Verify that the Opposite Angles Of a Cyclic Quadrilateral

OBJECTIVE

To verify that the opposite angles of a cyclic quadrilateral are supplementary.

Materials Required

1. Cardboard
2. White paper
3. Drawing sheet
4. Geometry box
5. Scissors
6. Sketch pens
7. Adhesive
8. Transparent sheet

Prerequisite Knowledge

1. Knowledge about the supplementary angles, linear pair axiom and interior opposite angles.
2. All the basic information related to cyclic quadrilateral.

Theory

1. Two angles are said to be supplementary, if the sum of their measures is 180°. In the figure given below, ∠BOC and ∠AOC are supplementary angles, (see Fig. 25.1)

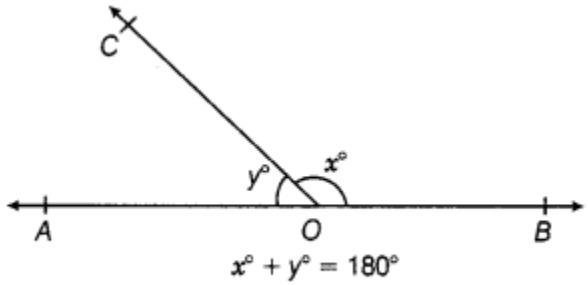

Fig. 25.1

2. If a ray stands on a line, then the sum of two adjacent angles so formed is 180°, i.e. the sum of the linear pair is 180°.
   In the above figure, ∠AOC + ∠BOC =180°.

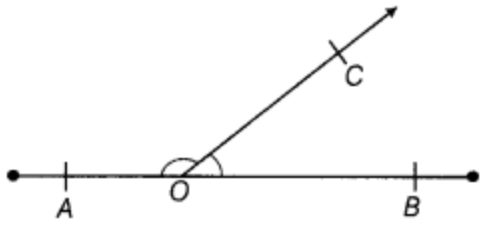

Fig. 25.2

If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line or two opposite rays. The two axioms given above together are called the linear pair axiom.

3. If the side BC of ΔABC is produced to D, then ∠AÇD is called an exterior angle of ΔABC at C, while ∠BAC and ∠ABC are called its interior opposite angles. It is denoted by exterior ∠ACD.

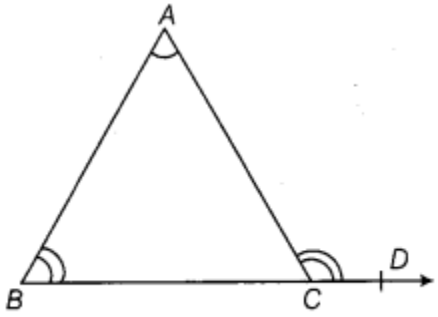

Fig. 25.3

4. A quadrilateral ABCD is called a cyclic quadrilateral, if all the four vertices A B, C and D are concyclic, i.e. A, B, C and D lie on a circle. In Fig. 25.4, ABCD is a cyclic quadrilateral. The sum of opposite angles of a cyclic quadrilateral is always 80°, i.e. they are supplementary.

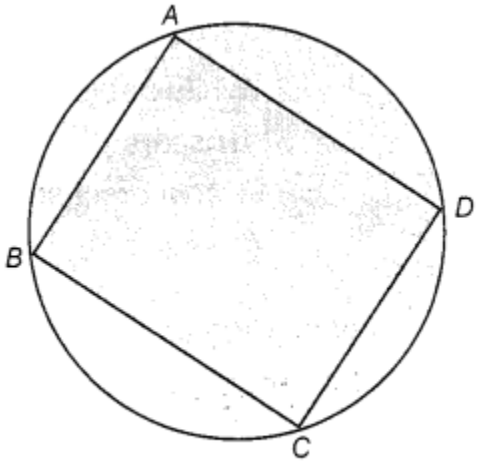

Fig. 25.4

Procedure

1. Take a cardboard of suitable size and paste a white paper on it.
2. Draw a circle of suitable radius on drawing sheet.
3. Cut out the circle and paste it on cardboard.
4. In the circle, draw a quadrilateral ABCD such that all the four vertices of quadrilateral lie on the circle. Name the angles as ∠A, ∠B, ∠C and ∠D. (see Fig. 25.5)

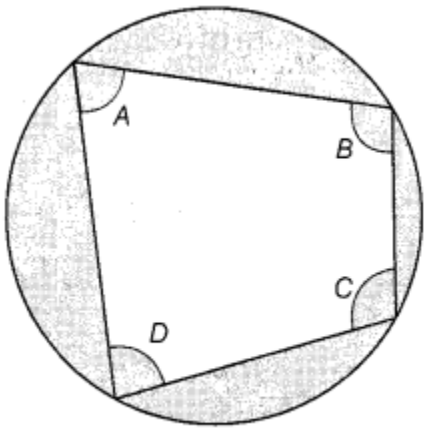

Fig. 25.5

5. Make the cut outs of the angles, (i.e. ∠A, ∠B, ∠C and ∠D) with the help of a transparent sheet, (see Fig. 25.6)

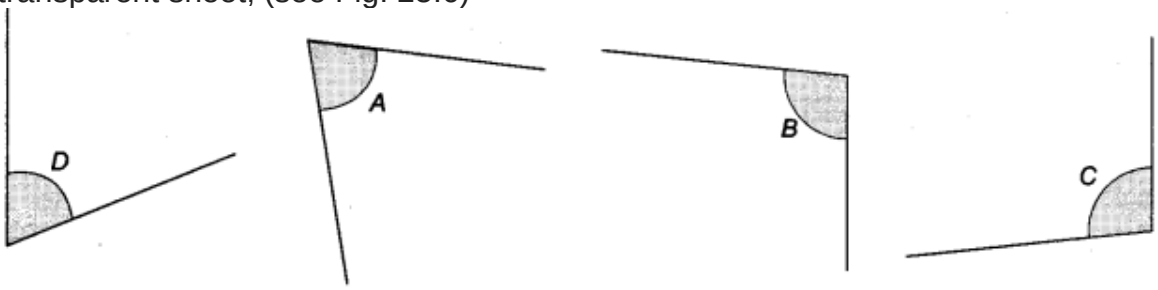

Fig. 25.6

6. By using adhesive, paste cut outs of the opposite angles ∠A and ∠C, ∠B and ∠D. (see Fig. 25.7)

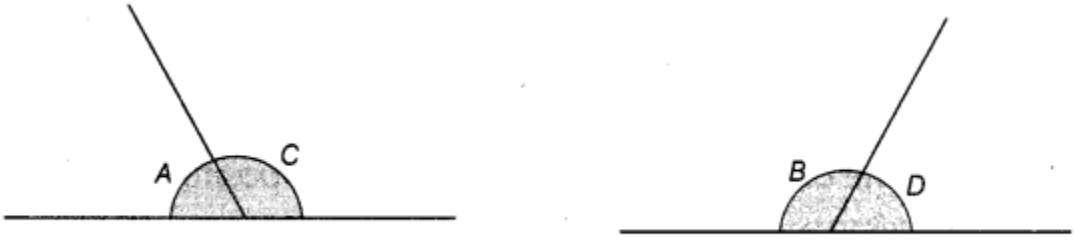

Fig. 25.7

Demonstration

Joining the opposite angles ∠A and ∠C, ∠B and ∠D, we get straight angles, (see Fig. 25.4) Hence, ∠A + ∠C = 180° and ∠B + ∠D = 180°.

Observation

By actual measurement, ∠A = …… , ∠B = …… ,
∠C = …… , ∠D = …… ,
So, ∠A + ∠C = …… , ∠B + ∠D = …… .
Hence, sum of each pair of the opposite angles of a cyclic quadrilateral is …… .

Result

We have verified that the opposite angles of a cyclic quadrilateral are always supplementary.

Application

This property can be used in solving various problems in geometry.

Viva Voce

Question 1:
What do you understand by the term a cyclic quadrilateral?
Answer:
A quadrilateral having all the vertices on the boundary of the circle is called a cyclic quadrilateral.

Question 2:
What is the sum of each pair of opposite angles of a cyclic quadrilateral?
Answer:
180°

Question 3:
If one of the angles of a cyclic quadrilateral is 40°, then what will be the value of its opposite angle?
Answer:
140°

Question 4:
If a cyclic quadrilateral is a parallelogram, then what is the type of parallelogram?
Answer:
Rectangle

Question 5:
Is the sum of adjacent angles of a cyclic quadrilateral 180°?
Answer:
No, only the sum of opposite angles of a cyclic quadrilateral is 180°.

Question 6:
What is the name of quadrilateral, if each pair of opposite angles is supplementary?
Answer:
Cyclic quadrilateral

Question 7:
What is the type of quadrilateral formed by the internal angle bisectors of cyclic .quadrilateral?
Answer:
Cyclic quadrilateral

Question 8:
Which property has to be added in a trapezium for making it a cyclic quadrilateral?
Answer:
Non-parallel sides of a trapezium should be equal.

Suggested Activity

Verify that the exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.