Chapter 15

Statistics

Exercise 15.2

Q. 1 Find the mean and variance for each of the data.

6, 7, 10, 12, 13, 4, 8, 12

Answer:

We know that Mean,

Mean, $\bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

From the given data, we can form the table:

x_i	Deviation forms mean $|x_i - \bar{x}|$	$(x_i - \bar{x})^2$
6	6 − 9 = - 3	9
7	7 − 9 = -2	4
10	10 − 9 = 1	1
12	12 − 9 = 3	9
13	13 − 9 = 4	16
4	4 − 9 = - 5	25
8	8 − 9 = - 1	1
12	12 − 9 = 3	9
		$\sum|x_i - \bar{x}|^2 = 74$

We know that Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8} \times 74 = 9.25$

Mean = 9 and Variance = 9.25

Q. 2 Find the mean and variance for each of the data.

First 10 multiples of 3

Answer:

The 10 multiples of 3 are:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here the number of observation, n = 10

We know that Mean,

Mean $\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$

From the given data, we can form the table:

x_i	Deviations from mean $(x_i - \bar{x})$	$(X_i - \bar{x})^2$
3	3 -16.5 = -13.5	182.25
6	6 -16.5 = -10.5	110.25
9	9 -16.5 = -7.5	56.25
12	12 -16.5 = -4.5	20. 25
15	15 -16.5 = -1.5	2. 25
18	18 -16.5 = 1.5	2. 25
21	21 -16.5 = 4.5	20.25
24	24 -16.5 = 7.5	56.25
27	27 -16.5 = 10.5	110.25
30	30 -16.5 = 13.5	182.25
		$\sum_{f=1}^{10} (x_i - \bar{x})^2$ $= 742.5$

We know that Variance, $\sigma^2 = 1\frac{1}{n}\sum_{i=1}^{10}(x_i - \overline{x})^2 = \frac{1}{10} \times 742.5 = 74.25$

Ans. Mean = 16.5 and Variance = 74.25

Q. 3 Find the mean and variance for each of the data.

x_i	6	10	14	18	24	28	30
y_i	2	4	7	12	8	4	3

Answer:

Presenting the data in the tabular form, we get

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	6-19 = -13	169	338
10	4	40	10-19 = -9	81	324
14	7	98	14-19 =-5	25	175
18	12	2136	18 − 19 = -1	1	12
24	8	192	24-19 =5	25	200
28	4	112	28-19=9	81	324
30	3	90	30-19 = 11	121	363
	$\Sigma f_i = N = 40$	$\Sigma f_i x_i$=760			$\Sigma f_i(x_i - \bar{x})^2$=1736

We know that Mean,

Where N = 40

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{760}{40} = 19$

We know that Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$

Mean = 19 and Variance = 43.4

Q. 4 Find the mean and variance for each of the data.

x_i	92	93	97	102	104	109
f_i	3	2	3	6	3	3

Answer:

Presenting the data in the tabular form, we get

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$\lvert x_i - \bar{x}\rvert^2$	$f_i \lvert x_i - \bar{x}\rvert^2$
92	3	276	$92 - 100$ $= -8$	64	192
93	2	186	$93 - 100$ $= -7$	49	98
97	3	291	$97 - 100$ $= -3$	9	27
98	2	196	$98 - 100$ $= -2$	4	8
102	6	612	$102 - 100$ $= 2$	4	24
104	3	312	$104 - 100$ $= 4$	16	48
109	3	327	$109 - 100$ $= 9$	81	243

We know that Mean,

Where N = 22

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$$

We know that Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$

Mean = 100 and Variance = 29.09

Q. 5 Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Answer:

Let the assumed mean, A = 64 and h = 1

We obtain the following table from the given data:

x_i	f_i	$y_i = \frac{x_i - A}{h}$	y_1^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80

We know that Mean,

$$\therefore \text{Mean, } \bar{x} = A \frac{\sum_{i=1}^{9} f_i x_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$$

We know that Variance $\sigma2 =$

$$\therefore \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^{9} f_i x_i{}^2 - \left(\sum_{i=1}^{9} f_i x_i \right)^2 \right]$$

$$= \frac{1}{100^2}[100 \times 286 - 0]$$

$$= 2.86$$

We know that Standard Deviation = σ

$$\therefore \sigma = \sqrt{2.86} = 1.691$$

Ans. Mean = 64 and Standard Deviation = 1.691

Q. 6 Find the mean and variance for the following frequency distributions.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Answer:

classes	frequency f_i	Midpoint X_i	$f_i x_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f_i(X_i - \bar{X})^2$
0-30	2	15	30	-92	8464	16928
30-60	3	45	135	-62	3844	11532
60-90	5	75	375	-32	1024	5120
90-120	10	105	1050	-2	4	40
120-150	3	135	405	28	784	2352
150-180	5	165	825	58	3364	16820
180-210	2	195	390	88	7744	15488
	$\Sigma f_i = N = 30$		$\Sigma f_i X_i = 3210$			$\Sigma f_i(X_i - \bar{X})^2 = 68280$

Presenting the data in the tabular form, we get

We know that Mean,

Where N = 30

$$\therefore \bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{20}{30} \times 30 = 105 + 2 = 107$$

We know that Variance, σ2 =

$$= \frac{h^2}{N^2} \left[N \sum_{l=1}^{7} f_i x_i{}^2 - \left(\sum_{i=1}^{7} f_i x_i \right)^2 \right]$$

$$= \frac{(30)^2}{(30)^2}[\ 30 \times 76 - (2)^2]$$

$$= 2280 - 4$$

$$= 2276$$

Ans. Mean = 107 and Variance = 2276

Q. 7 Find the mean and variance for the following frequency distributions.

Class	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Answer:

Presenting the data in the tabular form, we get

classes	frequency f_i	Midpoint X_i	$f_i x_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f_i(X_i - \bar{X})^2$
0-10	5	5	25	-22	484	2420
10-20	8	15	120	-12	144	1152
20-30	15	25	375	-2	4	60
30-40	16	35	560	8	64	1024
40-50	6	45	270	18	324	1944
	$\Sigma f_i = N = 50$		$\Sigma f_i X_i = 1350$			$\Sigma f_i(X_i - \bar{X})^2 = 6600$

We know that Mean,

$$\therefore \bar{x} = A + \frac{\sum_{i-1}^{3} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$$

We know that Variance

$$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i{}^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}\left[50 \times 68 - (10)^2\right]$$

$$= \frac{1}{25}\left[3400 - 100\right] = \frac{3300}{25}$$

$$= 132$$

Ans. Mean = 27 and Variance = 132

Q. 8 Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-110	105-110	105-115
No. of children	3	4	7	15	9	6	6	3

Answer:

Let the assumed mean, A = 92.5 and h = 5

We obtain the following table from the given data:

Height (class)	Number of children (frequency) f_i	Midpoint X_i	$y_i = \frac{x_{1-A}}{h}$	$y_i{}^2$	$f_i y_i$	$f_i y_i{}^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	72.5	-3	9	-12	36
80-85	7	38.5	-2	4	-14	28
85-90	7	42.5	-1	1	-7	7
90-95	15	46.5	0	0	0	0
95-100	9	50.5	1	1	9	9
100-105	6	102.5	2	4	12	24

105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	$\Sigma f_i = N$ $= 60$				$\Sigma f_i y_i$ $= 6$	$\Sigma f_i y_i{}^2 = 254$

We know that Mean,

$$\therefore \bar{x} = A + \frac{\Sigma_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$$

We know that Variance

$$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N \Sigma_{i=1}^{9} f_i y_i{}^2 - \left(\Sigma_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{(5)^2}{(60)^2}\left[60 \times 254 - (6)^2\right]$$

$$= \frac{25}{3600}(15204) = 105.58$$

We know that Standard Deviation = σ

$$\therefore \sigma = \sqrt{105.58} = 10.275$$

Ans. Mean = 93, Variance = 105.583 and Standard Deviation = 10.275

Q. 9 The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5,

40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Answer:

Let the assumed mean, A = 42.5 and h = 4

We obtain the following table from the given data:

Height (class)	Number of children (frequency)	Midpoint X_i	$y_i = \frac{x_{1-A}}{h}$	$f_i y_i$	$f_i y_i{}^2$
32.5-36.5	15	34.5	-2	-30	60
36.5-40.5	17	38.5	-1	-17	17
40.5-44.5	21	42.5	0	0	0
44.5-48.5	22	46.5	1	22	22
48.5-52.5	25	50.5	4	50	100
	$\Sigma f_i = N$ $= 100$			$\Sigma f_i y_i$ $= 25$	$\Sigma f_i y_i{}^2 = 199$

We know that Mean,

$$\therefore \bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$$

We know that Variance σ2 =

$$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N \sum_{i=1}^{5} f_i y_i{}^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{16}{10000}[100 \times 199 - (25)^2]$$

$$= \frac{16}{10000}[19900 - 625]$$

$$= \frac{16}{10000}(19275)$$

$= 30.84$

We know that Standard Deviation $= \sigma$

$\therefore \sigma = \sqrt{30.84} = 5.553$

Ans. Mean = 43.5, Variance = 30.84 and Standard Deviation = 5.553