Chapter 7. Solving Systems of Linear Equations and Inequalities

Answer 1PT3.

1.

A system of equations with two parallel lines is inconsistent (no solution)

Consider the following example,

$6x - y = 9$ ...... (1)

$6x - y = 11$ ...... (2)

Since the coefficients of the *y* terms, 6 and 6 and the coefficients of *x*, are the same, we can eliminate the *x* and *y* terms by subtracting the equations.

$$\begin{array}{rrrrrrr} & 6x & - & y & = & 9 \\ (-) & 6x & - & y & = & 11 \\ \hline & 0 & - & 0 & = & -2 \end{array}$$

Write the equations in column form and subtract

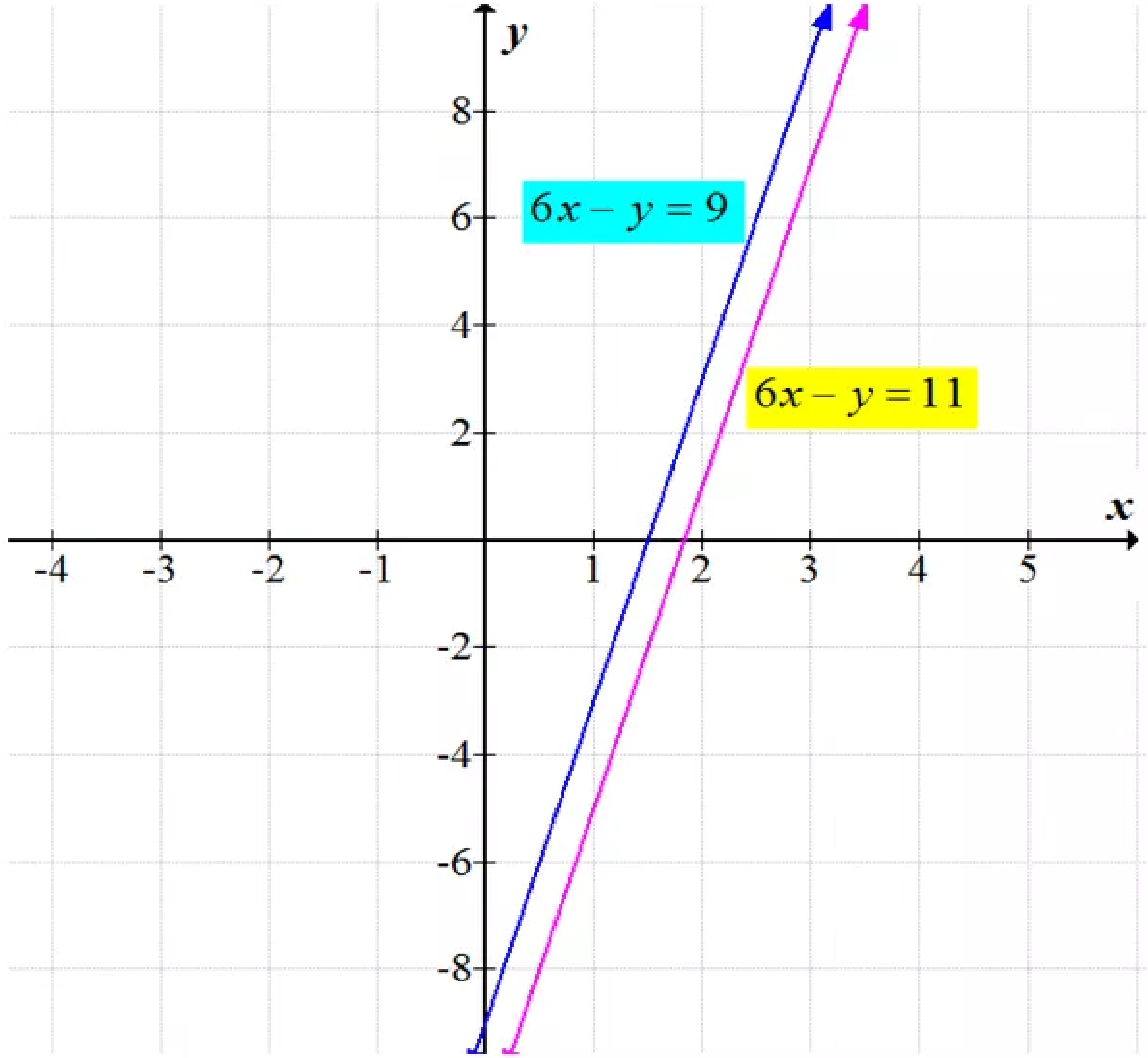
Notice that the x and y variable eliminated

$0 = -2$ Simplify

The result is a false statement. Hence the system has **no solution**.

The following graph supports our conclusion

The graph of the equations $6x-y=9$ and $6x-y=11$ is shown below:



From the graph, observe that the lines are non-intersecting, so the equation has **no** solution. That is system is **inconsistent**.

2.

A system of equations is said to be **consistent** if the system with at least one ordered pair that satisfies both the equations.

Consider the following example,

$-5x+3y=6$ ...... (1)

$x-y=4$ ...... (2)

Eliminate *x*

$$\begin{array}{llll} -5x+3y=6 & & -5x+3y=6 & \\ x-y=4 & \text{Multiply by 5} & (+)5x-5y=20 & \\ & & -2y=26 & \text{Add the equations} \\ & & \dfrac{-2y}{-2}=\dfrac{26}{-2} & \text{Divide each side with } -2 \\ & & y=-13 & \text{Simplify} \end{array}$$

Now substitute -13 for *y* in either equation to find the value of x

$x-y=4$	Second Equation
$x-(-13)=4$	Substitue -13 for y
$x+13=4$	Simplify
$x+13-13=4-13$	Subtract 13 from each side
$x=-9$	Simplify

The system is **consistent** and the solution is $\boxed{(-9,-13)}$

3.

A system of equations may be solved using **elimination** method

Consider the following examples,

$x+y=-3$ ...... (1)

$x-y=1$ ...... (2)

Since the coefficients of the *y* terms, -1 and 1, are additive inverses, we can eliminate the *y* terms by adding the equations.

$$\begin{array}{rcccccl} & x & + & y & = & -3 & \text{Write the equations in column form and add} \\ (+) & x & - & y & = & 1 & \\ \hline & 2x & & & = & -2 & \text{Notice that the } y \text{ variable eliminated} \end{array}$$

$\frac{2x}{2}=\frac{-2}{2}$ Divide each side with 2

$x=-1$ Simplify

Now substitute -1 for *x* in either equation to find the value of *y*.

$x-y=1$ Second equation

$-1-y=1 \quad x=-1$

$-1-y+1=1+1$ Add 1 to each side of the equation

$-y=2$ Simplify

$(-y)\times-1=2\times-1$ Multiply each side with -1

$y=-2$ Simplify

The solution is $\boxed{(-1,-2)}$

Answer 1STP.

Consider the equation

$4x-2(x-2)-8=0$ ...... (1)

$4x-2x+4-8=0$ Use the Distributive Property

$2x-4=0$ Combine Like terms

$2x=4$ Add 4 to each side

$\frac{2x}{2}=\frac{4}{2}$ Divide each side of the equation

$x=2$ Simplify

Hence the solution to the equation is $x=2$

The Correct Option is **B**

Substitute $x=-2$ in the equation (1)

$$\begin{aligned} 4x-2(x-2)-8&=0 && \text{First equation} \\ 4(-2)-2(-2-2)-8&=0 && \text{Substitute } -2 \text{ for } x \\ -8-2(-4)-8&=0 \\ -8+8-8&=0 \\ -8&=0 && \text{False} \end{aligned}$$

Option **A** is not Correct.

Substitute $x=2$ in the equation (1)

$$\begin{aligned} 4x-2(x-2)-8&=0 && \text{First equation} \\ 4(2)-2(2-2)-8&=0 && \text{Substitute 2 for } x \\ 8-2(0)-8&=0 \\ 8-8&=0 \\ 0&=0 && \text{True} \end{aligned}$$

Option **B** is Correct.

Substitute $x=5$ in the equation (1)

$$\begin{aligned} 4x-2(x-2)-8&=0 && \text{First equation} \\ 4(5)-2(5-2)-8&=0 && \text{Substitute 5 for } x \\ 20-2(3)-8&=0 \\ 20-6-8&=0 \\ 6&=0 && \text{False} \end{aligned}$$

Option **C** is not Correct.

Substitute $x=6$ in the equation (1)

$$\begin{aligned} 4x-2(x-2)-8&=0 && \text{First equation} \\ 4(6)-2(6-2)-8&=0 && \text{Substitute 6 for } x \\ 24-2(4)-8&=0 \\ 24-8-8&=0 \\ 8&=0 && \text{False} \end{aligned}$$

Option **D** is not Correct.

Answer 1VC.

If the graphs intersect or coincide, the system of equations is said to be consistent.

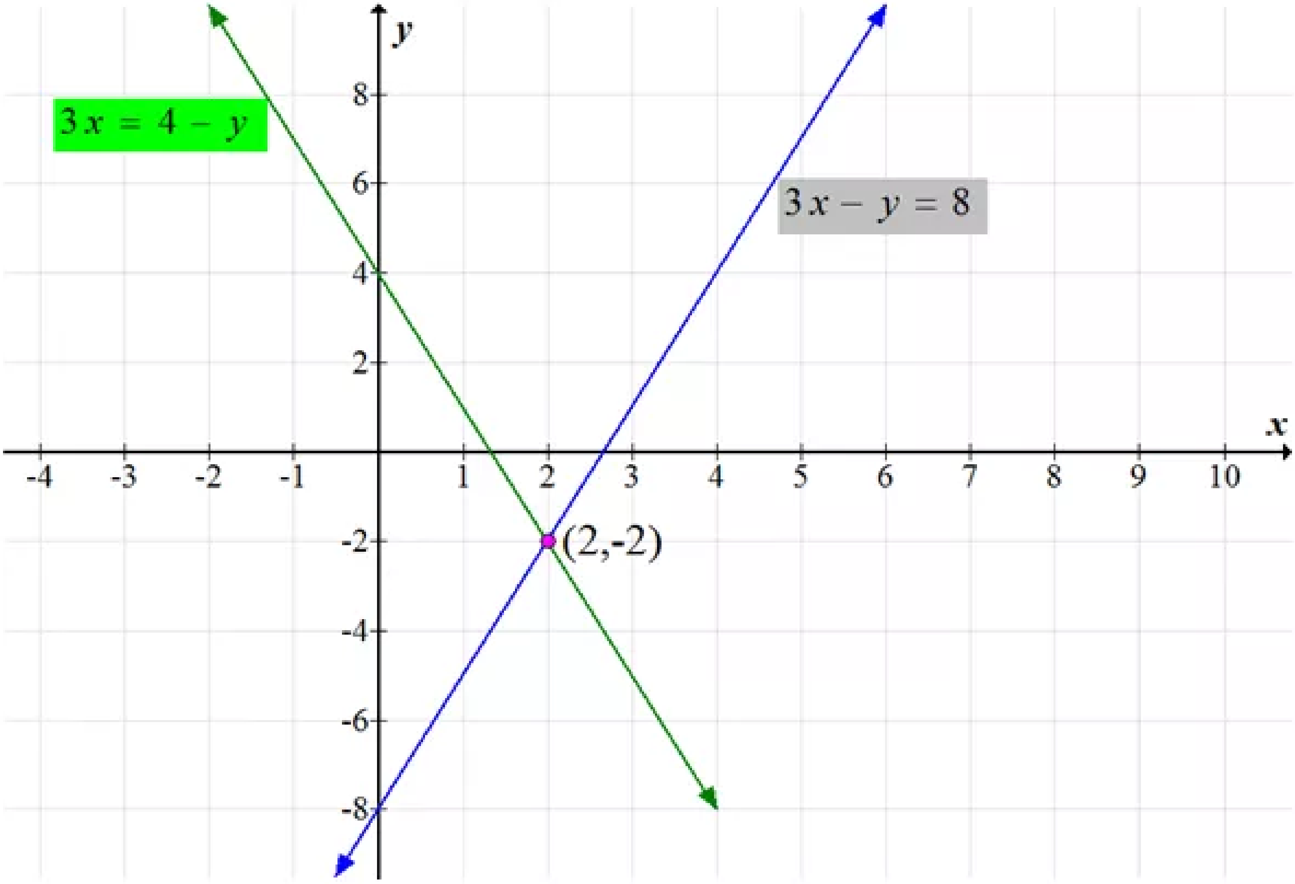
Consistent equations can be independent or dependent. If a system has exactly one solution, it is **independent**

Consider the following example,

$3x - y = 8$ ...... (1)

$3x = 4 - y$ ...... (2)

The graph of the equations is shown below:



The graphs appear to intersect at the point with coordinates $(2,-2)$

Since the system of equations has only one solution, so it is independent

Answer 2STP.

Let the cost of a CD is $\$x$

The cost of a CD (including 7% tax) is

$\$x + 7\% \text{ of } x = \$17.1$

$x + 0.07x = 17.1$

$x(1.07) = 17.1$ Combine like terms

$x = \dfrac{17.1}{1.07}$ Divide each side with 1.07

$x \approx 15.99$ Simplify

Hence the cost of the CD is \$15.99

The correct option is **C**

Answer 2VC.

If the graphs are parallel, the system of equations is said to be **inconsistent**.

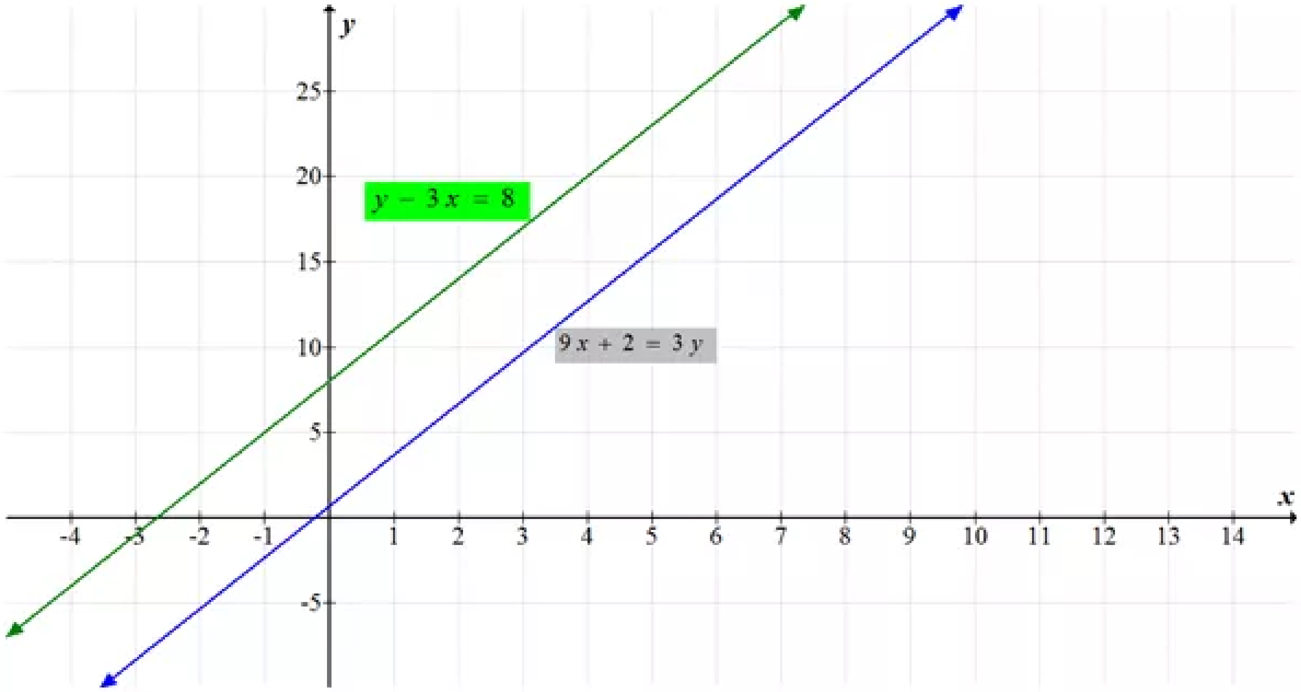
There are *no* order pairs that satisfy both equations.

Consider the following example,

$9x + 2 = 3y$ ...... (1)

$y - 3x = 8$ ...... (2)

The graph of the equations is shown below:



Since the graphs $9x + 2 = 3y$ and $y - 3x = 8$ are parallel, there are **no solutions.** That is system is **inconsistent**.

Answer 3STP.

Consider,

$f(x) = 2x - 3$

$f(3) = 2(3) - 3$ Substitute $x = 1$ in the function

$f(3) = 6 - 3$

$f(3) = 3$

$f(x) = 2x - 3$

$f(4) = 2(4) - 3$ Substitute $x = 4$ in the function

$f(4) = 8 - 3$

$f(4) = 5$

$f(x) = 2x - 3$

$f(5) = 2(5) - 3$ Substitute $x = 5$ in the function

$f(5) = 10 - 3$

$f(5) = 7$

The correct option is **B**

Answer 3VC.

If the graphs intersect or coincide, the system of equations is said to be consistent.

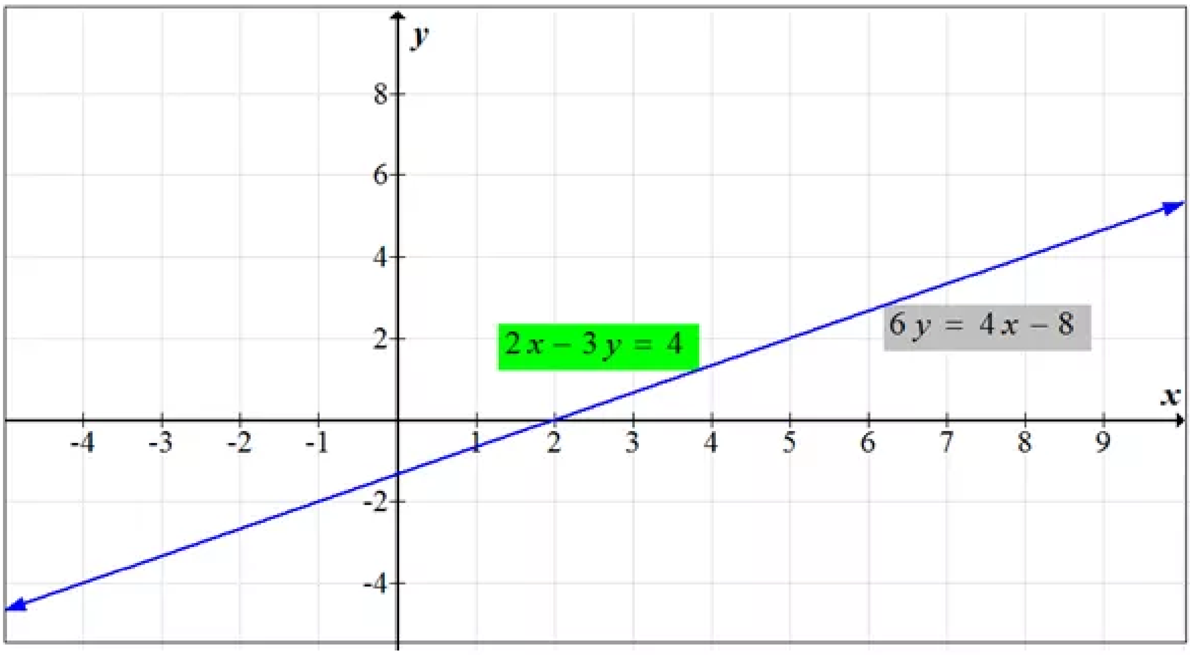
Consistent equations can be independent or dependent. If a system has infinitely many solutions, it is **dependent**

Consider the following example,

$2x-3y=4$ ...... (1)

$6y=4x-8$ ...... (2)

The graph of the equations is shown below:



Since the graphs $2x-3y=4$ and $6y=4x-8$ are coincide, there are infinitely many solutions.

Since the system of equations has infinitely many solutions, so it is **dependent**

Answer 4PT.

Consider the equations,

$y=x+2$ ...... (1)

$y=2x+7$ ...... (2)

Since $y=x+2$, substitute $x+2$ for y in the second equation

$2x+7=x+2$

$2x+7-x=x+2-x$ Subtract x from each side

$x+7=2$ Simplify

$x+7-7=2-7$ Subtract 7 from each side

$x=-5$ Combine like terms

Use $y=x+2$ to find the value of y

$y=x+2$ First equation

$y=-5+2$ $x=-5$

$y=-3$ Simplify

The solution is $\boxed{(-5,-3)}$

Answer 4STP.

Consider the following table,

Number of hours, x	1	3	4	6
Number of birds, y	6	14	18	26

Now we check each option

Option	Number of hours, x	1	3	4	6
A	$y = x + 5$	6	8	9	11
B	$y = 3x + 3$	6	12	15	21
C	$y = 3x + 5$	8	14	17	23
D	$y = 4x + 2$	6	14	18	26

The option A and B matches only at one point at $x = 1$ and for remaining points it does not match with the above table.

The option C matches only at one point at $x = 2$ and for remaining points it does not match with the above table.

The option D matches at each point.

The correct option is **D**

Answer 4VC.

If the graphs of the equations in a system have same slope and different y intercepts, the graph of the system is a pair of ***parallel lines***.

Consider the following example,

$9x+2=3y$ ...... (1)

$y-3x=8$ ...... (2)

Slope intercept form of equation (1)

$9x+2=3y$	First equation
$\frac{9x+2}{3}=y$	Divide each side with 3
$y=3x+\frac{2}{3}$	Slope intercept form

Slope $m=3$ and *y*-intercept is $\frac{2}{3}$

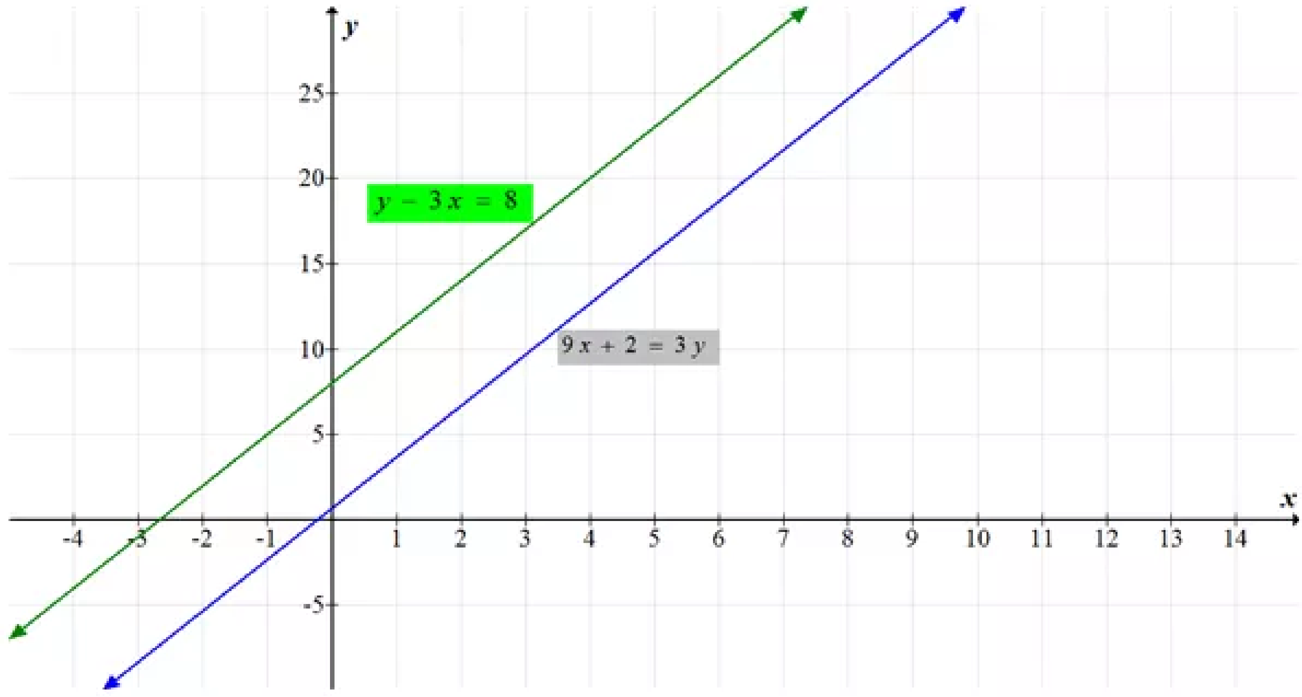
Slope intercept form of equation (2)

$y-3x=8$	First equation
$y=8+3x$	Add $3x$ to each side
$y=3x+8$	Slope intercept form

Slope $m=3$ and *y*-intercept is 8

The slopes of the two lines are the same but *y* intercepts are different.

The graph of the equations is shown below:



Since the graphs $9x+2=3y$ and $y-3x=8$ are parallel, there are **no solutions.** That is system is **inconsistent**.

Answer 5PT.

Consider the equations,

$x+2y=11$ ...... (1)

$x=14-2y$ ...... (2)

Since $x=14-2y$, substitute $14-2y$ for x in the First equation

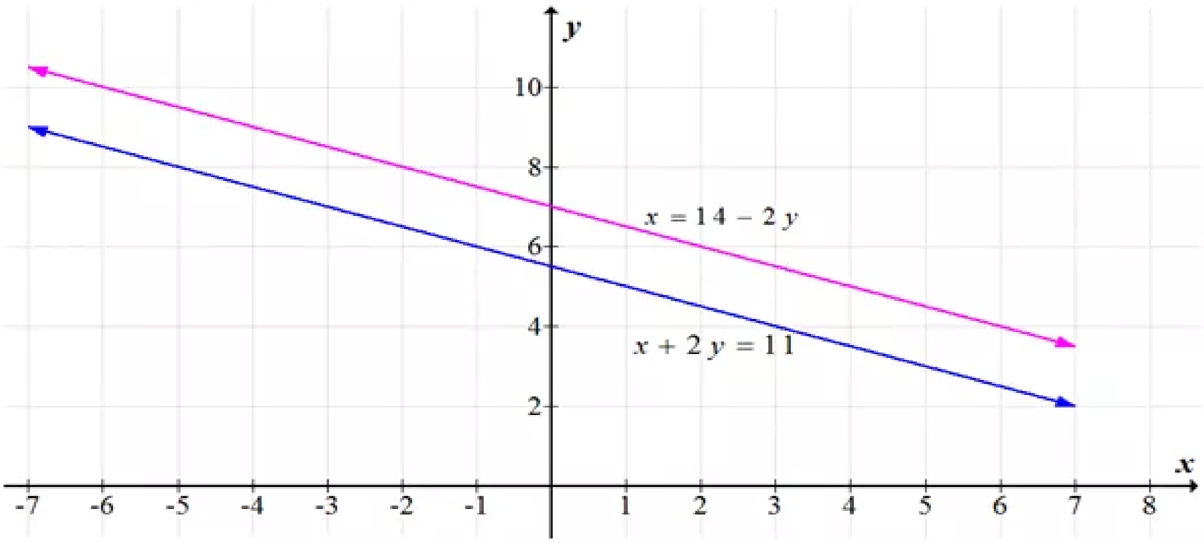
$x+2y=11$ First equation

$14-2y+2y=11$

$14=11$ Simplify

The result is false statement $(14=11)$, the system has **no solution**.

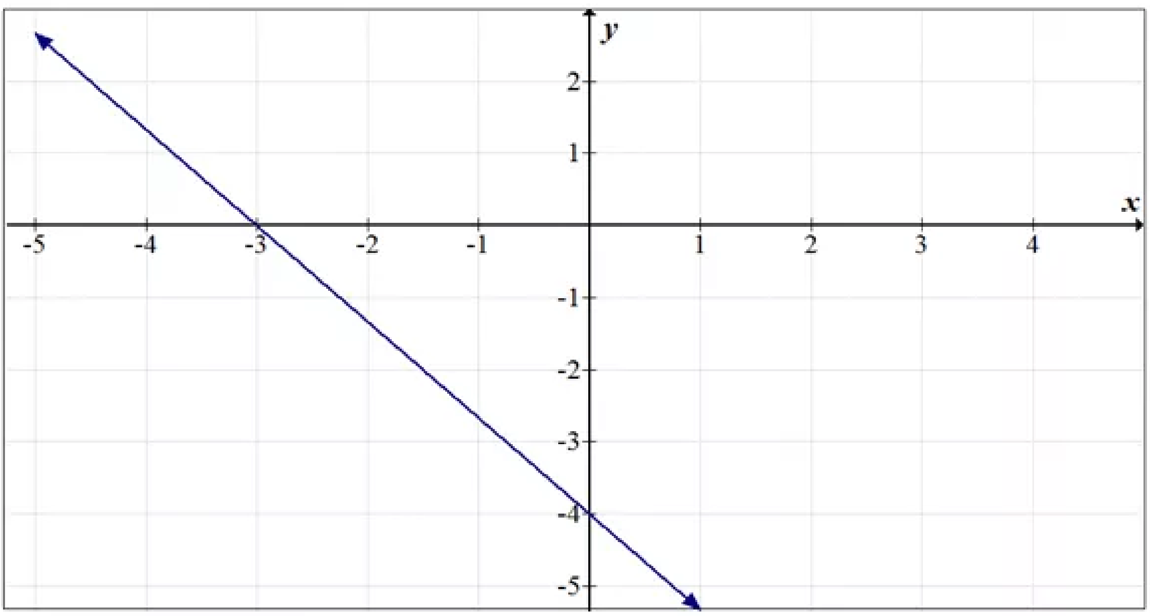
The following graph supports the above conclusion:



The two lines are parallel, and they never intersect. So, the system has no solution.

Answer 5STP.

Consider the following graph:



The *x* intercept of the line -3 is and *y* intercept of the line is -4

Now check each option whose *x* and *y* intercepts are -3 and -4 respectively

Option A

$$3y-4x=-12 \quad \text{First equation}$$
$$-4x+3y=-12$$
$$\frac{-4x+3y}{-12}=\frac{-12}{-12} \quad \text{Divide each side with } -12$$
$$\frac{x}{3}+\frac{y}{-4}=1 \quad \text{Intercept form of a line}$$

The *x* intercept of the line 3 is and *y* intercept of the line is -4 False Option

Option B

$$4y+3x=-16 \quad \text{First equation}$$
$$3x+4y=-16$$
$$\frac{3x+4y}{-16}=\frac{-16}{-16} \quad \text{Divide each side with } -16$$
$$\frac{x}{-\frac{16}{3}}+\frac{y}{-4}=1 \quad \text{Intercept form of a line}$$

The *x* intercept of the line $-\frac{16}{3}$ is and *y* intercept of the line is -4 False Option

Option C

$$3y+4x=-12 \quad \text{First equation}$$
$$4x+3y=-12$$
$$\frac{4x+3y}{-12}=\frac{-12}{-12} \quad \text{Divide each side with } -12$$
$$\frac{x}{-3}+\frac{y}{-4}=1 \quad \text{Intercept form of a line}$$

The *x* intercept of the line -3 is and *y* intercept of the line is -4 **True Option**

Option D

$$3y+4x=-9 \quad \text{First equation}$$
$$4x+3y=-9$$
$$\frac{4x+3y}{-9}=\frac{-9}{-9} \quad \text{Divide each side with } -9$$
$$\frac{x}{-\frac{9}{4}}+\frac{y}{-3}=1 \quad \text{Intercept form of a line}$$

The *x* intercept of the line $-\frac{9}{4}$ is and *y* intercept of the line is -3 False Option

Hence the correct option is **C**

Answer 5VC.

If the graphs of the equations in a system have same slope and *y* intercept, the system has **infinitely many solutions**.

Consider the following example,

$2x-3y=4$ ...... (1)

$6y=4x-8$ ...... (2)

Slope intercept form of equation (1)

$2x-3y=4$ First equation

$2x-4=3y$

$y=\frac{2}{3}x-\frac{4}{3}$ Slope intercept form

Slope $m=\frac{2}{3}$ and *y*-intercept is $-\frac{4}{3}$

Slope intercept form of equation (2)

$6y=4x-8$ First equation

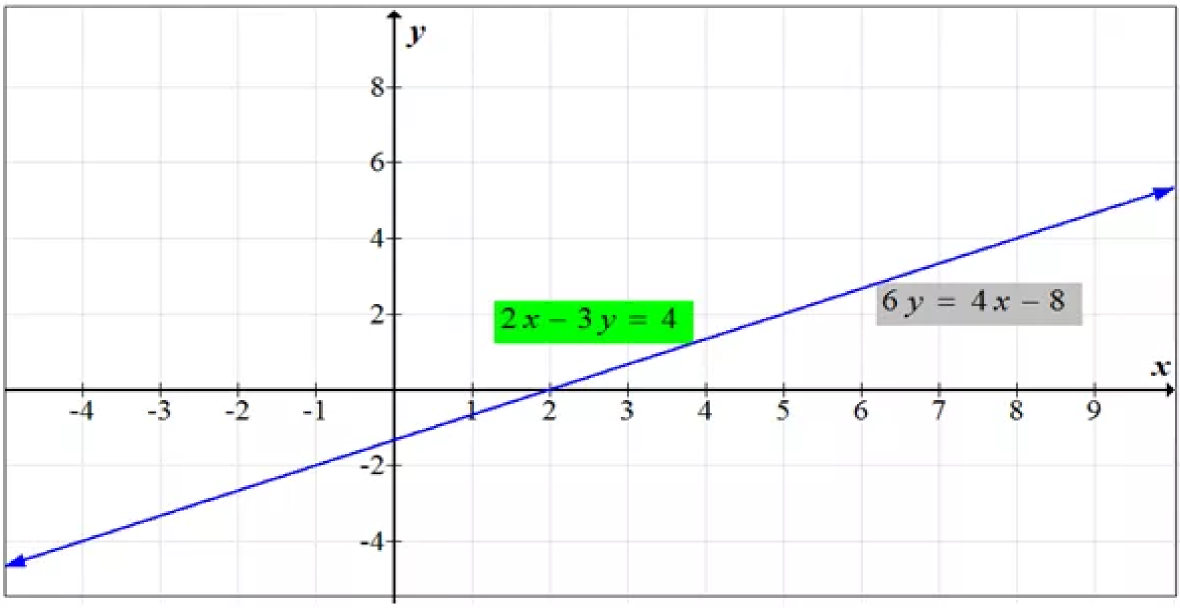
$y=\frac{4x-8}{6}$ Divide each side with 6

$y=\frac{2}{3}x-\frac{4}{3}$ Slope intercept form

Slope $m=\frac{2}{3}$ and *y*-intercept is $-\frac{4}{3}$

The slopes and *y* intercepts of the two lines are the same.

The graph of the equations is shown below:



Since the graphs $2x-3y=4$ and $6y=4x-8$ are coincide, there are infinitely many solutions.

Answer 6PT.

Consider the equations,

$3x + y = 5$ ...... (1)

$2y - 10 = -6x$ ...... (2)

From the equation (1)

$3x + y = 5$

$3x + y - 3x = 5 - 3x$ Subtract $3x$ from each side

$y = 5 - 3x$

Since $y = 5 - 3x$, substitute $5 - 3x$ for y in the Second equation

$2y - 10 = -6x$ First equation

$2(5 - 3x) - 10 = -6x$

$10 - 6x - 10 = -6x$ Use Distributive Property

$-6x = -6x$ Simplify

The statement is true. This means that there are **infinitely many solutions** of the system of equations.

Slope intercept form of equation (1) is $y = 5 - 3x$

From the equation (2)

$2y - 10 = -6x$

$2y = -6x + 10$ Add 10 to each side

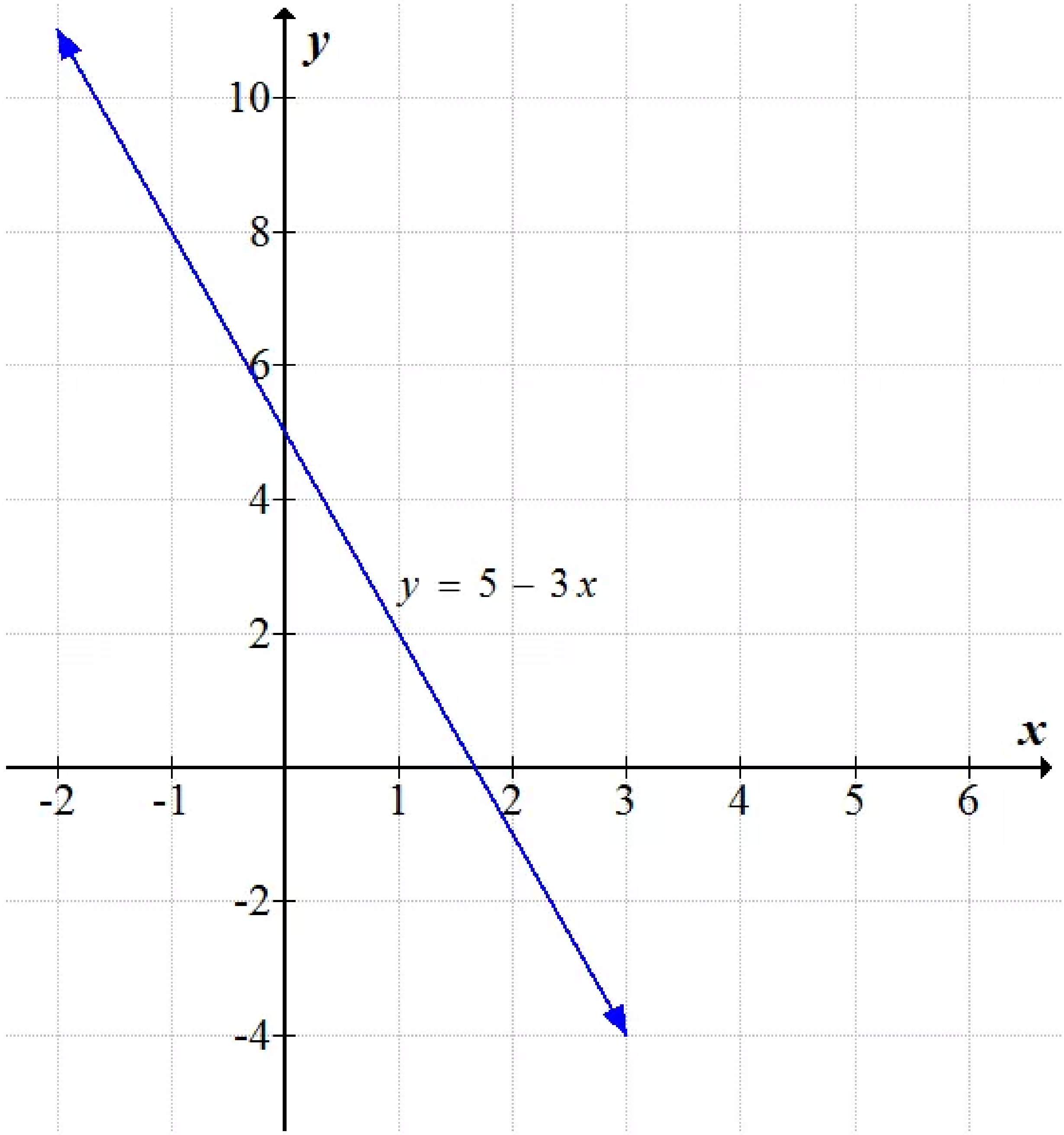
$y = 5 - 3x$ Divide each side with 2

Slope intercept form of equation (2) is $y = 5 - 3x$

This is because the slope intercept form of the both the equations is $y = 5 - 3x$

That is, the equations are equivalent, and they have the same graph.

The following graph supports the above conclusion:



The two lines are same, so the system has infinitely many solutions.

Answer 6STP.

The two lines are parallel if the lines having the same slope

Consider the equation,

$y-3x=6$ ...... (1)

The slope intercept form the equation (1), $y=3x+6$

The equation $y=3x+6$ is compared with $y=mx+c$

Then the slope of the line $y=3x+6$ is $m=3$

Now check each option whose slope is 3

Option A

The slope of the line $y=-3x+4$ is $m=-3$ False Option

Option B

The slope of the line $y=3x-2$ is $m=3$ **True Option**

Option C

The slope of the line $y=\frac{1}{3}x+6$ is $m=\frac{1}{3}$ False Option

Option D

The slope of the line $y=-\frac{1}{3}x+4$ is $m=-\frac{1}{3}$ False Option

Hence the correct option is **B**

Answer 6VC.

If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one order pair that satisfies both the equations.

If the graphs are parallel, the system of equations is said to be **inconsistent**. There are *no* ordered pairs that satisfy both equations.

Since the system of equations has one solution $(3,-5)$. The system is **consistent**.

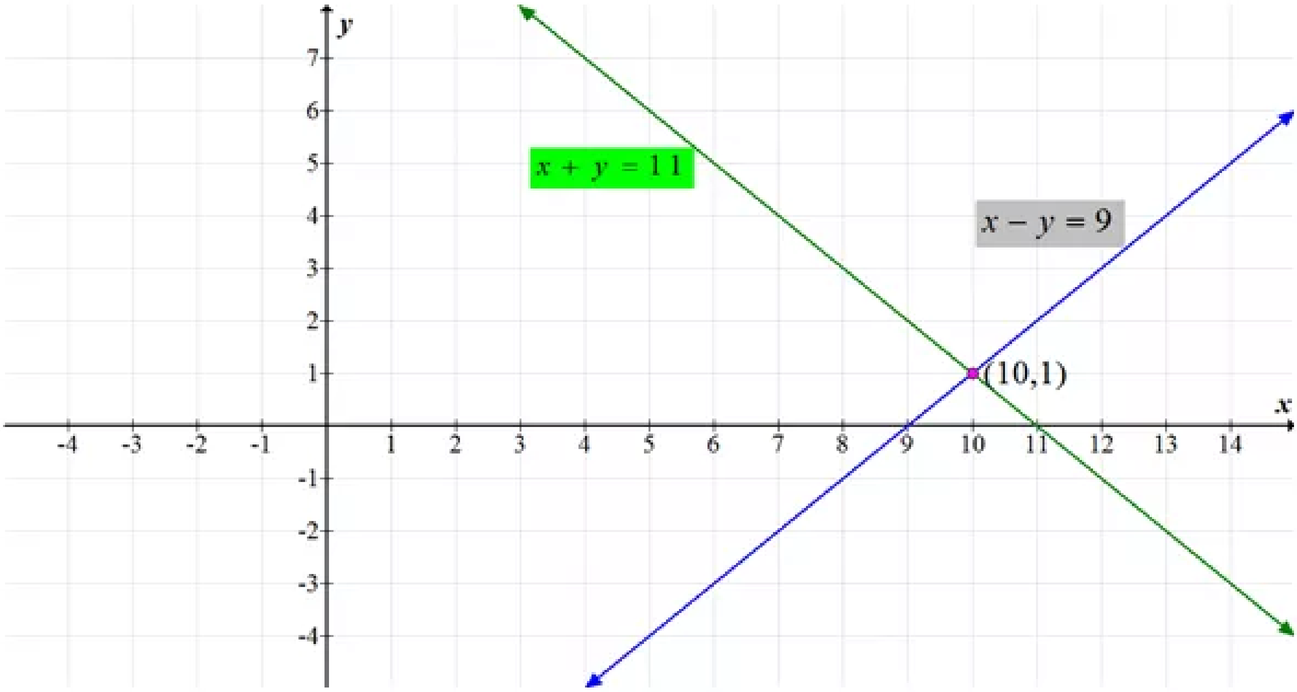
Answer 7E.

Consider the equations,

$x - y = 9$ ...... (1)

$x + y = 11$ ...... (2)

The graph of the equations is shown below:



The graphs appear to intersect at the point with coordinates $(10,1)$

Check:

$x - y = 9$ First equation

$10 - 1 = 9$ Substitue 10 for x and 1 for y

$9 = 9$

$x + y = 11$ First equation

$10 + 1 = 11$ Substitue 10 for x and 1 for y

$11 = 11$

Hence the solution to the system of equations is $\boxed{(10,1)}$

Answer 7PT.

Consider the equations,

$2x+5y=16$ ...... (1)

$5x-2y=11$ ...... (2)

Eliminate x

$$\begin{array}{lll} 2x+5y=16 & \text{Multiply by 5} & 10x+25y=80 \\ 5x-2y=11 & \text{Multiply by 2} & 10x-4y=22 \\ & & 29y=58 \quad \text{Subtract the equations} \end{array}$$

$$\begin{aligned} \frac{29y}{29} &= \frac{58}{29} && \text{Divide each side with 29} \\ y &= 2 && \text{Simplify} \end{aligned}$$

Now substitute 2 for y in either equation to find the value of x

$$\begin{aligned} 2x+5y &= 16 && \text{First Equation} \\ 2x+5(2) &= 16 && \text{Substitue 2 for } y \\ 2x+10 &= 16 && \text{Simplify} \\ 2x+10-10 &= 16-10 && \text{Subtract 10 from each side} \\ 2x &= 6 && \text{Simplify} \end{aligned}$$

$$\begin{aligned} \frac{2x}{2} &= \frac{6}{2} && \text{Divide each side with 2} \\ x &= 3 && \text{Simplify} \end{aligned}$$

The solution is $\boxed{(3,2)}$

Answer 7STP.

Let $*d*, the amount she needs to deposit

The amount required for car payment is $230

The amount in the bank is $185

Since she is withdrawing $230 from bank for car loan, the equation for minimum deposit would be

$185+d-230 \geq 200$

Hence the correct option is **D**

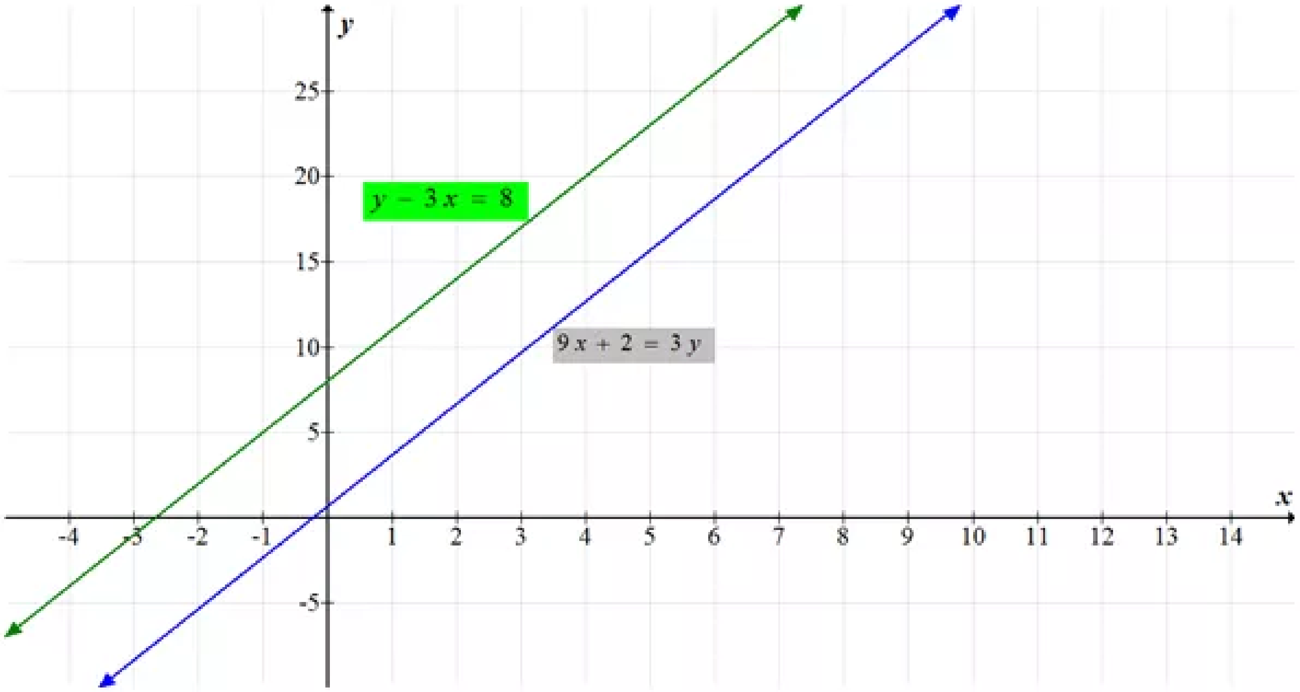
Answer 8E.

Consider the equations,

$9x+2=3y$ ...... (1)

$y-3x=8$ ...... (2)

The graph of the equations is shown below:



Since the graphs $9x+2=3y$ and $y-3x=8$ are parallel, there are **no solution**

Answer 8PT.

Consider the equations,

$y+2x=-1$ ...... (1)

$y-4=-2x$ ...... (2)

From the equation (2)

$y-4=-2x$

$y-4+4=-2x+4$

$y=4-2x$

Since $y = 4 - 2x$, substitute $4 - 2x$ for y in the first equation

$y + 2x = -1$ First equation

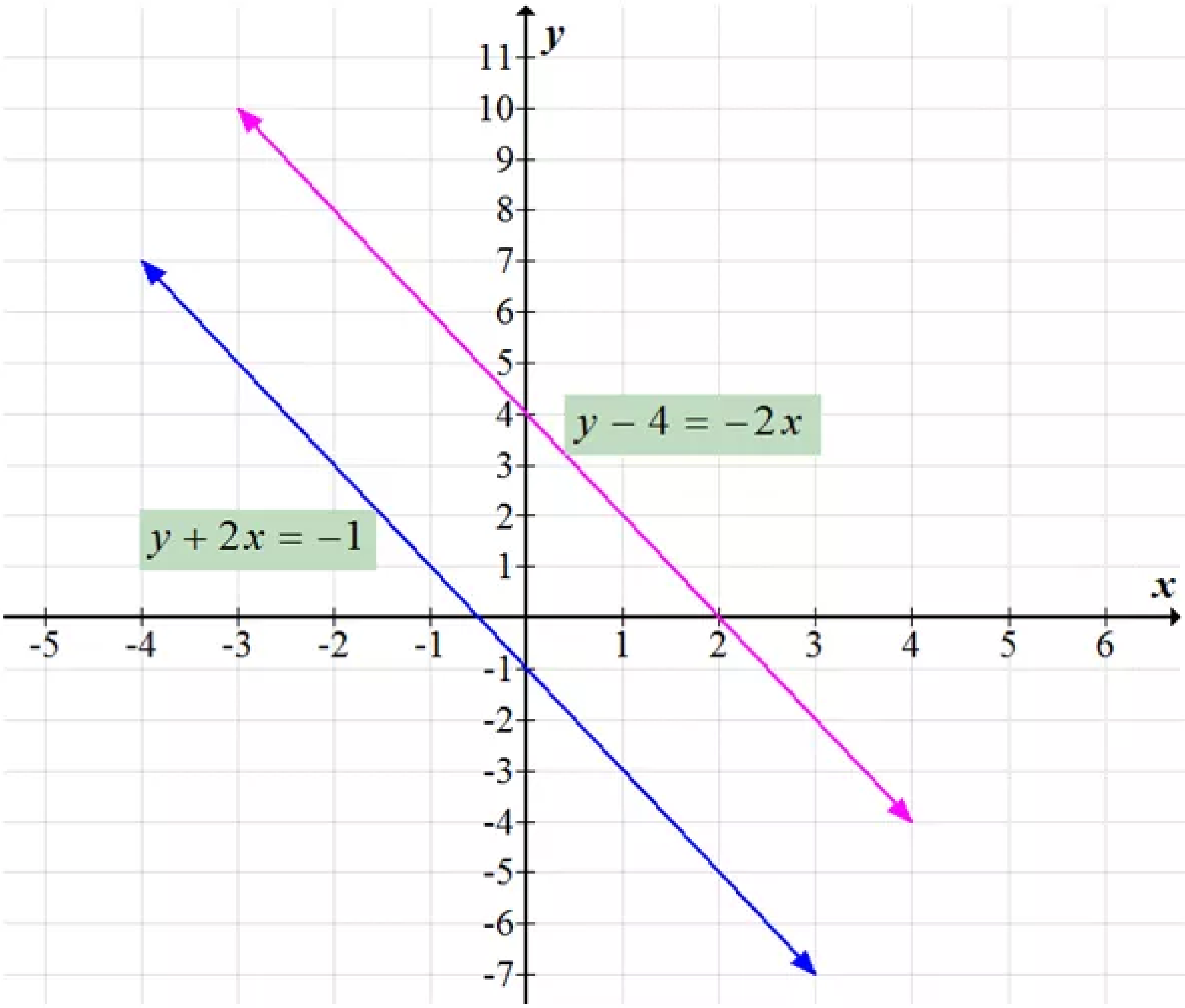
$4 - 2x + 2x = -1$ $y = 4 - 2x$

$4 - 0 = -1$ Combine like terms

$4 = -1$ Combine like terms

The result is false statement $(4 = -1)$, the system has **no solution**.

The following graph supports the above conclusion:



The two lines are parallel, and they never intersect. So, the system has no solution.

Answer 8STP.

Let the length of the rectangle be l feet

Let the width of the rectangle be w feet

The perimeter of the rectangle is $2l+2w$

Since the perimeter of the rectangle is 68 feet, so $\boxed{2l+2w=68}$ ...... (1)

The length of the garden is 4 more than twice the width.

That is $\boxed{l=2w+4}$ ...... (2)

From the first equation, the options may be **A** or **B** or **D**

From the equation (2), the correct option is **B**

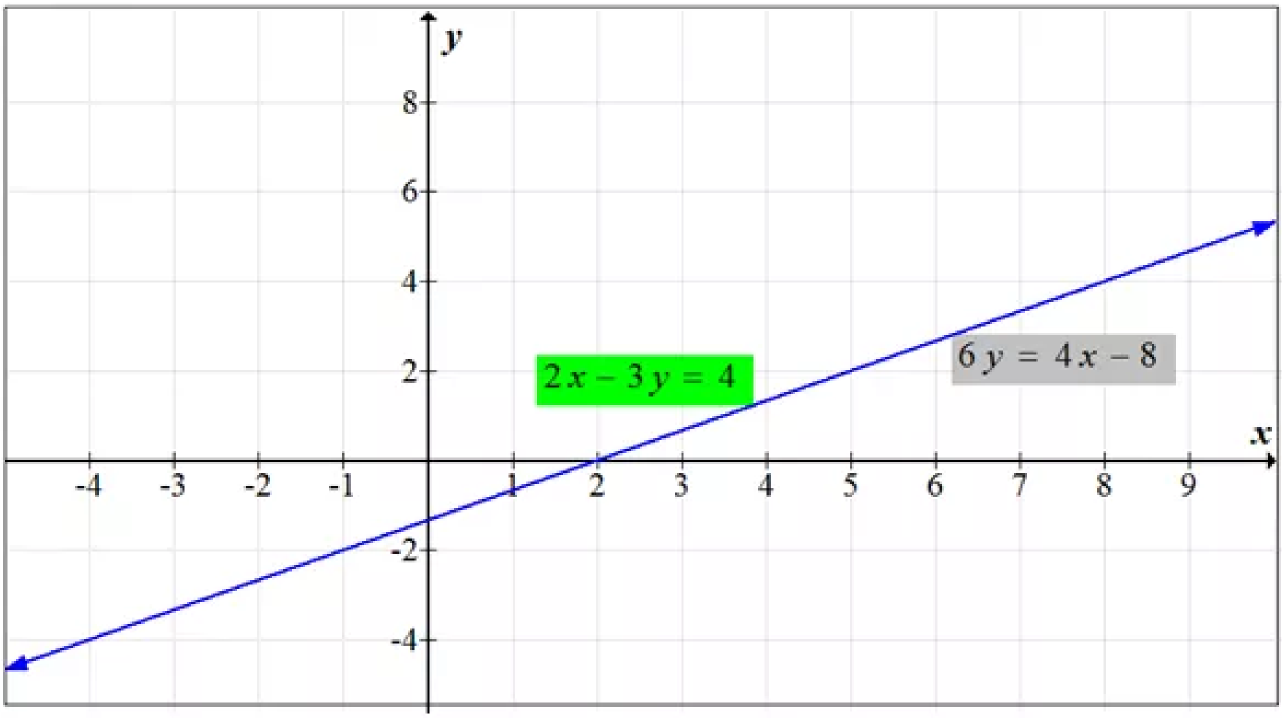
Answer 9E.

Consider the equations,

$2x-3y=4$ ...... (1)

$6y=4x-8$ ...... (2)

The graph of the equations is shown below:



Since the graphs $2x-3y=4$ and $6y=4x-8$ are coincide, there are infinitely many solutions.

Answer 9PT.

Consider the equations,

$2x + y = -4$ ...... (1)

$5x + 3y = -6$ ...... (2)

Eliminate *x*

$$\begin{array}{lll} 2x + y = -4 & \text{Multiply by 5} & 10x + 5y = -20 \\ 5x + 3y = -6 & \text{Multiply by 2} & 10x + 6y = -12 \\ & & -y = -8 \quad \text{Subtract the equations} \end{array}$$

$$\frac{-1y}{-1} = \frac{-8}{-1} \quad \text{Divide each side with } -1$$

$$y = 8 \quad \text{Simplify}$$

Now substitute 2 for *y* in either equation to find the value of x

$$\begin{array}{rl} 2x + y = -4 & \text{First Equation} \\ 2x + (8) = -4 & \text{Substitue 8 for } y \\ 2x + 8 = -4 & \text{Simplify} \\ 2x + 8 - 8 = -4 - 8 & \text{Subtract 8 from each side} \\ 2x = -12 & \text{Simplify} \end{array}$$

$$\frac{2x}{2} = \frac{-12}{2} \quad \text{Divide each side with 2}$$

$$x = -6 \quad \text{Simplify}$$

The solution is $\boxed{(-6, 8)}$

Answer 9STP.

Let the cost of the shirt be $\$x$

The cost of the jeans is $\$x+6$

The total cost of the pair of jeans and a shirt is $64

That is $x+6+x=64$

$$x+6+x=64$$
$$2x+6=64 \quad \text{Add}$$
$$2x-6=64-6 \quad \text{Subtract 6 from each side}$$
$$2x=58 \quad \text{Simplify}$$
$$x=29 \quad \text{Divide each side with 2}$$

The cost of the jeans is $\$29+6=\boxed{\$35}$

The cost of the shirt is $29, this is option B. But the question is the cost of jeans, that is $35, so the option is **C**

Hence the correct option is **C**

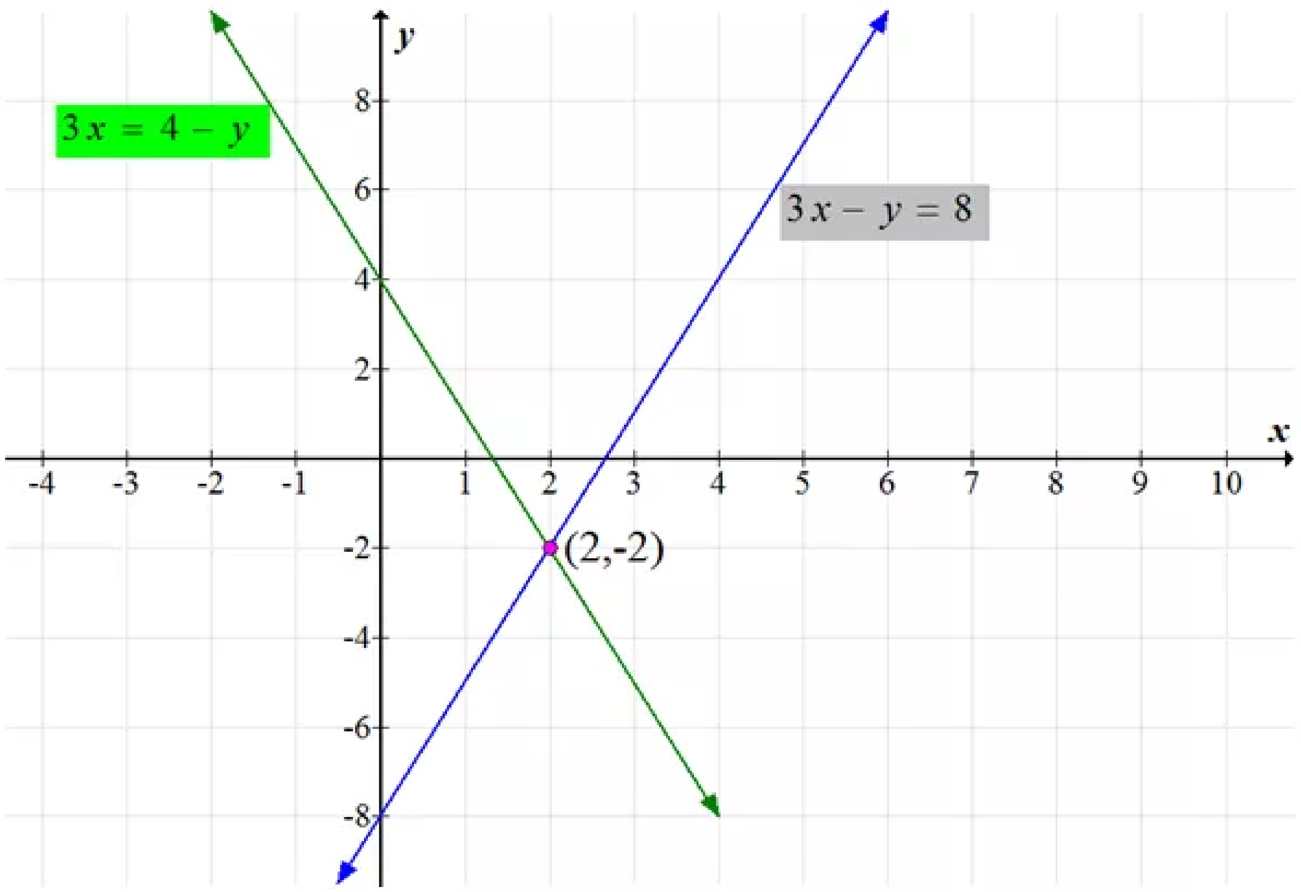
Answer 10E.

Consider the equations,

$3x-y=8$ ...... (1)

$3x=4-y$ ...... (2)

The graph of the equations is shown below:



The graphs appear to intersect at the point with coordinates $(2,-2)$

Check:

$3x - y = 8$ First equation

$6-(-2)=8$ Substitue 2 for x and -2 for y

$8=8$ Verified

$3x = 4 - y$ First equation

$3(2)=4-(-2)$ Substitue 2 for x and -2 for y

$6=6$ Verified

Hence the solution to the system of equations is $\boxed{(2,-2)}$

Answer 10PT.

Consider the equations,

$y = 7 - x$ ...... (1)

$x - y = -3$ ...... (2)

Since $y = 7 - x$, substitute $7 - x$ for y in the second equation

$x - y = -3$ Second equation

$x-(7-x)=-3$ $y = 7 - x$

$x - 7 + x = -3$ Use Distributive property

$2x - 7 = -3$ Subtract 7 from each side

$2x - 7 + 7 = -3 + 7$ Combine like terms

$2x = 4$ Simplify

$\frac{2x}{2} = \frac{4}{2}$ Divide each side with 2

$x = 2$ Simplify

Use $y = 7 - x$ to find the value of y

$y = 7 - x$ First equation

$y = 7 - 2$ $x = 2$

$y = 5$ Simplify

The solution is $\boxed{(2,5)}$

Answer 10STP.

Consider the equations,

$3x+4y=8$ ...... (1)

$3x+2y=-2$ ...... (2)

Since the coefficients of the *x* terms, 3 and 3, are the same, we can eliminate the *x* terms by subtracting the equations.

$$\begin{array}{rrcrcr} & 3x & + & 4y & = & 8 \\ (-) & 3x & + & 2y & = & -2 \\ \hline & & & 2y & = & 10 \end{array}$$

Write the equations in column form and subtract

Notice that the x variable eliminated

$\frac{2y}{2}=\frac{10}{2}$ Divide each side with 2

$y=5$ Simplify

However, the question asks for the value of *y*. The answer is **C**

Answer 11E.

Consider the equations,

$2m+n=1$ ...... (1)

$m-n=8$ ...... (2)

From the equation (2)

$m-n=8$ Second equation

$m-n+n=8+n$ Add n to each side

$m=8+n$ Simplify

$m=n+8$ Simplify

Since $m=n+8$, substitute $n+8$ for m in the first equation

$2m+n=1$ Second equation

$2(n+8)+n=1$ $m=n+8$

$2n+16+n=1$ Use Distributive property

$3n+16=1$ Combine like terms

$3n+16-16=1-16$ Subtract 16 from each side

$3n=-15$ Simplify

$\frac{3n}{3}=\frac{-15}{3}$ Divide each side with 3

$n=-5$ Simplify

Use $m = n + 8$ to find the value of *m*

$m = n + 8$

$m = -5 + 8$ Substitute -5 for n

$m = 3$ Simplify

Hence the solution is $\boxed{(3,-5)}$

Answer 11PT.

Consider the equations,

$x = 2y - 7$ ...... (1)

$y - 3x = -9$ ...... (2)

Since $x = 2y - 7$, substitute $2y - 7$ for x in the second equation

$y - 3x = -9$ Second equation

$y - 3(2y - 7) = -9$ $y = 7 - x$

$y - 6y + 21 = -9$ Use Distributive property

$-5y + 21 = -9$ Combine like terms

$-5y + 21 - 21 = -9 - 21$ Subtract 21 from each side

$-5y = -30$ Simplify

$\frac{-5y}{-5} = \frac{-30}{-5}$ Divide each side with -5

$y = 6$ Simplify

Use $x = 2y - 7$ to find the value of x

$x = 2y - 7$ First equation

$x = 2(6) - 7$ $y = 6$

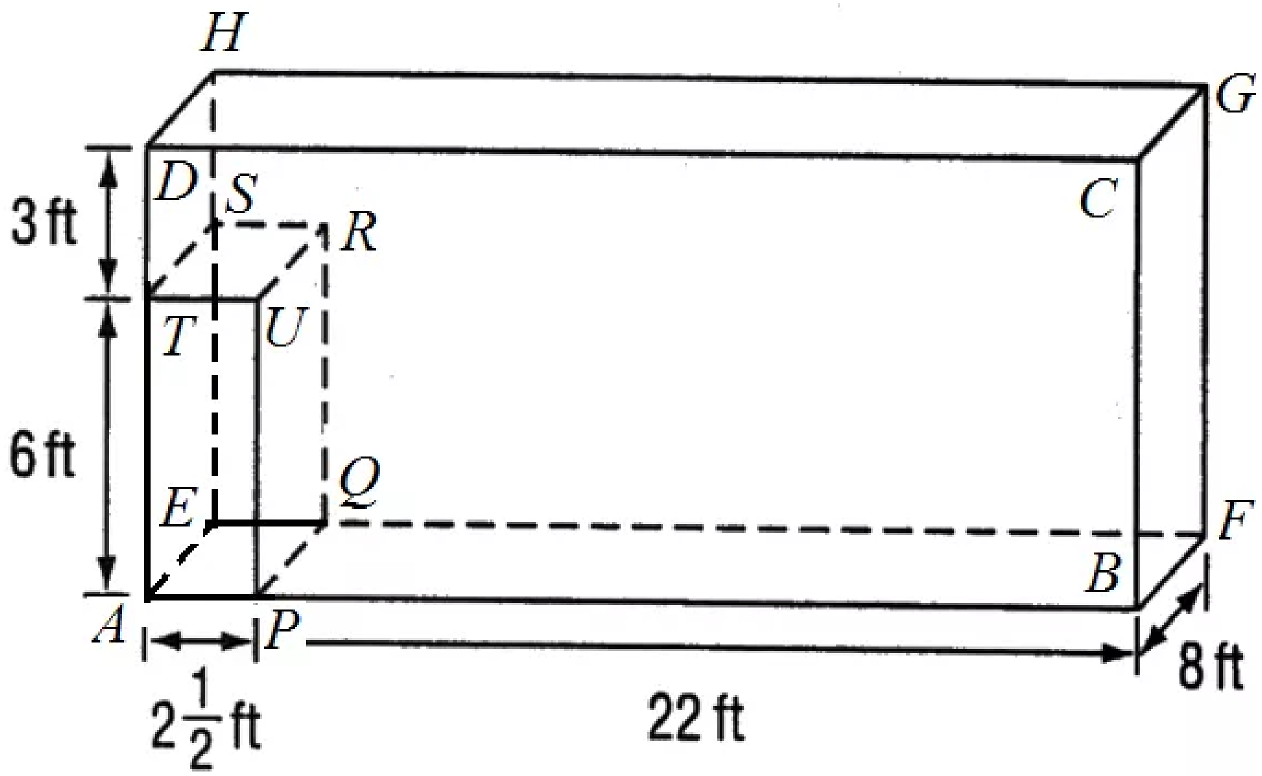
$x = 12 - 7$ Simplify

$x = 5$

The solution is $\boxed{(5,6)}$

Answer 11STP.

Consider the following figure,



From the picture

$AP = 2.5$ ft

$PB = 22$ ft

$BF = PQ = 8$ ft

$AT = 6$ ft

$TD = 3$ ft

Volume of the box is $V = l(\text{Length}) \times w(\text{Width}) \times h(\text{Height})$

Volume of the box is $ABCDEFGH$

$$\begin{aligned} V_1 &= (AP + PB) \times (BF) \times (AT + TD) \\ &= (2.5 + 22) \times (8) \times (6 + 3) \\ &= 24.5 \times 8 \times 9 \\ &= 1764 \end{aligned}$$

Volume of the box is $APQETURS$

$$\begin{aligned} V_2 &= (AP) \times (PQ) \times (AT) \\ &= (2.5) \times (8) \times (6) \\ &= 120 \end{aligned}$$

The required volume is $V = V_1 - V_2$

$V = 1764 - 120$

$V = \boxed{1644 \text{ cubic feet}}$

Answer 12E.

Consider the equations,

$x = 3 - 2y$ ...... (1)

$2x + 4y = 6$ ...... (2)

Since $x = 3 - 2y$, substitute $3 - 2y$ for x in the second equation

$2x + 4y = 6$ Second equation

$2(3 - 2y) + 4y = 6 \quad x = 3 - 2y$

$6 - 4y + 4y = 6$ Use Distributive property

$6 = 6$ Combine like terms

The statement $(6 = 6)$ is true. This means that there are infinitely many solutions of the system of equations.

From the equation (1)

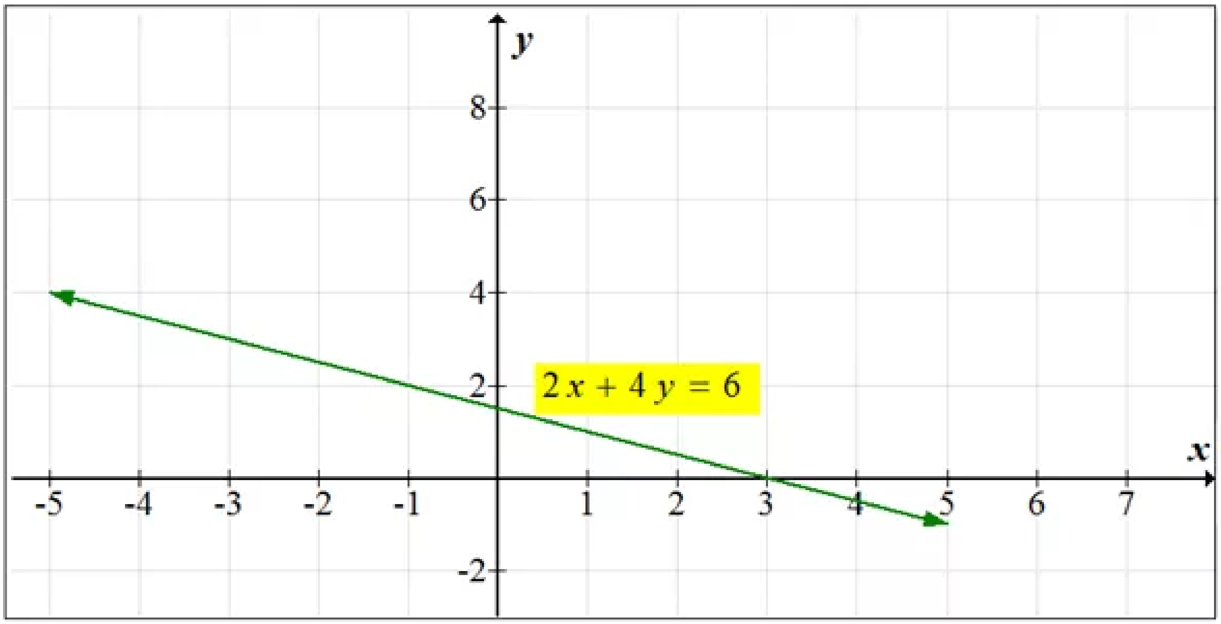
$x = 3 - 2y$	First equation
$x + 2y = 3$	Add $2y$ to each side
$2y = 3 - x$	Subtract x from each side
$y = -\frac{1}{2}x + \frac{3}{2}$	Divide each side with 2

From the equation (2)

$2x + 4y = 6$	First equation
$4y = 6 - 2x$	Subtract $2x$ from each side
$y = -\frac{1}{2}x + \frac{3}{2}$	Divide each side with 4

This is true because the slope intercept form of both the equations is

That is the equations are equivalent, and they have the same graph as shown below:



Answer 12PT.

Consider the equations,

$x + y = 10$ ...... (1)

$x - y = 2$ ...... (2)

Eliminate *y*

$$\begin{aligned} x + y &= 10 \\ x - y &= 2 \\ 2x &= 12 \quad \text{Add the equations} \end{aligned}$$

$$\frac{2x}{2} = \frac{12}{2} \quad \text{Divide each side with 2}$$

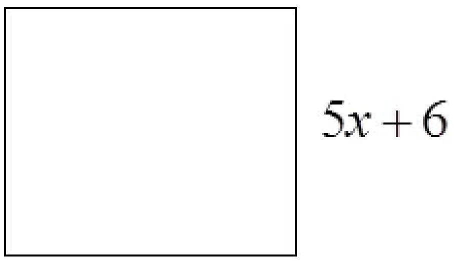
$$x = 6 \quad \text{Simplify}$$

Now substitute 3 for *x* in either equation to find the value of y

$$\begin{aligned} x + y &= 10 && \text{First Equation} \\ 6 + y &= 10 && \text{Substitue 6 for } x \\ 6 + y - 6 &= 10 - 6 && \text{Subtract 6 from each side} \\ y &= 4 && \text{Simplify} \end{aligned}$$

The solution is $\boxed{(6, 4)}$

Consider the following figure,



The Perimeter of the square is

$P = 4(\text{Length of the side})$

Given that the Perimeter of the Square is 204 feet

That is $4(\text{Length of the side}) = 204$

$4(5x + 6) = 204$ Substitute the length of the side

$20x + 24 = 204$ Use the Distributive Property

$20x = 180$ Subtract 24 from both sides

$x = \frac{180}{20}$ Divide each side with 20

$x = 9$ Simplify

Hence, the value of x is $\boxed{9}$

Answer 13E.

Consider the equations,

$3x - y = 1$ ...... (1)

$2x + 4y = 3$ ...... (2)

From the equation (1)

$$
\begin{aligned}
3x - y &= 1 && \text{First equation} \\
3x - y - 3x &= 1 - 3x && \text{Subtract } 3x \text{ from each side} \\
-y &= -3x + 1 && \text{Simplify} \\
y &= 3x - 1 && \text{Divide each side with } -1
\end{aligned}
$$

Since $y = 3x - 1$, substitute $3x - 1$ for y in the second equation

$2x + 4y = 3$ Second equation

$2x + 4(3x - 1) = 3 \quad x = 3 - 2y$

$2x + 12x - 4 = 3$ Use Distributive property

$14x - 4 = 3$ Combine like terms

$14x - 4 + 4 = 3 + 4$ Add 4 to each side of the equation

$14x = 7$ Simplify

$\frac{14x}{14} = \frac{7}{14}$ Divide each side with 14

$x = \frac{1}{2}$ Simplify

Use $y = 3x - 1$ to find the value of y

$$
\begin{aligned}
y &= 3x - 1 && \\
y &= 3\left(\frac{1}{2}\right) - 1 && \text{Substitute } \frac{1}{2} \text{ for } x \\
y &= \frac{3}{2} - 1 && \text{Simplify} \\
y &= \frac{1}{2} && \text{Simplify}
\end{aligned}
$$

Hence the solution is $\boxed{\left(\frac{1}{2}, \frac{1}{2}\right)}$

Answer 13PT.

Consider the equations,

$3x - y = 11$ ...... (1)

$x + 2y = -36$ ...... (2)

Eliminate *x*

$$\begin{array}{llll} 3x - y = 11 & & 3x - y = 11 & \\ x + 2y = -36 & \text{Multiply by 3} & 3x + 6y = -108 & \\ & & -7y = 119 & \text{Subtract the equations} \end{array}$$

$$\frac{-7y}{-7} = \frac{119}{-7} \quad \text{Divide each side with } -7$$

$$y = -17 \quad \text{Simplify}$$

Now substitute -17 for *y* in either equation to find the value of x

$$\begin{array}{rl} 3x - y = 11 & \text{First Equation} \\ 3x - (-17) = 11 & \text{Substitue } -17 \text{ for } y \\ 3x + 17 = 11 & \text{Simplify} \\ 3x + 17 - 17 = 11 - 17 & \text{Subtract 17 from each side} \\ 3x = -6 & \text{Simplify} \end{array}$$

$$\frac{3x}{3} = \frac{-6}{3} \quad \text{Divide each side with 3}$$

$$x = -2 \quad \text{Simplify}$$

The solution is $\boxed{(-2, -17)}$

Answer 13STP.

Consider the equation,

$4x + 3y = 12$

To find the *x* intercept of the equation $4x + 3y = 12$, substitute $y = 0$ in the given equation and solve for *x*

$$\begin{array}{rl} 4x + 3y = 12 & \\ 4x + 3(0) = 12 & \text{Substitute 0 for } y \\ 4x = 12 & \text{Simplify} \\ x = 3 & \text{Divide each side with 4} \end{array}$$

Hence, the *x* intercept is $\boxed{3}$

Answer 14E.

Consider the equations,

$0.6m-0.2n=0.9$ ...... (1)

$n=4.5-3m$ ...... (2)

Since $n=4.5-3m$, substitute $4.5-3m$ for n in the first equation

$0.6m-0.2n=0.9$ First equation

$0.6m-0.2(4.5-3m)=0.9$ $n=4.5-3m$

$0.6m-0.9+0.6m=0.9$ Use Distributive property

$1.2m-0.9=0.9$ Combine like terms

$1.2m-0.9+0.9=0.9+0.9$ Add 0.9 to each side

$1.2m=1.8$ Simplify

$\frac{1.2m}{1.2}=\frac{1.8}{1.2}$ Divide each side with 1.2

$m=1.5$ Simplify

Use $n=4.5-3m$ to find the value of m

$n=4.5-3m$	Second equation
$n=4.5-3(1.5)$	Substitute 1.5 for m
$n=4.5-4.5$	Simplify
$n=0$	Simplify

Hence the solution is $\boxed{(1.5,0)}$

Answer 14PT.

Consider the equations,

$3x + y = 10$ ...... (1)

$3x - 2y = 16$ ...... (2)

Eliminate *x*

$$\begin{aligned} 3x + y &= 10 \\ 3x - 2y &= 16 \\ 3y &= -6 \quad \text{Subtract the equations} \end{aligned}$$

$$\begin{aligned} \frac{3y}{3} &= \frac{-6}{3} \quad \text{Divide each side with 3} \\ y &= -2 \quad \text{Simplify} \end{aligned}$$

Now substitute -2 for *y* in either equation to find the value of x

$$\begin{aligned} 3x + y &= 10 && \text{First Equation} \\ 3x + (-2) &= 10 && \text{Substitue } -2 \text{ for } y \\ 3x - 2 &= 10 && \text{Simplify} \\ 3x - 2 + 2 &= 10 + 2 && \text{Add 2 to each side} \\ 3x &= 12 && \text{Simplify} \end{aligned}$$

$$\begin{aligned} \frac{3x}{3} &= \frac{12}{3} && \text{Divide each side with 3} \\ x &= 4 && \text{Simplify} \end{aligned}$$

The solution is $\boxed{(4, -2)}$

Answer 14STP.

Slope intercept form:

The equation $y = mx + c$ is called Slope intercept equation.

Here m is the slope of the line and c is *y* intercept of the line

Consider the equation,

$4x - 2y = 5$

To find the slope and *x* intercept of the equation $4x - 2y = 5$, reduce the equation to slope intercept form.

$$\begin{aligned} 4x - 2y &= 5 \\ 4x - 2y - 4x &= 5 - 4x && \text{Subtract } 4x \text{ from each side} \\ -2y &= -4x + 5 && \text{Combine like terms} \\ y &= 2x - \frac{5}{2} && \text{Divide each side with } -2 \end{aligned}$$

The equation $y = 2x - \frac{5}{2}$ is compared with slope intercept equation, $y = mx + c$.

Then $m = 2$ and $c = -\frac{5}{2}$

Hence, the **slope** of the line is $\boxed{2}$ and the ***y* intercept** of the line is $\boxed{-\frac{5}{2}}$

Answer 15E.

Consider the equations,

$x + 2y = 6$ ...... (1)

$x - 3y = -4$ ...... (2)

Since the coefficients of the *x* terms, 1 and 1, are the same, we can eliminate the *x* terms by subtracting the equations.

$$\begin{array}{rrcrcr} & x & + & 2y & = & 6 \\ (-) & x & - & 3y & = & -4 \\ \hline & & & 5y & = & 10 \end{array} \quad \begin{array}{l} \text{Write the equations in column form and subtract} \\ \\ \text{Notice that the } x \text{ variable eliminated} \end{array}$$

$\frac{5y}{5} = \frac{10}{5}$ Divide each side with 5

$y = 2$ Simplify

Now substitute 2 for *y* in either equation to find the value of *x*.

$x-3y=-4$ Second equation

$x-3(2)=-4 \quad y=2$

$x-6=-4$ Simplify

$x-6+6=-4+6$ Add 6 to each side of the equation

$x=2$ Simplify

The solution is $\boxed{(2,2)}$

Answer 15PT.

Consider the equations,

$5x-3y=12$ ...... (1)

$-2x+3y=-3$ ...... (2)

Eliminate *y*

$$\begin{aligned} 5x-3y&=12 \\ -2x+3y&=-3 \\ 3x&=9 \quad \text{Add the equations} \end{aligned}$$

$$\frac{3x}{3}=\frac{9}{3} \quad \text{Divide each side with 3}$$

$$x=3 \quad \text{Simplify}$$

Now substitute 3 for *x* in either equation to find the value of y

$$\begin{aligned} 5x-3y&=12 && \text{First Equation} \\ 5(3)-3y&=12 && \text{Substitue 3 for } x \\ 15-3y&=12 && \text{Simplify} \\ 15-3y-15&=12-15 && \text{Subtract 15 from each side} \\ -3y&=-3 && \text{Simplify} \end{aligned}$$

$$\frac{-3y}{-3}=\frac{-3}{-3} \quad \text{Divide each side with } -3$$

$$y=1 \quad \text{Simplify}$$

The solution is $\boxed{(3,1)}$

Answer 15STP.

Consider the equations,

$5x - y = 10$ ...... (1)

$7x - 2y = 11$ ...... (2)

From the equation (1)

$5x - y = 10$	First equation
$5x - y + y = 10 + y$	Add y to each side
$5x = 10 + y$	Simplify
$5x - 10 = y$	Subtract 10 from each side

$y = 5x - 10$

Since $y = 5x - 10$, substitute $5x - 10$ for y in the second equation

$7x - 2(5x - 10) = 11$

$7x - 10x + 20 = 11$ Use the Distributive property

$-3x + 20 = 11$ Combine like terms

$-3x + 20 - 20 = 11 - 20$ Subtract 20 from each side

$-3x = -9$ Combine like terms

$\frac{-3x}{-3} = \frac{-9}{-3}$ Divide each side with -3

$x = 3$ Simplify

Use $y = 5x - 10$ to find the value of y

$y = 5x - 10$

$y = 5(3) - 10$ $x = 3$

$y = 15 - 10$ Simplify

$y = 5$ Simplify

The solution is $\boxed{(3,5)}$

Answer 16E.

Consider the equations,

$2m-n=5$ ...... (1)

$2m+n=3$ ...... (2)

Since the coefficients of the *n* terms, -1 and 1, are additive inverses, we can eliminate the *n* terms by adding the equations.

$$\begin{array}{rrcrcr} & 2m & - & n & = & 5 \\ (+) & 2m & + & n & = & 3 \\ \hline & 4m & & & = & 8 \end{array}$$

Write the equations in column form and add

Notice that the n variable eliminated

$\frac{4m}{4}=\frac{8}{4}$ Divide each side with 4

$m=2$ Simplify

Now substitute 2 for *m* in either equation to find the value of *n*.

$2m+n=3$ Second equation

$2(2)+n=3$ $m=2$

$4+n=3$ Subtract 4 from each side of the equation

$4+n-4=3-4$ Simplify

$n=-1$ Simplify

The solution is $\boxed{(2,-1)}$

Answer 16PT.

Consider the equations,

$2x+5y=12$ ...... (1)

$x-6y=-11$ ...... (2)

Eliminate *x*

$$\begin{array}{llll} 2x+5y=12 & & 2x+5y=12 & \\ x-6y=-11 & \text{Multiply by 2} & 2x-12y=-22 & \\ & & 17y=34 & \text{Subtract the equations} \end{array}$$

$$\begin{aligned} \frac{17y}{17}&=\frac{34}{17} && \text{Divide each side with 17} \\ y&=2 && \text{Simplify} \end{aligned}$$

Now substitute 2 for *y* in either equation to find the value of x

$$\begin{aligned} 2x+5y&=12 && \text{First Equation} \\ 2x+5(2)&=12 && \text{Substitue 2 for } y \\ 2x+10&=12 && \text{Simplify} \\ 2x+10-10&=12-10 && \text{Subtract 2 from each side} \\ 2x&=2 && \text{Simplify} \end{aligned}$$

$$\begin{aligned} \frac{2x}{2}&=\frac{2}{2} && \text{Divide each side with 2} \\ x&=1 && \text{Simplify} \end{aligned}$$

The solution is $\boxed{(1,2)}$

Answer 16STP.

Let the first number be *x* and the second number is *y*

Two times one number minus three times another number is -11

That is $2x-3y=-11$ ...... (1)

The sum of the first number and three times the second number is 8

That is $x+3y=8$ ...... (2)

Eliminate y

$$\begin{aligned} 2x-3y&=-11 && \text{First equation} \\ x+3y&=8 && \text{Second equation} \\ 3x&=-3 && \text{Add the equations} \\ x&=-1 && \text{Divide each side with 3} \end{aligned}$$

Now substitute -1 for x in either equation to find the value of y

$$\begin{aligned} x+3y&=8 && \text{Second Equation} \\ -1+3y&=8 && \text{Substitue } -1 \text{ for } x \\ 3y&=9 && \text{Add 1 to each side} \\ \frac{3y}{3}&=\frac{9}{3} && \text{Divide each side with 3} \\ y&=3 && \text{Simplify} \end{aligned}$$

Hence the numbers are $x=\boxed{-1}$ and $y=\boxed{3}$

Answer 17E.

Consider the equations,

$3x-y=11$ ...... (1)

$x+y=5$ ...... (2)

Since the coefficients of the y terms, -1 and 1, are additive inverses, we can eliminate the y terms by adding the equations.

$$\begin{array}{rrcrcr} & 3x & - & y & = & 11 \\ (+) & x & + & y & = & 5 \\ \hline & 4x & & & = & 16 \end{array}$$

Write the equations in column form and add

Notice that the y variable eliminated

$\frac{4x}{4}=\frac{16}{4}$ Divide each side with 5

$x=4$ Simplify

Now substitute 4 for *x* in either equation to find the value of *y*.

$x+y=5$ Second equation

$4+y=5 \quad x=4$

$4+y-4=5-4$ Subtract 4 from each side of the equation

$y=1$ Simplify

The solution is $\boxed{(4,1)}$

Answer 17PT.

Consider the equations,

$x+y=6$ ...... (1)

$3x-3y=13$ ...... (2)

Eliminate *y*

$$\begin{array}{llr} x+y=6 & \text{Multiply by 3} & 3x+3y=18 \\ 3x-3y=13 & & 3x-3y=13 \\ & & 6x=31 \quad \text{Add the equations} \end{array}$$

$\frac{6x}{6}=\frac{31}{6}$ Divide each side with 6

$x=\frac{31}{6}$ Simplify

Now substitute $\frac{31}{6}$ for *x* in either equation to find the value of y

$x+y=6$ First Equation

$\frac{31}{6}+y=6$ Substitue $\frac{31}{6}$ for x

$y=6-\frac{31}{6}$ Subtract $\frac{31}{6}$ from each side

$y=\frac{36-31}{6}$ Simplify

$y=\frac{5}{6}$ Simplify

The solution is $\boxed{\left(\frac{31}{6},\frac{5}{6}\right)}$

Answer 18E.

Consider the equations,

$3x+1=-7y$ ...... (1)

$6x+7y=0$ ...... (2)

Since the coefficients of the *y* terms, 7 and 7, are the same, we can eliminate the *y* terms by subtracting the equations.

$$\begin{array}{rrrrrrl} & 3x & + & 7y & = & -1 & \quad \text{Write the equations in column form and subtract} \\ (-) & 6x & + & 7y & = & 0 & \\ \hline & -3x & & & = & -1 & \quad \text{Notice that the } y \text{ variable eliminated} \end{array}$$

$\frac{-3x}{-3}=\frac{-1}{-3}$ Divide each side with -3

$x=\frac{1}{3}$ Simplify

Now substitute $\frac{1}{3}$ for *x* in either equation to find the value of *y*.

$6x+7y=0$ Second equation

$6\left(\frac{1}{3}\right)+7y=0 \quad x=\frac{1}{3}$

$2+7y=0$ Simplify

$2+7y-2=0-2$ Subtract 2 from each side of the equation

$7y=-2$ Simplify

$\frac{7y}{7}=\frac{-2}{7}$ Divide each side with -7

$y=-\frac{2}{7}$ Simplify

The solution is $\boxed{\left(\frac{1}{3},-\frac{2}{7}\right)}$

Answer 18PT.

Consider the equations,

$$3x+\frac{1}{3}y=10 \quad \ldots\ldots (1)$$

$$2x-\frac{5}{3}y=35 \quad \ldots\ldots (2)$$

Eliminate *y*

$$3x+\frac{1}{3}y=10 \quad \text{Multiply by 5} \quad 15x+\frac{5}{3}y=50$$

$$2x-\frac{5}{3}y=35 \qquad\qquad 2x-\frac{5}{3}y=35$$

$$17x=85 \quad \text{Add the equations}$$

$$\frac{17x}{17}=\frac{85}{17} \quad \text{Divide each side with 17}$$

$$x=5 \quad \text{Simplify}$$

Now substitute 5 for x in either equation to find the value of y

$$3x+\frac{1}{3}y=10 \quad \text{First Equation}$$

$$3(5)+\frac{1}{3}y=10 \quad \text{Substitue 5 for } x$$

$$\frac{1}{3}y=10-15 \quad \text{Subtract 15 from each side}$$

$$\frac{1}{3}y=-5 \quad \text{Simplify}$$

$$y=-15 \quad \text{Simplify}$$

The solution is $\boxed{(5,-15)}$

Answer 18STP.

(a)

Let r represent the number of miles Mark ran and w represent the number of miles Mark walked.

The total distance is 20 miles

That is $r + w = 20$ ...... (1)

Mark either ran at a speed of 7 miles per hour or walked at a speed of 3 miles per hour.

He completed 20 miles in 3 hours.

That is $\frac{r}{7} + \frac{w}{3} = 3$ ...... (2)

$$\frac{r}{7} \cdot \frac{3}{3} + \frac{w}{3} \cdot \frac{7}{7} = 3$$

$$\frac{3r + 7w}{21} = 3$$

$$3r + 7w = 63$$

(b)

The graphs of $r + w = 20$ and $3r + 7w = 63$ are shown below:

The two lines $r + w = 20$ and $3r + 7w = 63$ intersect at the point $(19.25, 0.75)$

Hence the number of miles ran by Mark is **19.25** and number of miles walked is **0.75**

Answer 19E.

Consider the equations,

$x-5y=0$ ...... (1)

$2x-3y=7$ ...... (2)

Eliminate x

$$\begin{array}{lll} x-5y=0 & \text{Multiply by 2} & 2x-10y=0 \\ 2x-3y=7 & & 2x-3y=7 \\ & & -7y=-7 \quad \text{Subtract the equations} \end{array}$$

$$\begin{array}{rl} \frac{-7y}{-7}=\frac{-7}{-7} & \text{Divide each side with } -7 \\ y=1 & \text{Simplify} \end{array}$$

Now substitute 1 for y in either equation to find the value of x

$$\begin{array}{rl} x-5y=0 & \text{First Equation} \\ x-5(1)=0 & \text{Substitue 1 for } y \\ x-5=0 & \text{Simplify} \\ x-5+5=5 & \text{Add 5 to each side} \\ x=5 & \text{Simplify} \end{array}$$

The solution is $\boxed{(5,1)}$

Answer 19PT.

Let the unit's digit be *y* and 10's place digit is *x*. Hence the number is xy

The units digit of a two digit number exceeds twice the tens digit by 1. That is

$y = 2x + 1$ ...... (1)

The sum of its digits is 10. That is

$x + y = 10$ ...... (2)

Since $y = 2x + 1$, substitute $2x + 1$ for in the second equation

$x + y = 10$ First equation

$x + 2x + 1 = 10$ $y = 2x + 1$

$3x + 1 = 10$ Combine like terms

$3x + 1 - 1 = 10 - 1$ Subtract 1 from each side

$3x = 9$ Simplify

$\frac{3x}{3} = \frac{9}{3}$ Divide each side with 3

$x = 3$ Simplify

Use $y = 2x + 1$ to find the value of y

$y = 2x + 1$ First equation

$y = 2(3) + 1$ $x = 3$

$y = 6 + 1$ Simplify

$y = 7$

Hence the two digit number is $xy = \boxed{37}$

Answer 19STP.

(a)

Let a represent the number of adult tickets sold and c represent the number children tickets sold

The total **number of tickets sold** is 650

That is $a+c=650$ ...... (1)

The **total amount collected** from the adult and children tickets be $3675

That is $7.50a+4.50c=3675$ ...... (2)

(b)

Eliminate *a* from the equation (1),

$$\begin{aligned} a+c&=650 && \text{First equation} \\ a+c-c&=650-c && \text{Subtract } c \text{ from each side} \\ a&=650-c && \text{Simplify} \end{aligned}$$

Substitute $a=650-c$ in the equation (2)

$$\begin{aligned} 7.50a+4.50c&=3675 && \text{Second equation} \\ 7.50(650-c)+4.50c&=3675 && \text{Substitute } 650-c \text{ for } a \\ 4875-7.50c+4.50c&=3675 && \text{Simplify} \\ 4875-3c&=3675 && \text{Combine like terms} \\ 4875-3c-3675&=0 && \text{Subtract 3675 from each side} \end{aligned}$$

$$\begin{aligned} 1200-3c&=0 && \text{Simplify} \\ 3c&=1200 && \text{Add } 3c \text{ to each side} \\ c&=400 && \text{Divide each side with 3} \end{aligned}$$

Use $a=650-c$ to find the value of *a*

$$\begin{aligned} a&=650-c \\ a&=650-400 && \text{Substitute 400 for } c \\ a&=250 && \text{Simplify} \end{aligned}$$

Hence the number of adult tickets sold $\boxed{250}$ and the number of children tickets sold

$\boxed{400}$

Answer 20E.

Consider the equations,

$x-2y=5$ ...... (1)

$3x-5y=8$ ...... (2)

Eliminate *x*

$$\begin{array}{rll} x-2y=5 & \text{Multiply by 3} & 3x-6y=15 \\ 3x-5y=8 & & 3x-5y=8 \\ & & -y=7 \quad \text{Subtract the equations} \end{array}$$

$$\frac{-y}{-1}=\frac{7}{-1} \quad \text{Divide each side with } -1$$

$$y=-7 \quad \text{Simplify}$$

Now substitute -7 for *y* in either equation to find the value of x

$$\begin{array}{rl} x-2y=5 & \text{First Equation} \\ x-2(-7)=5 & \text{Substitue 1 for } y \\ x+14=5 & \text{Simplify} \\ x+14-14=5-14 & \text{Subtract 14 from each side} \\ x=-9 & \text{Simplify} \end{array}$$

The solution is $\boxed{(-9,-7)}$

Answer 20PT.

Let *x* be the length of the rectangle and *y* be the width of the rectangle

The difference between the length and width of the rectangle is 7 centimeters. That is

$x-y=7$ ...... (1)

The Perimeter of the rectangle is 50 centimeters. That is

$2x+2y=50$ ...... (2)

Eliminate *y*

$$\begin{array}{rll} x-y=7 & \text{Multiply by 2} & 2x-2y=14 \\ 2x+2y=50 & & 2x+2y=50 \\ & & 4x=64 \quad \text{Add the equations} \end{array}$$

$$\frac{4x}{4}=\frac{64}{4} \quad \text{Divide each side with 4}$$

$$x=16 \quad \text{Simplify}$$

Now substitute 16 for x in either equation to find the value of y

$x-y=7$	First Equation
$16-y=7$	Substitue 16 for x
$16-y-16=7-16$	Simplify
$-y=-9$	Subtract 16 from each side
$y=9$	Simplify

Hence the length of the rectangle, $x=\boxed{16\text{ cm}}$ and width of the rectangle, $y=\boxed{9\text{ cm}}$

Answer 21E.

Consider the equations,

$2x+3y=8$ ...... (1)

$x-y=2$ ...... (2)

Eliminate y

$$\begin{array}{lll} 2x+3y=8 & & 2x+3y=8 \\ x-y=2 & \text{Multiply by 3} & 3x-3y=6 \\ & & 5x=14 \quad \text{Add the equations} \end{array}$$

$\frac{5x}{5}=\frac{14}{5}$ Divide each side with 5

$x=\frac{14}{5}$ Simplify

Now substitute $\frac{14}{5}$ for x in either equation to find the value of y

$x-y=2$	Second Equation
$\frac{14}{5}-y=2$	Substitue $\frac{14}{5}$ for x
$\frac{14}{5}-y-\frac{14}{5}=2-\frac{14}{5}$	Subtract $\frac{14}{5}$ from each side
$-y=-\frac{4}{5}$	Simplify
$y=\frac{4}{5}$	Simplify

The solution is $\boxed{\left(\frac{14}{5},\frac{4}{5}\right)}$

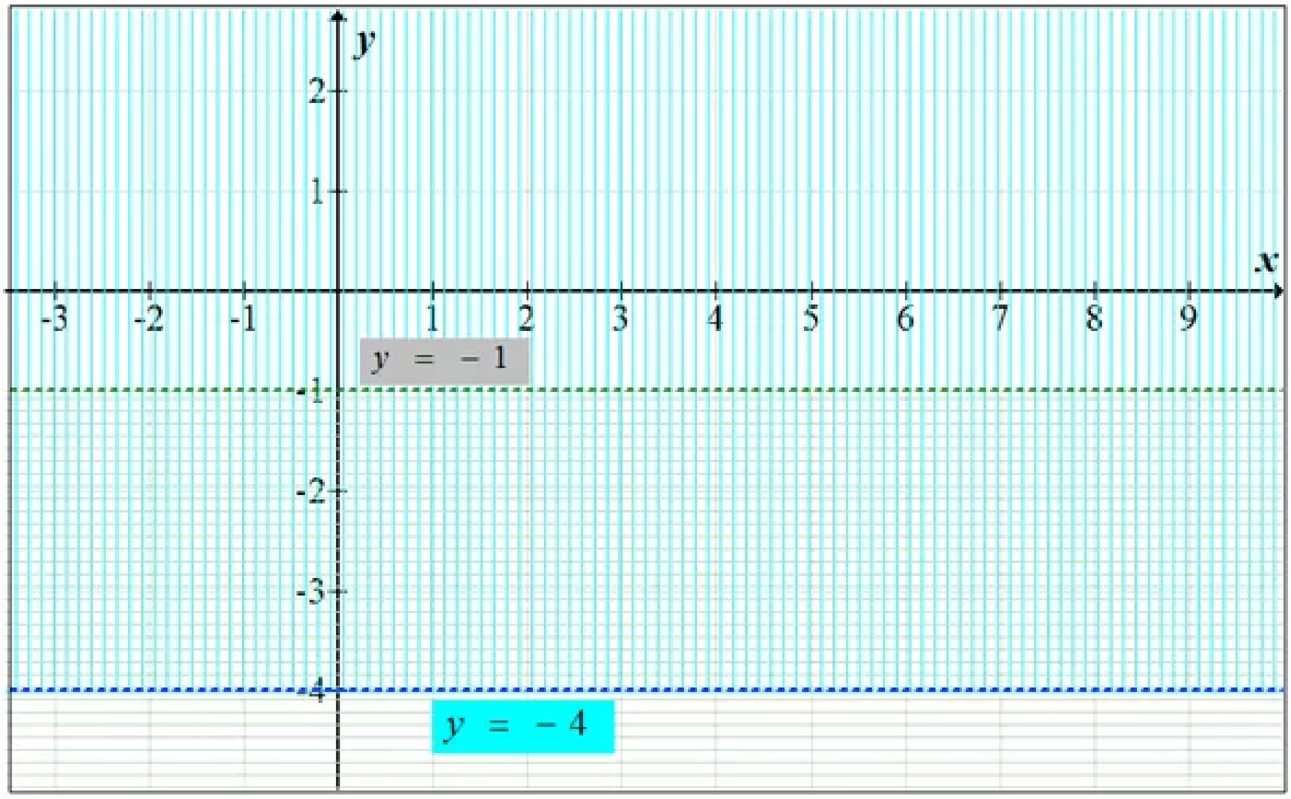
Answer 21PT.

Consider the inequalities,

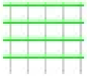
$y > -4$ ...... (1)

$y < -1$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $y > -4$ and $y < -1$

This solution region is shaded as  .

The graphs of $y = -4$ and $y = -1$ are boundaries of this region.

The graph of $y = -4$ and $y = -1$ is dashed and not included in the graph of $y > -4$ and $y < -1$

Answer 22E.

Consider the equations,

$-5x+8y=21$ ...... (1)

$10x+3y=15$ ...... (2)

Eliminate *x*

$$\begin{aligned} -5x+8y&=21 \quad \text{Multiply by 2} \qquad & -10x+16y&=42 \\ 10x+3y&=15 & 10x+3y&=15 \\ & & 19y&=57 \quad \text{Add the equations} \end{aligned}$$

$$\begin{aligned} \frac{19y}{19}&=\frac{57}{19} \quad \text{Divide each side with 19} \\ y&=3 \quad \text{Simplify} \end{aligned}$$

Now substitute 3 for *y* in either equation to find the value of x

$$\begin{aligned} 10x+3y&=15 && \text{Second Equation} \\ 10x+3(3)&=15 && \text{Substitue 3 for } y \\ 10x+9&=15 && \text{Simplify} \\ 10x&=6 && \text{Subtract 9 from each side} \\ x&=\frac{6}{10} && \text{Simplify} \end{aligned}$$

The solution is $\boxed{\left(\frac{3}{5},3\right)}$

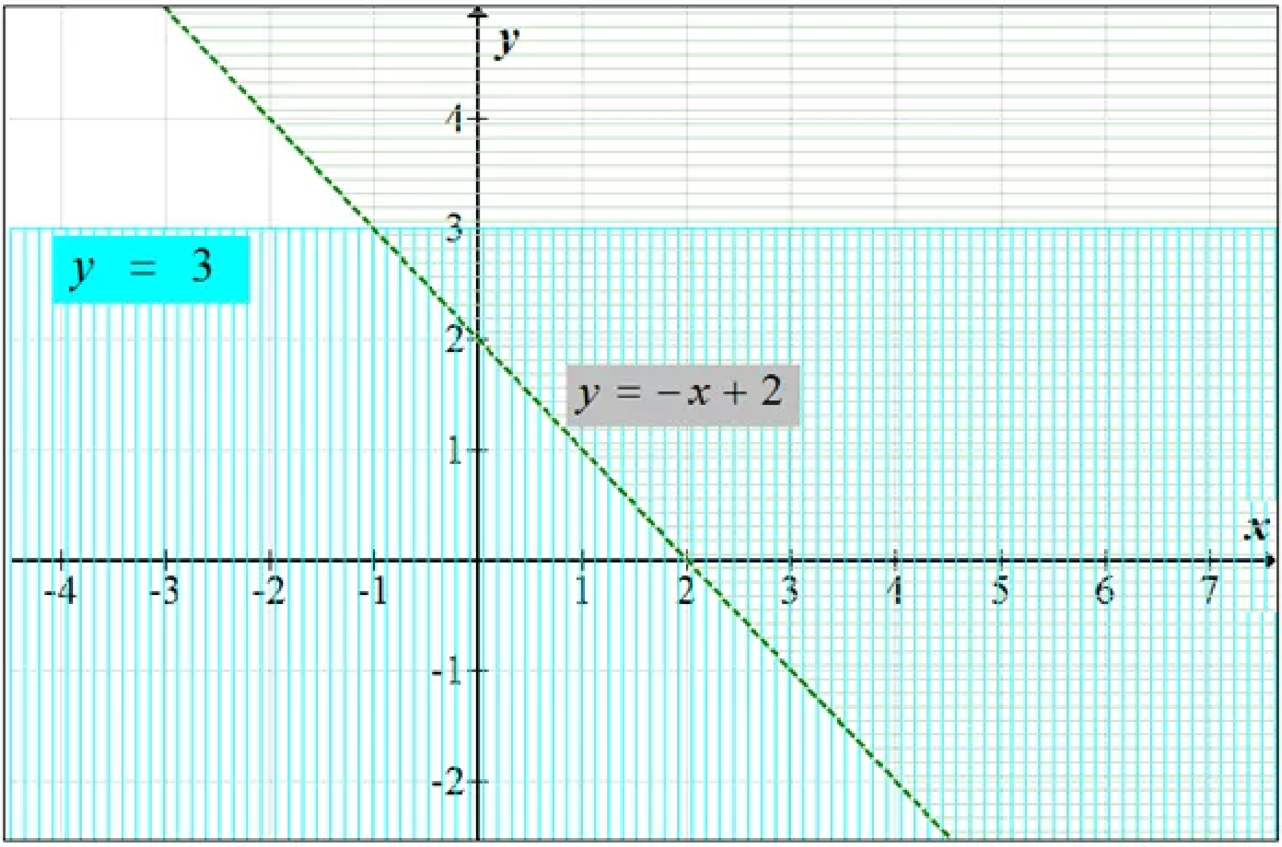
Answer 22PT.

Consider the inequalities,

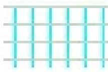
$y \le 3$ ...... (1)

$y > -x + 2$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $y \le 3$ and $y > -x + 2$

This solution region is shaded as  .

The graphs of $y = 3$ and $y = -x + 2$ are boundaries of this region.

The graph of $y = -x + 2$ is dashed and not included in the graph of $y > -x + 2$

The graph of $y = 3$ is included in the graph of $y \le 3$

Answer 23E.

Consider the equations,

$y = 2x$ ...... (1)

$x + 2y = 8$ ...... (2)

Since $y = 2x$, substitute $2x$ for y in the second equation

$x + 2y = 8$ Second equation

$x + 2(2x) = 8$ $y = 2x$

$x + 4x = 8$ Simplify

$5x = 8$ Combine like terms

$x = \frac{8}{5}$ Divide each side with 5

Use $y = 2x$ to find the value of y

$y = 2x$	First equation
$y = 2\left(\frac{8}{5}\right)$	Substitute $\frac{8}{5}$ for x
$y = \frac{16}{5}$	Simplify

Hence the solution is $\boxed{\left(\frac{8}{5}, \frac{16}{5}\right)}$

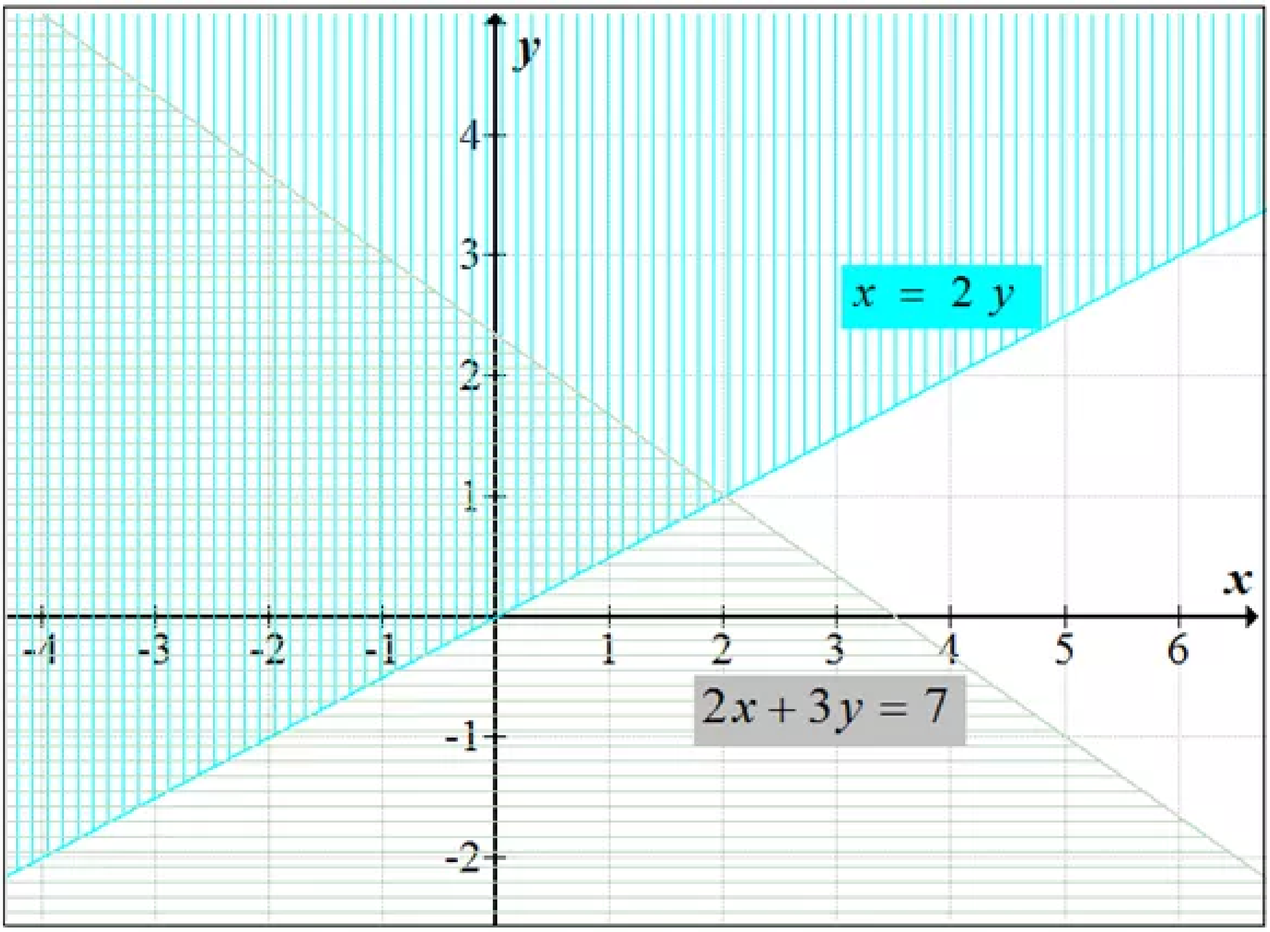
Answer 23PT.

Consider the inequalities,

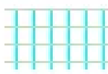
$x \le 2y$ ...... (1)

$2x + 3y \le 7$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $x \le 2y$ and $2x + 3y \le 7$

This solution region is shaded as  .

The graphs of $x = 2y$ and $2x + 3y = 7$ are boundaries of this region.

The graph of $x = 2y$ and $2x + 3y = 7$ is included in the graph of $x \le 2y$ and $2x + 3y \le 7$

Answer 24E.

Consider the equations,

$9x+8y=7$ ...... (1)

$18x-15y=14$ ...... (2)

Eliminate x

$$\begin{array}{rlr} 9x+8y=7 & \text{Multiply by 2} & 18x+16y=14 \\ 18x-15y=14 & & 18x-15y=14 \\ & & 31y=0 \quad \text{Subtract the equations} \end{array}$$

$$\begin{aligned} \frac{31y}{31}&=\frac{0}{31} && \text{Divide each side with 31} \\ y&=0 && \text{Simplify} \end{aligned}$$

Now substitute 0 for y in either equation to find the value of x

$$\begin{aligned} 18x-15y&=14 && \text{Second Equation} \\ 18x-15(0)&=14 && \text{Substitue 0 for } y \\ 18x-0&=14 && \text{Simplify} \\ 18x&=\frac{14}{18} && \text{Simplify} \\ x&=\frac{7}{9} && \text{Simplify} \end{aligned}$$

The solution is $\boxed{\left(\frac{7}{9},0\right)}$

Answer 24PT.

Suppose \$ x invested at the rate of 6% and \$ y invested at the rate of 8%.

Then 6% of x is $x\times\frac{6}{100}=0.06x$ and 8% of y is $y\times\frac{8}{100}=0.08y$

The total amount invested \$10,000. That is

$x+y=10000$ ...... (1)

Interest he got from the total amount is \$760. That is $6\% \text{ of } x+8\% \text{ of } x=\$760$

$0.06x+0.08y=760$ ...... (2)

Eliminate x

$$\begin{aligned} x+y&=10000 && \text{Multiply by } 100 && 6x+6y=60000 \\ 0.06x+0.08y&=760 && \text{Multiply by } 100 && 6x+8y=76000 \\ &&&& -2y=-16000 \quad \text{Subtract the equations} \end{aligned}$$

$$\begin{aligned} \frac{-2y}{-2}&=\frac{-16000}{-2} && \text{Divide each side with } -2 \\ y&=8000 && \text{Simplify} \end{aligned}$$

Now substitute 8000 for y in either equation to find the value of x

$$\begin{aligned} x+y&=10000 && \text{First Equation} \\ x+8000&=10000 && \text{Substitue 8000 for } y \\ x+8000-8000&=10000-8000 && \text{Subtract 8000 from each side} \\ x&=2000 && \text{Simplify} \end{aligned}$$

Hence **\$2000** invested at the rate of 6% and **\$8000** invested at the rate of 8%

Answer 25E.

Consider the equations,

$3x+5y=2x$ ...... (1)

$x+3y=y$ ...... (2)

From the equation (1)

$$\begin{aligned} 3x+5y&=2x \\ 3x+5y-2x&=2x-2x && \text{Subtract } 2x \text{ from each side} \\ x+5y&=0 && \text{Simplify} \end{aligned}$$

From the equation (2)

$$\begin{aligned} x+3y&=y \\ x+3y-y&=y-y && \text{Subtract } y \text{ from each side} \\ x+2y&=0 && \text{Simplify} \end{aligned}$$

Eliminate x

$$\begin{aligned} x+5y&=0 \\ x+2y&=0 \\ 3y&=0 \quad \text{Subtract the equations} \end{aligned}$$

$$\frac{3y}{3}=\frac{0}{3} \quad \text{Divide each side with 3}$$

$$y=0 \quad \text{Simplify}$$

Now substitute 0 for y in either equation to find the value of x

$$\begin{aligned} x+3y&=y && \text{Second Equation} \\ x+3(0)&=(0) && \text{Substitue 0 for } y \\ x+0&=0 && \text{Simplify} \\ x&=0 && \text{Simplify} \end{aligned}$$

The solution is $\boxed{(0,0)}$

Answer 26E.

Consider the equations,

$2x+y=3x-15$ ...... (1)

$x+y=4y+2x$ ...... (2)

From the equation (1)

$$\begin{aligned} 2x+y&=3x-15 \\ 2x+y-3x&=3x-15-3x && \text{Subtract } -3x \text{ from each side} \\ -x+y&=-15 && ......(3) \end{aligned}$$

From the equation (2)

$$\begin{aligned} x+5&=4y+2x \\ x+5-x&=4y+2x-x && \text{Subtract } x \text{ from each side} \\ 5&=x+4y && ......(4) \end{aligned}$$

Now solve the system of equations (3) and (4) by using elimination method:

Eliminate *x*

$$-x+y=-15 \quad \text{Multiply by 2} \qquad -x+y=-15$$
$$x+4y=5 \qquad\qquad\qquad\qquad x+4y=5$$
$$5y=-10 \quad \text{Add the equations}$$

$$\frac{5y}{5}=\frac{-10}{5} \quad \text{Divide each side with 5}$$
$$y=-2 \quad \text{Simplify}$$

Now substitute -2 for *y* in either equation to find the value of x

$$-x+y=-15 \quad \text{Second Equation}$$
$$-x-2=-15 \quad \text{Substitue } -2 \text{ for } y$$
$$-x=-13 \quad \text{Simplify}$$
$$x=13 \quad \text{Simplify}$$

The solution is $\boxed{(13,-2)}$

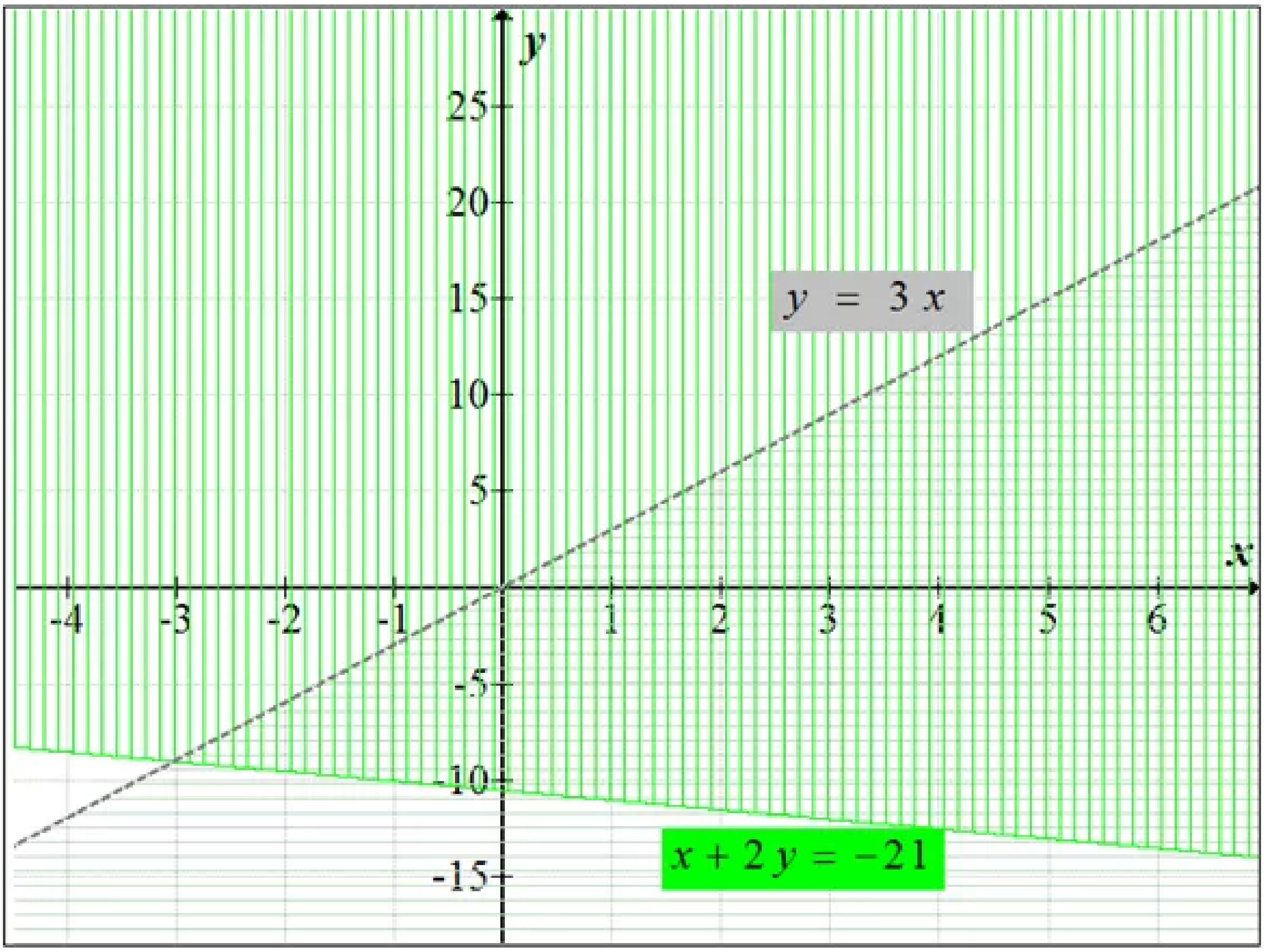
Answer 27E.

Consider the inequalities,

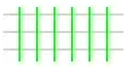
$y < 3x$ ...... (1)

$x + 2y \geq -21$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $y < 3x$ and $x + 2y \geq -21$

This solution region is shaded as  .

The graphs of $y = 3x$ and $x + 2y = -21$ are boundaries of this region.

The graph of $y = 3x$ is dashed and not included in the graph of $y < 3x$

The graph of $x + 2y = -21$ is included in the graph of $x + 2y \geq -21$

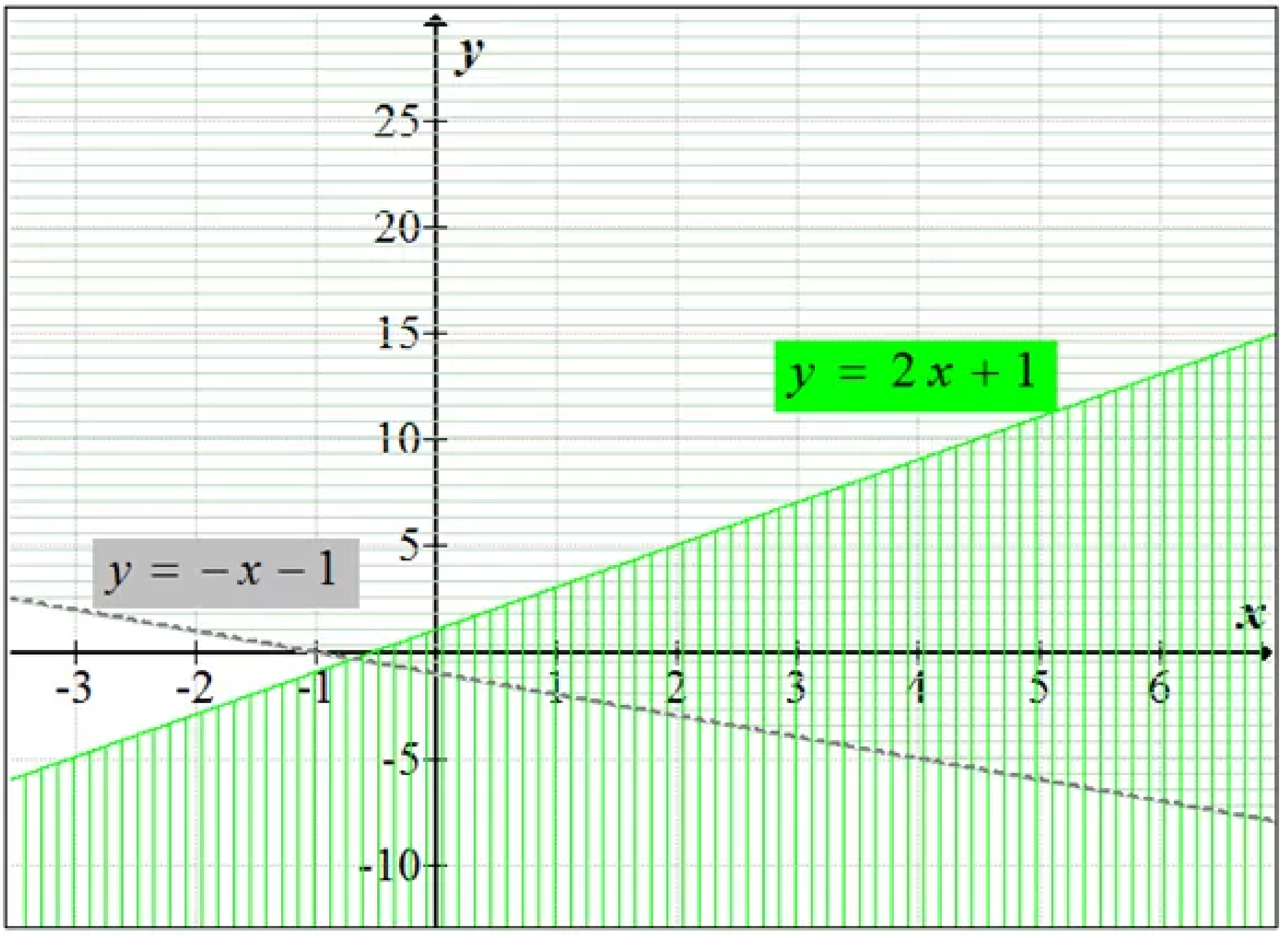
Answer 28E.

Consider the inequalities,

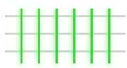
$y > -x - 1$ ...... (1)

$y \leq 2x + 1$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $y > -x - 1$ and $y \leq 2x + 1$

This solution region is shaded as  .

The graphs of $y = -x - 1$ and $y = 2x + 1$ are boundaries of this region.

The graph of $y = -x - 1$ is dashed and not included in the graph of $y > -x - 1$

The graph of $y = 2x + 1$ is included in the graph of $y \leq 2x + 1$

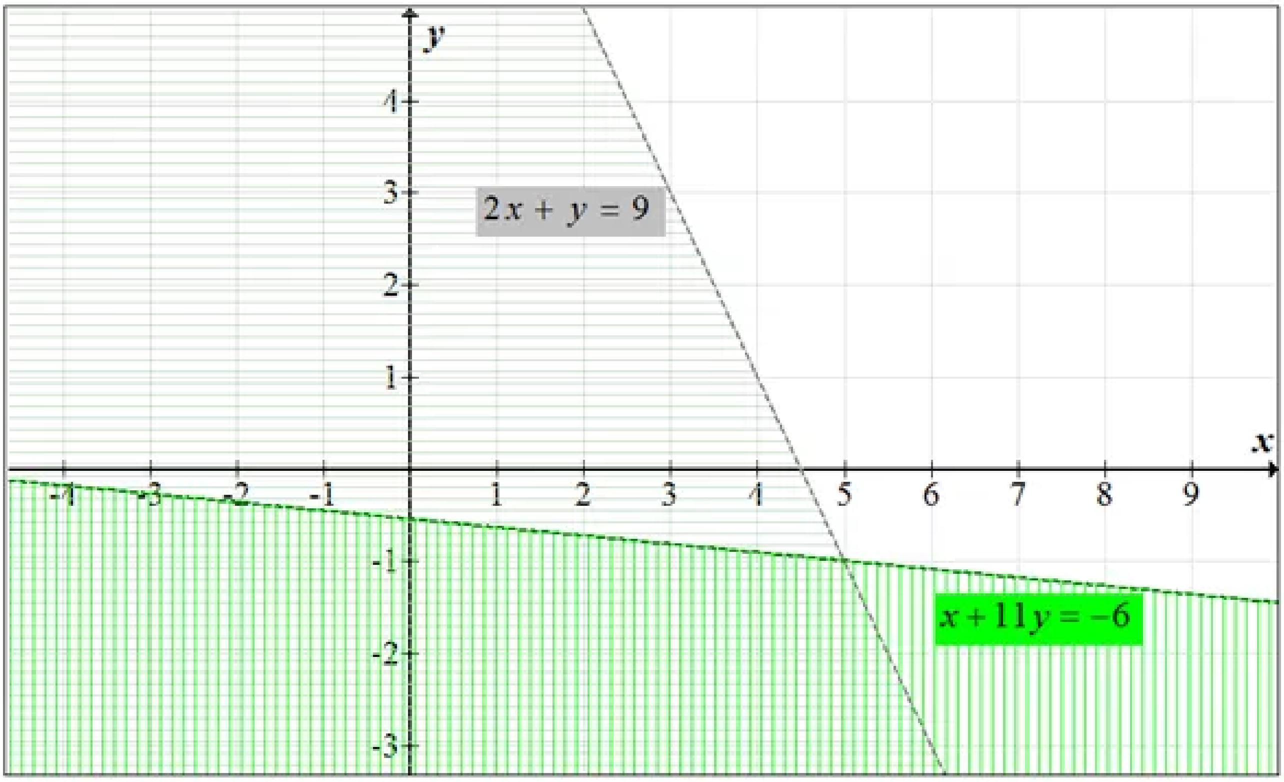
Answer 29E.

Consider the inequalities,

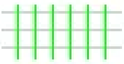
$2x + y < 9$ ...... (1)

$x + 11y < -6$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $2x + y < 9$ and $x + 11y < -6$

This solution region is shaded as  .

The graphs of $2x + y = 9$ and $x + 11y = -6$ are boundaries of this region.

The graph of $2x + y = 9$ and $x + 11y = -6$ is dashed and not included in the graph of $2x + y < 9$ and $x + 11y < -6$

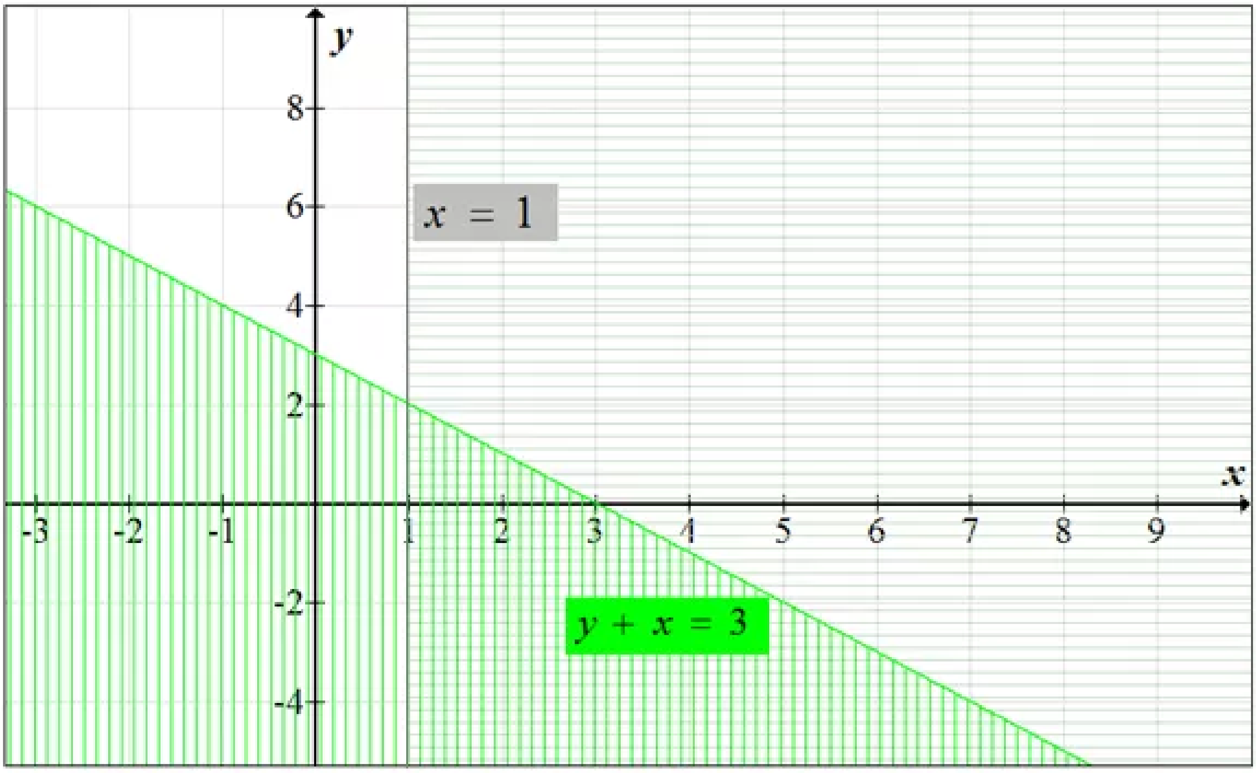
Answer 30E.

Consider the inequalities,

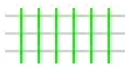
$x \geq 1$ ...... (1)

$y + x \leq 3$ ...... (2)

The graph of the inequalities is shown below:



The solution includes the order pairs in the intersection of the graphs of $x \geq 1$ and $y + x \leq 3$

This solution region is shaded as  .

The graphs of $x = 1$ and $y + x = 3$ are boundaries of this region.

The graph of $x = 1$ and $y + x = 3$ is dashed and not included in the graph of $x \geq 1$ and $y + x \leq 3$