CHAPTER 12

Suspended Cables

- Engineers have sought such structural forms which helps in bridging over increasing span.
- For suspension bridges, cables form an important structural element. A suspension bridge consists of two cables on either side of the roadway which are stretched over the span to be bridges.
- In suspension bridges, cables are load bearing elements. Figures of suspension bridge is shown below:

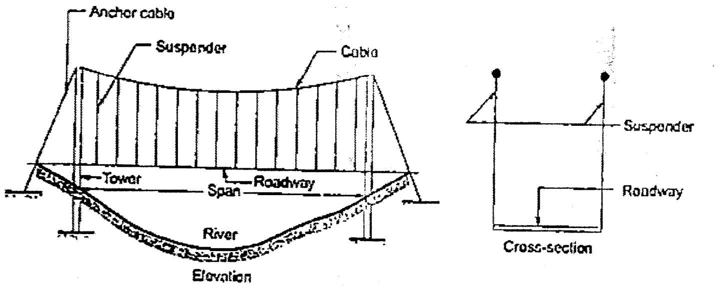


Fig. 12.1 A typical suspension bridge

A. Cables

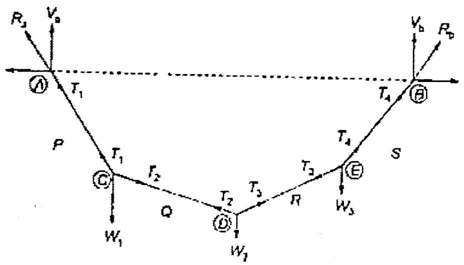


Fig. 12.2

- Loaded cable supported at the ends is a system in equilibrium, must satisfy the following two conditions:
  (i) The force polygon should be closed.
  (ii) The funicular polygon should be closed
- Let the loads W_1, W_2 and W_3 represented by *pq*, *qr* and *rs* to a suitable scale. Line '*po*' is drawn through *p* which parallel to *CA* and line '*so*' is drawn parallel to *EB* and hence point o is obtained.

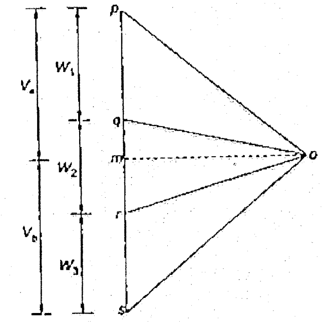


Fig. 12.3

- Now *pqrsop* is force polygon of the loaded cable (shown in figure above)
- *om* line is drawn perpendicular to *pqrs* line through *O*. Now in the figure above.
  mp is vertical reaction V_a
  sm is Vertical reaction V_b
- Now,
  T_1 is represented by *po*
  T_2 is represented by *qo*
  T_3 is represented by *ro*
  T_4 is represented by *so*

Note: Point *O* in the vector diagram is determinated to find tensions in various segments of the loaded cable.

- In figure, *OM* is horizontal reaction (*H*) or horizontal component of the tension in any section of cable.

Note: The funicular polygon coincides with the centre line of the loaded cord itself that is the shape of loaded cord is same as that of the funicular polygon.

12.1 Equation of the Cable

- Consider uniformly distributed cable. The loading on the cable determines the profile of the cable.
- The figure shown below is uniformly distributed cable suspended through simple supports at the same level.

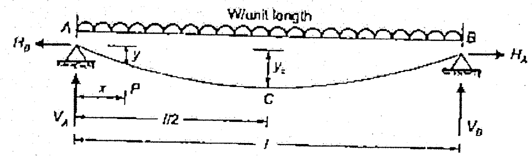


Fig. 12.4

where, $y_c \rightarrow$ sag of the cable.

- The vertical reactions at A and B.

$$V_A = V_B = \frac{wl}{2}$$

- The horizontal reactions

$$H_A = H_B = H$$

H is determined by taking moments about C and equating it to zero, that is $M_C = 0$

$$M_C = V_A\frac{l}{2} - w\frac{l}{2}\frac{l}{4} - Hy_c = 0$$

$$M_C = \frac{wl^2}{4} - \frac{wl^2}{8} - Hy_c = 0$$

$$\Rightarrow \quad H = \frac{wl^2}{8y_c} \quad \text{...(i)}$$

- To determine shape of the cable

we obtain the expression for moment at P (as shown in figure above), that is M_P

$$M_P = V_A x - \frac{wx^2}{2} - Hy = 0$$

$$\Rightarrow \quad \frac{wl}{2}x - \frac{wx^2}{12} = \frac{wl^2}{8y_c}y \quad \text{(Using eq. (i))}$$

$$\Rightarrow \quad y = \frac{4y_c}{l^2}x(l-x) \quad \text{...(ii)}$$

Here, x is measured from left side support A which is taken as origin.

NOTE

- Equation (ii) that is $y = \frac{4y_c}{l^2}x(l-x)$ is second order parabola. The deflected shape of the cable under UDL or due to its own weight is catenary and not parabola (in reality).
- Bending moment at any point on the cable is zero.

12.2 Horizontal Tension in Cables

- Horizontal force was algebraically determined in equation (i) for uniformly distributed suspended cable, that is

$$H = \frac{wl^2}{8y_c}$$

Note: In flat loaded cables, large horizontal force is generated at the supports.

12.3 Tension in Cable

- The tension in cable varies along its length.
- The maximum tension (T_{max}) occurs at supports as horizontal force (H) is constant all along the length of the cable and vertical force acting at the supports (V_A or V_B) is maximum.

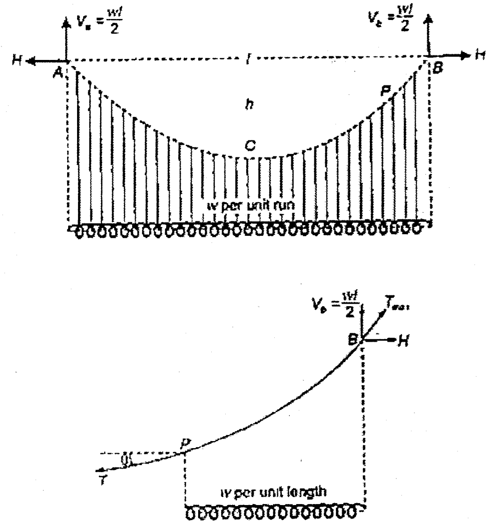


Fig. 12.5

NOTE

The tension in cable at any arbitrary point say P is evaluated with the help of vertical force at that section and constant horizontal force, that is

$$T_P = \sqrt{H^2 + V_P^2}$$

- T_{max} is evaluated as

$$T_{max_A} = \sqrt{V_A^2 + H^2}$$

or

$$T_{max_B} = \sqrt{V_B^2 + H^2}$$

Where maximum of T_{max_A} and T_{max_B} is design cable force for uniform cross-section of cables.

- Thus, $T_{max} = \left[\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8y_c}\right)^2\right]^{1/2}$

$$= \left[\left(\frac{W}{2}\right)^2 + \left(\frac{Wl}{8y_c}\right)^2\right]^{1/2}$$

$$= \frac{W}{2}\left[\left(1+\frac{l^2}{16y_c^2}\right)\right]^{1/2}$$

12.4 Cable Passed Over Guide Pulley at the Support

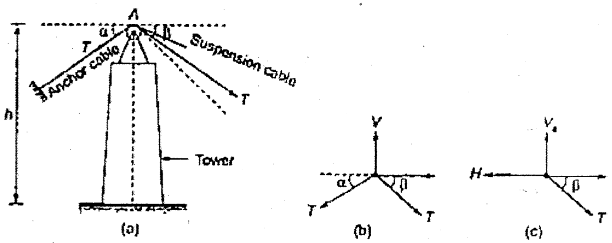


Fig. 12.6 (a) Guide pully support (b) FBD for Pier (c) FBD for cable at support

- Here, cable at the support is guided over pulley. Pulley is provided at top of the pier. This cable is anchored on the other side of pier.
- The tensions in both the cables is same, as suspended and anchor cable are same.
- Then tension in the cable at the end is

$$T = \sqrt{V_a^2 + H^2}$$

- $\tan\beta = \frac{V_a}{H}$, where β is the inclination of suspended cable.
- α is inclination of the anchor cable with the horizontal.
- Thus, total vertical load transmitted to the pier is

$$V = T\sin\alpha + T\sin\beta = T(\sin\alpha + \sin\beta)$$

- Net horizontal force transmitted to the top of the pier is,

$$H_1 = T(\cos\alpha - \cos\beta)$$

- Maximum bending moment for the pier, BM_{max} = Net horizontal force at the top of the pier × Height of the pier

that is, $BM_{max} = H \times \text{height}_{pier}$

12.5 Cable Clamped to Saddle Carried on Smooth Rollers on the Top of the Pier

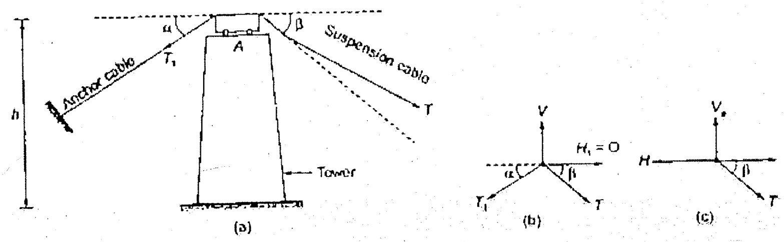


Fig. 12.7 (a) Roller support, (b) FBD for pier (c) FBD for cable at support

- Here, suspended cable and anchor cable are different, hence tension on both cable are different.
- Assume T as tension in suspended cable and T_1 as tension in anchor cable.
- $T = \sqrt{V_a^2 + H^2}$

  where, V_a and H are the vertical and horizontal reactions at the end of the suspension cable.
- $\tan\beta = \frac{V_a}{H}$
- For equilibrium of the saddle, we have

$$T_1\cos\alpha = T\cos\beta$$

And total vertical load transmitted to the pier is: $V = T_1\sin\alpha + T\sin\beta$

Note: There will be no bending moment for the pier since horizontal components of T_1 and T are equal.

12.6 Curved Length of the Cable

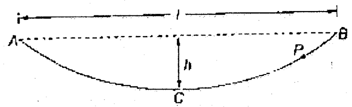


Fig. 12.8

Curved length of the cable, $L = l + \frac{8}{3}\frac{h^2}{l}$

where l = span length of cable, h = sag of the cable

Example 12.1 A cable supported at its ends 40 m apart carries load of 40 kN, 20 kN and 24 kN at distances of 10 m, 20 m and 30 m from the left end. The point where 20 kN is supported is 13 m below the level of end supports. Determine

(i) The reactions at supports
(ii) Tensions in different parts of the cable
(iii) Total length of the cable

Solution:

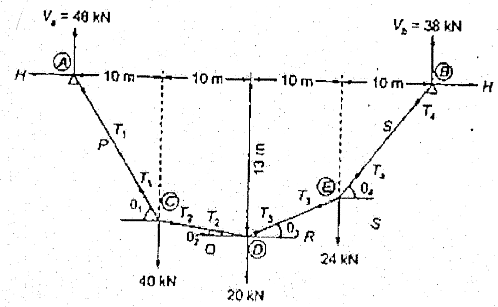
Taking moment about A,

$$M_A = 0$$

$$(40 \times 10) + (20 \times 20) + (24 \times 30) - V_b \times 40 = 0$$

$$\Rightarrow \quad V_b = \frac{400 + 400 + 720}{40} = 38 \text{ kN}$$

$$\Rightarrow \quad V_a = (40 + 20 + 24) - 38 = 46 \text{ kN}$$



Now,

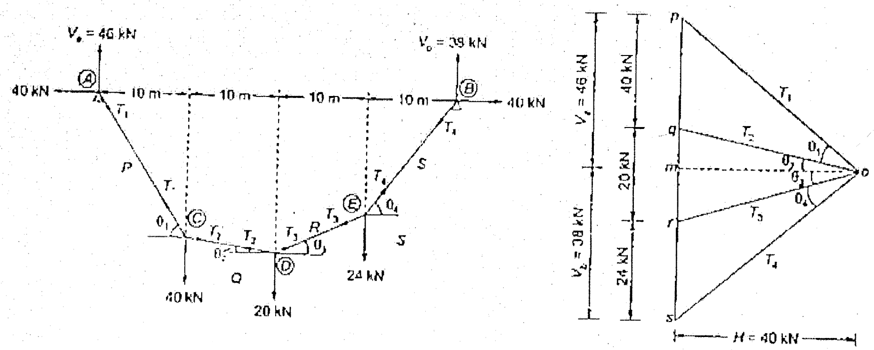
$$M_D = 0$$

$$46 \times 20 - H \times 13 - 40 \times 10 = 0$$

$$H = \frac{46 \times 20 - 40 \times 10}{13}$$

$$H = 40 \text{ kN}$$

Tension calculation



- *op*, *oq*, *or* and *os* represent the tensions T_1, T_2, T_3 and T_4 in the segments *AC*, *CD*, *DE* and *EB*
- $T_1 = \sqrt{40^2 + 46^2} = 60.96$ kN

  $T_2 = \sqrt{(46-40)^2 + 40^2} = 40.45$ kN

  $T_3 = \sqrt{(38-24)^2 + 40^2} = 42.38$ kN

  $T_4 = \sqrt{38^2 + 40^2} = 55.17$ kN

  and θ_1, θ_2, θ_3 and θ_4 are the inclinations of the different segments of the cable with the horizontal

$$\tan\theta_1 = \frac{46}{40} = 1.15$$

$$\Rightarrow \quad \theta_1 = 49°$$

$$\tan\theta_2 = \frac{46-40}{40} = 0.15$$

$$\Rightarrow \quad \theta_2 = 8°32'$$

$$\tan\theta_3 = \frac{38-24}{40} = 0.35$$

$$\Rightarrow \quad \theta_3 = 19°18'$$

$$\tan\theta_4 = \frac{38}{40} = 0.95$$

$$\Rightarrow \quad \theta_4 = 43°32'$$

- Length calculation

  AC = 10 sec 49° = 15.24 m

  CD = 10 sec 8° 32′ = 10.11 m

  DE = 10 sec 19° 18′ = 10.60 m

  EB = 10 sec 43° 32′ = 13.79 m

  ⇒ Total length = 15.24 + 10.11 + 10.60 + 13.79 = 49.74 meters

Example 12.2 A suspension cable of 150 m span and 15 m central dip carries a load of 2 kN/m. Calculate maximum and minimum tension in the cable, horizontal and vertical forces in each pier under the following conditions:

(i) Cable passes over a frictionless rollers on top of the piers.
(ii) Cable is firmly clamped to saddles carried on frictionless rollers on top of the piers.

For both the cases the back stay is inclined at 30° with the horizontal.

Solution:

Horizontal reaction, $H = \frac{wl^2}{8y_c} = \frac{2 \times 150^2}{8 \times 15} = 375$ kN

Vertical reaction, $V_a = \frac{wl}{2} = \frac{2 \times 150}{2} = 150$ kN

Maximum tension $T_{max} = \sqrt{150^2 + 375^2} = 403.89$ kN

Equation of shape of cable (parabola)

$$y = \frac{4y_c}{l^2}x(l-x)$$

Slope: $$\frac{dy}{dx} = \frac{4y_c}{l^2}(l-2x)$$

Slope of the cable at support: $$\frac{dy}{dx} = \frac{4y_c}{l} \text{ at } x = 0$$

$$\tan\theta = \frac{4\times 15}{150} = 0.40$$

$$\theta = 21°48'$$

(a) Horizontal force on the pier

$$H_1 = T_{max}(\cos 21°48' - \cos 30°)$$
$$= 403.89(\cos 21°48' - \cos 30°)$$
$$= 25.23 \text{ kN}$$

Vertical force on the pier

$$V = T_{max}(\sin 21°48' + \sin 30°)$$
$$= 403.89(\sin 21°48' + \sin 30°)$$
$$= 351.94 \text{ kN}$$

(b) In this case, $H_1 = 0$

$$T_{max}\cos 21°48' = T_2\cos 30°$$

(where, T_{max} is tension in suspension cable and T_2 is tension in anchor cable)

$$T_2 = 403.89\times\frac{\cos 21°48'}{\cos 30°} = 433 \text{ kN}$$

Vertical force on pier $= 403.89\times\sin 21°48' + 433\sin 30°$

$= 366.49$ kN

Example 12.3 **A cable is suspended between two points 30 m apart located at the same level. It carries uniformly distributed load of 20 kN per meter. The sag of the cable at mid span is 5 m. Calculate tension in the cable at the left quarter point and the curved length of the cable.**

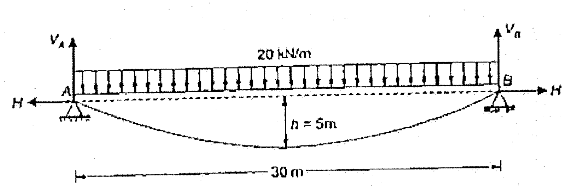
Solution:

The horizontal reaction:

$$H = \frac{wl^2}{8y_c} = \frac{20\times 30^2}{8\times 5} = 450 \text{ kN}$$

The vertical reaction at support is

$$V = V_A = V_B = \frac{wl}{2} = \frac{20\times 30}{2} = 300 \text{ kN} \quad \text{(because of symmetry)}$$



The vertical component of force at the left quarter point is

$$V' = V - \frac{wl}{4} = \frac{wl}{2} - \frac{wl}{4} = \frac{wl}{4} = \frac{20\times 30}{4} = 150 \text{ kN}$$

⇒ The tension in the cable at left quarter point is:

$$T = \sqrt{H^2 + V'^2} = \sqrt{450^2 + 150^2} = 474.34 \text{ kN}$$

The curved length of the cable is

$$L = l + \frac{8h^2}{3l} = 30 + \frac{8}{3}\times\frac{5^2}{30} = 32.22 \text{ m}$$

B. Cable Supports at Different Levels

12.7 Tension in Cable Supported at Different Levels

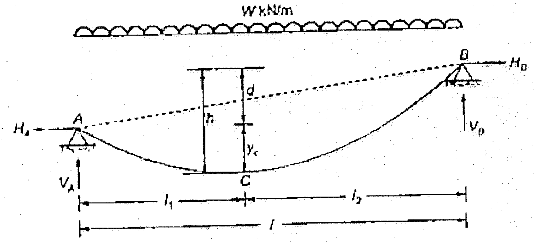


Fig. 12.9 Cable supported at ends at different levels

- Let C be the lowest point on the cable at a distance l_1 from support A or l_2 from support B.
- Now, from $M_C = 0$

$$H_A = \frac{w(2l_1)^2}{8y_c}$$

and $$H_B = \frac{w(2l_2)^2}{8(y_c+d)}$$

But $$H_A = H_B$$

$$\frac{w(2l_1)^2}{8y_c} = \frac{w(2l_2)^2}{8(y_c+d)}$$

$$\Rightarrow \qquad \frac{l_1^2}{l_2^2} = \frac{y_c}{y_c + d}$$

This relationship helps in locating the lowest point C on the cable.

- Vertical reactions at supports

  Taking moment at B,

$$M_B = 0$$

$$V_A l - \frac{wl^2}{2} + Hd = 0$$

$$\boxed{V_A = \frac{wl}{2} - H\frac{d}{l}}$$

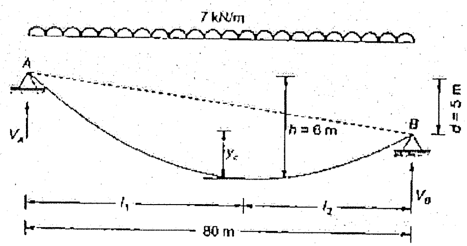
Similarly, $\qquad M_A = 0$

$$\boxed{V_B = \frac{wl}{2} + \frac{Hd}{l}}$$

- Tension in the cable at A, $\qquad T_A = \sqrt{V_A^2 + H^2}$

  and tension in the cable at B, $\qquad T_B = \sqrt{V_B^2 + H^2}$

Example 12.4 A cable of uniform cross-sectional area is stretched between two supports 80 m apart with one end 5 m above the other (as shown in figure below). The cable is loaded with uniformly distributed load of 7 kN/m and the sag of the cable measured from the higher end is 6 m. Calculate horizontal tension in the cable and maximum tension in the cable.



Solution:

$l = 80$ m, $d = 5$ m and $h = 6$ m

To locate lowest point

$$y_c = 6 - 5 = 1 \text{ m}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{y_c + d}{y_c}} = \left(\frac{6}{1}\right)^{1/2} = 2.45$$

$$\therefore \qquad l_1 = 2.45\, l_2$$

But $\qquad l_1 + l_2 = 80$

$$\Rightarrow \qquad 3.45\, l_2 = 80$$

$$\Rightarrow \qquad l_2 = 23.19 \text{ m}$$

$$l_1 = 56.81 \text{ m}$$

Now, $\qquad$ horizontal tension (H) $= \dfrac{w(2l_1)^2}{8(y_c + d)} = \dfrac{wl_1^2}{2(y_c + d)}$

$$= \frac{7 \times 56.81^2}{2 \times 6} = 1882.64 \text{ kN}$$

Maximum tension in the cable

$$V_A = \frac{wl}{2} + \frac{Hd}{l}$$

$$= \frac{7 \times 80}{2} + \frac{1882.64 \times 5}{80}$$

$$= 280 + 117.67 = 397.67 \text{ kN}$$

$$T_{max} = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{397.67^2 + 1882.64^2} = 1924.18 \text{ kN}$$

12.8 Curved Length of Cables

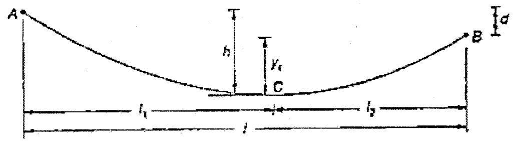


Fig. 12.10

Curved length of cable, $\qquad L = \dfrac{1}{2}\left(2l_1 + \dfrac{8}{3}\dfrac{h^2}{2l_1}\right) + \dfrac{1}{2}\left(2l_2 + \dfrac{8}{3}\dfrac{y_c^2}{2l_2}\right)$

$$L = l_1 + l_2 + \frac{2}{3}\left(\frac{h^2}{l_1} + \frac{y_c^2}{l_2}\right)$$

$$L = l + \frac{2}{3}\left(\frac{h^2}{l_1} + \frac{y_c^2}{l_2}\right)$$

C. Temperature Change Effect on Cable

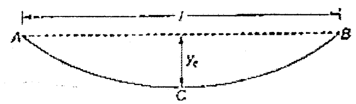


Fig. 12.11

- For the suspended cable shown above, the rise in temperature increases the length of the cable and the sag. Supports remains stationary
- Change in sag (δy_c)

$$L = l + \frac{8}{3}\frac{y_c^2}{l}$$

$$\delta L = \frac{16}{3}\frac{y_c}{l}\delta y_c$$

$$\delta y_c = \frac{3}{16}\times\frac{l}{y_c}\delta L$$

- Change in horizontal force with respect to dip

$$H = \frac{wl^2}{8y_c}$$

$$H \propto \frac{1}{y_c}$$

$$\frac{\delta H}{H} = -\frac{\delta y_c}{y_c}$$

Note: The minus sign in above formula signifies that if dip increases then horizontal force decreases..

- Change in stress with respect to dip

$$(\text{Stress})\ f = \frac{T_{max}}{A} = \frac{H}{A}$$

(assuming V to be very small compared to H)

$$\Rightarrow \quad f \propto H \propto \frac{1}{y_c}$$

$$\Rightarrow \quad \frac{\delta f}{f} \propto \frac{\delta H}{H}$$

$$\Rightarrow \quad \frac{\delta f}{f} \propto -\frac{\delta y_c}{y_c}$$

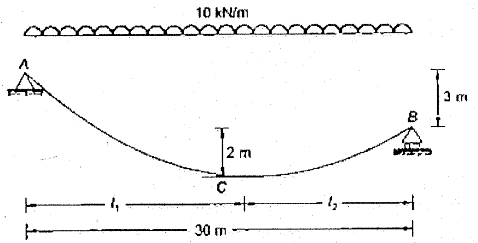
Note: The minus sign in above formula signifies that if cable dip increases then stress decreases.

D. Nonuniform Load

- Since throughout the cable, the horizontal force is constant, hence maximum tension will be evaluated at that support where vertical reaction is greater than the other

$$T_{max} = \sqrt{H^2 + V_{max}^2}$$

Example 12.5 A loaded suspended cable is shown in figure below. Determine the length of the cable.



Solution:

To locate lowest point C:

$$y_c = 2\text{ m},\ d = 3\text{ m},\ h = 2 + 3 = 5\text{ m}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{y_c + d}{y_c}} = \sqrt{\frac{5}{2}} = 1.58$$

$$l_1 = 1.58\, l_2$$

But $\quad l_1 + l_2 = 30$

$\Rightarrow \quad 2.58\, l_2 = 30$

$\Rightarrow \quad l_2 = 11.63\text{ m}$

$$l_1 = 18.37\text{ m}$$

Now, curved length of cable is

$$L = l + \frac{2}{3}\frac{h^2}{l_1} + \frac{2}{3}\frac{y_c^2}{l_2}$$

$$L = 30 + \frac{2}{3}\times\frac{5^2}{18.37} + \frac{2}{3}\times\frac{2^2}{11.63} = 31.14\text{ m}$$

Example 12.6 A steel cable of 8 mm diameter is stretched across two poles 50 m apart. The central dip is 1 m at a temperature of 20°C. Assume unit weight of steel as 78 kN/m³ and thermal coefficient as 11×10^{-6} °C. Determine

(i) Stress intensity in the cable

(ii) The fall in temperature necessary to raise the stress to 30 N/mm²

Solution:

Weight of the cable (W)

$$l = 50\text{ m}$$

$$y_c\ (\text{dip}) = 1\text{ m}$$

$$\gamma_c = 78\text{ kN/m}^3$$

$$W = Al\gamma_c = \frac{\pi}{4}\times 8^2\times 10^{-6}\times 50\times 78 = 0.196\text{ kN}$$

T_{max} Calculation

$$H = \frac{Wl}{8y_c} = \frac{0.196 \times 50}{8 \times 1} = 1.225 \text{ kN}$$

$$V = \frac{W}{2} = \frac{0.196}{2} = 0.098 \text{ kN}$$

$$T_{max} = \sqrt{H^2 + V^2} = \sqrt{1.225^2 + 0.098^2} = 1.23 \text{ kN}$$

$$f_{max}(\text{maximum stress}) = \frac{1.23}{\frac{\pi}{4} \times 8^2 \times 10^{-6}} = 24470 \text{ kN/m}^2 = 24.47 \text{ N/mm}^2$$

Change in dip

$$\delta y_c = \frac{3}{16}\frac{l}{y_c}\delta L$$

$$\delta y_c = \frac{3}{16}\frac{l^2}{y_c}\alpha t \qquad [\because \delta L \simeq l\alpha t]$$

$$= \frac{3}{16} \times \frac{50^2}{1} \times 11 \times 10^{-6} \times t$$

$$= 5.156 \times 10^{-3}\, t$$

Now,

$$\frac{\delta f}{f} = -\frac{\delta y_c}{y_c}$$

$$\left(\frac{30 - 24.47}{24.47}\right) = 5.156 \times 10^{-3} t$$

$$t = 43.83°\text{C}$$

∴ The fall in temperature necessary to raise the stress to 30 N/mm² is by 43.83°C.

E. Stiffening Girder

- The shape of the suspension cable is same as the bending moment diagram for the simply supported beam.
- As loads passes over the bridge, the shape of the bending moment diagram changes with the position of loads and thus the profile of the cable also changes.
- Therefore, to maintain the shape of cable that is parabolic shape throughout the passage of loads, the moving loads are transferred to the suspension cable as uniformly distributed load.
- This objective is obtained by providing stiffening girders which transmits the received load into the from of uniformly distributed load with the help of closely spaced hangers or suspenders.
- For transmitting received loads, stiffening girder have to resist bending moment and shear force. Whereas dead load is transmitted to the suspension cable without creating bending moment and shear force in the stiffening girder.

Note: The girders may be three hinged or two hinged. The girders are suspended from the cables through hanger cables. The roadway is then provided on stiffening girder.

12.9 Suspension Cable with Three Hinged Stiffening Girder

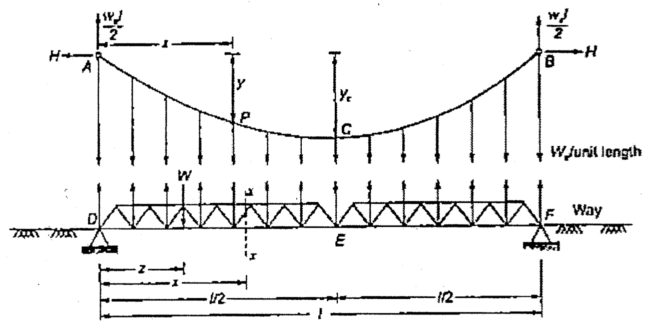


Fig. 12.12 Three-hinged stiffening girder

- A suspension bridge with three hinged stiffening girder is shown in figure above.

Note: Loads are entirely taken by the tension in the cables.

12.9.1 Single Concentrated Load

- Consider the above figure of suspended bridge on which concentrated load W is moving left to right. Here, ACB is suspension cable and DEF is the three hinged stiffening girder.
  Let the load W be at a distance Z from support D and consider a section at a distance of x from A or D.
  Cable: Assuming A as origin, the equation of shape of cable is

$$y = \frac{4y_c}{l^2}x(l - x)$$

- Let the equivalent UDL acting on the cable be w_e

$$\Rightarrow \quad V_A = V_B = V = \frac{w_e l}{2}$$

$$\Rightarrow \quad H = \frac{w_e l^2}{8y_c}$$

- $$T_{max} = \sqrt{V^2 + H^2}$$

- Consider a point $P(x, y)$ on the cable

$$M_p = 0$$

$$M_p = \frac{w_e l}{2}x - \frac{w_e x^2}{2} - Hy = 0$$

$$Hy = \frac{w_e l x}{2} - \frac{w_e x^2}{2}$$

Stiffening Girder:

- Vertical force at D,

$$V_D = \frac{W(l-z)}{l}, \text{ due to } W$$

$$V_{D_2} = \frac{w_e l}{2}, \text{ due to } w_e$$

Similarly

$$V_F = \frac{Wz}{l}, \text{ due to } W$$

$$V_{F_2} = \frac{w_e l}{2}, \text{ due to } w_e$$

- Bending moment at a section with x meters from the left

$$M_x = \{V_D x - W(x-z)\} - \left\{\frac{w_e l}{2}x - \frac{w_e x^2}{2}\right\}, \text{ for } z \le x$$

$$M_x = (V_D x) - \left\{\frac{w_e l}{2}(x) - \frac{w_e x^2}{2}\right\}, \text{ for } z \ge x$$

The second term in both the cases of bending moment above is equal to Hy.

$\Rightarrow$
$$M_x = \mu_x - Hy$$

where, μ_x = Moment due to W at section x

Expression H in terms of W

$$M_E = \mu_E - Hy_c = 0$$

($\because$ moment about hinge point at centre E of the girder is zero)

$\Rightarrow$
$$H = \frac{\mu_E}{y_c}$$

Thus,
$$H = \frac{Wz}{2y_c}, \text{ for } 0 \le z \le x$$

or
$$H = \frac{W(l-z)}{2y_c}, \text{ for } x \le z \le l$$

Also
$$H = \frac{w_e l^2}{8y_c} = \frac{\mu_E}{y_c} = \frac{Wz}{2y_c}; \text{ for } 0 \le z \le x$$

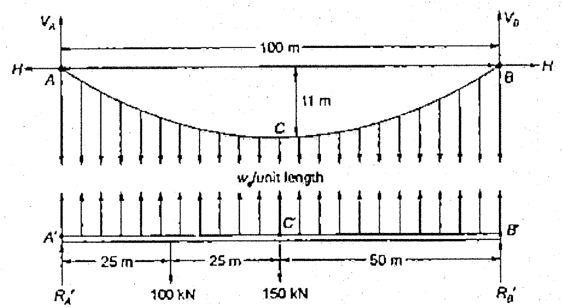
$\Rightarrow$
$$w_e = \frac{4Wz}{l^2}; \text{ for } 0 \le z \le x$$

or
$$w_e = \frac{4W(l-z)}{l^2}; \text{ for } x \le z \le l$$

Note: Due to w_e, the bending moment is $\frac{w_e x}{2}(l-x)$ at section x-x and maximum moment $\frac{w_e l^2}{8}$ at C. The nature of this moment is hogging. The shear force is $-w_e\left(\frac{l}{2}-x\right)$.

Example 12.7 A three hinged stiffening girder of a suspension bridge of span 100 m is subjected to two point load of 100 kN and 150 kN at a distance of 25 m and 50 m from the left end. Find the shear force and bending moment for the girder at a distence of 30 m from the let end. The supporting cable has a central dip of 11 m. Also find the maximum tension and its slope in the cable.

Solution:



Consider Girder:

Let the reactions at A' and B' due to given loading is R_A' and R_B' respectively.

Taking moment at A' and equating it to zero.

$$\Sigma M_A' = 0$$

$$R_B' \times 100 - 100 \times 25 - 150 \times 50 = 0$$

$$R_B' = 100\text{kN}$$

Also,
$$\Sigma M_C' = 0 \quad \text{(where } C \text{ is the mid point)}$$

$$R_B' \times \frac{l}{2} - \frac{w_e l^2}{8} = 0$$

$$w_e = \frac{4R_B'}{l} = \frac{4 \times 100}{100} = 4 \text{ kN/m}$$

$\Rightarrow$
$$R_A' = 250 - 100 = 150 \text{ kN/m}$$

At 30 m from left end:

Shear force = Shear force due to given loading + Shear force due to w_e

$$SF = R_A' - 100 - w_e\left(\frac{100}{2} - 30\right)$$

$$= 150 - 100 - 4 \times 20 = -30 \text{ kN}$$

M(bending moment) = Moment due to given loading + Moment due to w_o

$$M = 150\times30 - 100\times5 - 4\times\frac{30}{2}(100-30)$$

$$= -200\text{ kNm} = 200\text{ kNm (hogging)}$$

Consider cable:

$$V_A = w_o\frac{l}{2} = \frac{4\times100}{2} = 200\text{ kN}$$

$$H = \frac{w_o l^2}{8y_c} = \frac{4\times100^2}{8\times11} = 454.55\text{ kN}$$

$\Rightarrow$

$$T_{max} = \sqrt{V_A^2+H^2} = \sqrt{200^2+454.55^2} = 496.6\text{ kN}$$

Now, slope to horizontal is

$$T_{max}\cos\theta = H$$

$$\theta = \cos^{-1}\left(\frac{454.55}{496.6}\right) = 23°45'$$

12.10 Suspension Cable with Two Hinged Stiffening Girder

- Three hinged stiffening girder is a determinate structure, however two hinged stiffening girder is not.
- Consider a single rolling load W at a distance Z from D as shown in figure below. The load is transmitted to the cable as a UDL, irrespective of load position.

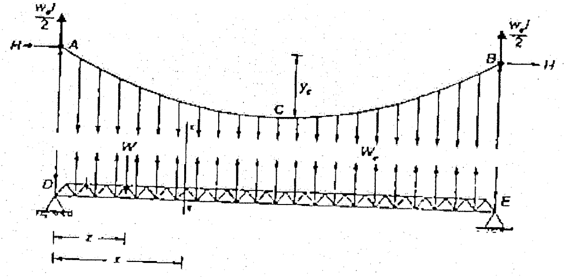


Fig. 12.13 Cable and the two hinged stiffening girder

$$W = w_o l$$

where, w_o is equivalent uniform distributed load.

- The horizontal reaction

$$H = \frac{w_o l^2}{8y_c} = \frac{Wl}{8y_c}$$

Note: The magnitude of horizontal force is constant irrespective of load position.

- Vertical reactions at the ends of the cables is

$$V_A = V_B = \frac{w_o l}{2}$$

$$T_{max} = \sqrt{V_A^2+H^2}$$

- Consider girder: M_x = Beam moment + Moment due to w_o

$$= \text{Beam moment} - w_o\times\frac{x(l-x)}{2}$$

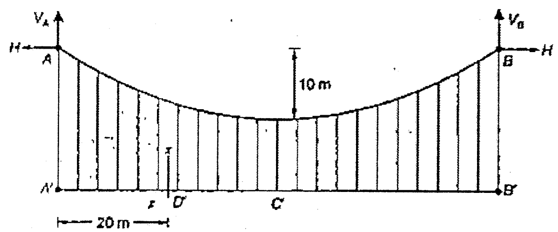
V_x (shear force at x)

$$= \text{Beam shear} + \text{Shear due to } w_e$$

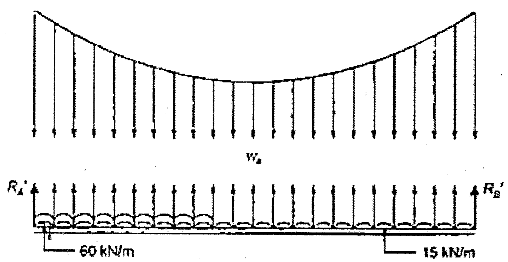
$$= \text{Beam shear} - w_o\left(\frac{l}{2}-x\right)$$

Example 12.8 A suspension bridge of span 100 m and width 6 m is having two cables stiffened with two hinged girders. The central dip is 10 m, the dead load on the bridge is 5 kN/m² and the live load is 15 kN/m² which covers the left half of the span. Determine the shear force and bending moment at 20 m from the left end. Also find the maximum tension in the cable.

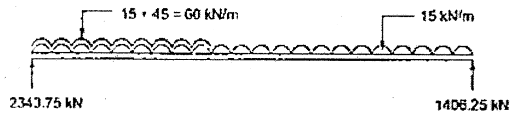
Solution:



(a)



(b) Loading for beam shear and beam moment



(c) Loading for beam shear and beam moment

(because there are two girders so load will be equally distributed to each of them)

Now, $l = 100\text{ m}, h = 10\text{ m}$

$$\text{Dead load per girder} = 5 \times \frac{6}{2} = 15\text{ kN/m}$$

$$\text{Live load per girder} = 15 \times \frac{6}{2} = 45\text{ kN/m on left half portion}$$

$\Rightarrow$ $$\text{Total load on girder} = 60 \times 50 + 15 \times 50 = 3750\text{ kN}$$

Considering cable:

$\therefore$ Equivalent UDL is transmitted to cables is

$$w_e = \frac{\text{Total load}}{\text{Span}} = \frac{3750}{100} = 37.5\text{ kN/m}$$

$$H = \frac{w_e l^2}{8h} = \frac{37.5 \times 100^2}{8 \times 10} = 4687.5\text{ kN}$$

$$V = w_e \frac{l}{2} = 37.5 \times \frac{100}{2} = 1875\text{ kN}$$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{1875^2 + 4687.5^2} = 5048.6\text{ kN}$$

Considering girder:

$$\text{Shear at } D(V_D') = \text{Beam shear} + \text{Shear due to } w_e$$

$$R_A' = \frac{M_B}{100} = \frac{15 \times 50 \times 25 + 60 \times 50 \times 75}{100}$$

$$R_A' = 2437.5\text{ kN}$$

$\therefore$ $$\text{Beam shear at } D' = R_A' - 60 \times 20$$

$$= 2437.5 - 1200 = 1237.5\text{ kN}$$

$\Rightarrow$ $$V_D' = 1237.5 - w_e\left(\frac{l}{2} - x\right)$$

$$= 1237.5 - 37.5(50 - 20) = 112.5\text{ kN}$$

Now, $$M_D = \text{Beam moment} + \text{Moment due to } w_e$$

$$= \text{Beam moment} - w_e \frac{x(l - x)}{2}$$

$$= [2437.5 \times 20 - 60 \times 20 \times 10] - 37.5 \times 20 \times \frac{80}{2}$$

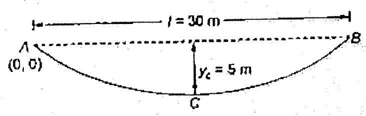
$$= 6750\text{ kNm}$$

Summary

- Suspension bridge makes use of suspended cables which in turn help for long span bridging.
- The shape of funicular polygon is same as the shape of a loaded cable.
- Under uniform distributed loading over entire span, the shape of the cable is assumed to be parabola.
- But under UDL or due to its own weight in reality deflected shape is catenary not parabola.
- For identical conditions and loading, horizontal force is comparatively much greater at the supports for flat cable (that is having lesser sag or dip)
- Tension in cable varies along its length whereas horizontal force in the cable is constant.
- Due to increase in temperature, dip in the cable increases and therefore horizontal force decreases.
- Due to increase in temperature, dip in the cable increases and therefore the stress decreases.
- The girders may be three hinged or two hinged, suspended from the cables through the hangers or suspenders. Then afterwards roadway is provided on the stiffening girder.
- Loads are entirely taken as tension in the cable.

Objective Brain Teasers

Q.1 Figure below show a cable suspended through simple supports. Assuming point A as origin, the equation of cable profile is



(a) $\frac{x}{45}(x - 30)$ (b) $\frac{x}{30}(45 - x)$

(c) $\frac{x}{30}(x - 45)$ (d) $\frac{x}{45}(30 - x)$

Q.2 Tension in cable under UDL
(a) varies along its length
(b) is constant
(c) has maximum value at the support
(d) Both (a) and (c)

Q.3 Horizontal force in the cable
(a) varies along its length
(b) maximum at support
(c) is constant
(d) maximum at mid span

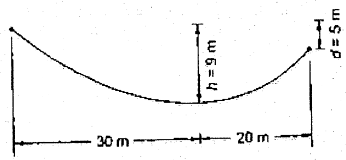
Q.4 A cable is suspended through simple supports (at same level) having span equal to 20 m and dip equal to 2 m. Then curved length of the cable is

(a) 18.52 m (b) 20.53 m
(c) 23.61 m (d) 24.82 m

Q.5 A cable stretched between two supports 50 m apart with one end above the other by 5 m. The left support is above the right support. The cable is loaded with uniform distributed load of 10 kN/m and dip in the cable is 8 m. The distance of the lowest point measured from right support is

(a) 31 m (b) 45 m
(c) 63 m (d) 89 m

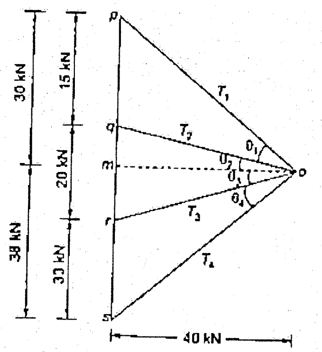
Q.6 For a suspended cable arrangement shown below, the curved length of cable is



(a) 28.68 m (b) 44.44 m
(c) 52.33 m (d) 68.81 m

Q.7 The bending moment for the pier in case of a cable clamped to saddle carried on smooth rollers on the top of the pier is
(a) depends on horizontal components of tensions of both sides of cable
(b) zero
(c) equal to any arbitrary value
(d) both (a) and (c)

Q.8 Figure below is of force polygon of a loaded suspend cable. op, oq, or and os represent the tensions T_1, T_2, T_3 and T_4 in the segments of the cable. The value of T_1 is



(a) 30 kN (b) 50 kN
(c) 60 kN (d) 90 kN

Q.9 Loaded cable supported at the ends satisfies the condition/conditions
(a) The force polygon should be open
(b) The force polygon should be closed
(c) The funicular polygon should be closed
(d) Both (b) and (c)

Q.10 The funicular polygon is
(a) same as the shape of loaded cord
(b) of a triangular shape
(c) of a parabolic shape
(d) a straight line

Q.11 A cable is suspended through simple support at the same level. The equivalent weight on the cable is 10 kN/m, distance between support is 50 m and the sag of the cable is 5 m. The horizontal reaction at the support is
(a) 500 kN (b) 625 kN
(c) 675 kN (d) 750 kN

Q.12 The equivalent uniform distributed load acting on the suspended cable with three hinged stiffening girder is 5 kN/m. The span of the cable is 10 m and sag of the cable is 1 m. The maximum tension acting on the cable is
(a) 46.2 kN (b) 67.3 kN
(c) 74.4 kN (d) 88.7 kN

Q.13 Degree of indeterminacy of a suspension bridge with a two hinged stiffening girder is x and that for a three hinged stiffening girder is y. Then x-y is
(a) 1 (b) 2
(c) 3 (d) 5

Q.14 The maximum tension occurring in a suspended cable is 5 kN and horizontal tension is 4 kN. The inclination of the cable with the horizontal at the support is
(a) 23° (b) 37°
(c) 53° (d) 90°

Answers

1. (d)	2. (d)	3. (c)	4. (b)	5. (a)
6. (c)	7. (b)	8. (b)	9. (d)	10. (a)
11. (b)	12. (b)	13. (a)	14. (b)	

Hints and Explanations:

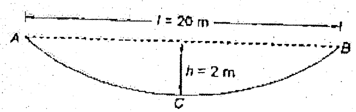
1. (d)

$$y = \frac{4y_c}{l^2}x(l-x) = \frac{4\times5}{30^2}\times x\times(30-x)$$

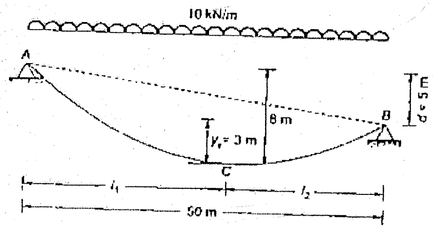
$$= \frac{x}{45}(30-x)$$

4. (b)

$$L = l+\frac{8}{3}\frac{h^2}{l} = 20.53\text{ m}$$



5. (a)



$$\frac{l_1}{l_2} = \left(\frac{y_c+d}{y_c}\right)^{1/2} = \left(\frac{8}{3}\right)^{1/2}$$

$l_1 = 1.633\, l_2$

But, $l_1 + l_2 = 50$ m

$\Rightarrow$ $l_1 = 31.01 \text{ m} \simeq 31$ m

6. (c)

$$L = l+\frac{2}{3}\left(\frac{h^2}{l_1}+\frac{y_c^2}{l_2}\right)$$

$$= 50+\frac{2}{3}\left[\frac{9^2}{30}+\frac{4^2}{20}\right]$$

$$= 52.33$$

8. (b)

$$T_1 = \sqrt{30^2+40^2} = 50\text{ kN}$$

11. (b)

$$H = \frac{w_e l^2}{8y_c} = \frac{10\times50^2}{8\times5} = 625\text{ kN}$$

12. (b)

$$V = V_A = V_B = \frac{w_e l}{2}$$

$$= \frac{5\times10}{2} = 25\text{ kN}$$

$$H = \frac{w_e l^2}{8y_c} = \frac{5\times10^2}{8\times1}$$

$$= \frac{500}{8} = 62.5\text{ kN}$$

$$T_{max} = \sqrt{V^2+H^2} = \sqrt{25^2+62.5^2}$$

$$= 67.315 \simeq 67.3\text{ kN}$$

13. (a)

$x = 1$
$y = 0$
$x - y = 1$

14. (b)

$T_{max} = 5$ kN
$H = 4$ kN
$H = T_{max}\cos\theta$

$$\theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$\theta = 36.87° \simeq 37°$

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