Playing with Numbers

General Form Of Numbers

A given number can be represented in various forms. One of the forms is the general form.

Lets now consider a four digit number *pqrs,* with the digits in the thousands, hundreds, tens and ones place respectively as *p*, *q*, *r* and *s* respectively. Then its generalised form is

$pqrs = (p \times 1000) + (q \times 100) + (r \times 10) + (s \times 1)$

Let us discuss some examples based on these concepts.

Example 1:

What is the general form of the following numbers?

(a) **375** **(b)** **901** **(c)** ***ps*** **(d)** **5*z*7**

Solution:

(a) General form of 375 = $300+70+5$ = $100 \times 3 + 10 \times 7 + 5$

(b) General form of 901 = $900+0+1$ = $100\times9+10\times0+1$

(c) General form of *ps* = $10 \times p + s$ = $10p+s$

(d) General form of 5*z*7 = $500+10z+7$ = $100 \times 5 + 10 \times z + 7$

Example 2:

Write the following expressions in their usual form.

1. **$100 \times x + 10 \times z + r$**
2. **$100 \times 7 + 10 \times 2 + 0$**

Solution:

(a) Usual form of $100 \times x + 10 \times z + r$ = $100x+10z+r$ = *xzr*

(b) Usual form of 100 × 7 + 10 × 2 + 0 = $700+20+0$ = 720

Example 3:

What is the generalised form of the number 4578?

Solution:

Generalised form of 4578 = 4 × 1000 + 5 × 100 + 7 × 10 + 8 × 1

Example 4:

What is the normal form of $(5\times1000)+(9\times100)+(3\times1)$?

Solution:

Normal form of $(5\times1000)+(9\times100)+(3\times1)=5000+900+3=5903$

Some Properties Of Numbers

Numbers have several interesting properties. When a number is combined with its reverse, several new and interesting results are obtained.

The reverse of a number simply means writing the digits of the number in the reverse order.

Now, let us learn the properties of three-digit numbers.

Properties exhibited by three-digit numbers when their digits are reversed

When we reverse the digits of a three-digit number, an important property of three-digit numbers can be observed.

Divisibility of a Number by 5 and 10

Hotel Grand Mansion has 432 rooms and 10 floors. Surbhi, who went to the hotel for the first time with her sister Shweta, was quite amazed by the numbers. She asked Shweta if she could tell whether each of the floors had the same number of rooms. Shweta replied that this is not possible as 432 is not exactly divisible by 10.

Surbhi wanted to check if what Shweta said was true. She quickly divided the numbers as

```
     43
10)432
   40
   --
    32
    30
    --
     2
```

She found that since the remainder is 2, 432 is not exactly divisible by 10.

Shweta had also said that the number 432 is not divisible by 10.

Do you want to know which trick Shweta used to give such a quick answer?

There is a rule to check if a number is divisible by 10. Similarly, we have a rule to check if a number is divisible by 5 or not.We can save the time used in division by applying this rule.

Example 1:

Use divisibility test to find which of the following numbers are divisible by 10.

(i) 11557468 (ii) 98746590 (iii) 95460 (iv) 74684255

Solution:

(i) 11557468 has 8 at its ones place. Therefore, it is not divisible by 10.

(ii) 98746590 has 0 at its ones place. Therefore, it is divisible by 10.

(iii) 95460 has 0 at its ones place. Therefore, it is divisible by 10.

(iv) 74684255 has 5 at its ones place. Therefore, it is not divisible by 10.

Example 2:

Which of the following numbers are divisible by 5 and 10 both?

(i) 8974 (ii) 5540 (iii) 58790 (iv) 57875

Solution:

(i) 8974 has 4 at its ones place. Therefore, it is neither divisible by 5 nor by 10.

(ii) 5540 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iii) 58790 has 0 at its ones place. Therefore, it is divisible by 5 and 10 both.

(iv) 57875 has 5 at its ones place. Therefore, it is divisible by 5 but not by 10.

Example 3:

Find the value of x and y such that 98x2y is divisible by 10.

Solution:

The given number is 98x2y.

We know that a number is divisible by 10 if the digit at its units place is 0. Hence, $y = 0$.

According to the rule of divisibility by 10, the digit at the units place of a number must be 0, while the rest of the digits can take any value from 0 to 9. Hence, x may take any value from 0 to 9.

Example 4:

Find the least number which, when

(i) subtracted from 624, gives a number divisible by 10

(ii) added to 624, gives a number divisible by 10

Solution:

(i) The given number is 624.

The number less than 624, which is divisible by 10 (which has a zero at its units place), is 620.

We can see that $624 - 620 = 4$.

Hence, 4 is subtracted from 624 to get a number that is divisible by 10.

(ii) Similarly, the number larger than 624, which is divisible by 10 (which has a zero at its units place), is 630.

We can see that $630 - 624 = 6$.

Hence, we need to add 6 to 624 to get a number that is divisible by 10.

Example 5:

How many numbers from 201 to 300 are divisible by 5?
Solution:
A number is divisible by 5 if and only if it ends with 0 or 5.
Now, the numbers from 201 to 300 which ends with 0 or 5 are 205, 210, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260, 265, 270, 275, 280, 285, 290, 295 and 300.
Thus, there are 20 such numbers.

Divisibility of a Number by 2, 4 and 8

Ashok was standing with his friend Nikhil at a bus stop. The first bus that came was of route 432. Ashok looked at it and said to Nikhil that the route number of the bus was divisible by 2. Nikhil wanted to verify what Ashok had said and started calculating in his notebook. This is what he did:

```
   216
2)432
  4
  --
   3
   2
  --
   12
   12
  ---
    ×
  ---
```

Although he verified what his friend had said, Nikhil was amazed at Ashok's quick calculation. He asked Ashok how he did it.

Ashok said he used a trick to check the divisibility of 432 by 2. He also told Nikhil that he knows divisibility rules for 4 and 8 also.

Note that **a number is also divisible by 4 if the number formed by its last two digits is 00.**

A number is also divisible by 8, if the number formed by its last three digits is 000.

For example, in the number 200, the number formed by the last two digits is 00. Therefore, 200 is divisible by 4. In the number 2000, the number formed by its last three digits is 000. Therefore, 2000 is divisible by 8.

Now, we do not have to divide a number by 2, 4, and 8 to check whether it is divisible by them or not. We can use the above stated divisibility rules.

Let us now look at some more examples to understand this concept better.

Example 1:

Which of the following numbers are divisible by 2?

(i) 48 (ii) 97 (iii) 345 (iv) 8460

Solution:

(i) The number 48 has 8 at its ones place, which is even. Therefore, 48 is divisible by 2.

(ii) The number 97 has 7 at its ones place, which is odd. Therefore, 97 is not divisible by 2.

(iii) The number 345 has 5 at its ones place, which is odd. Therefore, 345 is not divisible by 2.

(iv) The number 8460 has 0 at its ones place. Therefore, 8460 is divisible by 2.

Example 2:

Which of the following numbers are divisible by 4?

(i) 2348 (ii) 326

Solution:

(i) The given number is 2348.

The number formed by its last two digits is 48, which is divisible by 4.

Thus, the number 2348 is divisible by 4.

(ii) The given number is 326.

The number formed by its last two digits is 26, which is not divisible by 4.

Thus, the number 326 is not divisible by 4.

Example 3:

Find whether 56112 is divisible by 8 or not.

Solution:

The given number is 56112.

The number formed by its last three digits is 112, which is divisible by 8.

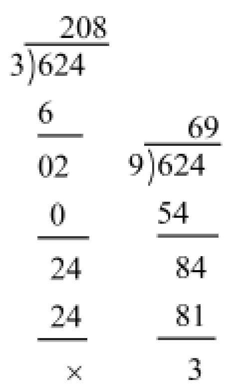
Thus, the given number is divisible by 8.

Divisibility of a Number by 3 and 9

Imagine if you had to check if numbers such as 27, 18, or 33 were divisible by 3 and 9. You will obviously check it mentally and give the answer without performing any long division. But what if you are asked to check if the number 123456789 is divisible by 3 and 9? You will take a lot of time calculating the divisibility of the number with both 3 and 9 before you give the answer.

A teacher asked the students in a class to check whether 624 is divisible by 3 and 9. Mayank said that 624 is divisible by 3 but not divisible by 9 by just looking at the number. Shikhar, another student, started performing the divisions.

This is what Shikhar did.



Although Shikhar and Mayank gave the same answer, Mayank took very little time to give the answer, while Shikhar had to perform the division. Do you know which trick Mayank used to give such a quick answer?

Mayank used two rules of divisibility to get the answer.

It can be observed that a number which is divisible by 9 is also divisible by 3.

For example, consider the number 252. The sum of its digits is 9, which is divisible by 3 and 9 both.

Let us discuss a few more examples to understand the concept better.

Example 1:

Which of the following numbers are divisible by 3?

(i) 163 (ii) 276

Solution:

(i) Sum of the digits of the number 163 = 1 + 6 + 3 = 10

Here, the sum of the digits (i.e., 10) is not divisible by 3. Therefore, 163 is not divisible by 3.

(ii) Sum of the digits of the number 276 = 2 + 7 + 6 = 15

Here, the sum of the digits (i.e., 15) is divisible by 3. Therefore, 276 is divisible by 3.

Example 2:

Find whether 477918 is divisible by 9 or not.

Solution:

The given number is 477918.

The sum of its digits is 4 + 7 + 7 + 9 + 1 + 8 = 36, which is divisible by 9.

Therefore, the given number is divisible by 9.

Example 3:

Fill in the blank space in the number 6587_41 so that it is divisible by 3.

Solution:

The given number is 6587_41.

The sum of the given digits is 6 + 5 + 8 + 7 + 4 + 1 = 31

We know that a number is divisible by 3 if the sum of its digits is also divisible by 3.

Therefore, the unknown digit can be 2 or 5 or 8 as then, the sum of the digits will be 33 or 36 or 39, and each of these sums is divisible by 3.

Example 4:

Check the divisibility of 388 by 3 and 9.

Solution:

Sum of the digits of the number 388 = 3 + 8 + 8 = 19

Here, the sum of the digits is not divisible by either 3 or 9.

Hence, 388 is not divisible by 3 or 9.

Example 5:

If x is a digit, then what are its possible values if the number $3x38$ is divisible by 3?

Solution:

We know that x is a digit (i.e., x is a number between 0 and 9).

We also know that the number $3x38$ is divisible by 3.

Thus, the sum of its digits will also be divisible by 3. [Divisibility rule of 3]

Hence, $3 + x + 3 + 8 = 14 + x$ is also divisible by 3.

The numbers larger than 14 and divisible by 3 are

15, 18, 21, and 24

To find the least value of x,

$14 + x = 15$

$x = 15 - 14$

$\therefore x = 1$

Similarly, $14 + x = 18$

$x = 18 - 14$

$\therefore x = 4$

Similarly, $14 + x = 21$

$x = 21 - 14$

$\therefore x = 7$

Similarly, $14 + x = 24$

$x = 24 - 14$

$\therefore x = 10$.

This is a two-digit number.

Hence, x cannot take this value.

Hence, the values of x are 1, 4, and 7 such that $3x38$ is divisible by 3.

Example 6:

The number $10x8$ is divisible by 9. If x is a digit, then find its possible values.

Solution:

$10x8$ is divisible by 9.

Hence, the sum of its digits is also divisible by 9.

$\therefore 1 + 0 + x + 8 = 9 + x$ is divisible by 9

$\therefore 9 + x = 9$ or 18 or 27 ….

If we take

$9 + x = 9$

Then, $x = 0$

And, if we take

$9 + x = 18$

Then, $x = 9$

Here, we cannot take $9 + x = 27$ because it gives $x = 18$, which is a two-digit number.

Hence, the values of x are 0 or 9.

Example 7:

How many numbers from 130 to 200 are divisible by 5 but not by 3?
Solution:
A number is divisible by 3 if and only if the sum of its digits is divisible by 3 whereas a number is divisible by 5 if and only if it ends with 0 or 5.
The numbers from 130 to 200 which are divisible by 5 are 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195 and 200. Now, the sum of digits of these numbers is 4, 9, 5, 10, 6, 11, 7, 12, 8, 13, 9, 14, 10, 15 and 2 respectively. Among these sum of digits, only 6, 9, 12 and 15 are divisible by 3.
Thus, the numbers which are divisible by 5 but not by 3 are 130, 140, 145, 155, 160, 170, 175, 185, 190 and 200.
Hence, there are 10 such number.

Letters for Digits

We all are familiar with puzzles in books, magazines, newspapers etc. Here, we introduce a new type of puzzle where you will need to find some missing numbers using the basic properties of mathematics.

Let us look at the following puzzle.

$$\begin{array}{r} \_\ \ 7 \\ +2\ \ \_ \\ \hline 6\ \ 2 \\ \hline \end{array}$$

How can we find the missing numbers?

In the first column, the sum of 7 and another digit cannot be 2, as 2 is less than 7. Hence, it is logical that the number may be 12 as $7 + 5 = 12$.

Thus, in the first column the missing number is 5.

$$\begin{array}{rr} (1) \rightarrow \text{carry} & \\ \_ & 7 \\ +2 & 5 \\ \hline 6 & 2 \\ \hline \end{array}$$

The second column has a carryover value of 1.

Thus, it becomes $1+2+\_=6$

Now, it is clear that the missing number is 3.

Therefore, we can write

$$\begin{array}{rr} (1) & \\ \underline{3} & 7 \\ +2 & 5 \\ \hline 6 & 2 \\ \hline \end{array}$$

We can also write the same problem as

$$\begin{array}{rr} B & 7 \\ +2 & A \\ \hline 6 & 2 \\ \hline \end{array}$$

Finding the missing values in the previous example is the same as finding the values of letters in the given problem.

Here, the digits are denoted by the letters of the English alphabet and only one digit is denoted by one letter.

For example, let us try to find the value of A in the following example.

$$\begin{array}{r} A \\ A \\ +A \\ \hline 1A \\ \hline \end{array}$$

Here, we add a digit three times and get the same digit at the units place.

Such digits are 0 and 5.

If we take A as 0, then the addition of three 0s equals 0. However, we cannot get any number in the tens place. In the problem, we can see that the sum has 1 in its tens place. Let us now take the digit 5 as the value of A.

If A = 5, then

$$\begin{array}{r} 5 \\ 5 \\ +\ 5 \\ \hline 15 \\ \hline \end{array}$$

Therefore, the value of A is 5.

Let us discuss some more examples to better understand this concept.

Example 1:

Find the values of the letters in the following puzzles:

(i)
$$\begin{array}{r} 1\ B\ A \\ +\ A\ B\ 2 \\ \hline 5\ 6\ 6 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 3\ A \\ \times\ \ 3 \\ \hline 1\ 0\ A \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} P\ Q \\ \times\ 6 \\ \hline Q\ Q\ Q \\ \hline \end{array}$$

Solution:

(i) We have

$$\begin{array}{r} 1\ B\ A \\ +A\ B\ 2 \\ \hline 5\ 6\ 6 \\ \hline \end{array}$$

Here, the sum of A and 2 is 6.

Thus, A is 4.

On replacing letter A with its value i.e., 4, the puzzle becomes

$$\begin{array}{r} 1\,B\,4 \\ +4\,B\,2 \\ \hline 5\,6\,6 \\ \hline \end{array}$$

We can see that the sum of B and B is 6 and at hundreds place 1+4=5.

Thus, B is 3.

$$\therefore \quad \begin{array}{r} 1\,3\,4 \\ +4\,3\,2 \\ \hline 5\,6\,6 \\ \hline \end{array}$$

Hence, the value of A is 4 and the value of B is 3.

(ii) We have

$$\begin{array}{r} 3\,A \\ \times\,3 \\ \hline 10A \\ \hline \end{array}$$

Here, the ones digit of A × 3 = A.

Thus, A must be 0 or 5.

If we take A = 0, then the puzzle becomes

$$\begin{array}{r} 3\,0 \\ \times\,3 \\ \hline 100 \\ \hline \end{array}$$

But 30 × 3 = 90 ≠ 100

Therefore, we take A = 5.

$$\begin{array}{r} 3\,5 \\ \times\,3 \\ \hline 105 \\ \hline \end{array}$$

This gives us the correct product, as given in the puzzle.

Hence, the answer is A = 5.

(iii) We have

$$\begin{array}{r} P\;Q \\ \times\;6 \\ \hline Q\,Q\,Q \\ \hline \end{array}$$

In the puzzle, we have 6 × Q = Q at the units place.

Hence, Q may be 0, 2, 4, 6, or 8.

If we take Q = 0, then the puzzle becomes

$$\begin{array}{r} P\;\;0 \\ \times\;6 \\ \hline 0\;0\;0 \\ \hline \end{array}$$

This can only be possible if P = 0.

Since P and Q are different numbers, this answer is not acceptable.

Let us now take Q = 2.

On doing so, the puzzle becomes

$$\begin{array}{r} P\;\;2 \\ \times\;6 \\ \hline 2\;2\;2 \\ \hline \end{array}$$

We know that 37 × 6 = 222. However, this is not our answer because 2 is not at the units place in the number 37.

Let us now take Q = 4.

On doing so, the puzzle becomes

$$\begin{array}{r} P\ 4 \\ \times\ 6 \\ \hline 4\ 4\ 4 \\ \hline \end{array}$$

We know that 74 × 6 = 444. Hence, the value of P is 7.

If we take Q = 6, then the puzzle becomes

$$\begin{array}{r} P\ 6 \\ \times\ 6 \\ \hline 6\ 6\ 6 \\ \hline \end{array}$$

However, 111 × 6 = 666

This cannot be our answer because 6 is not at the units place of 111 and 111 is a three-digit number.

If we take Q = 8, then the puzzle becomes

$$\begin{array}{r} P\ 8 \\ \times\ 6 \\ \hline 8\ 8\ 8 \\ \hline \end{array}$$

However, 148 × 6 = 888

This cannot be our answer because 148 is a three-digit number.

Hence, the only values that can complete the puzzle correctly are P = 7 and Q = 4.

Example 2:

Find the digits represented by the letter *A* and *B* in the following addition.

$$\begin{array}{ccc} & 2 & A \\ + & B & 2 \\ + & 3 & A \\ \hline & 9 & 8 \\ \hline \end{array}$$

Solution:
On observing the addition, we have

$$2A+2=8$$
$$2A=8-2$$
$$2A=6$$
$$A=\frac{6}{2}$$
$$A=3$$
And,
$$2+B+3=9$$
$$B=9-5$$
$$B=4$$
Verification:
$$\begin{array}{ccc} & 2 & 3 \\ + & 4 & 2 \\ + & 3 & 3 \\ \hline & 9 & 8 \\ \hline \end{array}$$
Example 3:
Find the digits *A*, *B* and *C* in the following addition, where *A*, *B* and *C* are three consecutive digits.
$$\begin{array}{cccc} & A & B & C \\ + & B & C & A \\ + & C & A & B \\ \hline 1 & 6 & 6 & 5 \\ \hline \end{array}$$
Solution:
A, *B* and *C* are three consecutive digits. Therefore,
$B = A + 1$
$C = B + 1 = (A + 1) + 1 = A + 2$
On observing the given addition, we have
$$C+A+B=15 \text{ or } 25$$
$$(A+2)+A+(A+1)=15 \text{ or } 25$$
$$3A+3=15 \text{ or } 25$$
$$3A=12 \text{ or } 22$$
$$A=4 \text{ or } \frac{22}{3} (\text{Not possible})$$
Thus, $A = 4$, $B = 5$ and $C = 6$.
Verification:

$$\begin{array}{r} 456 \\ 564 \\ +\ 645 \\ \hline 1665 \\ \hline \end{array}$$