefore$ Area $= \frac{1}{2}|a(b - y) + 0(y - 0) + x(0 - b)| = 0$

   $\Rightarrow \frac{1}{2}|ab - ay - bx| = 0$

   $\Rightarrow ab - ay - bx = 0$

   $\Rightarrow ay + bx = ab$

7. **(b)** 15 cm

Explanation: 

$\triangle \mathrm{DEF} \sim \triangle \mathrm{ABC}$

AB = 3CM, BC = 2 CM, CA = 2.5 CM, EF = 4CM

Since $\triangle'$s are similar, we have

$\frac{\mathrm{DE}}{\mathrm{AB}}=\frac{\mathrm{EF}}{\mathrm{BC}}=\frac{\mathrm{FD}}{\mathrm{CA}}$

$\Rightarrow \frac{\mathrm{DE}}{3}=\frac{4}{2}=\frac{\mathrm{FD}}{2.5}$

Now $\frac{\mathrm{DE}}{3}=\frac{4}{2}$

$\Rightarrow \mathrm{DE}=\frac{3 \times 4}{2}=6 \mathrm{cm}$

and $\mathrm{FD}=\frac{4}{2} \Rightarrow \mathrm{FD}=\frac{4 \times 2.5}{2}=5 \mathrm{cm}$

perimeter of $\triangle DEF$

= 6 + 4 + 5 = 15cm

8.

(d) 2.25 cm

Explanation: By BPT

$\frac{AD}{DB}=\frac{AE}{EC}$

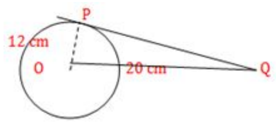
$\frac{3}{4}=\frac{AE}{3}$

AE = $\frac{9}{4}$

AE = 2.25 cm

9.

(d) 16 cm

Explanation: 

Since op is perpendicular to PQ, the $\angle$OPQ = 90°

Now, in right angled triangle OPQ,

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow (20)^2 = (12)^2 + PQ^2$

$\Rightarrow PQ^2 = 400 - 144$

$\Rightarrow PQ^2 = 256$

$\Rightarrow$ PQ = 16 cm

10. **(a)** 30^o

Explanation: $\sqrt{3} \tan 2\theta-3=0$

$\Rightarrow \sqrt{3} \tan 2\theta=3$

$\Rightarrow \tan 2\theta=\frac{3}{\sqrt{3}}$

$\Rightarrow \tan 2\theta=\sqrt{3}$

$\Rightarrow \tan 2\theta=\tan 60^{\circ}$

$\Rightarrow 2\theta=60^{\circ}$

$\Rightarrow \theta=30^{\circ}$

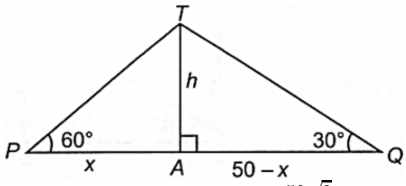
11.

(d) 21.65 m

Explanation: Let the height of the tree be h.

In $\triangle$PAT, tan 60^o = $\frac{h}{x}$ $\Rightarrow$ $\sqrt{3}$ = $\frac{h}{x}$ $\Rightarrow$ h = $\sqrt{3}$ x

In $\triangle$QAT, tan 30^o = $\frac{h}{50-x}$ $\Rightarrow$ $\frac{1}{\sqrt{3}}$ = $\frac{h}{50-x}$



$\Rightarrow \sqrt{3}$h = 50 - $\frac{h}{\sqrt{3}}$ $\Rightarrow$ h = $\frac{50\sqrt{3}}{4}$ = 21.65 m [$\because$ x = $\frac{h}{\sqrt{3}}$]

$\Rightarrow$ The height of the tree is 21.65 m

12.

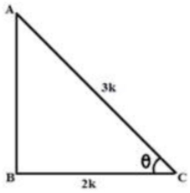
(c) 0

Explanation: Given,

cos θ = $\frac{2}{3}$ = $\frac{b}{h}$ = k

2sec^2 θ + 2 tan^2 θ - 7

b = 2k, h = 3k



In $\triangle$ABC,

h^2 = p^2 + b^2

$\Rightarrow$ (3k)^2 = p^2 + (2k)^2

$\Rightarrow$ 9k^2 = p^2 + 4k^2

$\Rightarrow$ p^2 = 9k^2 – 4k^2

$\Rightarrow$ p^2 = 5k^2

$\Rightarrow$ p = √5k

Then,

sec θ = $\frac{3k}{2k} = \frac{3}{2}$ and tan θ = $\frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$

⇒ 2 sec^2 θ + 2 tan^2 θ - 7

$\Rightarrow 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7$

$\Rightarrow 2 \times \frac{9}{4} + 2 \times \frac{5}{4} - 7$

$\Rightarrow \frac{9}{2} + \frac{5}{2} - 7$

$\Rightarrow \frac{9+5-14}{2} = 0$

13. **(a)** 7546 cm^2

Explanation: 2πR = 616

R = $\frac{(616\times7)}{(2\times22)}$

R = 98 cm

Area of quadrant = $\frac{\pi r^2}{4}$

= $\frac{(22\times98\times98)}{(7\times4)}$

= 7546 cm^2

14. **(a)** 10.90 cm

Explanation: The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{50^\circ}{360^\circ} \times \frac{22}{7} \times 5^2$

= 10.90 cm

15.

(c) $\frac{3}{26}$

Explanation: Total number of cards = 52.

Number of black face cards = 6

(2 kings + 2 queens + 2 jacks).

$\therefore$ P (getting a face card) = $\frac{6}{52} = \frac{3}{26}$

16. **(a)** 67.5

Explanation: Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= 60 + $\frac{16-6}{2\times16-6-6} \times 15$

= 60 + $\frac{10}{32-12} \times 15$

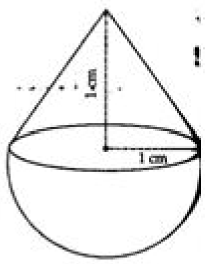
= 60 + $\frac{10}{20} \times 15$

= 60 + 7.5

= 67.5

17. **(a)** $\pi\ cm^3$

Explanation:



Radii of cone = r = 1 cm

Radius of hemisphere = r = 1 cm (h) = 1cm

Height of cone $(h) = 1$ h =1 cm

Volume of solid = Volume of cone + Volume of a hemisphere

= $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ = $\frac{1}{3}\pi r^2 (h + 2r)$

= $\frac{1}{3} \times \pi \times (1)^2 (1 + 2 \times 1)$

= $\frac{1}{3} \times \pi \times 3$ = π cm^3

18.

(b) 60 - 80

Explanation: Total frequencies (N) = 10 + 12 + 14 + 13 + 17

= 66

So, $\frac{N}{2}$ = $\frac{66}{2}$ = 33

c.f. Just greater than 33 is 36 and the corresponding class is 60 - 80

hence, median class = 60 - 80

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: It will be $\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: It is a result.

Section B

21. Given equations are

6x − 3y + 10 = 0

2x – y + 9 = 0

Comparing equation 6x − 3y + 10 = 0 with $a_1x + b_1y + c_1 = 0$

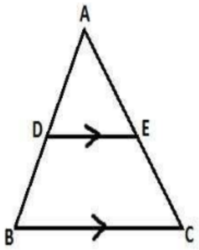
and 2*x* – *y* + 9 = 0 with

$a_2x + b_2y + c_2 = 0$,

We get, a_1 = 6, b_1 = -3, c_1 = 10, a_2 = 2, b_2 = -1, c_2 = 9

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9} \Rightarrow \frac{3}{1} = \frac{3}{1} \neq \frac{10}{9}$

Hence, lines are parallel to each other.

22. 

Given: In $\triangle$ABC, DE || BC. Also AD = 6x - 7, DB = 4x - 3, AE = 3x - 3 and EC = 2x - 1

By basic proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{6x-7}{4x-3} = \frac{3x-3}{2x-1}$

$\Rightarrow$ (6x - 7)(2x - 1) = (3x - 3)(4x - 3)

$\Rightarrow$ 12x^2 - 6x - 14x + 7 = 12x^2 - 9x - 12x + 9

$\Rightarrow$ -20x + 7 = -21x + 9

$\Rightarrow$ -20x + 21x = 9 - 7

$\Rightarrow$ x = 2

OR

It is given that, AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm

We have to check whether AD is bisector of $\angle A$

First we will check proportional ratio between sides

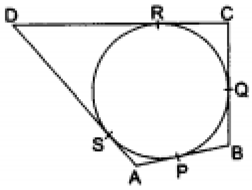
So, $\frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \frac{6}{8} = \frac{1.5}{2}$

$\Rightarrow \frac{3}{4} = \frac{3}{4}$

Therefore, the sides are proportional.

Hence, AD is bisector of $\angle A$

23. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

AP = AS, ... (i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]

DR = DS. ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC.

Hence, AB + CD = AD + BC.

24. cosec α = $\frac{1}{\sin\alpha}$ = $\sqrt{2}$

cosec β = $\sqrt{1+\cot^2\beta}$ = $\sqrt{1+3}$ = 2

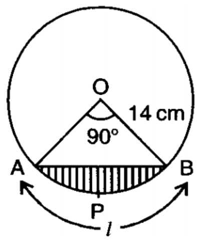
$\therefore$ cosec α + cosec β = $\sqrt{2}+2$ or $\sqrt{2}(\sqrt{2}+1)$

25. Here, r = 45 cm and $\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}$ cm ^{2}

OR



l = APB = 22 cm

$\frac{\theta}{180^\circ} \times \frac{22}{7} \times 14 = 22\text{cm}$

$\Rightarrow \quad \theta = 90^\circ$

Area of the sector = $\frac{lr}{2} = \frac{22 \times 14}{2}$ = 154 cm^2

Area of triangle AOB= $\frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times 14 \times 14$ = 98 cm^2

Area of the segment = (154 - 98) cm^2 = 56 cm^2

Section C

26. Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

7 × 11 × 13 + 13 = 13 × (7 × 11 + 1)

= 13 × (77 + 1)= 13 × 78= 13 ×13 × 6

The given expression has 6 and 13 as its factors.

Therefore, it is a composite number.

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5

= 5 ×(7 × 6 × 4 × 3 × 2 × 1 + 1)

= 5 × (1008 + 1)= 5 ×1009

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

Hence, it is a composite number.

27. Let p(x) = $6x^2$ - 3 - 7x

For zeroes of p(x),

p(x) = 0

⇒ $6x^2$ - 3 - 7x = 0

⇒ $6x^2$ - 7x - 3 = 0

⇒ $6x^2$ - 9x + 2x - 3 = 0

⇒ 3x(2x - 3) + (2x - 3) = 0

⇒ (2x - 3) (3x + 1) = 0

⇒ 2x - 3 = 0 or 3x + 1 = 0

$\Rightarrow x = \frac{3}{2} \text{ or } x = -\frac{1}{3} \Rightarrow x = \frac{3}{2}, -\frac{1}{3}$

So, the zeroes of p(x) are $\frac{3}{2}$ and $-\frac{1}{3}$

We observe that Sum of its zeroes

$= \frac{3}{2} + \left(-\frac{1}{3}\right) = \frac{3}{2} - \frac{1}{3}$

$= \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of its zeroes = $\left(\frac{3}{2}\right) \times \left(-\frac{1}{3}\right)$

= -$\frac{1}{2} = -\frac{3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

28. Let the unit's digit and the ten's digit in the two-digit number be x and y respectively.

Then the number = 10y + x

Also, the number obtained by reversing the order of the digits = 10x + y

According to the question,

x + y = 9...............(1)

9(10y + x) = 2(10x + y)

⇒ 90y + 9x = 20x + 2y

⇒ 11x - 88y = 0

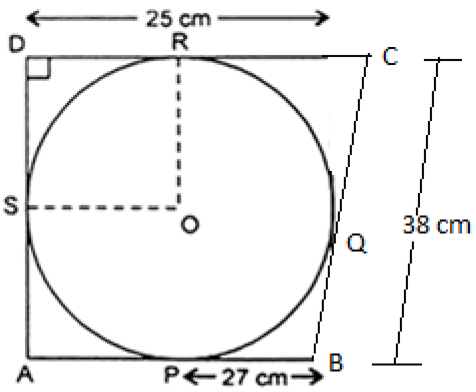
⇒ x - 8y = 0 ....(2)
Subtracting equation(2) from equation(1), we get
9y = 9
⇒ $y = \frac{9}{9} = 1$
Substituting this value of y in equation (1), we get
x + 1 = 9
⇒ $x = 9 - 1 = 8$
Hence, the required number is 18.
Verification: substituting x = 8 and y = 1,
we find that both the equations (1) and (2) are satisfied as shown below:
x + y = 8 + 1 = 9
x - 8y = 8 - 8(1) = 0
Hence, the solution is correct.

OR

Let the digits at units and tens place of the given number be x and y respectively.
Then,
Number =$10y + x$ ....(i)
Number obtained by reversing the order of the digits = $10x + y$
According to the question,
$(10y + x) + (10x + y) = 165$
⇒ $x + y = 15$
and, $x - y = 3$
Thus, we obtain the following systems of linear equations.

i. $x + y = 15$
   $x - y = 3$
ii. $x + y = 15$
   $y - x = 3$

Solving first system of equations, we get
$x = 9, y = 6$
Solving second system of equation, we. get
$x = 6, y = 9$
Substituting the values of x and y in equation (i), we have
Number = $69\ or,\ 96$.

29. 

Given that ABCD is a quadrilateral such that ∠D = 90°.
BC = 38 cm,CD = 25cm and BP = 27 cm
∴ From the figure,
BP = BQ = 27 cm [Tangents from an external point are equal]
Now, BC = 38
⇒ BQ + QC = 38
⇒ 27 + QC = 38
⇒ QC = 38 - 27
⇒ QC = 11 cm

∴ QC = 11 cm = CR [Tangents from an external point are equal]

CD = 25 cm

CR + RD=25

⇒ 11+ RD = 25

⇒ RD = 25-11

⇒ RD = 14 cm

Also,

RD = DS = 14 cm [Tangents from an external point are equal]

OR and OS are radii of the circle.

From tangents R and S, ∠ORD = ∠OSD = 90°

Thus, ORDS is a square.

OR = DS = 14 cm

Hence, the radius of the circle, r = OR = 14 cm

30. We have to prove that, $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$ using identity $sec^2\theta=1+tan^2\theta$

LHS = $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]

$=\frac{(\tan\theta+\sec\theta)-1}{(\tan\theta-\sec\theta)+1}=\frac{\{(\tan\theta+\sec\theta)-1\}(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ [Multiplying and dividing by $(\tan\theta-\sec\theta)$]

$=\frac{(\tan^2\theta-\sec^2\theta)-(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ $[\because (a-b)(a+b)=a^2-b^2\ ]$

$=\frac{-1-\tan\theta+\sec\theta}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$ $[\because \tan^2\theta-\sec^2\theta=-1\ ]$

$=\frac{-(\tan\theta-\sec\theta+1)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}=\frac{-1}{\tan\theta-\sec\theta}$

$=\frac{1}{\sec\theta-\tan\theta}$ =**RHS**

Hence Proved.

OR

To prove-

$\frac{1}{(\text{cot A})(\text{sec A})-\text{cot A}}-\text{cosec A}=\text{cosec A}-\frac{1}{(\cot A)(\text{sec A})+\cot A}$

Taking LHS

$=\frac{1}{(\text{cot A})(\text{sec A})-\text{cot A}}-\text{cosec A}$

$=\frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})-\left(\frac{\cos A}{\sin A}\right)}-\frac{1}{\sin A}$

$\frac{1}{(\frac{1}{\sin A})-\left(\frac{\cos A}{\sin A}\right)}-\frac{1}{\sin A}=\frac{1}{\frac{1-\cos A}{\sin A}}-\frac{1}{\sin A}=\frac{\sin A}{1-\cos A}-\frac{1}{\sin A}=\frac{\sin^2 A-1+\cos A}{(1-\cos A)\sin A}$

$=\frac{-\cos^2 A+\cos A}{(1-\cos A)\sin A}=\frac{\cos A(1-\cos A)}{(1-\cos A)\sin A}$ $\{\because \sin^2\text{A} + \cos^2\text{A} = 1\}$

$=\frac{\cos A}{\sin A}=cotA$

Now, taking RHS

$=\text{cosec A}-\frac{1}{(\cot A)(\text{sec A})+\cot A}$

$=\frac{1}{\sin A}-\frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})+\frac{\cos A}{\sin A}}$

$=\frac{1}{\sin A}-\frac{1}{(\frac{1}{\sin A})+\frac{\cos A}{\sin A}}=\frac{1}{\sin A}-\frac{\sin A}{(1+\cos A)}$

$=\frac{1+\cos A-\sin^2 A}{(1+\cos A)\sin A}=\frac{\cos^2 A+\cos A}{(1+\cos A)\sin A}$

$=\frac{\cos A(\cos A+1)}{(1+\cos A)\sin A}=\frac{\cos A}{\sin A}$

= cotA = LHS

31. When two dice are thrown simultaneously, all possible outcomes are

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

Number of all possible outcomes = 36.

i. Let E_1 be the event of getting two numbers whose sum is 5.

Then, the favourable outcomes are (1,4) (2,3), (3,2), (4,1). Number of favourable outcomes = 4.

$\therefore$ P(getting two numbers whose sum is 5)$= P\left(E_2\right) = \frac{4}{36} = \frac{1}{9}$

ii. Let E_2 be the event of getting even numbers on both dice.

Then, the favourable outcomes are

(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6). Number of favourable outcomes = 9.

$\therefore$ P(getting even number on both dice) $= P\left(E_2\right) = \frac{9}{36} = \frac{1}{4}$

iii. Let E_3 be the event of getting a doublet.

Then, the favourable outcomes are

(1,1), (2,2), (3,3), (4,4), (5,5), (6,6).

Number of favourable outcomes = 6.

$\therefore$ P(getting a doublet) = P) $= P\left(E_3\right) = \frac{6}{36} = \frac{1}{6}$.

Section D

32. Let the width of the path be x m

Length of the field including the path = (20 + 2x) m

Breadth of the field including the path = (14 + 2x) m.

Area of rectangle = L $\times$ B

Area of the field including the path = (20 + 2x) (14 + 2x) m^2 .

Area of the field excluding the path = (20 $\times$ 14) m^2= 280 m^2.

$\therefore$ Area of the path = (20 + 2x)(14 + 2x) - 280

(20 + 2x) (14 + 2x) - 280 = 111

$\Rightarrow 4x^2 + 68x - 111 = 0$

Factorise the equation,

$\Rightarrow 4x^2 + 74x - 6x - 111 = 0$

$\Rightarrow$ 2x(2x + 37) - 3(2x + 37) = 0

$\Rightarrow (2x + 37)(2x - 3) = 0$

$\Rightarrow$ x = $-\frac{37}{2}$ or x = $\frac{3}{2}$

As width can't be negative.

$\Rightarrow$ x = $\frac{3}{2}$ = 1.5

Therefore, the width of the path is 1.5 m.

OR

Let the original list price be Rs x

$\therefore$ No. of books bought for Rs 300 = $\frac{300}{x}$

Reduced list price of the book = Rs (x - 5)

No. of books bought for Rs 300 = $\frac{300}{x-5}$

According to question,

$\frac{300}{x-5} - \frac{300}{x} = 5$

$\Rightarrow \frac{300x - 300x + 1500}{x^2 - 5x} = 5$

$\Rightarrow x^2 - 5x = 300 \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow x^2 - 20x + 15x - 300 = 0$

$\Rightarrow (x - 20)(x + 15) = 0$

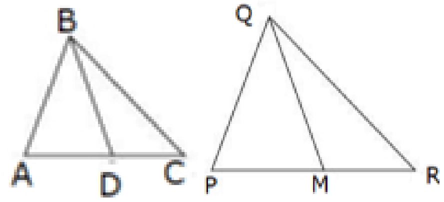
$\Rightarrow x = 20 \quad \text{or} \quad x = -15$

$\Rightarrow x = 20$

The negative sign is rejected.

Therefore x = 20

Therefore the original price list is Rs. 20

33. 

Given: $\triangle ABC \sim \triangle PQR$ and BD, QM are medians

To prove: $\frac{AB}{PQ} = \frac{BD}{QM}$

Proof: $\triangle ABC \sim \triangle PQR$ (given)

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{2AD}{2PM}$ (BD and QM are medians)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

In $\triangle ABD$ and $\triangle PQM$

$\frac{AB}{PQ} = \frac{AD}{PM}$ (proved above)

$\angle A = \angle P$ ($\triangle ABC \sim \triangle PQR$)

$\therefore \triangle ABD \sim \triangle PQM$ (SAS criteria)

$\therefore \frac{AB}{PQ} = \frac{BD}{QM}$ (C.P.S.T)

34. Radius of hemisphere = radius of cone = $\frac{7}{2} cm$

Height of cone = $\frac{7}{2} cm$

Volume of the solid = Volume of hemisphere + Volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(2 \times \frac{7}{2} + \frac{7}{2}\right)$

$= \frac{539}{4}\ cm^3$ or $134.75\ cm^3$

OR

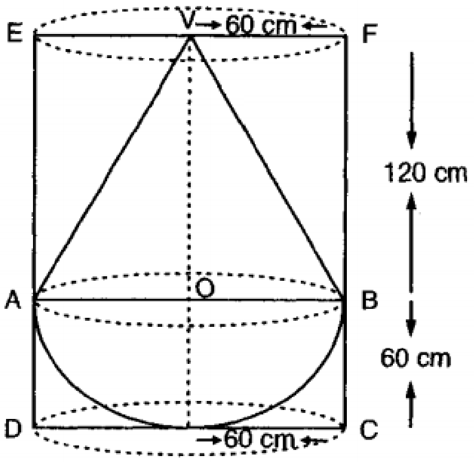
We have radius of cylinder = radius of cone = radius of hemisphere = 60 cm

Height of cone = 120 cm

$\therefore$ Height of cylindrical vessel = 120 + 60 =180 cm

$\therefore$ V = Volume of water that the cylinder contains = $\pi r^2 h = \left\{\pi \times (60)^2 \times 180\right\} cm^3$

Let V_1 be the volume of the conical part. Then,



$V_1 = \frac{1}{3}\pi r^2 h_1$

$\Rightarrow \quad V_1 = \frac{1}{3} \times \pi \times 60^2 \times 120 cm^3 = \left\{\pi \times 60^2 \times 40\right\} cm^3$

For hemispherical part r = Radius = 60 cm

Let V_2 be the volume of the hemisphere. Then,

$V_2 = \left\{\frac{2}{3}\pi \times 60^3\right\} cm^3$

$\Rightarrow \quad V_2 = \left\{2\pi \times 20 \times 60^2\right\} cm^3 = \left\{40\pi \cdot 60^2\right\} cm^3$

Let V_3 the the volume of the water left-out in the cylinder. Then,

V_3= V - V_1 - V_2

$V_3 = \left\{\pi \times 60^2 \times 180 - \pi \times 60^2 \times 40 - 40\pi \times 60^2\right\} cm^3$

$V_3 = \pi \times 60^2 \times \{180 - 40 - 40\}\text{cm}^3$

$V_3 = \frac{22}{7} \times 3600 \times 100\text{cm}^3$

$\Rightarrow \quad V_3 = \frac{22 \times 360000}{7}\text{cm}^3 = \frac{22 \times 360000}{7 \times (100)^3}\text{m}^3 = \frac{22 \times 36}{700}\text{m}^3 = 1.1314\text{m}^3.$

35.

Life time	**Number of lamps (f_i)**	**Cumulative frequency**
1500-2000	14	14
2000-2500	56	14 + 56 = 70
2500-300	60	70 + 60 = 130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290
4000-4500	62	290 + 62 = 352
4500-5000	48	352 + 48 = 400
	400	

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

Median = l + $\left(\frac{\frac{n}{2} - cf}{f}\right) \times$ h

Here,

l = Lower limit of median class

F = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

Lower limit (l) of median class = 3000

Frequency (f) of median class 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median = 3000 + $\left(\frac{200-130}{86}\right) \times$ 500

= 3000 + $\frac{70 \times 500}{86}$

= 3406.98

Section E

36. i. 8 coins

ii. Money in the piggy bank day wise 5, 10, 15, 20 ...

Money after 8 days = ₹ 180

iii. a. We can have at most 120 coins.

$\frac{\text{n}}{2}[2(1) + (\text{n} - 1)1] = 120$

$\text{n}^2 + \text{n} - 240 = 0$

Solving for n, we get, n = 15 as n $\neq$ -16

$\therefore$ Number of days = 15

OR

b. Total money saved = 120 $\times$ 5 = ₹ 600

37. i.

Middle point of QS = $\left(\frac{10+3}{2}, \frac{6+2}{2}\right)$

= (6.5, 4)

ii. Length = RS = $\sqrt{(10-3)^2 + (2-2)^2}$

RS = $\sqrt{7^2 + 0}$

RS = 7 m

Breadth = RQ = $\sqrt{(10-10)^2 + (2-6)^2}$

$= \sqrt{0 + 16}$

= 4 m

iii. Area of rectangle = l $\times$ b

= 7 $\times$ 4

= 28 m^2

OR

Diagonal = $\sqrt{l^2 + b^2}$

$= \sqrt{7^2 + 4^2}$

$= \sqrt{49 + 16}$

$= \sqrt{65}$

38. i. In $\triangle$DCX

tan 60^o = $\frac{DC}{CX}$

$\sqrt{3} = \frac{DC}{8}$

DC = $8\sqrt{3}$ m

DX = $\sqrt{DC^2 + CX^2}$

$= \sqrt{(8\sqrt{3})^2 + 8^2}$

$= \sqrt{192 + 64}$

$= \sqrt{256}$

= 16 m

Hence, distance between X and top of smaller tree is 16 m.

ii. In $\triangle$BAX

cos 60^o = $\frac{AX}{BX}$

$\frac{1}{2} = \frac{AC+8}{36}$

36 = 2AC + 16

20 = 2AC

$\frac{20}{2}$ = 10 AC

AC = 10

$\therefore$ horizontal distance between both trees is 10 m.

iii. Height of big tree = AB

$\therefore$ In $\triangle$BAX

tan 60^o = $\frac{AB}{AX} = \frac{AB}{18}$

AB = $18\sqrt{3}$ m

OR

Height of small tree = CD

In $\triangle$ CDX

tan 60^o = $\frac{CD}{CX}$

$\sqrt{3} = \frac{CD}{8}$

CD = $8\sqrt{3}$ m