Chapter 3

Numbers

CHAPTER HIGHLIGHTS

- *Rule of Signs*
- *Classification of Real Numbers*
- *Perfect Numbers*
- *Hierarchy of Arithmetic Operations*
- *Number of Factors of a Number*
- *Involution and Evolution*
- *LCM and HCF Models*
- *Successive Division*
- *Factorial*
- *Number Systems*
- *Conversions*
- *Binary Arithmetic*

NUMBERS

NUMBERS is one of the most important topics required for competitive entrance exams—particularly, the MBA entrance exams. In this chapter, we have put together a number of models of problems—mainly based on various problems that have been appearing in different exams.

ADDITION is the process of finding out single number or fraction equal to two or more quantities taken together.

SUBTRACTION is the process of finding out the quantity left when a smaller quantity (number/fraction) is reduced from a larger one.

MULTIPLICATION signifies repeated addition. If a number has to be repeatedly added then that number is multiplicand. The number of multiplicands considered for addition is multiplier. The sum of repetition is product. For example, in the multiplication $3 \times 4 = 12$, 3 is the multiplicand, 4 is the multiplier and 12 is product.

DIVISION is a reversal of multiplication. In this we find how often a given number called divisor is contained in another given number called dividend. The number expressing this is called the quotient and the excess of the dividend over the product of the divisor and the quotient is called remainder.

For example, in the division 32/5, 32 is dividend, 5 is divisor, 6 is quotient, and 2 is remainder.

RULE OF SIGNS

The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.

Example: $-1 \times -1 = +1$;

$+1 \times -1 = -1$;

$+1 \times +1 = +1$;

$-1 \times +1 = -1$;

CLASSIFICATION OF REAL NUMBERS

Real numbers are classified into rational and irrational numbers.

Rational Numbers: A number which can be expressed in the form p/q where p and q are integers and $q \neq 0$ is called a rational number.

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator $1 \neq 0$. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly (For example, 0.333…, 0.111111…, 0.166666…, etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form p/q, and hence it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

Irrational Numbers: Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt[4]{5}$, $\sqrt[3]{9}$, etc.

Numbers like π, e are also irrational numbers.

Between any two numbers, there are infinite number of irrational numbers.

Another way of looking at rational and irrational numbers is terminating decimals and recurring decimals are both rational numbers.

Any non-terminating, non-recurring decimal is an irrational number.

Integers: All integers are rational numbers. Integers are classified into negative integers, zero, and positive integers. Positive integers can be classified as prime numbers and composite numbers. In problems on numbers, we very often use the word 'number' to mean an 'integer.'

Prime Numbers: A number other than 1 which does not have any factor apart from one and itself is called a prime number.

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. **Every prime number greater than 3 can be written in the form of ($6k + 1$) or ($6k - 1$) where k is an integer.** For the proof of this, refer to 4th point under 'Some important points to note' given later on in this chapter.

Composite Numbers: Any number other than 1, which is not a prime number is called a composite number. In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

NOTE

The number 1 is neither prime nor composite.

The only prime number that is even is 2.

There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

Relative Primes: Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1. For example, the numbers 15 and 16 do not have any common factors, and, hence, they are relative primes. Please note that none of the two numbers may individually be prime and still they can be relative primes. Unity is a relative prime to all numbers.

Multiples: If one number is divisible exactly by a second number, then the first number is said to be a multiple of the second number. For example, 15 is a multiple of 5; 24 is a multiple of 4.

Factors: If one number divides a second number exactly, then the first number is said to be a factor of the second number. For example, 5 is a factor of 15; 3 is a factor of 18. Factors are also called sub-multiples or divisors.

Even and odd numbers: Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

NOTE

Every even number ends in 0, 2, 4, 6, or 8.

The sum of any number of even numbers is always even.

The sum of odd number of odd numbers (i.e. the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.

The product of any number of odd numbers is always odd.

The product of any number of numbers where there is at least one even number is even.

Perfect Numbers

A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

For example, 6 is a perfect number because the factors of 6, i.e. 1, 2, and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

Rules for Divisibility

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

Divisibility by 2: A number divisible by 2 will have an even number as its last digit (128, 246, 2346, etc.).

Divisibility by 3: A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is $9 + 1 + 2 + 3 = 15$, which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789, etc. are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence, the number 74549 is not divisible by 3.

Divisibility by 4: A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 1785$\overline{64}$, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence the number 476854 is not divisible by 4.

Divisibility by 5: A number is divisible by 5 if its last digit is 5 or zero (15, 40, etc.).

Divisibility by 6: A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.).

Divisibility by 7: If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

Col(1) Number	Col(2) Twice the units digit	Col(3) Remaining part of the number	Col(3) – Col(2)
456745	10	45674	45664
45664	8	4566	4558
4558	16	455	439
439	18	43	25

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

Divisibility by 8: A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc. are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc. are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence, the number 4763 is not divisible by 9.

Divisibility by 10: A number divisible by 10 should end in zero.

Divisibility by 11: A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11—that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

Recurring Decimals: A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, thus

$$\frac{8}{3} = 2.666..... = 2.\dot{6} \text{ or } 2.\overline{6};$$

$$\frac{1}{7} = 0.142857142857142857... = 0.142857$$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that recur.

A recurring decimal like $0.\overline{3}$ is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\overline{6}$ (which is equal to 0.16666...) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a 'circulator'. The digit, or set of digits, which is repeated is called the 'period' of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form p/q, where p and q are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

Solved Examples

Example 1

Express $0\cdot\overline{4}$ in the form of a fraction.

Solution

Let $x = 0\cdot\overline{4} = 0.444\ldots$ (1)

$$10x = 4.444\ldots = 4.\overline{4} \quad (2)$$

Subtracting (1) from (2),

$$9x = 4$$

$$\Rightarrow \quad x = \frac{4}{9}.$$

Example 2

Express $0\cdot\overline{63}$ in the form of a fraction.

Solution

Let $x = 0\cdot\overline{63} = 0.636363\ldots$ (3)

$$100x = 63.636363\ldots = 63\cdot\overline{63} \quad (4)$$

Subtracting (3) from (4),

$$99x = 63$$

$$\Rightarrow \quad x = \frac{7}{11}.$$

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits (called the period of the decimal) for its numerator, and for its denominator, the number which has for its digits as many nines as there are digits in the period.

Thus, $0.\overline{37}$ can be written as equal to $\frac{37}{99}$; $0.\overline{225}$ can be written as equal to $\frac{225}{999} = \frac{25}{111}$;

$$0.\overline{63} = \frac{63}{99} = \frac{7}{11}.$$

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

Now we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below:

In the numerator, write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e. as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e., $$0.1\overline{56} = \frac{156-1}{990} = \frac{155}{990} = \frac{31}{198}$$

$$0.7\overline{3} = \frac{73-7}{90} = \frac{66}{90} = \frac{11}{15}$$

Number of Factors of a Number

If N is a composite number such that $N = a^p \,.\, b^q \,.\, c^r \ldots$ where a, b, c are prime factors of N and p, q, r ... are positive integers, then the number of factors of N is given by the expression

$$(p + 1)\,(q + 1)\,(r + 1)\ldots$$

For example $140 = 2^2 \times 5^1 \times 7^1$.

Hence, 140 has $(2 + 1)(1 + 1)(1 + 1)$, i.e. 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So if you want to find the number of factors the given number has excluding 1 and the number itself, we find out $(p + 1)\,(q + 1)\,(r + 1)$ and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and itself.

Number of Ways of Expressing a Given Number as a Product of Two Factors

The given number N (which can be written as equal to $a^p.\, b^q.\, c^r \ldots$ where a, b, c are prime factors of N and p, q, r ... are positive integers) can be expressed as the product of two factors in different ways.

The number of ways in which this can be done is given by the expression $1/2\ \{(p + 1)(q + 1)(r + 1)\ldots\}$

So, 140 can be expressed as a product of two factors in 12/2 or 6 ways {because $(p + 1)\,(q + 1)\,(r + 1)$ in the case of 140 is equal to 12}

If p, q, r, etc. are all even, then the product $(p + 1)\,(q + 1)\,(r + 1)\ldots$ becomes odd and the above rule will not be valid since we cannot take 1/2 of an odd number to get the number of ways. If p, q, r, ... are all even, it means that the number N is a perfect square. This situation arises in the specific cases of perfect squares because a perfect square can also be written as {square root × square root}. So, two different cases arise in case of perfect squares depending on whether we would like to consider writing the number as {square root x square root} also as one of the ways.

Thus, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following two rules.

1. as a product of two DIFFERENT factors: $\frac{1}{2}$ $\{(p+1)(q+1)(r+1)\ \ldots\ -1\}$ ways (excluding $\sqrt{N}\times\sqrt{N}$).
2. as a product of two factors (including $\sqrt{N}\times\sqrt{N}$) in $\frac{1}{2}\{(p+1)(q+1)(r+1)\ \ldots\ +1\}$ways.

Example 3

Find the number of factors of 3025.

Solution

$3025 = (5)\,(605) = (5)\,(5)\,(121) = 5^2\,11^2$

Number of factors of 3025

$$= (2+1)\,(2+1) = 9$$

Example 4

In how many ways can 22500 be written as a product of two different factors?

Solution

$22500 = 150^2 = ((2)\,(5)\,(3)\,(5))^2 = 2^2 5^4 3^2$

Number of ways

$$= \frac{1}{2}\{(2+1)\,(4+1)\,(2+1)-1\} = 22.$$

Sum of all the Factors of a Number

If a number $N = a^p \,.\, b^q \,.\, c^r \ldots$ where $a, b, c \ldots$ are prime numbers and $p, q, r \ldots$ are positive integers, then, the sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1}-1}{a-1}\right).\left(\frac{b^{q+1}-1}{b-1}\right).\left(\frac{c^{r+1}-1}{c-1}\right)\ldots$$

The above can be verified by an example.

Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here $a = 2$, $b = 3$, $p = 4$, $q = 1$.

Hence, sum of all the factors

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1}\times\frac{8}{2} = 124$$

The list of factors of 48 is:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

Product of all the Factors of a Number

We shall now consider another kind of question which has also appeared frequently in exams. These questions refer to the ‘structure’ of numbers, i.e. the prime factors of a number and the canonical representation of a number. We shall begin by working out the product of all the factors of a given number.

Example 5

What is the product of all the factors of 180?

Solution

$180 = 4(45) = 2^2 3^2 5^1$. There are $(2+1)\,(2+1)\,(1+1)$ or 18 factors.

If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).

∴ The required product is 180^9.

In general, if $N = p^a\, q^b\, r^c$ (where at least one of a, b, c is odd), the product of all the factors of N is $N^{\frac{d}{2}}$, where d is the number of factors of N and is given by $(a+1)(b+1)(c+1)$.

Example 6

Let us see what happens when N is a perfect square, say 36.

We want the product of all the factors of 36.

Solution

$36 = 2^2 3^2$ (there are 9 factors)

$$1(36) = 2(18) = 3(12) = 4(9) = 6(6)$$

∴ The product of all the factors is 36^4 (6).

In general, let $N = p^a\, q^b\, r^c$ where each of a, b, c is even.

There are $(a+1)(b+1)(c+1)$ say d factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e. $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}}\,(\sqrt{N}) = N^{\frac{d}{2}}$

∴ Whether or not N is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where d is the number of factors of N.

Number of Ways of Writing a Number as Product of Two Co-primes

Using the same notation and convention used earlier.

If $N = a^p \,.\, b^q \,.\, c^r \ldots$, then, the number of ways of writing N as a product of 2 co-primes is 2^{n-1}, where ‘n’ is the number of distinct prime factors of the given number N.

Taking the example of 48, which is $2^4 \times 3^1$, the value of ‘n’ is 2 because only two distinct prime factors (i.e. 2 and 3 only) are involved.

Hence, the number of ways $= 2^{2-1} = 2^1 = 2$ i.e., 48 can be written as product of 2 coprimes, in two different ways. They are (1 and 48) and (3, 16).

Number of Co-primes to *N*, That are Less than *N*

If N is a number that can be written as $a^p \,.\, b^q \,.\, c^r \ldots$, then, the number of co-primes of N, which are less than N, represented by $\phi(N)$ is,

$$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

For example if, 48 is considered,

$$N = a^p \,.\, b^q \,.\, c^r \ldots$$

i.e., $48 = 2^4 \,.\, 3^1$.

Hence, $a = 2$, $b = 3$, $p = 4$, $q = 1$.

$$\phi(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$$

$$= 48\times\frac{1}{2}\times\frac{2}{3} = 16.$$

NOTE

If numbers less than 48 are listed, and co-primes to 48 are picked up, the count of co-primes will be 16.

Sum of Co-primes to *N* That Are Less Than *N*

The sum of the co-primes of N, that are less than N is $\frac{N}{2}\,.\,\phi(N)$. If we consider the above example, already we have $\phi(48) = 16$.

Hence, sum of co-primes of 48 that are less than $48 = \frac{N}{2}\,.\,\phi(N) = \frac{48}{2}\times 16 = 384$.

NOTE

After picking out the co-primes of 48 that are less than 48, they can be added and the sum can be verified.

Least Common Multiple (LCM) and Highest Common Factor (HCF)

Least common multiple (LCM) of two or more numbers is the least number that is divisible by each of these numbers (i.e. leaves no remainder or remainder is zero). The same can be algebraically defined as 'LCM of two or more expressions is the expression of the lowest dimension which is divisible by each of them, i.e. leaves no remainder or remainder is zero'.

Highest common factor (HCF) is the largest factor of two or more given numbers. The same can be defined algebraically as 'HCF of two or more algebraical expressions is the expression of highest dimension which divides each of them without remainder'.

HCF is also called GCD (greatest common divisor).

Product of two numbers = LCM × HCF

LCM is a multiple of HCF

For finding **LCM and HCF of fractions,** first reduce each fraction to its simplest form, i.e. cancel out any common factors between the denominator and numerator and then apply appropriate formula from the following:

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

LCM and HCF Models

LCM—Model 1: In this model of problem, you will need to find out the smallest number (or number in a specified range like the largest five-digit number) which when divided by 2 or more other numbers (i.e. divisors) leaves the same remainder in all cases.

The basic distinguishing feature of this model of problems is that the remainder will be the **same** in all the cases (and that remainder will also be given).

The smallest such number will be the remainder itself. The next higher number that satisfies the given conditions is the LCM of the given numbers (i.e. divisors) plus the remainder given, i.e. add the remainder (which is the same in all cases) to the LCM of the given numbers (i.e. divisors).

To find any larger number that satisfies a given condition, we will first need to find out a multiple of the LCM in that range and add the remainder to this multiple of the LCM.

The general rule can be written as follows:

Any number which when divided by p, q, or r leaving the same remainder s in each case will be of the form ***k (LCM of p, q, and r) + s*** *where* $k = 0, 1, 2, \ldots$

If we take $k = 0$, *then we get the smallest such number.*

Example 7

Find the HCF of 1363 and 1457.

Solution

```
              1
1363)1457
     1363        14
       94)1362
          1316    2
            47)94
               94
               --
                0
               --
```

∴ HCF (1367, 1457) = 47

Example 8

Find the smallest number which when divided by 5 or 11 leaves a remainder of 4 and is greater than the remainder.

Solution

Set of such numbers are of the form K [LCM (5, 11)] + 4 where K is a whole number.

We get the required number when $K = 1$

∴ Smallest number = LCM (5, 11) + 4

$$= 55 + 4 = 59.$$

LCM—Model 2: In this model, the remainders in the divisions given will not be the same but the difference between the divisor and the remainder (i.e. the complement of the remainder) will be the same in each case. For example, you may be asked to find out 'the smallest number which when divided by 4 or 6 gives respective remainders of 3 and 5'. Here, the remainders are not the same as in LCM —Model 1; but the difference between the divisor and the remainder is same in each case. In the first case, the difference between the divisor and the remainder is 1(= 4 – 3). In the second case, also the difference between the divisor and the remainder is 1(= 6 – 5).

The smallest such number is LCM minus constant difference (the constant difference being the difference between the divisor and the corresponding remainder in all cases).

Similarly, any multiple of the LCM minus the constant remainder also will satisfy the same condition.

In the aforementioned example, the LCM of 4 and 6 is 12, and hence the required number is 11 (which is equal to 12 – 1).

The general rule can be written as follows:

Any number which when divided by p, q, or r leaving respective remainders of s, t, and u where (p – s) = (q – t) = (r – u) = v (say) will be of the form ***k (LCM of p, q, and r)*** *– v*

The smallest such number will be obtained by substituting k = 1.

Example 9

Find the smallest number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

Solution

Required number = LCM (9, 11) – 2 = 97.

Example 10

Find the largest four-digit number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

Solution

Required number must be in the form LCM (9, 11) k – 2, i.e. $99k - 2$, where k is the largest natural number satisfying $99k - 2 < 10000$.

$$\therefore \quad k < 101\frac{1}{33}$$

$$\therefore \quad k = 101$$

∴ Largest number = 9997.

LCM—Model 3: In this model, the remainders will not be the same and even the differences between each of the given divisors and the corresponding remainders also will not remain the same.

Let us take an example and see how to solve this type of problem.

Example 11

Find the smallest number which leaves a remainder of 7 when divided by 11 and leaves a remainder of 12 when divided by 13.

Solution

Let the number be in the forms $11k_1 + 7$ and $13k_2 + 12$ where k_1 and k_2 have the least possible values.

$$11k_1 + 7 = 13k_2 + 12$$

$$k_1 = k_2 + \frac{2k_2 + 5}{11}$$

As k_1 is an integer, $2k_2 + 5$ must be divisible by 11.

Hence $k_2 = 3$.

∴ Smallest number = 51.

HCF—Model 1: In this model, we have to identify the largest number that exactly divides the given dividends (which are obtained by subtracting the respective remainders from the given numbers).

The largest number with which the numbers p, q, or r are divided giving remainders of s, t, and u, respectively, will be the ***HCF of the three numbers (p – s), (q – t), and (r – u).***

Let us understand this model with an example.

Example 11

Find the largest number which leaves remainders of 2 and 3 when it divides 89 and 148, respectively.

Solution

Largest number = HCF (89 – 2, 148 – 3) = 29

HCF—Model 2: In this model, the problem will be as follows:

'Find the largest number with which if we divide the numbers p, q and r, the remainders are the same'.

Take the difference between any two pairs out of the three given numbers. Let us say we take the two differences $(p - q)$ and $(p - r)$. The HCF of these numbers will be the required number.

Here, the required number = HCF of $(p \sim q)$ and $(p \sim r)$ = HCF of $(p \sim q)$ and $(q \sim r)$ = HCF of $(q \sim r)$ and $(p \sim r)$

Let us take an example and look at this model.

Example 12

Find the largest number which divides 444, 804, and 1344 leaving the same remainder in each case.

Solution

Largest number

= HCF (804 – 444, 1344 – 804)

= HCF (360, 540) = 180.

Successive Division

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called 'successive division'. A successive division process can continue upto any number of steps—until the quotient in a division becomes zero for the first time, i.e. the quotient in the first division is taken as dividend and divided in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

If we say that 2479 is divided successively by 3, 5, 7, and 2, then the quotients and remainders are as follows in the successive division.

Dividend	Divisor	Quotient	Remainder
2479	3	826	1
826	5	165	1
165	7	23	4
23	2	11	1

Here we say that when 2479 is successively divided by 3, 5, 7, and 2 the respective remainders are 1, 1, 4 and 2.

Example 13

A number when divided by 6 and 4 successively leaves remainders of 5 and 2, respectively. Find the remainder when the largest such two digit number is divided by 9.

Solution

Let the quotients obtained when the number is divided by 6 and 4 successively be q_1 and q_2, respectively.

$$\text{Number} = 6q_1 + 5$$

Its successive division, the quotient obtained for each division starting from the first, forms the dividend for the next division.

$\therefore$ $q_1 = 4q_2 + 2$

$\therefore$ number = $6(4q_2 + 2) + 5 = 24q_2 + 17$

Largest two-digit number satisfying the given conditions is obtained when $24q_2 + 17 < 100$ and q_2 is maximum, i.e. $q_2 < 3\frac{11}{24}$ and it is maximum, i.e. $q_2 = 3$.

$\therefore$ number = 89. required remainder = 8

Alternative method:

Divisors: 6 × 4

Remainders: 5 + 2

The smallest number satisfying the given conditions is found using the following method. Each divisor and the remainder it leaves are written as shown above. Starting with the last remainder, each remainder is multiplied with the previous divisor and added to that divisor's remainder. This procedure is carried out until the divisor's remainder is the first remainder.

Smallest possible value of the number

$$= (6)(2) + 5 = 17$$

General form of the number = $k(6 \times 4) + 17 = 24k + 17$ where k is any whole number.

The number would be the largest two-digit number when $24k + 17 < 100$ and k is maximum, i.e. $k < 3\frac{11}{24}$ and k is maximum, i.e. $k = 3$.

$\therefore$ Largest two-digit number = 89

$\therefore$ Required remainder = 8

Example 14

A number when divided by 3, 5, and 6 successively leaves remainders of 1, 3, and 2, respectively. Find the number of possible values it can assume which are less than 1000.

Solution

Let the quotients obtained when the number is divided by 3, 5, and 6 successively be q_1, q_2, and q_3, respectively.

Number = $3q_1 + 1$

$q_1 = 5q_2 + 3$

$q_3 = 6q_1 + 2$

$\therefore$ number = $3(5q_2 + 3) = 1$

$= 3(5(6q_1 + 2) + 3) + 1 = 90q_1 + 40$

$90q_1 + 40 < 1000$

$q_1 < 10\frac{2}{3}$

$\therefore$ q_1 has 11 possibilities, i.e. 0 to 10.

Alternative method:

Divisors: 3 × 5 × 6

Remainders: 1 + 3 + 2

Smallest possible value of the number

$$= ([(5 \times 2) + 3] \times 3) + 1 = 40$$

General form of the number = $k \times (3 \times 5 \times 6) + 40 = 90k + 40$, where k is any whole number.

If $\quad 90k + 40 < 1000, k < 10\frac{2}{3}$

$\therefore$ k has 11 possibilities (i.e. 0 to 10).

Factorial

Factorial is defined for any positive integer. It is denoted by $\angle$ or !. Thus, 'Factorial n' is written as $n!$ or $\angle n$. $n!$ is defined as the product of all the integers from 1 to n.

Thus $n! = 1, 2, 3, \ldots (n-1)\, n$.

0! is defined to be equal to 1.

0! = 1 and 1! is also equal to 1.

Largest Power of a Number in *N*!

There is a specific model of problems relating to factorial which appeared about 3 to 4 times in CAT papers. This involves finding the largest power of a number contained in the factorial of a given number. Let us understand this type of problem with the help of an example.

Example 15

Find the number of zeros that 324! ends with.

Solution

The largest power of A in $B!$ can be found using the method below when A is composite.

The largest power of each prime factor of A in $B!$ is found. The minimum of these results is the required power.

In the given problem, 10 = (2) (5). The required power is the minimum of the largest power of 2 in 324! and the largest power of 5 in 324!. Using the approach shown in the previous example, the largest power of 2 in 324! is 32!. From the previous example, the largest power of 5 in 324! is 78.

$\therefore$ Required power = min (321, 781) = 78.

Alternative method:

Largest power of 10 = Largest power of (2) (5)

As 5 > 2, the largest power of 5 which can divide 324! < the largest power of 2 which can divide 324!

$\therefore$ Largest power of (2) (5) which can divide 324! = largest power of 5 which can divide 324! is 10^{78}.

$\therefore$ 324! ends with 78 zeros.

Some Important Points to Note

Please note the following points also which will be very useful in solving problems on numbers.

1. When any two consecutive integers are taken, one of them is odd and the other is even. Hence, the product of any two consecutive integers is always even, i.e. divisible by 2.

   Two consecutive integers can be written in the form of n and $n-1$ or n and $n+1$. Hence, any number of the form $n(n-1)$ or $n(n+1)$ will always be even.
2. Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6.

   Three consecutive integers can be of the form $(n-1)$, n, and $(n+1)$. The product of 3 consecutive integers will be of the form $(n-1)n(n+1)$ or $n(n^2-1)$ or $(n^3-\text{n})$. Hence, any number of the form $(n-1)n(n+1)$ or $n(n^2-1)$ or (n^3-n) will always be divisible by 6.
3. Out of any n consecutive integers, exactly one number will be divided by n and the product of n consecutive integers will be divisible by $n!$
4. Any prime number greater than 3 can be written in the form of $6k+1$ or $6k-1$. The explanation is:

   Let p be any prime number greater than 3. Consider the three consecutive integers $(p-1)$, p, and $(p+1)$. Since p is a prime number greater than 3, p CANNOT be even. Since p is odd, both $(p-1)$ and $(p+1)$ will be even, i.e. both are divisible by 2.

   Also, since, out of any three consecutive integers, one number will be divisible by 3, one of the three numbers $(p-1)$, p, or $(p+1)$ will be divisible by 3. But, since p is prime number—that too greater than 3—p cannot be divisible by 3. Hence, either $(p-1)$ or $(p+1)$, one of them—and only one of them—is definitely divisible by 3.

   If $(p-1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e. it will be of the form $6k$. If $(p-1)$ is of the form $6k$, then p will be of the form $(6k+1)$.

   If $(p+1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e. it will be of the form $6k$. If $(p+1)$ is of the form $6k$, then p will be of the form $(6k-1)$.

   Hence any prime number greater than 3 will be of the form $(6k+1)$ or $(6k-1)$.

Example 16

Find the HCF of $\frac{3}{5}, \frac{6}{10}$, and $\frac{9}{20}$.

Solution

To find the LCM or HCF of fractions, first empress all the fractions in their simplest term.

$$\text{HCF (fractions)} = \frac{\text{HCF(numerators)}}{\text{LCM}(\text{denominators})}$$

$$= \frac{\text{HCF}(3,3,9)}{\text{LCM}(5,5,20)} = \frac{3}{20}$$

Example 17

Find the LCM of $\frac{3}{5}, \frac{6}{10}$, and $\frac{9}{20}$.

Solution

To find the LCM or HCF of fractions, first empress all the fractions in their simplest term.

LCM (fractions)

$$= \frac{\text{LCM (numerators)}}{\text{HCF (denominators)}} = \frac{\text{LCM}(3,3,9)}{\text{HCF}(5,5,20)} = \frac{9}{5}$$

Number Systems

The numbers that are commonly used are the decimal numbers, which involve ten symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we consider the number 526 in the decimal system, it means $5 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$. Likewise, 85.67 means $8 \times 10^1 + 5 \times 10^0 + 6 \times 10^{-1} + 7 \times 10^{-2}$. The role played by '10' in the decimal system is termed as the 'base' of the system. In this chapter, we see the numbers expressed in various other bases.

Base: It is a number which decides the place value of a symbol or a digit in a number. Alternatively, it is the number of distinct symbols that are used in that number system.

NOTES

1. The base of a number system can be any integer greater than 1.
2. Base is also termed as radix or scale of notation.

The following table lists some number systems along with their respective base and symbols.

Number System	Base	Symbols
Binary	2	0, 1
Septenuary	7	0, 1, 2, 3, 4, 5, 6
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Duo-decimal	12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
Hexa-decimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$A = 10, B = 11, C = 12, D = 13, E = 14, F = 15$. Some books denote ten as 'E' and eleven as 'e'.

Representation: Let N be any integer, r be the base of the system, and $a_0, a_1, a_2, \ldots, a_n$ be the required digits by which N is expressed. Then, $N = a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \ldots + a_1 r + a_0$, where $0 \le a_i < r$.

We now look into some representations and their meaning in decimal system.

Examples:

1. $(100011)_2$
   $= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
   $= 32 + 0 + 0 + 2 + 1 = (35)_{10}$
2. $(1741)_8$
   $= 1 \times 8^3 + 7 \times 8^2 + 4 \times 8^1 + 1 \times 8^0$
   $= 512 + 448 + 32 + 1 = 993_{10}$
3. $(A3D)_{16}$
   $= A \times 16^2 + 3 \times 16^1 + D \times 16^0$
   $= 10 \times 256 + 48 + 13 = 2621_{10}$

Conversions

1. **Decimal to binary:**

   (a) $(253)_{10} = (11111101)_2$

   Working:

2	253
2	126 – 1
2	63 – 0
2	31 – 1
2	15 – 1
2	7 – 1
2	3 – 1
	1 – 1

NOTE

The remainders are written from bottom to top.

(b) $(37.3125)_{10} = (100101.0101)_2$

Working: The given decimal number has 2 parts:

(i) Integral part 37,

(ii) Fractional part 0.3125.

(i) Conversion of integral part:

2	37	
2	18	– 1
2	9	– 0
2	4	– 1
2	2	– 0
1		– 0

$\therefore (36)_{10} = (100100)_2$

(ii) Conversion of the fractional part:

Multiply the decimal part with 2 successively and take the integral part of all the products starting from the first.

	Binary digits
$0.3125 \times 2 = 0.6250$	0
$0.6250 \times 2 = 1.2500$	1
$0.2500 \times 2 = 0.500$	0
$0.5000 \times 2 = 1.0$	1

$\therefore (0.3125)_{10} = (0.0101)_2$

NOTE

We should stop multiplying the fractional part by 2, once we get 0 as a fraction or the fractional part is non-terminating. It can be decided depending on the number of digits in the fractional part required.

2. **Binary to decimal:**
   (i) $(101011011)_2 = (347)_{10}$
   Working: $(101011001)_2$
   $= 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
   $= 256 + 0 + 64 + 0 + 16 + 8 + 0 + 2 + 1$
   $= (347)_{10}$
   (ii) $(0.11001)_2 = (0.78125)_{10}$
   Working: $(0.11001)_2$
   $= 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$
   $= 1/2 + 1/4 + 1/32 = 25/32 = (0.78125)_{10}$

3. **Decimal to octal:**
   (i) $(2595)_{10} = (5043)_8$
   Working:

8	2595	
8	324	– 3
8	40	– 4
	5	– 0

$\therefore (2595)_{10} = (5043)_8$

4. **Octal to decimal:**
   (i) $(4721)_8 = (2513)_{10}$
   Working: $(4721)_8$
   $= 4 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$
   $= 2048 + 448 + 16 + 1 = (2513)_{10}$
   (ii) $(365.74)_8 = (245.9375)_{10}$
   Working:
   (a) Integral part:
   $(365)_8 = 3 \times 8^2 + 6 \times 8^1 + 5 \times 8^0$
   $= 192 + 48 + 5 = 245$
   $\therefore (365)_8 = (245)_{10}$
   (b) Fractional part:
   $(0.74)_8 = 7 \times 8^{-1} + 4 \times 8^{-2}$
   $= \frac{56+4}{64} = \frac{60}{64} = 0.9375$
   $\therefore (365.74)_8 = (245.9375)_{10}$

5. **Decimal to hexa-decimal:**
   (i) $(47239)_{10} = (B887)_{16}$
   Working:

16	47239	
16	2952	– 7
16	184	– 8
	11	– 8

Recall: 11 is B, in hexa-decimal system.
$\therefore (47239)_{10} = (B887)_{16}$

   (ii) $(30014)_{10} = (753E)_{16}$
   Working:

16	30014	
16	1875	– 14 = E
16	117	– 3
	7	– 5

$\therefore (30014)_{10} = (753E)_{16}$

6. **Hexa-decimal to decimal:**
   $(52B)_{16} = (1323)_{10}$
   Working: $(52B)_{16}$
   $= 5 \times 16^2 + 2 \times 16^1 + B \times 16^0$
   $= 1280 + 32 + 11 = (1323)_{10}$
   $\therefore (52B)_{16} = (1323)_{10}$

7. **Decimal to duo-decimal or duodenary (base 12):**
   $(948)_{10} = (66C)_{12}$
   Working:

12	948	
12	78	– 12 or C
	6	– 6

$\therefore (948)_{10} = (66C)_{12}$

8. **Duo-decimal to decimal:**
   $(5BC)_{12} = (864)_{10}$
   Working: $(5BC)_{12}$
   $= 5 \times 12^2 + B \times 12^1 + C \times 12^0$
   $= 720 + 132 + 12 = (864)_{10}$

9. **Binary to octal:**
   8 being the base of octal system and 2 being the base of binary system, there is a close relationship between both the systems. One can just club three digits of a binary number into a single block and write the decimal equivalent of each group (left to right).
   Example:
   (i) $(100101111)_2 = (100)_2\ (101)_2\ (111)_2$
   $= (457)_8$
   $\therefore (100101111)_2 = (457)_8$
   (ii) $(11111110)_2 = (011)_2\ (111)_2\ (110)_2$
   $= (376)_8$
   $\therefore (11111110)_2 = (376)_8$

NOTE

Introduce leading zeros to form a block of 3 without changing the magnitude of the number.

10. **Binary to hexa-decimal:**
   This is similar to the method disscused for octal; instead of clubbing 3, we club 4 digits.

Example:

$(10111110)_2 = (1011)_2\ (1110)_2 = (11)\ (14)$

$= (BE)_{16}$

$\therefore (10111110)_2 = (BE)_{16}$

NOTE

If the number of digits is not a multiple of 4, introduce leading zeros as done earlier for octal conversion.

Binary Arithmetic

Addition: Elementary Rules

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$ (1 will be regarded as carry

$1 + 1 + 1 = 11$ as we do in decimal system)

Examples of Binary Addition

1. $(110101)_2 + (110)_2$

```
    1    →  carry
1 1 0 1 0 1
0 0 0 1 1 0   (Introduce leading zeros)
-----------
1 1 1 0 1 1
```

2. $(101111)_2 + (111011)_2$

```
  1 1 1 1 1 →  carry
  1 0 1 1 1 1
  1 1 1 0 1 1
-------------
1 1 0 1 0 1 0
```

3. $(110)_2 + (100)_2 + (010)_2$

```
1  →  carry
1 1 0
1 0 0
0 1 0
------
11 0 0
```

Subtraction: Subtract 1101 from 11010.

1.

```
            2
          0 0 2 0 2
          ---------
          1 1 0 1 0
          – 1 1 0 1
          ---------
result → 1 1 0 1
```

Explanation: Say $N = 11010$,

As 1 cannot be subtracted from 0, we borrow 2 from the next place. This gives 2 – 1 = 1, as the right most digit of the result. The penultimate digit of N would become 0. A similar calculation gives the 3rd digit of the result from the right as 1 and the 4th digit of N from the right becomes 0.

We now borrow a 2 from the 5th digit of N, this makes the 4th digit of N as 2, thereby resulting in 2 – 1 = 1 as the 4th digit of the result.

2. Subtract 11011 from 111001

```
  2 2 1
  0 0 2 2 →   Borrow
-----------
1 1 1 0 0 1
–1 1 0 1 1
-----------
 1 1 1 1 0
```

Example 18

If $(624)_7 = 312_k$ then find K.

Solution

$(624)_7 = (6)\ (7^2) + (2)\ (7) + (4)\ (7)^0 = 312.$

$(312)_k = 3k^2 + k + 2$

Given $(312)_k = (624)_7$

$$3k^2 + k + 2 = 312$$

$$3k^2 + k - 310 = 0$$

$$(k - 10)\ (3k + 31) = 0$$

$$k > 0$$

$$\therefore \quad k = 10.$$

Example 19

Find the hexadecimal equivalent of the number $(234567)_8$.

Solution

$(234567)_8$

$= (10\ \ 011\ \ 100\ \ 101\ \ 110\ \ 111)_2$

$= (0001\ \ 0011\ \ 1001\ \ 0111\ \ 0111)_2$

$= (1\ \ 3\ \ 9\ \ 7\ \ 7)_{16}$

$= (13977)_{16}.$

Example 20

A non-zero number in base 8 is such that twice the number is the number formed by reversing its digits. Find it.

Solution

Let the number be $(xy)_8$,
where $0 \le x, y < 8$.

The number formed by reversing its digits is $(yx)_8$.

$$2(xy)_8 = (yx)_8$$

$$2(8x + y) = 8y + x$$

$$\frac{x}{y} = \frac{2}{5}$$

$x = 2$ and $y = 5$ is the only possibility.

$\therefore \quad (xy)_8 = (25)_8.$

EXERCISES

Direction for questions 1 to 50: Select the correct alternative from the given choices.

1. If x and y are irrational numbers, then $x + y - xy$ is ________.

(A) a real number (B) a complex number
(C) a rational number (D) an irrational number

2. Which of the following is a prime number?

(A) 851 (B) 589
(C) 429 (D) 307

3. Which of the following pairs of numbers are not twin primes?

(A) 131 and 133
(B) 191 and 193
(C) 157 and 159
(D) More than one of above

4. Which of the following is divisible by 11?

(A) 8787878
(B) 7777777
(C) 1234567
(D) More than one of the above

5. What is the least natural number that should be added to 52341693 so that the sum is a multiple of 8?

(A) 3 (B) 9 (C) 5 (D) 7

6. The product of 7 consecutive natural numbers is always divisible by

(A) 5040 (B) 10080
(C) 3430 (D) 6860

7. How many odd natural numbers have the same parity as their factorials?

(A) 1 (B) 2 (C) 0 (D) 3

8. N is a perfect number. What is the ratio of the sum of the factors of N and N?

(A) 1 (B) 2 (C) 3 (D) 4

9. $0.2\overline{55} =$

(A) $\frac{23}{90}$ (B) $\frac{23}{99}$
(C) $\frac{253}{990}$ (D) $\frac{253}{900}$

10. $0.3\overline{21} =$

(A) $\frac{53}{165}$ (B) $\frac{106}{333}$
(C) $\frac{10}{11}$ (D) None of these

11. $0.32\overline{1} =$

(A) $\frac{289}{900}$ (B) $\frac{289}{990}$
(C) $\frac{32}{99}$ (D) $\frac{16}{45}$

12. The least natural number that must be added to 599 so that the sum is a perfect cube is

(A) 120 (B) 125 (C) 130 (D) 135

13. There are 15 consecutive odd numbers. The sum of the first ten of those odd numbers is 200. What is the sum of the last five odd numbers?

(A) 125 (B) 175 (C) 150 (D) 200

14. Find the number of prime factors of 19019.

(A) 1 (B) 2 (C) 3 (D) 4

15. If $N = 2^a \times 3^b \times 5^c$, how many numbers (in terms of N) are less than N and are co-prime to it?

(A) $\frac{2}{15}N$ (B) $\frac{4}{15}N$
(C) $\frac{8}{15}N$ (D) $\frac{2}{5}N$

16. Which of the following numbers is divisible by 40 and 72?

(A) 7560 (B) 3840 (C) 5670 (D) 3780

17. What is the least whole number that should be added to 723111 to make the resultant is a multiple of 11?

(A) 4 (B) 8 (C) 7 (D) 3

18. (a) Prime factorize: 9000

(A) $2^2 \times 3^2 \times 5^2$ (B) $2^4 \times 3 \times 5^2$
(C) $2^3 \times 3^2 \times 5^3$ (D) $2^3 \times 3 \times 5^4$

(b) Prime factorize: 1936

(A) $2^2 \times 3 \times 11^3$ (B) $2^3 \times 11^3$
(C) $2^4 \times 11^2$ (D) $2^2 \times 3^2 \times 11^2$

(c) Write 3969 as a product of prime factors.

(A) $3^5 \times 7$ (B) $3^3 \times 7^3$
(C) $3^4 \times 7^2$ (D) $3^2 \times 7^4$

(d) Write 14553 as a product of prime numbers

(A) $3 \times 7^3 \times 11$ (B) $3^2 \times 7 \times 11^3$
(C) $3^3 \times 7^2 \times 11$ (D) $3 \times 7^2 \times 11^2$

19. Simplify the following:

(a) $248 \times 555 + 148 \times 445$

(A) 203500 (B) 302500
(C) 205300 (D) 305200

(b) $4\frac{1}{2} + 3\frac{1}{5} - 2\frac{1}{10} - 4\frac{1}{20}$

(A) $1\frac{1}{10}$ (B) $1\frac{11}{20}$
(C) $1\frac{1}{5}$ (D) $1\frac{11}{40}$

(c) $\dfrac{(3.37)^3 + 10.11(6.63)^2 + 19.89(3.37)^2 + (6.63)^3}{(3.37)^2 + 2 \times (6.63)(3.37) + (6.63)^2}$

(A) 3.26 (B) 6.74 (C) 10 (D) 8

20. Find the square root of 17689

(A) 143 (B) 137 (C) 133 (D) 147

21. The number of positive integers which are co-prime to 349247 is ______.
(A) 4 (B) 5
(C) 3 (D) infinite

22. The sum of the first N natural numbers is equal to x^2 where x is an integer less than 100. What are the values that N can take?
(A) 1, 9, 27 (B) 1, 7, 26
(C) 1, 8, 48 (D) 1, 8, 49

23. What is the unit's place of $(5^n + 4^{2n} + 7^{4n})^{4n}$?
(A) 4 (B) 8 (C) 2 (D) 6

24. What is the highest power of 5 in 240!?
(A) 58 (B) 17 (C) 116 (D) 39

25. The least possible number which when successively divided by 10, 7, and 6 leaves remainders of 8, 4, and 5 respectively is
(A) 256 (B) 148 (C) 398 (D) 198

26. The LCM and HCF of a pair of numbers is 1232 and 14, respectively. How many such pairs are possible?
(A) 3 (B) 2 (C) 1 (D) None

27. Find the square root of 12345654321.
(A) 1111 (B) 11111
(C) 111111 (D) 1111111

28. There are four prime numbers written in ascending order. The product of the first three prime numbers is 2431 and that of the last three is 4199. Find the greatest of them.
(A) 17 (B) 19 (C) 23 (D) 13

29. Find the minimum number of coins required to pay three persons 69 paise, 105 paise, and 85 paise, respectively, using coins in the denominations of 2 paise, 5 paise, 10 paise, 25 paise, and 50 paise.
(A) 9 (B) 10 (C) 14 (D) 11

30. If a, b, and c are prime numbers satisfying $a = b - 2 = c - 4$. How many possible combinations exist for a, b, and c?
(A) 4 (B) 3 (C) 2 (D) 1

31. Let p, q, and r be distinct positive integers that are odd. Which of the following statements cannot always be true?
(A) $pq^2 r^3$ is odd.
(B) $(p + q)^2 r^3$ is even
(C) $(p - q + r)^2 (q + r)$ is even.
(D) If p, q, and r are consecutive odd integers, the remainder of their product when divided by 4 is 3.

32. If *abcde* is a five-digit number the difference of *abcde* and *acdbe* would always be divisible by which of the following for all values of a, b, c, d and e?
(A) 9
(B) 18
(C) 99
(D) Both (A) and (B)

33. Find the value of the expression below

$$\frac{(0.68)^3 + (0.67)^3 - (0.5)^3 + (0.68)(0.67)(1.5)}{(0.68)^2 + (0.67)^2 + (0.5)^2 - (0.68)(0.67) + (0.67)(0.5) + (0.68)(0.5)}$$

(A) 1.85 (B) 0.51 (C) 0.49 (D) 0.85

34. Find the sum of all possible distinct remainders which are obtained when squares of a prime numbers are divided by 6.
(A) 7 (B) 8 (C) 9 (D) 10

35. The least number, which when successively divided by 2, 3, and 7 leaves respective remainders of 1, 2, and 3, is
(A) 56 (B) 130
(C) 68 (D) 23

36. Find the GCD of the numbers p and q where $p = 2^3. 3^2. 7^2. 11^6$ and $q = 2^2. 3^1 . 5^4. 11^2. 13^2$.
(A) 776 (B) 1452
(C) 1164 (D) 2028

37. Which of the following sets of numbers are relative primes?
(a) 57,61
(b) 396,455
(c) 693,132
(d) 6561,1024
(e) 384,352
(A) (c), (e) (B) (a), (b), (d)
(C) (a), (c), (d) (D) (b), (e)

38. (a) Find the units digit of 8^{173}.
(A) 2 (B) 4 (C) 8 (D) 6
(b) What is the last digit of $518^{163} + 142^{157}$?
(A) 2 (B) 4 (C) 6 (D) 8
(c) Find the last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$
(A) 2 (B) 3 (C) 4 (D) 6

39. If n is a positive integer, then $43^{5n} - 21^{5n}$ is always divisible by
(A) 11 (B) 18 (C) 25 (D) 64

40. Find the greatest number which when divides 6850 and 2575 leaving respective remainders of 50 and 25.
(A) 425 (B) 850 (C) 1700 (D) 1275

41. Find the least number which when divided by 12, 18, and 33 leaves a remainder of 5 in each case.
(A) 394 (B) 396 (C) 391 (D) 401

42. Find the smallest number that must be added to 1994 such that a remainder of 28 is left when the number is divided by 38 and 57.
(A) 66 (B) 68 (C) 86 (D) 98

43. Find the greatest number which divides 3300 and 3640 leaving respective remainders of 23 and 24.
(A) 13 (B) 113 (C) 339 (D) 226

44. Find the greatest number which divides 68, 140, and 248 leaving the same remainder in each case.
(A) 36 (B) 18 (C) 72 (D) 108

45. Five bells toll at intervals of 5, 6, 10, 12, and 15 seconds respectively. If they toll together at the same time, after how many seconds will they toll together again, for the first time?
(A) 300 (B) 120 (C) 60 (D) 30

46. If three numbers are in the ratio 3 : 4 : 5, and their LCM is 480, then find the sum of the three numbers.
(A) 96 (B) 72 (C) 84 (D) 108

47. If $(121)_8 = (x)_2$, then $x =$
(A) 101001 (B) 1010011
(C) 1010001 (D) 1011001

48. If $(ACD)_{16} = (x)_{10}$, then $x =$
(A) 2765 (B) 6725 (C) 5672 (D) 7625

49. Find the digit in the unit's place, in the product of $(25)^7 \times (37)^{12} \times (123)^9$.
(A) 1 (B) 5 (C) 3 (D) 9

50. What is the remainder when 3^{86} is divided by 6?
(A) 2 (B) 3 (C) 4 (D) 0

Answer Keys

1. A	**2.** D	**3.** D	**4.** A	**5.** A	**6.** A	**7.** A	**8.** B	**9.** A	**10.** A
11. A	**12.** C	**13.** B	**14.** D	**15.** B	**16.** A	**17.** C	**18.** (a) C	(b) C	(c) C (d) C
19. (a) A	(b) B	(c) C	**20.** C	**21.** D	**22.** D	**23.** D	**24.** A	**25.** C	**26.** B
27. C	**28.** B	**29.** D	**30.** D	**31.** D	**32.** D	**33.** D	**34.** B	**35.** D	**36.** B
37. B	**38.** (a) C	(b) B	(c) A	**39.** A	**40.** B	**41.** D	**42.** C	**43.** B	**44.** A
45. C	**46.** A	**47.** C	**48.** A	**49.** B	**50.** B				