CHAPTER 01

Stability and Indeterminacy

1.1 Support System

1.1.1 2-D Supports

(a) Fixed Support

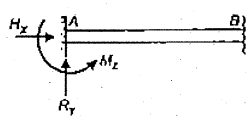


Fig. 1.1 (i) Number of reaction = 3

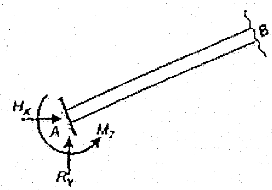


Fig. 1.1 (ii) Number of reactions = 3

At 2-D fixed support, there can be three reactions:

(*i*) one vertical reaction (R_y)

(*ii*) one horizontal reaction (H_x)

(*iii*) one moment reaction (M_z)

(b) Hinge Support

Hinge support is represented by the symbol $\triangle$.

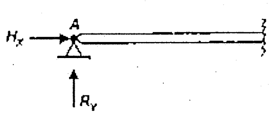


Fig. 1.2 (i) Number of reactions = 2

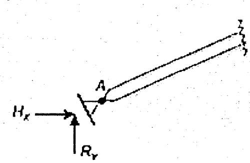


Fig. 1.2 (ii) Number of reactions = 2

At hinged support, there can be two reactions:

(*i*) one horizontal reaction (H_x)

(*ii*) one vertical reaction (R_y)

(c) Roller Support

Roller support is represented by the symbol or .

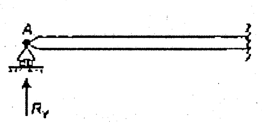


Fig. 1.3 (i) Number of reactions = 1

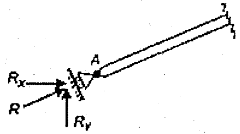


Fig. 1.3 (ii) Number of reactions = 1

At roller support there can be only one externally independent reaction which is normal to the contact surface.

(d) Guided Roller Support

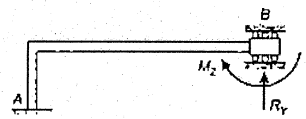


Fig. 1.4 Number of reactions = 2

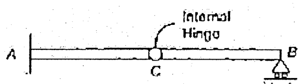
At guided roller supports there can be two reactions:

(*i*) one vertical reaction (R_y)   (*ii*) one moment reaction (M_z)

1.1.2 2-D Internal Joints

(a) Internal Hinge

At internal hinge bending moment will be zero.



$BM_C = 0$

Fig. 1.5

NOTE: An internal hinge provides one additional equilibrium equation for structures.

(b) Internal Roller

At internal roller either axially force or shear force will be zero.

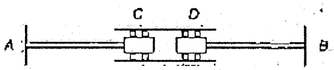


Fig. 1.6

In fig. 1.6, axially force at *C* and *D* is zero.

Fig. 1.7

In fig. 1.7, shear force at *C* and *D* will be zero i.e., $S_C = S_D = 0$

(c) Internal Link

If any member is connected by hinges at its end and subjected to no external loading in between then it can be termed as internal link and carry axial force only.

Here *BC* is a link, link *BC* carry only axial force

Also $BM_B = 0$ and $BM_C = 0$

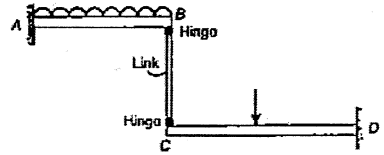


Fig. 1.8

NOTE: Internal release also provides additional equation for analysis of structure.

1.1.3 3-D Supports

(a) Fixed Support

At 3-D fixed support there can be six reactions:

(*i*) three reactions R_x, R_y and R_z

(*ii*) three moment reactions M_x, M_y and M_z

The fixed support are also called *Built-in support*.

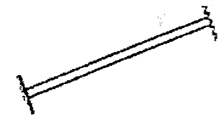


Fig. 1.9: Number of reactions = 6

(b) 3-D Hinged Support

At 3-D hinged support there can be three reactions

(*i*) R_x

(*ii*) R_y

(*ii*) R_z

The 3-D hinged support is also called 'ball and socket joint'.

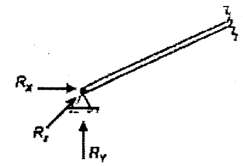


Fig. 1.10 Number of reactions = 3

(c) Roller Support

At 3-D roller support there can be only one externally independent reaction which is perpendicular to the contact surface

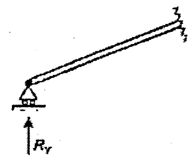


Fig. 1.11 Number of reactions = 1

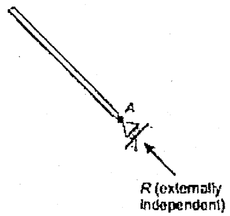


Fig. 1.12 (i)

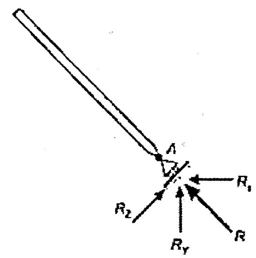


Fig. 1.12 (ii)

In figure 1.12.(ii), reactions at roller support *A*, R_x, R_y and R_z are externally dependent reactions which depends on reaction *R*.

1.2 Structure

1.2.1 Elements of Structure

Some of the major elements of structure by which structures are fabricated are as follows:

(a) Beams: Beams are structural members which is predominantly subjected to bending. On the basic of support system beams can be classified as:

(*i*) Simply supported beam

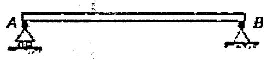


Fig. 1.13

(*ii*) Cantilever beam

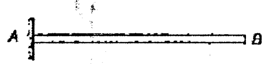


Fig. 1.14

(*iii*) Propped cantilever

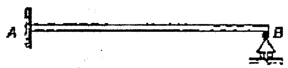


Fig. 1.15

(*iv*) Fixed beam

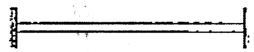


Fig. 1.16

(*v*) Continuous beam

Fig. 1.17

(b) Columns: A column is a vertical compression member which is slender and straight. Generally columns are subjected to axial compression and bending moment as shown in figure.

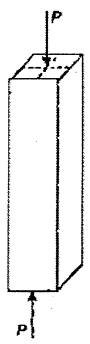


Fig. 1.18 (i)

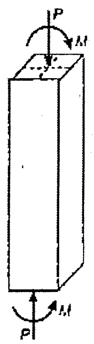


Fig. 1.18 (ii)

(c) Tie Members: Tie members are tension members of trusses and frame, which are subjected to axial tensile force. (Figure : 1.19)

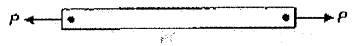


Fig. 1.19 Tie Rod

1.2.2 Types of Structures

(a) Trusses: A truss is constructed from pin jointed slender members, usually arranged in triangular manner. In trusses, loads are applied on joints due to which each member of truss subjected to only axial forces i.e . either axial compression or axial tension. Generally trusses are used when span of structure is large.

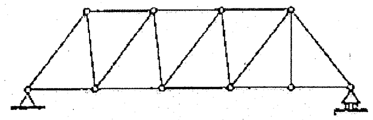


Fig. 1.20 Truss

(b) Frames: A frame is constructed from either pin jointed or fixed jointed beam and columns. Generally loads are applied on beams and this loading causes axial force, shear force and bending to the members of frame.

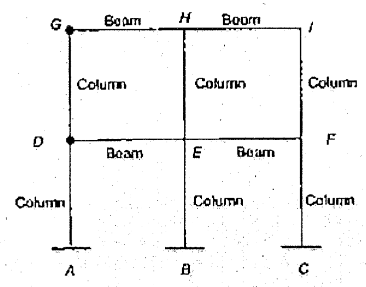


Fig. 1.21 Frames

(c) Arches: Arches are used in bridges, dome roof, auditorium, where span of structures are relatively more due to external loading, Arch can be subjected to axial compression, shear force or bending moment

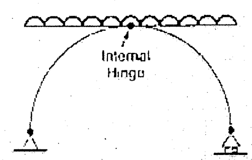


Fig. 1.22 (I) Three Hinge Arch

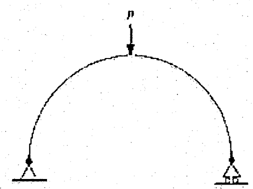


Fig. 1.22 (II) Two Hinge Arch

(d) **Cables:** Cables are used to support long span bridges. Cables are flexible members and due to external loading it is subjected to axial tension only.

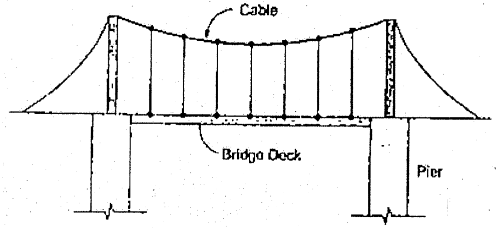


Fig. 1.23 Cable and Bridge

1.3 Types of Loading

(a) **Point load:** A point load is considered to be acting at a point. It is also called concentrated load. In actual practice point loads are distributed load which are distributed over very small area.

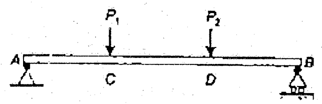


Fig. 1.24 Point Load

(b) **Distributed loads:** Distributed loads are those loads, which acts over some measurable area. Distributed loads are measured by the intensity of loading per unit length along the beam.

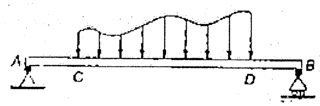


Fig. 1.25 Distributed Loads

(c) **Uniformly distributed loads:** Uniformly distributed loads are those distributed loads which have uniform intensity of loading over the area.

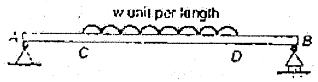


Fig. 1.26 Uniformly Distributed Loads

(d) **Uniformly varying loads :** A uniformly varying load, commonly abbreviated as UVL, is the one in which the intensity of loading varies from one end to other. For example, intensity is zero at one end and *w* at other end.

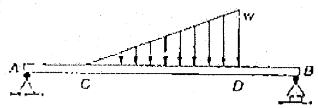


Fig. 1.27 Uniformly Varying Loads

(e) **Couple :** A system of forces with resultant moment, but no resultant force is called couple. It is statically equivalent to force times the offset distance.

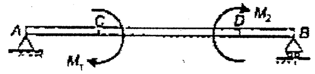


Fig. 1.28 Couple

1.4 Stability of Structures

Structural stability is the major concern of the structural designer. To ensure the stability, a structure must have enough support reaction along with proper arrangement of members. The overall stability of structures can be divided into

(*i*) External stability (*ii*) Internal stability

1.4.1 External Stability

(a) **2-D Structures:** For stability of 2-D structures there should be no rigid body movement of structure due to loading so, it should have support in *x*-direction, *y*-direction and no rotation in *x*-*y* plane. So there should be enough reactions to restrain the rigid body motion.

For stability of 2-D structures, following three conditions of static equilibrium should be satisfied.

(*i*) $\Sigma F_x = 0$ (To prevent Δ_x)

(*ii*) $\Sigma F_y = 0$ (To prevent Δ_y)

(*iii*) $\Sigma M_z = 0$ (To prevent θ_z)

For stability in 2-D structures following conditions also be satisfied:

(*i*) There should be minimum three number of externally independent support reaction.

(*ii*) All reactions should not be parallel, otherwise linearly unstability will set up.

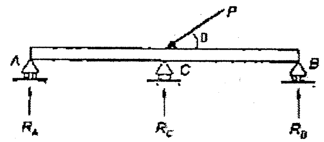


Fig. 1.29 Unstable

(*iii*) All reactions should not be linearly concurrent otherwise rotational unstability will setup.

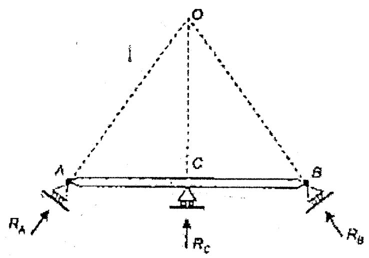


Fig. 1.30 (i) Unstable

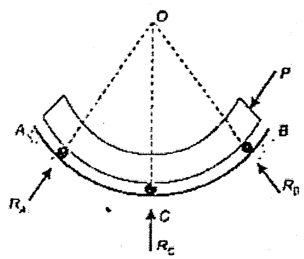


Fig. 1.30 (ii) Unstable

(iv) Reactions should be non-trival i.e. there should be enough magnitude and enough difference between them.

(b) 3-D Structures: In case of 3-D structures, there should be a minimum of six independent external reactions to prevent rigid body displacement of structure. The displacement to be prevented are: $\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y$ and θ_z. Therefore, there will be six equation of static equilibrium.

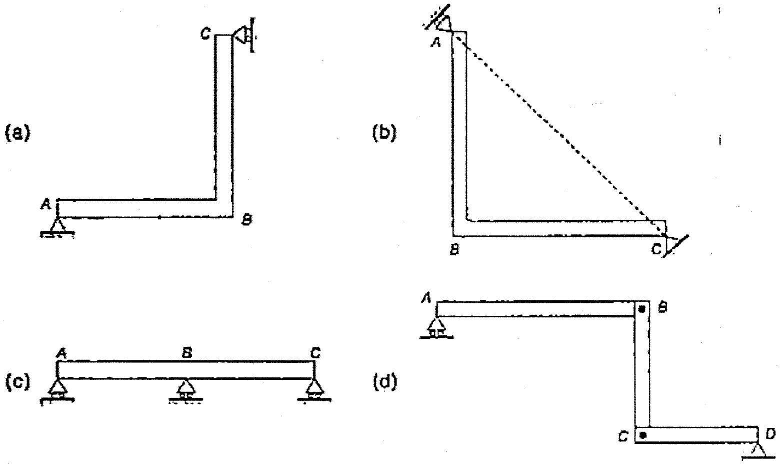
(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$ (iii) $\Sigma F_z = 0$

(iv) $\Sigma M_x = 0$ (v) $\Sigma M_y = 0$ (vi) $\Sigma M_y = 0$

For stability in 3-D structures, all the reactions should be non-coplanar, non-concurrent and non-parallel.

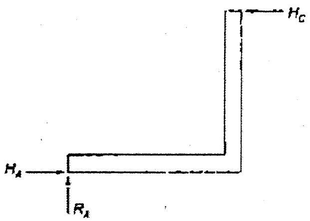
Remember: If a structure is constructed from elastic members then small elastic displacement may be permitted but small rigid body displacement will not be permitted.

Example 1.1 Which one of the following structures is stable?

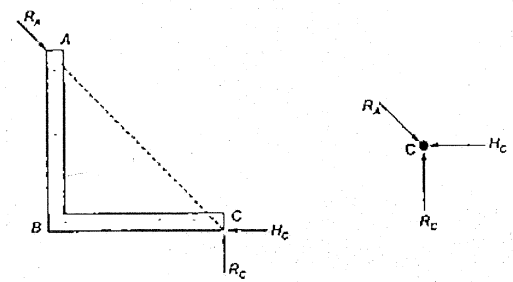


Ans. (a)

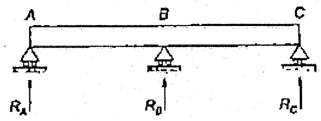
Member (a) is stable, since reactions are non-parallel and non-concurrent.



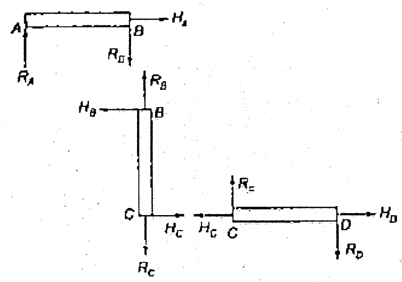
Member (b) is unstable since all the reactions are concurrent at C.



Beam (c) is unstable, since all three reactions are parallel.



Structure (d) is unstable, since the member AB can move horizontally without any restrain, i.e. $\Sigma F_x \neq 0$



1.4.2 Internal Stability

For the internal stability, no part of the structure can move rigidly relative to the other part so that geometry of the structure is preserved, however small elastic deformations are permitted. To preserve geometry, enough number of members and their adequate arrangement is required. For the geometric stability, there should not be any condition of mechanism. Mechanism is formed when there are three collinear hinges, hence to preserve geometric stability there should not be three collinear hinges.

Fig. 1.31

For 2-D truss the minimum number of members needed for geometric stability are:

$$m = 2j - 3$$

and for 3-D truss,

$$m = 3j - 6$$

where, j = Number of joint in truss

m = Member required for geometrical stability.

All the members should be arranged in such a way that truss can be divided into triangular blocks. i.e. no rectangular or polygonal blocks.

Hence, for overall geometrical stability of truss:

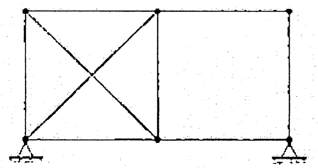
(*i*) Minimum number of member should be present

$m = 2j - 3$ (2-D truss)

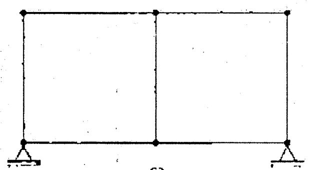
and $m = 3j - 6$ (3-D truss)

(*ii*) There should be no condition of mechanism i.e. no three collinear hinges.

Example 1.2 Check geometrical stability for given trusses.



(*i*)



(*ii*)

Solution:

(*i*) In case (*i*), arrangement of members is not adequate, hence right panel is unstable and left panel is over stiff. For geometric stability, all panels of truss should be stable so given truss is geometrical unstable.

For right panel: $j = 4$

Number of member present, $m = 4$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence Right panel is deficient.

For left panel: $j = 4$

Number of member present, $m = 6$

But minimum number of member needed $= 2j - 3 = 2 \times 4 - 3 = 5$

Hence left panel is over stiff.

(*ii*) $j = 6$

Number of members present, $m = 7$

But minimum number of member needed $= 2j - 3 = 2 \times 6 - 3 = 9$

Hence, above truss is geometrically unstable and it can be called 'deficient structure'.

∴ Number of deficiency = 2

1.4.3 Overall Stability

For overall stability, external stability is compulsory. In some cases structure is overall stable but it may be over stiff externally or deficient internally. It mean support reactions are more than three and number of member are less then $2j - 3$.

Consider a truss shown in figure 1.32.

Here,

External reaction, $r_e = 4$

Number of member present, $m = 10$

But min. number of members needed

$= 2j - 3 = 2 \times 7 - 3$

$= 11$

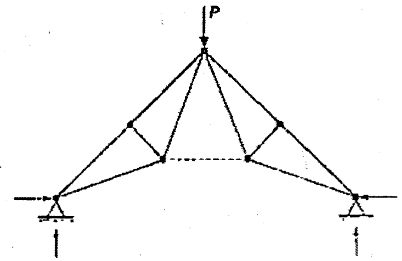


Fig. 1.32

It means truss is deficient to 1 degree.

But above truss is overall stable because there is one extra redundant reaction which prevent geometric deficiency.

In above fig. 1.33, external reactions,

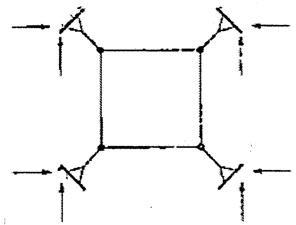


Fig. 1.33

$r_e = 8$

Number of member present, $m = 8$

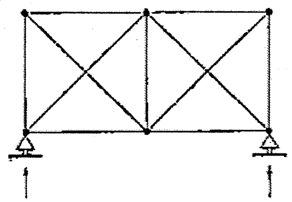
But min. no. of member needed$= 2j - 3 = 2 \times 8 - 3 = 13$

it means truss is deficient to 5 degree

But above structure is overall stable because there are five extra redundant reaction which prevent geometric deficiency.

Example 1.3 Comment on the stability of pin-jointed frame shown in figure.

Solution:



External reaction, $r_e = 2$

No. of member present = 11

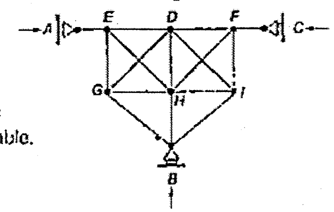
No. of joints = 6

Min. no. of members needed $= 2j - 3 = 2 \times 6 - 3 = 9$

Above truss is internally stable (over stiff) but externally unstable. Hence this truss is overall unstable.

Example 1.4 Comment on the stability of pin-jointed frame shown in figure.

Solution:



External stability:

Number of external reaction = 3

All three reactions are nonparallel but all three reactions are concurrent at point *D*, hence given frame is externally unstable.

Internal stability:

Number of joint, $j = 9$

Number of member present, $m = 16$

Number of member needed $= 2j - 3$

$= 2 \times 9 - 3 = 15$

Above frame is internally stable (over stiff).

Since frame is externally unstable. Hence given frame is overall unstable.

Remember: It is desirable for overall stability, structure should be stable externally and internally both.

1.5 Statically Determinate and Indeterminate Structures

1.5.1 Statically Determinate Structures

A structure is said to be determinate if conditions of static equilibrium are sufficient to analyse the structure.

- In determinate structures, bending moment and shear force are independent of properties of material and cross-sectional area.
- No stresses are induced due to temperature changes.
- No stresses are induced due to lack of fit and support settlement.

1.5.2 Statically Indeterminate Structures

A structure is said to be statically indeterminate if conditions of static equilibrium are not sufficient to analyse the structure. To analyse these structures, additional compatibility conditions are required.

- In indeterminate structures, bending moment and shear force depends upon the properties of material and cross-sectional area.
- Stresses are induced due to temperature variation.
- Stresses are induced due to lack of fit and support settlement.

1.6 Degree of Indeterminacy

The degree of indeterminacy can be divided into:

1. Static indeterminacy, which can be classified as
   (a) external indeterminacy
   (b) internal indeterminacy
2. Kinematic indeterminacy

1.6.1 Static Indeterminacy

Those structures which can not be analyse using equations of static equilibrium alone are called indeterminate structures or hyper static structures. To analyse these structures extra equation are required which is called compatibility equation.

(a) External Static Indeterminacy (D_{Se}):

It is related to support system of the structure. External static indeterminacy is equal to number of independent external reactions in excess to available equilibrium condition for static equilibrium.

$$D_{Se} = r_e - r$$

where, r_e = Total number of independent support reaction

r = Total number of available equations of static equilibrium

$= 3$ [2-D] ... [2-D]

$= 6$ [3-D] ... [3-D]

Case-1: (2-D beam subjected to general loading)

Here, $r_e = 6$

$r = 3$ ... (2-D)

Therefore, $D_{Se} = r_e - 3$

$D_{Se} = 6 - 3 = 3$

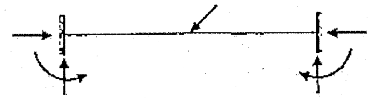


Fig. 1.34

For general loading system, a fixed beam is statically indeterminate to 3rd degree.

However for vertical loading system.

Case-2: (2-D beam vertical loading)

$r_e = 4$

and equations of static equilibrium available,

$r = 2$

therefore, $D_{Se} = r_e - r$

$= 4 - 2 = 2$

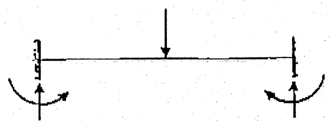


Fig. 1.35

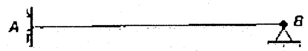
Here beam indeterminate to 2nd degree.

Hence, for general loading, the external indeterminacy is given by

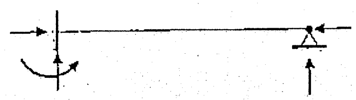
$D_{Se} = r_e - 3$ [For 2-D]

and $D_{Se} = r_e - 6$ [For 3-D]

Example 1.5 For the structure shown in figure. Determine degree of external static indeterminacy (D_{Se})



Solution:



$r_e = 5$

For general loading, $D_{Se} = r_e - 3$

$= 5 - 3 = 2$

Hence given beam is externally indeterminate to 2nd degree.

Example 1.6 For the space frame shown in figure determine D_{Se}.

Solution:

Total $r_e = r_{e1} + r_{e2} + r_{e3} + r_{e4}$

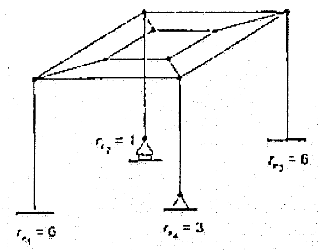
$= 6 + 1 + 3 + 6$

$= 16$

For general loading, $D_{Se} = r_e - 6$ ... (3-D)

$= 16 - 6 = 10$

Since all reactions are nonparallel and nonconcurrent, hence given frame is stable and indeterminate to 10th degree.



(b) Internal Static Indeterminacy (D_{si}):

Case-I: Pin jointed plane frame (2-D Truss):

In trusses, all joints are hinged and loading is applied at joint only, the self weight of members are neglected. Hence all member of truss will carry only axial force either tension or compression. If there are m members in the truss, then there will be m internal member force (axial force in each member). At each joint in the truss, there are two equilibrium conditions i.e. $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Let there are j number of joint. Hence total equilibrium conditions available on all joint will be $2j$, out of $2j$ equilibrium conditions, three equations are used to determine external support reactions. Hence net available equations to determine internal reactions will be $2j - 3$.

Therefore, D_{Si} = Total number of internal reactions – Available equation of equilibrium

$D_{Si} = m - (2j - 3)$

if $D_{Si} = 0$ Truss is internally determinate

Such trusses are called perfect trusses

if $D_{Si} > 0$ It will be internally indeterminate and over stiff.

if $D_{Si} < 0$ Internally deficient and geometrically unstable

Case-II: 3-D Truss (Pin-jointed space frame)

In 3-D truss, each member is having one internal force i.e. axial force but each joint has three condition of equilibrium i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$. Therefore, total condition of equilibrium at j number of joint will be $3j$. Out of $3j$ equilibrium conditions six conditions are used to find external support reactions. Hence,

D_{Si} = Total number of internal reactions – Available equation of equilibrium

$D_{Si} = m - (3j - 6)$

where, m = total number of members

j = total number of joints

Case-III: (2-D and 3-D Rigid Frames)

In rigid frames, internal indeterminacy will not exist if it forms an open configuration like a tree. To check internal indeterminacy following thumb rule may be applied.

(i) If structure is internally determinate then it is impossible to make a cut anywhere in structure without splitting the structure into two free bodies.

(ii) In case of internally determinate structure, it is impossible to return back at same point without retracing the path. It mean internally determinate structures do not have any cyclic loop.

In two dimensional (2-D) rigid frame, each member has three-internal forces i.e. R_x, R_y and M_z and in 3-D rigid frame each member has six internal force i.e. R_x, R_y, R_z, M_x and M_z. It means each closed loop in 2-D has three internal indeterminacy and each closed loop in 3-D has six internal indeterminacy. Hence

For 2-D rigid frames, $D_{Si} = 3C$

For 3-D rigid frames, $D_{Si} = 6C$

where, C = Number of closed loop

In above analysis all the joints are considered rigid. If some of the joints are hybrid (hinged) then some of the internal reactions will released. Hence D_{Si} will reduced.

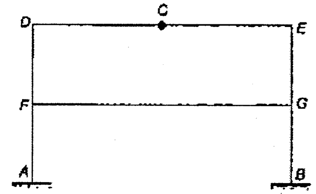
Therefore, $D_{Si} = 3C - r_r$ For 2-D

and $D_{Si} = 6C - r_r$ For 3-D

where, r_r = Total number of internal reaction released.

For example:

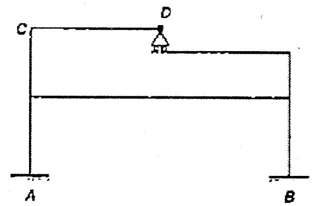
(i)



at C, $M_c = 0$, hence, one internal force (M_c) is released

$\therefore$ $r_r = 1$

(ii)



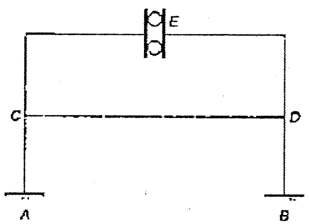
At D, two internal forces are released

(a) Axial Force (AF)

(b) Bending Moment (BM)

$\therefore$ $r_r = 2$

(iii)



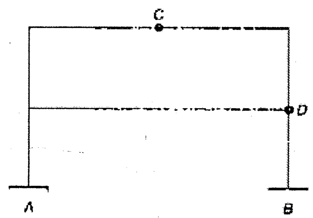
At E, Shear Force (SF) is released

$\therefore$ $r_r = 1$

The number of released forces (r_r) depends upon number of members meeting at hybrid joints.

For example:

(iv)



At C, two members meets. Hence one internal reaction will released.

$\therefore \quad r_r = 1$

At D, three members meets. Hence, two internal reactions will released.

$\therefore \quad r_r = 2$

Hence, total, $\quad r_r = 1 + 2 = 3$

We can generalize internal reaction released as follows:

For plane structures, $\quad r_r = \Sigma(m' - 1)$ (2-D frame)

For space structures, $\quad r_r = \Sigma 3(m' - 1)$

where, $\quad m'$ = Number of members meeting at hybrid joint

Hence D_{S_i} can be written as

$$D_{S_i} = 3C - \Sigma(m' - 1) \quad \text{...(2-D)}$$
$$= 6C - \Sigma 3(m' - 1) \quad \text{...(3-D)}$$

where, $\quad C$ = Number of closed loops

m' = Number of members meeting at hybrid joint

Overall Degree of Static Indeterminacy (D_S)

D_S = External static indeterminacy + Internal static indeterminacy

$$D_S = D_{So} + D_{Si}$$

Alternative Approach to Find D_S

(a) Plane Truss (2-D Truss)

D_S = Total unknown forces (External + Internal) – Total equation of equilibrium available

$= (m + r_e) - 2j$

where, $\quad m$ = Number of members (Number of internal reactions)

r_e = Number of external reactions

j = Number of joints

if $\quad D_S = 0 \quad$ Truss is statically determinate

$D_S > 0 \quad$ Truss is statically indeterminate

$D_S < 0 \quad$ Truss is statically unstable

(b) Space Truss (3-D Truss)

$$D_S = (m + r_e) - 3j$$

(c) 2-D Rigid Frames

$D_S = (3m + r_e) - 3j \quad$ (When all joints are rigid)

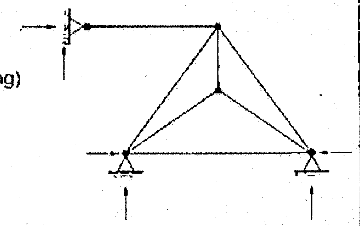
$D_S = (3m + r_e) - 3j - r_r \quad$ (When some joints are hybrid)

(d) 3-D Rigid Frames

$D_S = (6m + r_e) - 6j \quad$ (When all joints are rigid)

$D_S = (6m + r_e) - 6j - r_r \quad$ (When some joints are hybrid)

Example 1.7 For 2-D truss shown in figure, find degree of static indeterminacy.



Solution:

First approach:

$D_{So} = r_o - 3$ (For general loading)

$= 6 - 3 = 3$

$D_{Si} = m - (2j - 3)$

$= 7 - (2 \times 5 - 3) = 0$

$\therefore$ Degree of static indeterminacy,

$D_S = D_{Si} + D_{So}$

$D_S = 3 + 0 = 3$

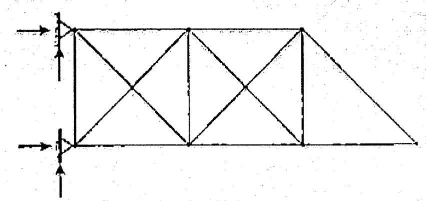
Second approach:

$D_S = m + r_o - 2j$

$= 7 + 6 - 2 \times 5$

$= 13 - 10 = 3$

Example 1.8 What is the total degree of static indeterminacy (both internal and external) of the cantilever plane truss shown in the figure below?



(a) 2 (b) 3

(c) 4 (d) 5

Ans. (b)

$m = 13$

$j = 7$

$r_e = 4$

First approach:

$D_{So} = r_o - 3 = 4 - 3 = 1$

$D_{Si} = M - (2j - 3)$

$= 13 - (2 \times 7 - 3) = 2$

$\therefore \quad D_S = D_{So} + D_{Si}$

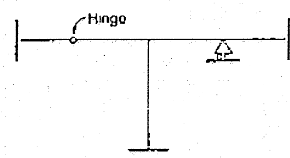
$D_S = 1 + 2 = 3$

Second approach:

$D_S = m + r_e - 2j$

$= 13 + 4 - 2 \times 7 = 3$

Example 1.9 The degree of static indeterminacy for the rigid frame as shown below is



(a) 6 (b) 4

(c) 8 (d) 10

Ans. (a)

First approach:

$m = 5$

$j = 6$

$r_o = 10$

$r_r = 1$

$D_{S_o} = r_o - 3 = 10 - 3 = 7$

$D_{S_i} = 3C - r_r = 0 - 1 = -1$

$D_S = D_{S_o} + D_{S_i}$

$D_S = 7 - 1 = 6$

Second approach:

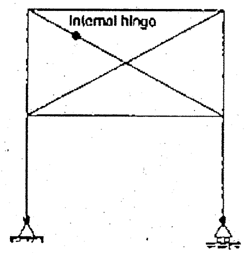
$D_S = 3m + r_o - 3j - r_r$

$= 3 \times 5 + 10 - 3 \times 6 - 1 = 6$

Example 1.10 For 2-D frame shown in figure find D_S.

(a) 9 (b) 8

(c) 10 (d) 11



Ans. (d)

First approach:

$m = 11$

$j = 8$

$r_o = 3$

$r_r = 1$

$C = 4$

$D_{S_o} = r_o - 3 = 3 - 3 = 0$

$D_{S_i} = 3C - r_r = 3 \times 4 - 1 = 11$

$D_S = D_{S_o} + D_{S_i} = 0 + 11 = 11$

Second approach:

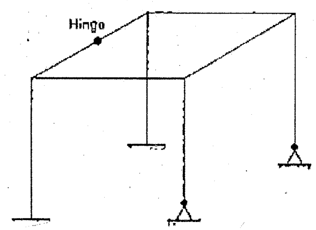
$D_S = 3m + r_o - 3j - r_r$

$= 3 \times 11 + 3 - 3 \times 8 - 1 = 11$

Example 1.11 For 3-D hybrid frame shown in figure, find D_S.

(a) 12 (b) 15

(c) 11 (d) 17



Ans. (b)

$m = 9, r_o = 18, j = 9$

Total reactions released, $r_r = 3(m' - 1)$

$= 3(2 - 1) = 3$

First approach:

$D_{S_o} = r_o - 6$

$= 18 - 6 = 12$

$D_{S_i} = 6C - r_r$

$= 6 \times 1 - 3 = 3$

Total, $D_S = D_{S_o} + D_{S_i} = 12 + 3$

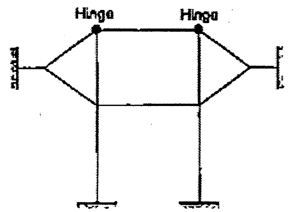
$D_S = 15$

Second approach:

$D_S = 6m + r_o - 6j - r_r$

$= 6 \times 9 + 18 - 6 \times 9 - 3 = 15$

Example 1.12 What is the total degree of static indeterminacy, both internal and external of the plane frame shown below?



(a) 10 (b) 11

(c) 12 (d) 14

Ans. (d)

$m = 12$

$r_o = 12$

$j = 10$

$r_r = \Sigma(m' - 1)$

$= 2 + 2 = 4$

First approach:

$D_{S_o} = r_o - 3$

$= 12 - 3 = 9$

$D_{S_i} = 3C - r_r$

$= 3 \times 3 - 4 = 5$

$\therefore$ $D_S = D_{S_o} + D_{S_i}$

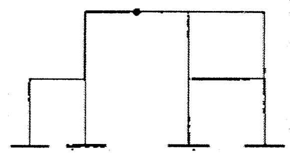
$= 9 + 5 = 14$

Second approach:

$D_S = 3m + r_o - 3j - r_r$

$= 3 \times 12 + 12 - 3 \times 10 - 4 = 14$

Example 1.13 What is the static indeterminacy for the frame shown below:



(a) 12 (b) 15

(c) 11 (d) 14

Ans. (c)

$m = 12$

$j = 12$

$r_o = 12$

$r_r = (m' - 1) = 2 - 1 = 1$

First approach:

$D_{S_o} = r_o - 3$

$= 12 - 3 = 9$

$D_{S_i} = 3C - r_r$

$= 3 \times 1 - 1$

$D_{S_i} = 2$

$\therefore$ $D_S = D_{S_o} + D_{S_i}$

$= 9 + 2 = 11$

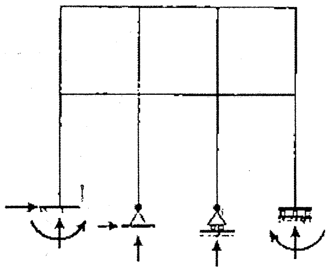
Second approach:

$D_S = 3m + r_o - 3j - r_r$

$= 3 \times 12 + 12 - 3 \times 12 - 1$

$= 11$

Example 1.14 For rigid plane frame shown in figure. Determine degree of static indeterminacy.



Solution:

$m = 14, j = 12, r_e = 8, C = 3$

First approach:

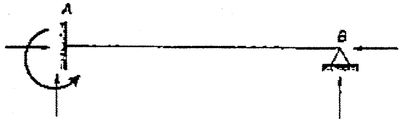
$$D_{S_e} = r_e - 3$$
$$= 8 - 3 = 5$$
$$D_{S_i} = 3C$$
$$= 3 \times 3 = 9$$
$$\therefore \quad D_S = D_{S_e} + D_{S_i}$$
$$= 5 + 9 = 14$$

Second approach:

$$D_S = 3m + r_e - 3j - r_r$$
$$= 3 \times 14 + 8 - 3 \times 12 - 0$$
$$= 14$$

Static Indeterminacy for Beams

Approach-1: Beams are treated like rigid jointed plane frame of open configuration



$$D_S = D_{S_e} + D_{S_i}$$
$$D_{S_e} = r_e - 3$$

and $\quad D_{S_i} = 3C = 0 \quad$ [No closed loop]

$\therefore \quad D_S = r_e - 3$...... if no hybrid joints

D_S for above beam

$$r_e = 5$$
$$\therefore \quad D_S = 5 - 3 = 2$$

Alternatively, D_S can be found by relation,

$$D_S = 3m + r_e - 3j - r_r$$

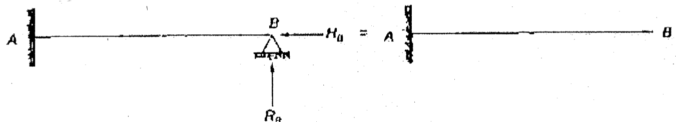
here, $m = 1, r_e = 5, j = 2$ and $r_r = 0$

$$\therefore \quad D_S = 3 \times 1 + 5 - 3 \times 2$$
$$D_S = 2$$

Approach-2: In this approach beam are converted into cantilever by removing support reactions and constraint are added to Hybrid joints, then degree of static indeterminacy is given by

D_S = Support Reactions Removed - Constraint add to Hybrid joints

For above beam,



(i) Given Beam (ii) Converted cantilever

Here, support reactions removed to make above beam cantilever are

(i) H_B

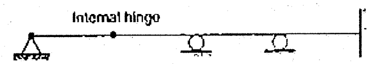
(ii) R_B

There is no Hybrid joint, so no constraint is added.

$$\therefore \quad D_S = 2 - 0$$
$$= 2$$

NOTE: This approach is applicable only for general cases of loading.

Example 1.15 The static indeterminacy of the beam is



(a) 1 (b) 2

(c) 3 (d) 4

Ans. (c)

Approach-I

$$D_S = r_e - 3 - r_r$$

Here, $\quad r_e = 2 + 1 + 1 + 3 = 7$

$$r_r = 1$$
$$\therefore \quad D_S = 7 - 3 - 1 = 3$$

Approach-II

$$D_S = 3m + r_e - 3j - r_r$$

Here, $m = 4, r_e = 7, j = 5$ and $r_r = 1$

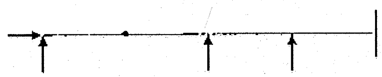
$$\therefore \quad D_S = 3 \times 4 + 7 - 3 \times 5 - 1 = 3$$

Approach-III

$$D_S = R_R - C_A$$

Here, R_R = reaction removed to make cantilever

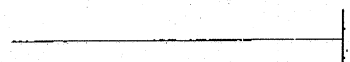
$$= 2 + 1 + 1$$
$$= 4$$



(i) Given Beam

C_A = constraint added to Hybrid joints

$$= 1$$

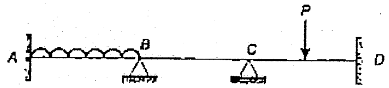


(ii) Converted cantilever

$$\therefore \quad D_S = 4 - 1$$
$$= 3$$

Hence option (c) is correct

Example 1.16 What is the total degree of static indeterminacy in the continuous prismatic beam shown in the figure below?



(a) 1
(b) 2
(c) 3
(d) 4

Ans. (d)

$D_S = r_o - E - r_r$

r_r = Total external unknown reaction

$= R_A, M_A, R_B, R_C, R_D$ and $M_D = 6$

E = Equation of equilibrium available

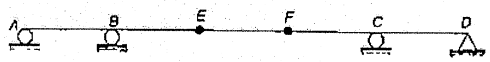
(i) $\Sigma f_y = 0$

(ii) $\Sigma M_z = 0$

$r_r = 0$

$\therefore$ $D_S = 6 - 2 = 4$

Example 1.17 What is the total degree of static indeterminacy for the continuous beam shown in figure?



(a) 1
(b) 0
(c) 3
(d) 2

Ans. (b)

Approach-I:

$D_S = r_o - 3 - r_r$

Here, $r_o = 1 + 1 + 1 + 2 = 5$

$r_r = 1 + 1 = 2$

$\therefore$ $D_S = 5 - 3 - 2 = 0$

Approach-II:

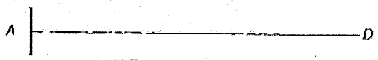
$D_S = 3m + r_e - 3j - r_r$

Here, $m = 5$, $r_o = 5$, $j = 6$ and $r_r = 2$

$\therefore$ $D_S = 3 \times 5 + 5 - 3 \times 6 - 2$

$D_S = 0$

Approach-III:



(i) Converted Cantilever

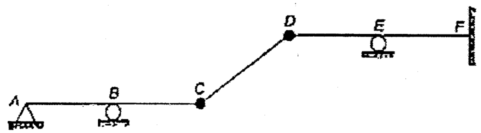
$R_H = 4$ (one each at B and E and two at F)

$C_A = 2 + 1 + 1 = 4$ (two at A, one each at E and F)

$D_S = R_H - C_A$

$= 4 - 4 = 0$

Example 1.18 The static indeterminacy for the beam is



(a) 2
(b) 1
(c) 3
(d) 4

Ans. (a)

$D_S = r_o - 3 - r_r$

Here, $r_o = 2 + 1 + 1 + 3 = 7$

$r_r = 2$

$\therefore$ $D_S = 7 - 3 - 2$

$= 2$

Alternate Approach:

$D_S = 3m + r_o - 3j - r_r$

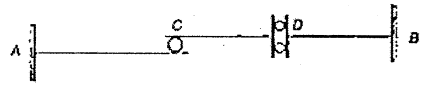
Here, $m = 5$, $r_o = 7$, $j = 6$ and $r_r = 2$

$\therefore$ $D_S = 3 \times 5 + 7 - 3 \times 6 - 2$

$= 2$

Hence option (a) is correct.

Example 1.19 The degree of statically indeterminacy for the beam shown in figure is



(a) 0
(b) 1
(c) 2
(d) 3

Ans. (a)

$D_S = r_e - 3 - r_r$

Here, $r_o = 3 + 3 = 6$

$r_r = (AF)_C, (SF)_D$ and M_C

$= 3$

$\therefore$ $D_S = 6 - 3 - 3$

$= 0$

Alternative Approach:

$D_S = 3m + r_o - 3j - r_r$

Here, $m = 3$, $r_e = 6$, $j = 4$, $r_r = 3$

$\therefore$ $D_S = 3 \times 3 + 6 - 3 \times 4 - 3$

$= 0$

Hence option (a) is correct.

1.6.2 Degree of Kinematic Indeterminacy

Degree of kinematic indeterminacy (D_K) refers to the total number of independent available degree of freedom at all joints. The degree of kinematic indeterminacy may be defined as the total number of unrestraint displacement component at all joints.

S.No.	Type of joint	Possible degree of freedom
1.	2-D Truss joint	• Two degree of freedoms are available 1. Δx 2. Δy
2.	3-D Truss joint	• Three degree of freedoms are available 1. Δx 2. Δy 3. Δz
3.	2-D Rigid joint	• Three degree of freedoms are available 1. Δx 2. Δy 3. θz
4.	3-D Rigid joint	• Six degree of freedoms are available 1. Δx 2. Δy 3. Δz 4. θx 5. θy 6. θz

Degree of kinematic indeterminacy for:

(a) Plane truss (2-D Truss):

In a pin-jointed plane truss each joint is having two degree of freedom (Δx and Δy) therefore j number of joint, will be having $2j$ degree of freedom. We know r_e be the support reactions, hence at supports joint displacements will not be available in the direction of reaction. Therefore total number of unrestraint displacement component at all joint (D_K) will be

$$D_K = 2j - r_e$$

(b) Space truss (3-D truss):

Similarly, $D_K = 3j - r_e$

(c) Rigid jointed plane frame:

Similarly, $D_K = 3j - r_e$

(d) Rigid jointed space frame:

Similarly, $D_K = 6j - R_e$

In above analysis all members are considered axially inextensible and all above displacements are elastic displacements.

Special Case-1

In rigid frame, if some of the members are axially rigid then axial displacements in such members may not be available, hence degree of freedom will be reduced.

For 2-D rigid jointed frame,

$$D_K = 3j - r_e - m''$$

For 3-D rigid jointed frame,

$$D_K = 6j - r_e - m''$$

where, m'' = Number of axially rigid members

Example 1.20 Determine degree of kinematic indeterminacy for the given cantilever if

(*i*) beam is axially flexible

(*ii*) beam is axially rigid

Solution:

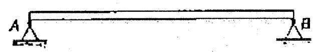
(*i*) $j = 2,\ r_e = 3$

$$D_K = 3j - r_e$$
$$= 3 \times 2 - 3$$
$$= 6 - 3 = 3$$
$$\therefore \quad D_K = 3$$

(*ii*) If beam *AB* is axially rigid

$$D_K = 3j - r_e - m''$$
$$= 3 \times 2 - 3 - 1$$
$$= 2$$

Example 1.21 Determine degree of kinematic indeterminacy for the given beam.



Also, find D_K when member *AB* is axially rigid.

Solution:

$j = 2,\ r_e = 4$

$$D_K = 3j - r_e$$
$$= 3 \times 2 - 4$$
$$= 2 \quad (\theta_A \text{ and } \theta_B)$$

If we consider *AB* as axially rigid, then D_K will be same as previous case because axial displacement already restrained by reactions

$$\therefore \quad D_K = 2 \quad (\theta_A \text{ and } \theta_B)$$

Special Case-2

In rigid jointed frames, some of the joints may be hybrid then additional degree of freedom will be available. Hence D_K will increase.

For 2-D rigid jointed frame,

$$D_K = 3j - r_e - m'' + r_r$$

For 3-D rigid jointed frame,

$$D_K = 6j - r_e - m'' + r_r$$

where, j = Total number of joints including rigid joint, hybrid joint, supported joint and unsupported joint

m'' = Number of axialy rigid members

r_r = $\Sigma(m' - 1)$ ...(2-D)

m' = Number of members meeting at hybrid joint

Example 1.22 What is the number of independent degree of freedom of the two span continuous beam of uniform sections shown in the figure below?

(a) 1 (b) 2
(c) 3 (d) 4

Ans. (d)

Beams are considered as 2-D rigid jointed open frame.

$\therefore$ $D_K = 3j - r_e$

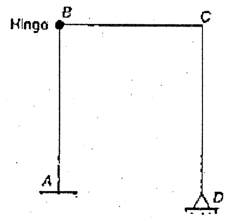
Here, $j = 3$

$r_e = 1 + 1 + 3 = 5$

$\therefore$ $D_K = 3 \times 3 - 5$

$= 4$

Example 1.23 The kinematic indeterminacy of the frame is



(a) 4 (b) 6
(c) 8 (d) 10

Ans. (c)

Here, $j = 4$

$r_e = 3 + 2 = 5$

$r_r = \Sigma(m' - 1)$

$= 2 - 1 = 1$

$D_K = 3j - r_e + r_r$

$\therefore$ $D_K = 3 \times 4 - 5 + 1$

$= 8$

Example 1.24 For the beam shown in figure. Find degree of kinematic indeterminacy considering beam as flexible.

Solution:

Here, $j = 3$

$r_e = 5$

$m'' = 0$

$r_r = \Sigma(m' - 1) = 2 - 1$

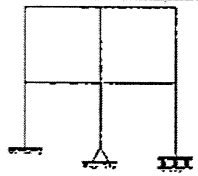
$= 1$

$D_K = 3j - r_e - m'' + r_r$

$\therefore$ $D_K = 3 \times 3 - 5 - 0 + 1$

$= 5$

Example 1.25 Rigid jointed frame shown in figure. Find degree of kinematic indeterminacy. Assuming beams are axially inextensible.



Solution:

Here, $j = 9$

$r_e = 3 + 2 + 2 = 7$

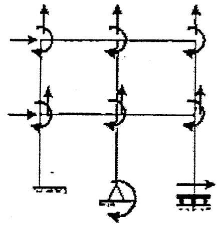
m'' = Axialy rigid members = 4

$D_K = 3j - r_e - m'' + r_r$

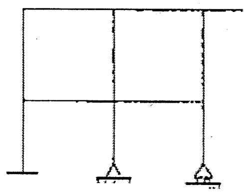
$\therefore$ $D_K = 3 \times 9 - 7 - 4$

$= 27 - 7 - 4$

$= 16$



Example 1.26 For 2-D rigid frame shown in figure. Find D_K and show all displacement component at respective joints.



Solution:

Here, $j = 10$

$r_e = 3 + 2 + 1$

$= 6$

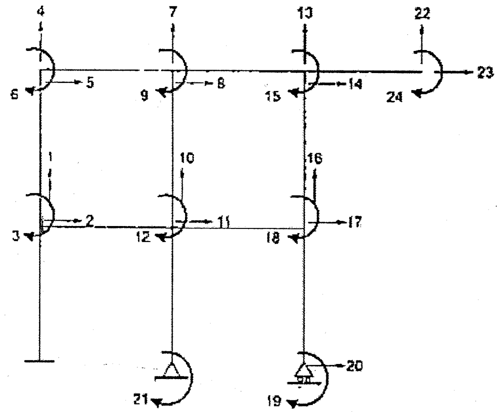
$m'' = 0$

and $r_r = 0$

$D_K = 3j - r_e - m'' + r_r$

$\therefore$ $D_K = 3 \times 10 - 6$

$= 24$



Summary

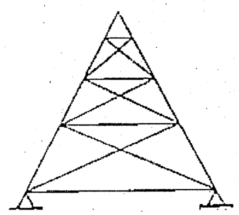
- Some of major elements of structures from which structures are fabricated are:
  (*i*) beams (*ii*) columns (*iii*) tie members
- Common types of structures are
  (*i*) trusses (*ii*) frames (*iii*) arches and cables
- For the external stability of structures following conditions should be satisfied
  (*i*) there should be minimum number of externally independent support reactions.
  (*ii*) all reactions should not be parallel.
  (*iii*) all reactions should not be concurrent.
  (*iv*) reactions should be nontrivial.
  (*v*) for stability in 3-D structures all reactions should be non-coplanar, nonconcurrent and nonparallel.
- To preserve geometric stability, the minimum number of members needed are
  $m = 2j - 3$ (2D-Truss)
  $m = 3j - 6$ (3D-Truss)
- It is desirable for overall stability, structure should be stable externally and internally both.
- D_S for plane truss, is given by
  $$D_S = m + r_e - 2j$$
- D_S for space truss is given by
  $$D_S = m + r_e - 3j$$
- D_S for rigid jointed frame is given by
  $D_S = 3m + r_e - 3j$ (When all joints are rigid)
  $D_S = 3m + r_e - 3j - r_r$ (When some joints are hybrid)
- D_S for 3-D rigid jointed frame is given by
  $D_S = 6m + r_e - 6j$ (When all joints are rigid)
  $D_S = 6m + r_e - 6j - r_r$ (When some joints are hybrid)
- Degree of kinematic indeterminacy for
  (*i*) Plane truss
  $$D_K = 2j - r_e$$
  (*ii*) Space truss
  $$D_K = 3j - r_e$$
  (*iii*) 2-D rigid jointed frame
  $$D_K = 3j - r_e - m'' + r_r$$
  (*iv*) 3-D rigid jointed frame
  $$D_K = 6j - r_e - m'' + r_r$$
  where, m'' = Number of axialy rigid members
  r_r = Number of reactions released
  $= \Sigma(m' - 1)$ ...(2D)
  $= \Sigma\, 3(m' - 1)$ ...(3D)
  where, m' = Number of members meeting at hybrid joint

Objective Brain Teasers

Q.1 The degree of static indeterminacy of the beam given below is

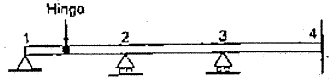
(a) zero (b) one
(c) two (d) three

Q.2 What is the total degree of static indeterminacy (both internal and external) of the triangular planar truss shown in the figure below?



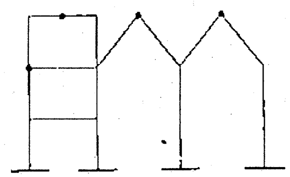
(a) 2 (b) 4
(c) 5 (d) 6

Q.3 The kinematic indeterminacy of the beam is



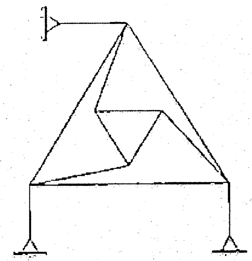
(a) 5 (b) 9
(c) 14 (d) 15

Q.4 For rigid frame shown in figure. Determine total degree of static indeterminacy.



(a) 10 (b) 11
(c) 13 (d) 8

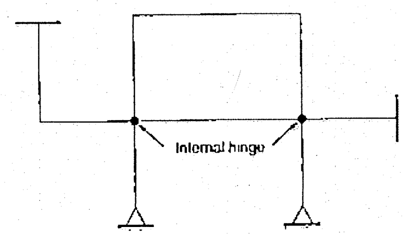
Q.5 The following two statements are made with reference to the plane truss shown below:



I. The truss is statically determinate
II. The truss is kinematically determinate

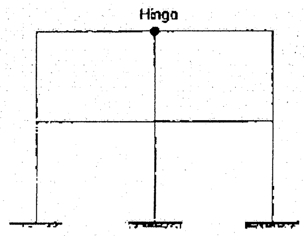
With reference to the above statements, which of the following applies?
(a) Both statements are true
(b) Both statements are false
(c) II is true but I is false
(d) I is true but II is false

Q.6 Find static Indeterminacy of the Frame shown in figure



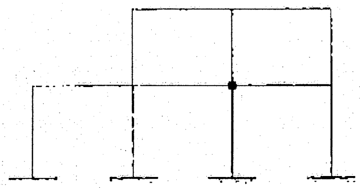
(a) 5 (b) 4
(c) 6 (d) 8

Q.7 The rigid Frame shown in figure, the statical indeterminacy is



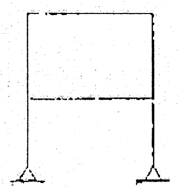
(a) 8 (b) 12
(c) 10 (d) 14

Q.8 The total degree of static indeterminacy of the plane frame shown in given figure is



(a) 10 (b) 11
(c) 12 (d) 15

Q.9 The degree of kinematic indeterminacy of frame shown in the figure ignoring the axial deformation is given by

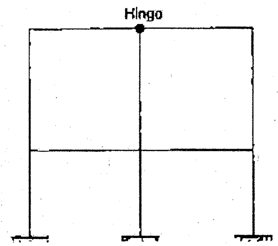


(a) 8 (b) 10
(c) 12 (d) 14

Q.10 The degree of static indeterminacy of a rigid jointed space frame is
(a) $m + r - 2j$ (b) $m + r - 3j$
(c) $3m + r - 3j$ (d) $6m + r - 6j$
where, m, r and j have their usual meanings

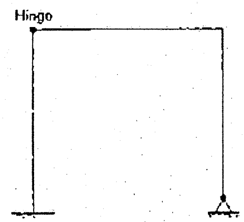
Q.11 A plane frame is statically determinate if
(a) $3m + r = 3j + c$ (b) $3m + c = 3j + r$
(c) $3m + c < 3j + r$ (d) $3m + c > 3j + r$
Where,
m = no. of members
j = no. of joints
r = no. of reactions
c = no. of equations of conditions

Q.12 The rigid frame shown in figure, the statical indeterminacy is



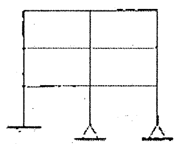
(a) 8 (b) 14
(c) 10 (d) 12

Q.13 The kinematic indeterminacy (Degree of Freedom) of the frame given below is



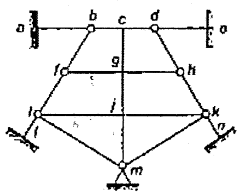
(a) 4 (b) 6
(c) 8 (d) 10

Q.14 The total degree of kinematic indeterminacy of the plane frame shown in the given figure considering columns to be axially rigid is



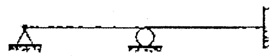
(a) 20 (b) 37
(c) 44 (d) 28

Q.15 The degree of static indeterminacy of the hybrid plane frame as shown in figure is



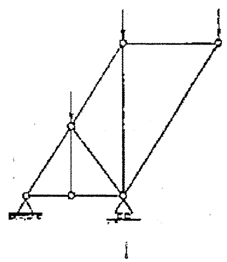
(a) 10 (b) 11
(c) 12 (d) 13

Q.16 Degree of freedom for the structure shown in figure is



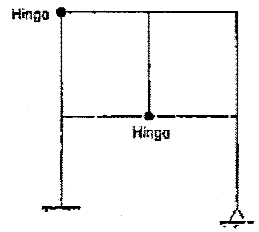
(a) 2 (b) 3
(c) 4 (d) 5

Q.17 The pin jointed frame shown in figure is



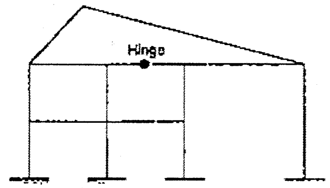
(a) a perfect frame
(b) a redundant frame
(c) a deficient frame
(d) None of these

Q.18 Number of static indeterminacy for the structure shown below is



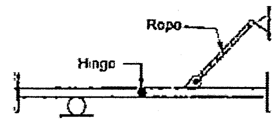
(a) 3 (b) 4
(c) 5 (d) 6

Q.19 The degree of static and kinematic indeterminacy of the plane frame as shown in the figure is (Assume members are axially extensible)



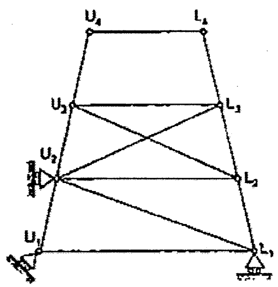
(a) 15, 21 (b) 17, 18
(c) 18, 27 (d) 17, 28

Q.20 The degree of static indeterminacy for the beam as shown in figure is



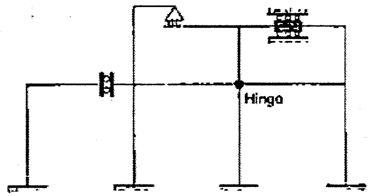
(a) 1 (b) 4
(c) 5 (d) 3

Q.21 Which of the following statement is true for the pin jointed frame as shown in the figure below?



(a) Statically determinate and internally unstable.
(b) Statically indeterminate and internally stable.
(c) Statically indeterminate and internally unstable.
(d) Statically determinate and internally stable.

Q.22 The total (both internal and external) degree of static indeterminacy of the plane frame shown in the given figure is ________.



Answers

1. (b)	2. (b)	3. (b)	4. (a)	5. (d)
6. (b)	7. (c)	8. (c)	9. (a)	10. (d)
11. (a)	12. (c)	13. (c)	14. (a)	15. (d)
16. (b)	17. (a)	18. (c)	19. (d)	20. (b)
21. (a)	22 (8)			

Hints and Explanations:

1. (b)

$$D_S = r_e + 3m - r_r - 3(j + j')$$
$$r_e = 3 + 1 + 1 = 5$$

$m = 3$, $j = 3$, $j' = 1$

The hinge will create 2 members.

Number of internal reaction components released.

$$r_r = 1.0$$
$$\therefore \quad D_S = 5 + 9 - 1.0 - 3 \times (3 + 1) = 1.0$$

2. (b)

The total degree of indeterminacy is given by

$$D_S = m + r_e - 2j$$

Where,

m = number of members = 18

r_e = number of external reactions = 4

j = number of joints = 9

$$\therefore \quad D_S = 18 + 4 - 2 \times 9 = 4$$

3. (b)

The kinematic indeterminacy of the beam is

$$D_K = 3j - r_e + r_R$$

Given, $j = 5$

$r_e = 7$

$r_R = \Sigma(m' - 1) = \Sigma(2 - 1) = 1$

$$\therefore \quad D_K = 3 \times 5 - 7 + 1 = 9$$

$(\theta_1, \theta_{H1}, \theta_{H2}, \Delta_{Hx}, \Delta_{Hy}, \theta_2, \Delta_{2x}, \theta_3, \Delta_{3x})$

4. (a)

$m = 16$, $j = 15$, $r_e = 12$

$$r_r = \Sigma(m' - 1) = (3 - 1) + (2 - 1) + (2 - 1) + (2 - 1) = 2 + 1 + 1 + 1 = 5$$

First approach:

$$D_{Se} = r_e - 3 = 12 - 3 = 9$$
$$D_{Si} = 3C - r_r = 3 \times 2 - 5 = 1$$
$$D_S = D_{Se} + D_{Si} = 9 + 1 = 10$$

Second approach:

$$D_S = 3m + r_e - 3j - r_r = 3 \times 16 + 12 - 3 \times 15 - 5 = 48 + 12 - 45 - 5 = 10$$

5. (d)

Degree of static indeterminacy

$$D_s = m + r_e - 2j$$

Here, $m = 12$, $j = 9$ and $r_e = 6$

$$\therefore \quad D_s = 12 + 6 - 2 \times 9 = 0$$

Degree of kinematic indeterminacy

$$D_K = 2j - r_e - m = 2 \times 9 - 6 - 0 = 12$$

$\therefore$ Thus truss is statically determinate and kinematically indeterminate.

6. (b)

$$D_S = D_{Se} + D_{Si}$$

D_{Se} = external static indeterminacy

$$D_{Se} = r_e - 3$$

Here, $r_e = 3 + 2 + 2 + 3 = 10$

$$\therefore \quad D_{Se} = 10 - 3 = 7$$

and D_{Si} = Internal static indeterminacy

$$D_{Si} = 3C - r_r$$

Here, C = Number of closed loop = 1

r_r = internal reactions released $= \Sigma(m' - 1) = (4 - 1) + (4 - 1) = 6$

$$\therefore \quad D_{Si} = 3 \times 1 - 6 = -3$$

Hence, $D_S = D_{Se} + D_{Si} = 7 - 3$

$$\therefore \quad = 4$$

Alternative approach:

$$D_S = 3m + r_e - 3j - r_r$$

Here, $m = 9$, $j = 9$, $r_e = 10$ and $r_r = 6$

$$\therefore \quad D_S = 3 \times 9 + 10 - 3 \times 9 - 6$$
$$D_S = 4$$

7. (c)

$$D_S = D_{Se} + D_{Si}$$
$$D_{Se} = r_e - 3$$

Here, $r_e = 3 + 3 + 3 = 9$

$$\therefore \quad D_{Se} = 9 - 3 = 6$$

and $D_{Si} = 3C - r_r$

Here, C = Number of closed loop = 2

r_r = internal reactions released $= \Sigma(m' - 1) \Rightarrow (3 - 1) = 2$

$$\therefore \quad D_{Si} = 3 \times 2 - 2 = 4$$

Hence, $D_S = D_{Se} + D_{Si} = 6 + 4 = 10$

Alternative approach:

$$D_S = 3m + r_e - 3j - r_r$$

Here, $m = 10$, $j = 9$, $r_e = 9$ and $r_r = 2$

$$\therefore \quad D_S = 3 \times 10 + 9 - 3 \times 9 - 2$$
$$\Rightarrow \quad D_S = 10$$

8. (c)

$$D_S = 3m + r_e - 3j - r_r$$

Here, $m = 12$, $j = 11$, $r_e = 3 + 3 + 3 + 3 = 12$

r_r = reaction released $= \Sigma(m - 1) = (4 - 1) = 3$

$$\therefore \quad D_S = 3 \times 12 + 12 - 3 \times 11 - 3$$
$$D_S = 12$$

Alternative Approach:

$$D_S = D_{Se} + D_{Si}$$
$$D_{Se} = r_e - 3$$
$$D_{Se} = 12 - 3 = 9$$
$$D_{Si} = 3C - r_r = 3 \times 2 - 3 = 6 - 3 = 3$$
$$\therefore \quad D_S = D_{Se} + D_{Si} = 9 + 3 = 12$$

Hence option (c) is correct.

9. (a)

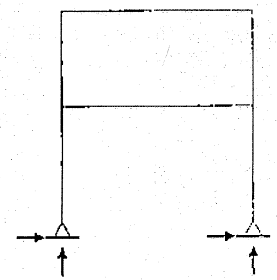
For 2D-rigid frame, the degree of kinematic indeterminacy is given by

$$D_k = 3j - r_e - m''$$

where j = No. of joint

r_e = external support reactions

m'' = axially rigid members



Here, $j = 6$, $m'' = 6$, $r_e = 4$

$$\therefore \quad D_k = 3 \times 6 - 4 - 6 = 8$$

Hence option (a) is correct.

10. (d)

D_S = Total unknown reaction – Total equation of equilibrium

In rigid frame each member have six internal reactions i.e. F_x, F_y, F_z, M_x, M_y and M_z. Therefore total internal reaction will be 6 m. At each joint there are six equations of equilibrium available i.e., $\Sigma f_x = 0$, $\Sigma f_y = 0$, $\Sigma f_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$ and $\Sigma M_z = 0$. If there are r unknown external support reactions then D_s is given by

$D_S = (6m + r) - 6j$

Hence option (d) is correct.

11. (a)

A plane frame is determinate if degree of static indeterminacy is zero.

D_S = Total unknown reactions – Total equation of equilibrium – Additional condition of equilibrium like r_r

$D_S = (3m + r) - 3j - C$

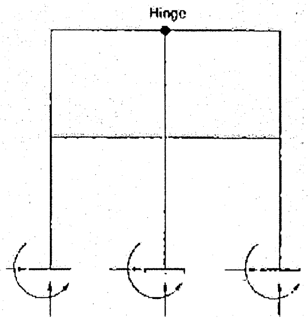
For frame to be determinate,

$D_S = 0$

$3m + r = 3j + C$

Hence option (a) is correct.

12. (c)



First approach:

$D_S = D_{Se} + D_{Si}$

$D_{Se} = r_e - 3$

Here, $r_e = 9$

$\therefore$ $D_{Se} = 9 - 3 = 6$

and $D_{Si} = 3C - r_r$

Here, $C = 2$ and $r_r = \Sigma(m - 1) = (3 - 1) = 2$

$\therefore$ $D_{Si} = 3 \times 2 - 2 = 4$

Hence, $D_S = 6 + 4$

$D_S = 10$

Second approach:

$D_S = 3m + r_e - 3j - r_r$

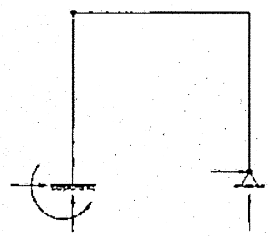
Here, $m = 10$, $r_e = 9$, $j = 9$ and $r_r = 2$

$\therefore$ $D_S = 3 \times 10 + 9 - 3 \times 9 - 2$

$D_S = 10$

Hence option (c) is correct.

13. (c)



$D_k = 3j - r_e - m'' + r_r$

where

j = no. of joints

r_e = external supports reactions

m'' = axially rigid members

r_r = No. of reactions released

Here, $j = 4$, $r_e = 5$, $m'' = 0$ and $r_r = 1$

$\therefore$ $D_k = 3 \times 4 - 5 - 0 + 1$

$D_K = 8$

14. (a)

$D_K = [3j - r_e] - m$

where j = total number of rigid joints = 12

r_e = total number of external reactions = 3 + 2 + 2 = 7

m = total number of axially rigid members = 9

$\therefore$ $D_k = [3 \times 12 - 7] - 9 = 20$

15. (d)

$D_s = D_{se} + D_{si}$

$D_{se} = r_e - 3 = 14 - 3 = 11$

$D_{si} = 3c - r_r$

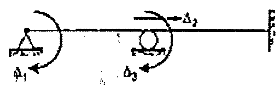
$= 3 \times 6 - \Sigma(m_j - 1)$

$= 18 - (2 + 2 + 3 + 2 + 3 + 2 + 2)$

$= 18 - (16) = 2$

$\therefore$ $D_s = 11 + 2 = 13$

16. (b)



$\therefore$ $D_k = 3$

Alternate Method:

$D_k = 3j - r_e - m + r_r$

$= 9 - 6 - 0 + 0 = 3$

17. (a)

$D_s = m + r_e - 2j$

$= 9 + 3 - 2 \times 6 = 0$

So, it is a perfect frame.

18. (c)

$D_{Se} = r_e - 3 = 5 - 3 = 2$

$D_{Si} = 3c - r_r$

$= 3 \times 2 - (1 + 2)$

$= 6 - 3 = 3$

$\therefore$ $D_S = D_{Se} + D_{Si}$

$= 2 + 3 = 5$

19. (d)

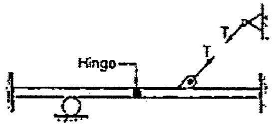
$D_S = 3m + r_e - 3j - r_r$

$= 3 \times 15 + 12 - 3 \times 13 - 1 = 17$

$D_k = 3j - r_e + r_r$

$= 3 \times 13 - 12 + 1 = 28$

20. (b)



$D_s = 3m + r_e - 3j - r_r$

$= 3 \times 4 + 8 - 3 \times 5 - 1$

$= 20 - 16 = 4$

21. (a)

$D_s = m + r_e - 2j$

$= 13 + 3 - 2 \times 8 = 0$

The frame has 8 joints and consequently requires 13 members. The frame does have 13 members but even then it is not stable. Actually, the frame is a combination of a stable panel $U_1U_2L_2L_1$, as overstiff panel $U_2U_3L_3L_2$ and as unstable panel $U_3U_4L_4L_3$. When the internal stability of the frame as a whole is considered, the frame will have to be designated as unstable.

22 (8)

$D_s = 3m + r_e - 3j - r_r$

$m = 15$

$r_e = 12$

$j = 14$

$r_r = (3 + 1 + 1 + 2)$

$D_s = 15 \times 3 + 12 - 3 \times 14 - 7 = 8$

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