CONDITIONAL - PROBABILITY

- Conditional probability –

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

→ Consider two events A and B defined on a sample space S. The probability of occurence of event A given that event B has already occurred is known as conditional probability of A relative to B.

Ex – 1

- Rolling a dice :

→ Sample space(S) = $\{1, 2, 3, 4, 5, 6\}$
$\quad\quad\quad\quad\quad\quad\quad\quad\ \ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}$

event(E) = $\{1\}$

Probability of E $\boxed{P(E) = \frac{n(E)}{n(S)}} \Rightarrow \frac{1}{6}$.

→ If you know some thing about the output,

→ for example – The dice are rolled and the output will be even number. In this condition the probability of getting output will change.

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$\downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow$$
$$0\ \ \tfrac{1}{3}\ \ 0\ \ \tfrac{1}{3}\ \ 0\ \ \tfrac{1}{3}$$

New Sample space, $S_N = \{2, 4, 6\}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

→ The dice are rolled and the outcome will be prime number. so,

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$\downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow$$
$$0\ \ \tfrac{1}{3}\ \ \tfrac{1}{3}\ \ 0\ \ \tfrac{1}{3}\ \ 0$$

new S going to change = $\{2, 3, 5\}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

** → We have a sample space and they have perform a experiment and said that something have occured then the new probability to the simple event should changed.

- $E = \{2, 4, 6\}$ (event to getting even number)

$$P(E) = \frac{n(E)}{n(S)}$$

$$\Rightarrow \frac{3}{6} \Rightarrow 1/2$$

- $F = \{2, 3, 5\}$ (event to getting prime no)

$$P(F) \Rightarrow \frac{n(F)}{n(S)} \Rightarrow \frac{3}{6} \Rightarrow 1/2.$$

- $E \cap F = \{2\}$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} \Rightarrow \frac{\cancel{2}\,1}{6}.$$

Q. What is the probability of getting a prime number given that an even number showed up.

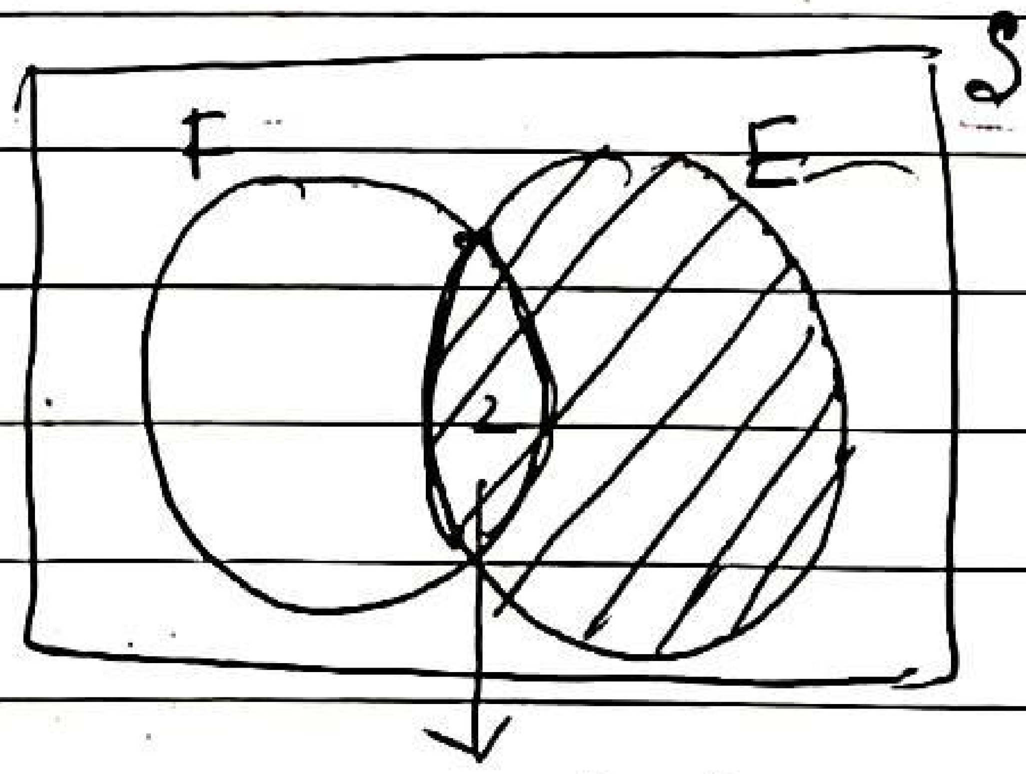
$$\rightarrow P(F/E) = \frac{n(F \cap E)}{n(E)} = \frac{1}{3}$$

$S = \{1, 2, 3, 4, 5, 6\}$

$1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$0 \quad \underline{1/3} \quad 0 \quad 1/3 \quad 0 \quad 1/3$



required probability

$$P(F/E) \Rightarrow \frac{\frac{n(F \cap E)}{n(S)}}{\frac{n(E)}{n(S)}}$$

**

$$\boxed{P(F/E) \Rightarrow \frac{P(F \cap E)}{P(E)}} \Rightarrow \frac{1/6}{1/2} \Rightarrow 1/3$$

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$\boxed{EX-2}$

Rolling two dice. It is already known that there is 4 on the first dice.

$\rightarrow S = \{6 \times 6\} = 36$

- $E = \{41, 42, 43, 44, 45, 46\}$ (4 on first dice)
  $\quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$$P(E) \Rightarrow \frac{n(E)}{n(S)} \Rightarrow \frac{6}{36} \Rightarrow \frac{1}{6}.$$

- $F = \{15, 51, 24, 42, 33\}$ [event the sum of out come 6

$$P(F) = \frac{5}{36}.$$

$$P(E/E) \Rightarrow \frac{P(E \cap E)}{P(F)} \Rightarrow \frac{\frac{n(E \cap E)}{n(S)}}{\frac{n(E)}{n(S)}} \Rightarrow \frac{1/36}{6/36} \Rightarrow 1/6.$$

- $\boxed{\text{Example}-1}$

A family has two children. If one of them is known to be ~~a~~ boy, then what is the probability that both are boy.

$\rightarrow S = \{BB, BG, GB, GG\}$

$\quad 1/4 \quad 1/4 \quad 1/4 \quad 1/4$
$\quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
$\quad 1/3 \quad 1/3 \quad 1/3 \quad 0$

$E = \{BB, BG, GB\}$ (event one of them are boy)

$F = \{BB\}$ (Both are boy) $\quad E \cap F = \{BB\}$

$$P(F/E) = \frac{P(F \cap E)}{P(E)} \Rightarrow \frac{\frac{n(F \cap E)}{n(S)}}{\frac{n(E)}{n(S)}}$$

$$= \frac{1/4}{3/4} \Rightarrow 1/3$$

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$\boxed{\text{Example-2}}$

performing a experimen In which flipping 3 - coins. Known that at least ~~2 Head turn~~ two of them is turnout to be head, then ~~what is the prob~~ at least one of them → turnout to be tail.

$S \Rightarrow \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
$\quad 1/8 \quad 1/8 \quad 1/8 \quad 1/8 \quad 1/8 \quad 1/8 \quad 1/8 \quad 1/8$

$E = \{HHT, HHT, HTH, THH\}$

$F = \{\cancel{HHT}, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \Rightarrow \frac{3/8}{7/8} \Rightarrow 3/7.$$

$$F(F/E) = \frac{P(F \cap E)}{P(E)} \Rightarrow \frac{3/8}{4/8} \Rightarrow 3/4$$

$\boxed{\text{Ex-3}}$

given 10 cards which are no from 1-10 ~~and~~ then we ~~have~~ are drawn one of them, and found that already the number is at least 5. therefore What is the probability that It is actually 10.

$\rightarrow \quad S = \{1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10\}$
$\quad 1/10 \quad 1/10 \quad \ldots \quad 1/10$
$\quad 0 \quad 0 \quad 0 \quad 0 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$E = \{5, 6, 7, 8, 9, 10\}$ (atleast 5)

$F = \{10\}$

$(F \cap F) = \{10\}$

$$P(F/E) = \frac{P(E \cap F)}{P(E)} \Rightarrow \frac{1/10}{6/10} \Rightarrow 1/6$$