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In previous chapters, we have studied that the motion is the change in position of an object with respect to time and its reference point. On the basis of path traced and various other properties, motion can be classified into several categories. These motions are given as below,

(i) Non-repetitive motions like rectilinear motion and projectile motion, etc.
(ii) Periodic motion like circular motion or orbital motion, motion of a bouncing ball, etc.
(iii) Oscillatory motion, like the motion of a pendulum.

OSCILLATIONS

CHAPTER CHECKLIST

- Periodic and Oscillatory Motion
- Period and Frequency
- Periodic Function
- Simple Harmonic Motion
- Simple Harmonic Motion and Uniform Circular Motion
- Force Law for Simple Harmonic Motion
- Energy in SHM
- Some Systems Executing SHM

|TOPIC 1|
Introduction to Oscillation

We can notice a number of phenomena in our daily life, illustrating oscillatory motion, e.g. motion of vibrating strings of sitar or guitar, vibration of air molecules helping in propagation of sound, vibration of atoms about their equilibrium positions in a solid, oscillatory motion of AC voltage (going positive and negative about the mean value).

PERIODIC AND OSCILLATORY MOTION

Periodic Motion

A motion that repeats itself after regular intervals of time is called **periodic motion**. e.g. Revolution of planets around the sun, rotation of the earth about its polar axis, etc.

Oscillatory Motion

A motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a regular interval of time is called **oscillatory motion**. This fixed point is known as mean/equilibrium position or centre of oscillation.

e.g. Motion of a simple pendulum as it moves to and fro about its mean position.

Periodic Motion *vs* Oscillatory Motion

Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. e.g. Motion of the planet around the sun is periodic but not oscillatory, body undergoing periodic motion has an equilibrium position somewhere on its path. The body is at this position during its motion, no net force acts on it. So, if it is left there at rest, it remains there forever.

If the body is given a small displacement from its mean position, a force called restoring one comes into play which tries to bring the body back to the equilibrium point. This process repeats itself again and again causing oscillation.

e.g. A ball placed in a bowl will be in equilibrium at the bottom. If it is displaced a little from the mean position, it will start oscillatory in the bowl.

Period and Frequency

Period

The smallest interval of time after which a periodic motion is repeated, is called its **period**. Its SI unit is second and it is denoted by the symbol T.

Period of oscillation should have a very high range of accuracy. So, its unit varies with the necessity.

e.g.

(i) Period of vibration of a quartz crystal is expressed in units of microsecond (10^{-6} s) i.e. μs.

(ii) Period of revolution of the planet mercury is 88 earth days.

(iii) Period of revolution of Halleys Comet is 76 years, so it appears after every 76 years.

Frequency

The number of oscillations/vibrations preformed by a oscillating body about its mean position in a unit time is known as its frequency. The SI unit of frequency of oscillation is hertz (Hz) on the name of Heinrich Rudolph Hertz (1857-1894) who discovered radio waves.

Frequency can also be defined as reciprocal of time period, it is denoted by ν.

Thus, $$\text{Frequency}, \nu = \frac{1}{T}$$

and 1 hertz = 1 Hz = 1 oscillation per sec = $1\,s^{-1}$

This will go down, after frequency. When frequency of an oscillatory motion is very high, then motion is called **vibrational motion**.

EXAMPLE |1| Human Heart

On an average, a human heart is found to beat 75 times in a minute. Calculate its beat frequency and period. **[NCERT]**

Sol. The beat frequency of heart

$$= \frac{\text{Number of beats}}{\text{Time taken}} = \frac{75}{60} = 1.25\,s^{-1} = 1.25\text{ Hz}$$

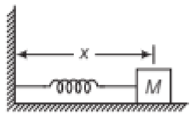
$$\text{Beat period, } T = \frac{1}{\text{frequency}} = \frac{1}{1.25\,s^{-1}} = 0.8\text{ s}$$

Displacement

In general, the name displacement is given to a change in position of a particle with respect to origin. In oscillation, we use term displacement as the change in position with respect to mean position (or equilibrium position) or reference position.

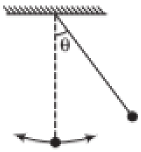
Illustrations for Displacement of a Particle

(i) Consider a block is attached to a spring at one of its end, the other end of which is fixed to a rigid wall. Generally, it is convenient to measure the displacement variable (x) as the deviation of the block from the mean position of the oscillation with time, as given in the diagram below.



Displacement variable as the deviation of block

(ii) In a simple pendulum, the displacement variable is its angular deviation (θ) from the vertical during oscillations with time as given in the diagram below.



Displacement variable as the deviation of simple pendulum

Note

Displacement variable is measured as a function of time having both positive and negative values.

Periodic Function

The displacement can be represented by a mathematical function of time. The function which repeats its value in regular interval of time or period is called **periodic function**. One of the simplest periodic function is given by

$$f(t) = A\cos\omega t \quad \text{...(i)}$$

The time period for this function will be, $T = 2\pi/\omega$ because when we add 2π with the argument ωt of the function, then its value remains same.

Thus, $f(t) = f(t+T)$

Similarly, if we consider a sine function then

$$f(t) = A\sin\omega t$$

Also, $f(t) = A\sin\omega t + B\cos\omega t$ ...(ii)

is a periodic function with time period T.

Now taking, $A = D\cos\phi$

and $B = D\sin\phi$

Thus, Eq. (ii) can be written as

$$f(t) = D\sin(\omega t + \phi)$$

where, $D = \sqrt{A^2 + B^2}$ and $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

An important result given by the French mathematician, Jean Baptiste Joseph Fourier (1768-1830) is that **any periodic function can be expressed as a superposition of sine and cosine function of different time periods with suitable coefficients.**

EXAMPLE |2| Periodic and Non-periodic Motion

Which of the following functions of time represent (a) periodic and (b) non-periodic motion? Give the period for each case of periodic motion. [ω is any positive constant]. **[NCERT]**

(i) $\sin\omega t + \cos\omega t$

(ii) $\sin\omega t + \cos 2\omega t + \sin 4\omega t$

(iii) $e^{-\omega t}$

(iv) $\log(\omega t)$

Sol. (i) Given, $x(t) = \sin\omega t + \cos\omega t$

$$= \sqrt{2}\left[\sin\omega t \cos\frac{\pi}{4} + \cos\omega t \sin\frac{\pi}{4}\right]$$

$$x(t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$$

Moreover, $x\left(t + \frac{2\pi}{\omega}\right) = \sqrt{2}\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \frac{\pi}{4}\right]$

$$= \sqrt{2}\sin\left(\omega t + 2\pi + \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) = x(t)$$

[$\because \sin(2\pi + \theta) = \sin\theta$]

Hence, $\sin\omega t + \cos\omega t$ is a periodic function with time period equal to $\frac{2\pi}{\omega}$.

(ii) Given, $x(t) = \sin\omega t + \cos 2\omega t + \sin 4\omega t$

$\sin\omega t$ is a periodic function with period

$$= \frac{2\pi}{\omega} = T$$

$\cos 2\omega t$ is a periodic function with period

$$= \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{T}{2}$$

$\sin 4\omega t$ is a periodic function with period

$$= \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} = \frac{T}{4}$$

Clearly, the function $x(t)$ repeats after a minimum time, $T = \frac{2\pi}{\omega}$. Hence, the given function is periodic.

(iii) The function $e^{-\omega t}$ decreases monotonically to zero as $t \to \infty$. It is an exponential function with a negative exponent of e, where $e = 2.71828$. It never repeat its value. So, it is non-periodic.

(iv) The function $\log(\omega t)$ increases monotonically with time. As $t \to \infty$, $\log(\omega t) \to \infty$. It never repeat its value. So, it is non-periodic.

EXAMPLE |3| Standard Equation of Waves

The equation of a wave is given by $y = 6\sin 10\pi t + 8\cos 10\pi t$, where y is in centimetre and t in second. Determine the constants involved in the standard equation of the wave.

Sol. Given, $y = 6\sin 10\pi t + 8\cos 10\pi t$ ...(i)

The general equation of wave is

$$y = A\sin(\omega t + \phi)$$

$$= A\sin\omega t\cos\phi + A\cos\omega t\sin\phi$$

$$= (A\cos\phi)\sin\omega t + (A\sin\phi)\cos\omega t \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$A\cos\phi = 6 \quad \text{...(iii)}$$

$$A\sin\phi = 8 \quad \text{...(iv)}$$

and $\omega t = 10\pi t$ or $\omega = 10\pi$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2\text{s}$

Squaring and adding Eqs. (iii) and (iv), we get

$$A^2(\cos^2\phi + \sin^2\phi) = 6^2 + 8^2$$

$$= 36 + 64 = 100$$

or $A^2 = 100$

$\therefore$ $A = 10\text{ cm}$

Dividing Eq. (iv) by Eq. (iii), we get

$$\tan\phi = \frac{8}{6} = 1.3333$$

$\therefore$ $\phi = \tan^{-1}(1.3333) = 53°8'$

SIMPLE HARMONIC MOTION

A special type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is known as **Simple Harmonic Motion** (SHM).

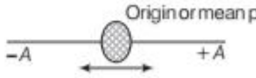
Characteristic of Restoring Force in SHM

The restoring force is always directed towards the mean position and its magnitude at any instant is directly proportional to the displacement of the particle from its mean position at that instant.

As the restoring force is directed towards the mean position at any point in its oscillation, thus, displacement of a simple harmonic motion is always a sinusoidal function of time.

Equation of SHM

Consider a particle oscillating back and forth about the origin of an x-axis between the limits $+A$ and $-A$ as given in the figure below.



A particle executing SHM

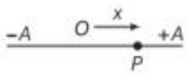
This oscillatory motion of the particle is said to be SHM if the displacement x of the particle from the origin varies with time as

$$\text{Displacement, } x(t) = A\cos(\omega t + \phi)$$

Where, A, ω and ϕ are constants.

Given, the time t is taken as zero when the particle is at position $+A$ and it returns to same point with position $+A$ at time $t = T$.

Let at an instant t, the particle be at P, if O is taken as mean position of the particle, then $OP = x$ (say), i.e. the displacement of the particle from the mean position.



The restoring force F acting on the particle at that instant is

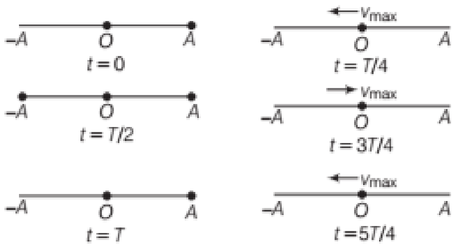
$$F = -kx$$

where, k is a force constant having SI unit N/m. The negative sign shows that the restoring force F is always directed towards the mean position.

The relation $F = -kx$ is known as **force law** for SHM.

Location of the Particle Executing SHM at the Discrete Value of Time t

The figure below shows the positions of a particle executing SHM are at discrete values of time. It must be noted that each interval of time being $\frac{T}{4}$.



Location of a particle executing SHM with different time

Observations from the figures are

(i) The time after which motion repeats itself is T.

(ii) T will remain fixed, i.e. it doesn't vary with variation in initial location ($t = 0$).

(iii) The speed is maximum for zero displacement at $x = 0$ and zero at the extremes of motion.

EXAMPLE |4| Periodicity of an SHM

Which of the following functions of time represent (a) simple harmonic motion (b) periodic but not simple harmonic? Give the period for each function when functions are

(i) $\sin\omega t - \cos\omega t$ (ii) $\sin^2\omega t$ **[NCERT]**

Sol. (i) Given, function is

$$\sin\omega t - \cos\omega t = \sin\omega t - \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$= 2\sin\left(\frac{\omega t + \frac{\pi}{2} - \omega t}{2}\right)\cdot\sin\left(\frac{\omega t - \frac{\pi}{2} + \omega t}{2}\right)$$

$$\text{Given function} = 2\sin\left(\frac{\pi}{4}\right).\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents a simple harmonic motion having a period of $T = \frac{2\pi}{\omega}$ and a phase angle $\left(-\frac{\pi}{4}\right)$ or $\left(-\frac{7\pi}{4}\right)$.

(ii) Also, $\sin^2\omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

The function is periodic having a period of $T = \frac{\pi}{\omega}$. It also represents a harmonic motion with the point of equilibrium occurring at $\frac{1}{2}$ instead of zero.

EXAMPLE |5| SHM on a Straight Line

Two particles execute SHM of the same amplitude and frequency along close parallel lines. They pass each other moving in opposite directions, each time their displacement is half their amplitude. What is their phase difference?

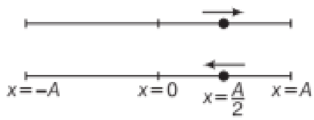
Sol. In SHM, $x = A\sin(\omega t + \phi)$ ...(i)

Velocity, $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$ ...(ii)

At $t = 0$, $x = \frac{A}{2}$, then from Eq. (i), $\frac{A}{2} = A\sin\phi$

or $\sin\phi = \frac{1}{2} = \sin\frac{\pi}{6}$ or $\sin\frac{5\pi}{6}$

$\therefore$ $\phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$



If $\phi = \frac{\pi}{6}$, displacement and velocity both are positive.

When $\phi = \frac{5\pi}{6}$, displacement is positive and velocity is negative. Therefore, displacement-time equations of two particles will be

$$x_1 = A \sin\left(\omega t + \frac{\pi}{6}\right)$$

and $$x_2 = A \sin\left(\omega t + \frac{5\pi}{6}\right)$$

$\therefore$ Phase difference

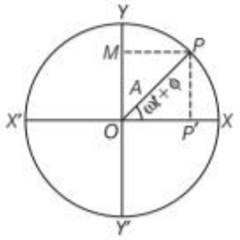
$$\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad}$$

SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

(GEOMETRICAL INTERPRETATION OF SHM)

Consider a particle P starting from X is moving with a uniform speed along the circumference of a circle of radius A, with centre at O. This circle is known as **circle of reference**, while the particle P is known as **reference particle**.

Let P' be the foot of perpendicular drawn from the point P to the diameter XOX'. P' is known as **projection** of the particle P at diameter XOX'.



Circular representation of SHM

As P moves along the circle from X to Y, Y to X', X' to Y' and Y' to X, the projection of the particle P i.e. P' moves from X to O, O to X', X' to O and O to X, respectively.

Thus, P revolves along the circumference of the circle while its projection P' moves to and fro about the point O along the diameter XOX'. Hence, the motion of P' about point O is said to be **simple harmonic**.

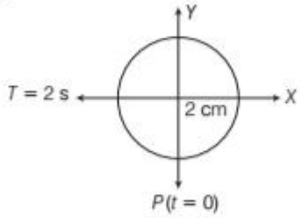
Therefore, the projection of uniform circular motion upon a diameter of the circle executes simple harmonic motion.

The motion of projection of reference particle along any other diameter of the circle of reference will also be simple harmonic motion.

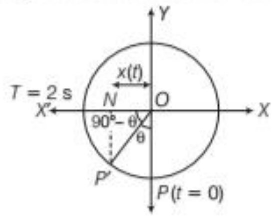
Hence, **SHM can be geometrically defined as the projection of a uniform circular motion on any diameter of the circle of reference.**

EXAMPLE |6| Circular Motion of a Particle Executing SHM

Figure below shows a circular motion of a particle. All the parameters are labelled in the figure. Obtain the corresponding equation for simple harmonic motion of the revolving particle P.



Sol. Suppose the particle moves from P to P' in time t.



The angle swept by the radius vector

$$\theta = \omega t = \frac{2\pi}{T} t = \frac{2\pi}{2} \cdot t = \pi t \text{ rad}$$

Displacement $ON = OP' \cos\left(\frac{\pi}{2} - \theta\right) = 2\cos\left(\frac{\pi}{2} - \theta\right)$

$$-x(t) = 2\sin\theta$$

[negative sign shows displacement being to left from O]

$\Rightarrow$ $$x(t) = 2\sin \pi t$$

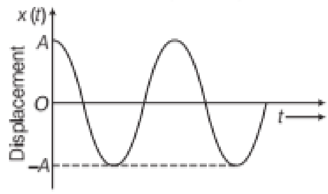
CHARACTERISTICS OF SIMPLE HARMONIC MOTION

Some of the important parameters which define the characteristics of a simple harmonic motion are given below.

(i) Displacement

The displacement of a particle executing SHM at an instant is given by the distance of the particle from the mean position at that instant.

The values of displacement as a continuous function of time can be represented as a graph given below.



Displacement as a continuous function of time

Here, $\boxed{\text{Displacement, } x(t) = A\cos(\omega t + \phi)}$

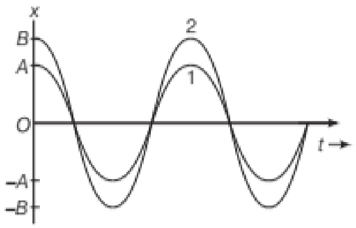
The function containing sine or cosine term is known as sinusoidal function.

(ii) Amplitude

The magnitude of maximum displacement of a particle executing SHM is called **amplitude** of the oscillation of that particle. Amplitude is measured on either side of mean position.

The displacement varies between the extremes $+A$ and $-A$ because the sinusoidal function of time varies from $+1$ to -1.

Two SHM may have same ω (angular frequency) and ϕ (phase constant) but different amplitudes A and B.



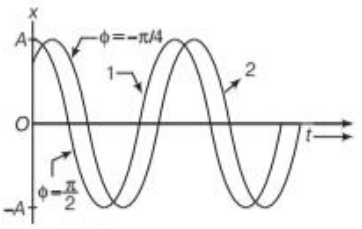
Different amplitudes in SHM

(iii) Phase

If amplitude A is fixed for a given SHM, then the state of motion i.e. position and velocity of the particle at any time t is determined by the argument $(\omega t + \phi_0)$ in the sinusoidal function. This quantity $(\omega t + \phi_0)$ is called phase of the motion.

For $t = 0$, phase $\omega t + \phi_0 = \phi_0$. Thus, ϕ_0 is called **phase constant** or **phase angle**. Two SHM may have the same A (amplitude) and ω (angular frequency) but different phase angle ϕ.

This can be shown in the graph below



Phase determination

As in above graph, the curve 1 has phase constant of $\pi/2$ (i.e. $\phi = \pi/2$) and amplitude of A, the curve 2 has phase constant of $\frac{\pi}{4}$ $\left(\text{i.e.} \, \phi = +\frac{\pi}{4}\right)$ and amplitude of A.

(iv) Angular Frequency

Angular frequency of a body executing periodic motion is equal to the product of frequency of the particle with factor 2π.

It is denoted by ω and its SI unit is radian per second.

We know that, $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = 2\pi\nu$

where, ν = frequency of the particle.

Since, the motion is periodic with a period T, the displacement $x(t)$ must return to its initial value after one period of the motion i.e. $x(t)$ must be equal to $x(t+T)$ for all t.

Consider the equation, $x(t) = A\cos\omega t$

$$A\cos\omega t = A\cos\omega(t+T)$$

Now, the cosine function is periodic with period 2π i.e. it first repeats itself when the argument changes by 2π. Therefore,

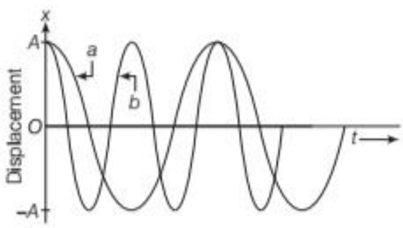
$$\Rightarrow \quad \omega(t+T) = \omega t + 2\pi$$

$$\Rightarrow \quad \boxed{\text{Angular frequency, } \omega = \frac{2\pi}{T}}$$

Thus, angular frequency ω is 2π times the frequency of oscillation $\frac{1}{T}$.

Two SHM may have the same amplitude (A) and phase angle (ϕ) but different angular frequency ω.

This can be represented on the graph as below



Two SHM having same amplitude and phase

In above wave diagram, curve (b) has half the period and twice the frequency of the curve (a).

(v) Velocity

The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$\boxed{\text{Velocity, } v = \omega\sqrt{A^2 - x^2}}$$

At mean position, $x = 0$

$\therefore \quad v = \omega A \quad$ [maximum velocity]

At extreme position $\quad x = A$

$\therefore \quad v = 0 \quad$ [minimum velocity]

The velocity can also be calculated as

$$v(t) = \frac{d}{dt}[x(t)] = \frac{d}{dt}[A\cos(\omega t + \phi)]$$

$$\boxed{\text{Velocity, } v(t) = -\omega A \sin(\omega t + \phi)}$$

(vi) Acceleration

The acceleration of the particle executing SHM at any instant is defined as the time rate of change of its velocity at that instant.

$$\boxed{\text{Acceleration, } a = -\omega^2 x}$$

At mean position, $x = 0$

$\therefore \quad a = 0 \quad$ [minimum acceleration]

At extreme position $x = A$,

$\therefore \quad a = -\omega^2 A \quad$ [maximum acceleration]

The acceleration can also be calculated as

i.e. $$a(t) = \frac{d}{dt}[v(t)]$$

$$= \frac{d}{dt}[-\omega A \sin(\omega t + \phi)]$$

$$\boxed{\text{Acceleration, } a(t) = -\omega^2 A\cos(\omega t + \phi)}$$

EXAMPLE |7| Period and Velocity

Consider an SHM as $x(t) = 2\cos(4\pi t + \pi/6)$ where x is in metres and t in seconds. Determine the time period and initial velocity of the oscillating body.

Sol. Given equation of SHM

$$x(t) = 2\cos(4\pi t + \pi/6) \quad \text{...(i)}$$

We know that, $x(t) = A\cos(\omega t + \phi) \quad$ ...(ii)

Comparing Eq. (i) with Eq. (ii), we get

$\therefore$ Amplitude, $A = 2$

$$\omega = 4\pi \quad \text{or} \quad \frac{2\pi}{T} = 4\pi$$

$$\Rightarrow \quad T = \frac{1}{2} = 0.5 \text{ s}$$

The velocity of the particle can be found by differentiating displacement w.r.t time

$$\therefore \text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}[2\cos(4\pi t + \pi/6)]$$

$$= -2\sin\left(4\pi t + \frac{\pi}{6}\right) \times 4\pi$$

or $\quad$ velocity, $v = -8\pi\sin\left(4\pi t + \frac{\pi}{6}\right)$

The initial velocity is the velocity of the oscillating body at time $t = 0$

$$\therefore \text{Initial velocity} = \left.\frac{dx}{dt}\right|_{t=0} = -8\pi\sin\left(4\pi \times 0 + \frac{\pi}{6}\right)$$

$$= -8\pi\sin\left(\frac{\pi}{6}\right)$$

Initial velocity $= -4\pi$ m/s $\quad \left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$

EXAMPLE |8| Finding the Characteristics of SHM

A simple harmonic motion is represented by

$$x = 12\sin(10t + 0.6)$$

Find out the amplitude, angular frequency, frequency, time period and initial phase if displacement is measured in metre and time in seconds.

Sol. Given equation, $x = 12\sin(10t + 0.6)$

On comparing with $x(t) = A\sin(\omega t + \phi)$

We have,

(i) Amplitude, $A = 12$ m

(ii) Angular frequency, $\omega = 10$ rad/s

(iii) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.59$ Hz

(iv) Time period, $T = \frac{2\pi}{\omega} = \frac{1}{1.59} = 0.628$ s

(v) Initial phase, $\omega t + \phi|_{t=0} = 10t + 0.6|_{t=0}$

$= 0.6$ rad

EXAMPLE |9| A Periodic Motion

A particle executes SHM with a time period of 2 s and amplitude 5 cm. Find

(i) displacement $\quad$ (ii) velocity and

(iii) acceleration after 1/3 s starting from the mean position.

Sol. Here, $T = 2$ s, $A = 5$ cm, $t = \frac{1}{3}$ s

(i) For the particle starting from mean position, (i.e. $\phi = 0$) displacement,

$$x = A\sin\omega t = A\sin\frac{2\pi}{T}t$$

$$= 5\sin\frac{2\pi}{2} \times \frac{1}{3} = 5\sin\frac{\pi}{3}$$

$$= 5 \times \frac{\sqrt{3}}{2} = 4.33 \text{ cm}$$

(ii) Velocity, $v = \frac{dx}{dt} = \frac{d(A\sin\omega t)}{dt} = A\omega\cos\omega t$

$$= \frac{2\pi A}{T}\cos\frac{2\pi}{T}t = \frac{2\pi \times 5}{2}\cos\frac{\pi}{3}$$

$$= 5 \times 3.14 \times 0.5 \qquad \left[\because \cos\frac{\pi}{3} = 0.5\right]$$

$$= 7.85 \text{ cm s}^{-1}$$

(iii) Acceleration, $a = \frac{dv}{dt} = \frac{d(A\omega\cos\omega t)}{dt} = -A\omega^2 \sin\omega t$

$$= -\frac{4\pi^2 A}{T^2}\sin\frac{2\pi}{T}t$$

$$= -\frac{4 \times 9.87 \times 5}{4}\sin\frac{\pi}{3}$$

$$= -9.87 \times 5 \times \frac{\sqrt{3}}{2}$$

$$= -42.73 \text{ cm s}^{-2}$$

$$\therefore \quad |a| = 42.73 \text{ cm s}^{-2}$$

EXAMPLE |10| Displacement, Speed and Acceleration

A body oscillates with SHM according to the equation,

$$x = (5.0 \text{ m}) \cos[(2\pi \text{ rad s}^{-1})t + \pi/4].$$

At $t = 1.5$ s, calculate displacement, speed and acceleration of the body. **[NCERT]**

Sol. Equation of SHM,

$$x(t) = (5.0 \text{ m}) \cos[(2\pi \text{ rad s}^{-1})t + \pi/4]$$

time, $t = 1.5$ s

$$x(t) = 5.0 \times \cos[2\pi \times 1.5 + \pi/4]$$

$$= 5.0 \times \cos\left(3\pi + \frac{\pi}{4}\right)$$

$$= -5.0 \cos\frac{\pi}{4}$$

$$= -5.0 \times 0.707$$

$$x(t) = -3.535 \text{ m}$$

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[5.0\cos\left(2\pi t + \frac{\pi}{4}\right)\right]$$

$$= -5.0 \times \sin\left(2\pi t + \frac{\pi}{4}\right) \times 2\pi$$

$$= -10\pi \sin\left(2\pi t + \frac{\pi}{4}\right)$$

$$\therefore \quad \left.\frac{dx}{dt}\right|_{t=1.5} = -10\pi\sin\left(3\pi + \frac{\pi}{4}\right) = 10\pi \times 0.707$$

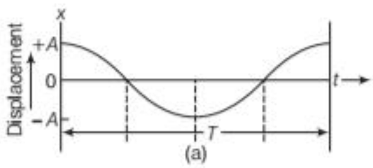
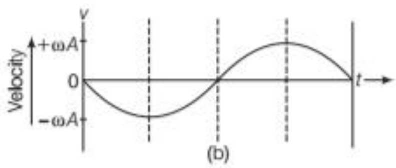
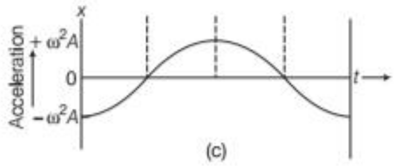
$$= 22.22 \text{ m/s} \qquad [\because \sin(3\pi + \theta) = -\sin\theta]$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[-10\pi\sin\left[2\pi t + \frac{\pi}{4}\right]\right]$$

$$= -10\pi \times 2\pi\cos\left(2\pi t + \frac{\pi}{4}\right)$$

$$\left.\frac{dv}{dt}\right|_{t=1.5} = -20\pi^2\cos\left(3\pi + \frac{\pi}{4}\right) = 139.56 \text{ m/s}^2$$

Displacement, Velocity and Acceleration of a Body Executing SHM

Displacement $x(t) = A\cos\omega t$	Velocity $v(t) = -\omega A\sin\omega t$	Acceleration $a(t) = -\omega^2 A\cos\omega t$
 (a)	 (b)	 (c)
Sinusoidal in nature, x varies from $-A$ to A. It has zero phase difference.	Sinusoidal in nature. $v(t)$ varies from $-\omega A$ to ωA. It has a phase difference of $\frac{\pi}{2}$ w.r.t. $x(t)$	Sinusoidal in nature. $a(t)$ varies from $-\omega^2 A$ to $\omega^2 A$ It has a phase difference of π w.r.t. $x(t)$.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The motion of satellites and planets is

(a) periodic (b) oscillatory
(c) simple harmonic (d) non-periodic

Sol. (a) The motion of planets and satellites are repetitive and repeats itself after a fixed interval of time. These type of motions are known as periodic motion.

2. The periodic function $f(t) = A\sin(\omega t)$ repeats itself with periodic function of

(a) 2π (b) 3π
(c) π (d) $\pi/2$

Sol. (a) A periodic function repeats itself after a time period T.

and $f(t) = f(t+T)$

As, $\sin(\omega t) = \sin(\omega t + 2\pi)$

$\therefore$ Period of function is 2π.

3. SHM could be related to

(a) non-uniform circular motion
(b) uniform circular motion
(c) straight line motion
(d) projectile motion

Sol. (b) SHM could be related to uniform circular motion. The projection of uniform circular motion on a diameter of the circle follows simple harmonic motion.

4. The displacement of a particle in SHM varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is

(a) -4 (b) 4 (c) $4\sqrt{2}$ (d) 8

Sol. (c) Given, equation $x(t) = 4(\cos\pi t + \sin\pi t)$

Now, comparing above equation with general form

$$x(t) = A\cos\omega t + B\sin\omega t$$

We get, $A = 4$ and $B = 4$

As, the resultant amplitude for such a equation is

$$= \sqrt{A^2 + B^2}$$

$\therefore$ Amplitude $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$

5. At extreme position, velocity of the particle executing SHM that has amplitude A is

(a) $\omega^2 A$ (b) 0
(c) ωA (d) $\frac{\omega A}{2}$

Sol. (b) Velocity of the particle executing SHM is given as

$$v = \omega\sqrt{A^2 - x^2}$$

At extreme position, $x = A$

$\Rightarrow$ $v = 0$

VERY SHORT ANSWER Type Questions

6. What are the basic properties required by a system to oscillate?

Sol. Inertia and elasticity are the properties which are required by a system to oscillate.

7. All oscillatory motions are periodic and *vice-versa*. Is it true?

Sol. No, there are other types of periodic motions also. Circular motion and rotatory motion are periodic but non-oscillatory.

8. State the conditions when motion of a particle can be an SHM.

Sol. For SHM, the restoring force on the particle must be proportional to its displacement and directed towards mean position.

9. Give the name of three important characteristics of a SHM.

Sol. Three important characteristics of an SHM are amplitude, time period (or frequency) and phase.

10. If the body is given a small displacement from the mean position, a force comes in to play which tends to bring the body back to the mean point, this give rise to vibrations. Define phase of a vibrating particle.

Sol. The phase of a vibrating particle at any instant of time is the state of particle as regards to its position and state of motion.

11. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston executes simple harmonic motion with an angular frequency of 200 rad/min, then what is its maximum speed? **[NCERT]**

Sol. Given, angular frequency of the piston, $\omega = 200$ rad/min

Stroke length = 1 m

$\therefore$ Amplitude of SHM,

$$A = \frac{\text{Stroke length}}{2} = \frac{1}{2} = 0.5 \text{ m}$$

Now, $v_{max} = \omega A = 200 \times 0.5 = 100$ m/min

SHORT ANSWER Type Questions

12. Which of the following examples represent periodic motion?

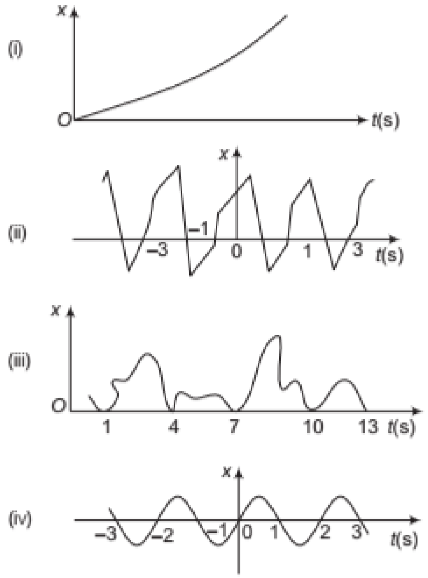
(i) A swimmer completing one (return) trip from one bank of a river to the other and back.
(ii) A freely suspended bar magnet displaced from its N-S direction and released.
(iii) A hydrogen molecule rotating about its centre of mass.

(iv) An arrow released from a bow. **[NCERT]**

If the motion is repeated after certain interval of time, then, it is periodic. Circular motion is periodic.

Sol. (i) There is no repetition of the motion as the swimmer just completes one trip, hence not periodic.
(ii) The motion is repeated after a certain interval of time, hence periodic. In fact, the bar magnet oscillates about its mean position with a definite period of time.
(iii) Rotatary motion is periodic as repeating after fixed time-interval.
(iv) There is no repetition, hence not periodic.

13. Figures depict four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)? **[NCERT]**



As the graphs are plot between displacements *versus* time, we shall check for repetition to ascertain, whether periodic or not. Also, the minimum time span after which the plot repeats itself will be time period of the motion.

Sol. (i) No repetition of motion. Its a unidirectional, linear but non-uniform motion of the particle.
(ii) Motion repeats after every 2 s. Hence, periodic with time period 2 s.
(iii) Repetition of one position or a few positions (but not all) is not enough for motion to be periodic, the entire motion during one period must be repeated successively. Hence, the given x-t plot is not periodic, though there is repetition of a single position ($x = 0$ at every 3 s) but other positions are not repeated.
(iv) Clearly, the motion repeats itself after every 2 s. Hence, periodic motion having a time-period of 2s.

14. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(i) The rotation of earth about its axis.
(ii) Motion of an oscillating mercury column in a U-tube.
(iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(iv) General vibrations of a polyatomic molecule about its equilibrium position. **[NCERT]**

Every SHM is periodic but every periodic motion is not SHM. Only that periodic motion which are governed by the force law i.e. $F = -kx$, is simple harmonic.

Sol. (i) There is no to and fro motion which is a must for a periodic motion to be SHM. Hence, rotation of earth about its axis is not SHM.
(ii) This is a periodic motion and as it follows $F = -kx$ (about mean position, to and fro motion) hence SHM.
(iii) A periodic motion, oscillatory in nature about lower most point as mean position following SHM force law, hence, it is SHM.
(iv) A polyatomic molecule has a number of natural frequencies. So, in general, its vibration is a superposition of SHMs of a number of different frequencies. Thus, superposition is periodic but not necessarily SHM.

15. Every SHM is periodic motion, but every periodic motion need not to be a simple harmonic motion. Do you agree? Give an example to justify your statement.

Sol. Yes, every periodic motion need not to be SHM. e.g. the motion of the earth round the sun is a periodic motion, but not simple harmonic motion as the back and forth motion is not taking place.

16. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion? **[NCERT]**

(i) $a = 0.7x$ (ii) $a = -200x^2$
(iii) $a = -10x$ (iv) $a = 100x^3$

All SHM follows the condition acceleration $\propto -$displacement or acceleration $= -$ constant $\times$ displacement.

Sol. (i) No negative sign on RHS, hence, not SHM.
(ii) Displacement on RHS is squared, hence not SHM.
(iii) $a = -10x$ follows the condition of SHM, acceleration $\propto$ − displacement hence, SHM.
(iv) No negative sign on RHS and displacement appears as cubed, hence, not SHM.

17. **The maximum acceleration of a simple harmonic oscillator is a_0 and the maximum velocity is v_0. What is the displacement amplitude?**

Sol. Let A be the displacement amplitude and ω be the angular frequency of the simple harmonic oscillator.

Then, $a_0 = \omega^2 A$ and $v_0 = \omega A$

On dividing, $$\frac{v_0^2}{a_0} = \frac{\omega^2 A^2}{\omega^2 A} = A$$

or $$A = \frac{v_0^2}{a_0}$$

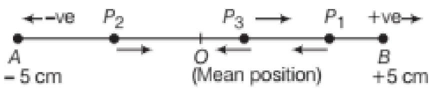
LONG ANSWER Type I Questions

18. **A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is**

(i) at the end A.
(ii) at the end B.
(iii) at the mid-point of AB going towards A.
(iv) at 2 cm away from B going towards A.
(v) at 3 cm away from A going towards B.
(vi) at 4 cm away from B going towards A. **[NCERT]**

At either extreme positions in SHM, the velocity is 0 but not the acceleration and force. Acceleration is always direct towards the mean position (point about which SHM is taking place). Further, acceleration decides the direction of force.

Sol. Visualise the situation using the diagram below



Now,

		Velocity	Acceleration	Force
(i)	A	0	+	+
(ii)	B	0	–	–
(iii)	O	–	0	0
(iv)	P_1	–	–	–
(v)	P_2	+	+	+
(vi)	P_3	–	–	–

19. **A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (i) 5 cm, (ii) 3 cm and (iii) 0 cm.** **[NCERT]**

Sol. Given, amplitude, $A = 5 \text{ cm} = 0.05 \text{ m}$,
Time period, $T = 0.2 \text{ s}$

(i) When displacement is $x = 5 \text{ cm} = 0.05 \text{ m}$

Acceleration, $$a = -\omega^2(x) = -\left(\frac{2\pi}{T}\right)^2 (x)$$
$$= -\left(\frac{2\pi}{0.2}\right)^2 (0.05) = -5\pi^2 \text{ m/s}^2$$

Velocity, $$v = \omega\sqrt{A^2 - x^2}$$
$$= \left(\frac{2\pi}{T}\right)\sqrt{(0.05)^2 - (0.05)^2}$$
$$= \left(\frac{2\pi}{T}\right) \times 0 = 0$$

(ii) When displacement is $x = 3 \text{ cm} = 0.03 \text{ m}$

Acceleration, $$a = -\left(\frac{2\pi}{0.2}\right)^2 (0.03) = -3\pi^2 \text{ m/s}^2$$

and velocity, $$v = \omega\sqrt{A^2 - x^2}$$
$$= \left(\frac{2\pi}{0.2}\right)\sqrt{(0.05)^2 - (0.03)^2} = 0.4\pi \text{ m/s}$$

(iii) When displacement is $x = 0$

Acceleration, $a = -\omega^2 x = 0$

Velocity, $$v = \omega\sqrt{A^2 - x^2}$$
$$= \left(\frac{2\pi}{0.2}\right)\sqrt{(0.05)^2 - (0)^2}$$
$$= 0.5\pi \text{ m/s}$$

20. **A particle performs SHM on a rectilinear path. Starting from rest, it travels x_1 distance in first second and in the next second, it travels x_2 distance. Find out the amplitude of this SHM.**

Sol. Because the particle starts from rest, so its starting point will be extreme position.

Thus, the displacement of the particle from the mean position after one second

$$A - x_1 = A\cos\omega t = A\cos\omega \quad \text{...(i) [putting } t = 1 \text{ s]}$$

where, A is amplitude of the SHM and for next second

$$A - (x_1 + x_2) = A\cos\omega t$$
$$= A\cos 2\omega \quad \text{[putting } t = 2 \text{ s]}$$
$$= A[2\cos^2\omega - 1] \quad \text{...(ii)}$$
$$[\because \cos 2\omega = 2\cos^2\omega - 1]$$

From Eq. (i) and Eq. (ii), we have

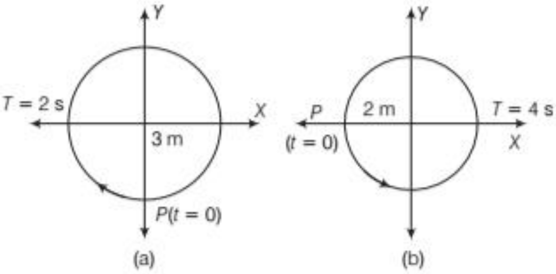
$$A - (x_1 + x_2) = A\left[2\cdot\left(\frac{A - x_1}{A}\right)^2 - 1\right]$$
$$= \frac{1}{A}[2A^2 + 2x_1^2 - 4Ax_1 - A^2]$$

$$\Rightarrow A^2 - A(x_1 + x_2) = A^2 + 2x_1^2 - 4Ax_1$$

$$\Rightarrow A[3x_1 - x_2] = 2x_1^2$$

$$\therefore A = \frac{2x_1^2}{3x_1 - x_2}$$

21. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.

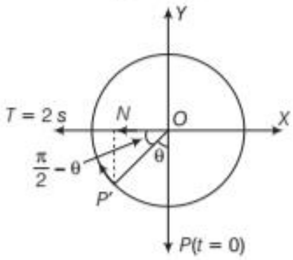


Obtain the corresponding simple harmonic motions of the X-projection of the radius vector of the revolving particle P, in each case. **[NCERT]**

The projection on the required axis gives the displacement. Initial phase is found from the position of the particle at $t = 0$.
The displacement is considered as a function of time and equated to the graphical displacement to get the required equation.

Sol. If the particle moves from P to P' in time-interval t, then angle moved by position vector (or radius vector)

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{2}t = \pi t \text{ rad}$$



Displacement suffered by the particle

$$ON = OP'\cos\left(\frac{\pi}{2}-\theta\right) = OP'\sin\theta$$

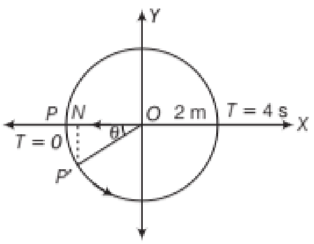
But $\quad ON = -x(t)$

As it is left to mean position O, hence, negative sign.

$\Rightarrow \quad -x(t) = 3\sin\theta \qquad [\because OP = 3\text{ cm}]$

or $\quad x(t) = -3\sin\theta$

or $\quad x(t) = -3\sin\pi t$ as $\theta = \pi t$



If the particle moves from P to P' in the time-interval t.

Angle swept by radius vector

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{4}t = \frac{\pi t}{2}\text{ rad}$$

Displacement, $\quad ON = OP'\cos\theta$

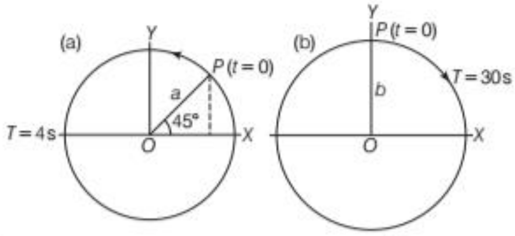
or $\quad -x(t) = 2\cos\frac{\pi t}{2} \qquad [\because OP = 1\text{cm}]$

Negative sign to indicate left to mean position.

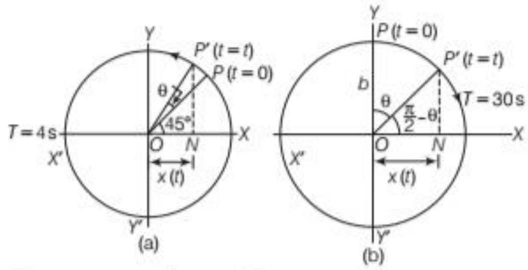
$$\Rightarrow \quad x(t) = -2\cos\left(\frac{\pi t}{2}\right)$$

22. The following figures depict two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures.

Obtain the simple harmonic motion of the x-projection of the radius vector of the rotating particle P in each case. **[NCERT]**



Sol.



In figure, suppose the particle moves in the anti-clockwise sense from P to P' in time t as shown in Fig.(a). Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{4}t \qquad [\because T = 4\text{ s}]$$

N is the foot of perpendicular drawn from P' on the XOX'-axis.

Displacement, $\quad ON = OP'\cos(\theta + \pi/4)$

or $\quad x(t) = a\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$

This equation represents SHM of amplitude a, period 4 s and an initial phase of $\frac{\pi}{4}$ rad.

Now, in Fig. (b), suppose the particle moves in the clockwise sense from P to P' in time t.

Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{30}t \qquad [\because T = 30\text{ s}]$$

Displacement,

$$ON = OP' \cos\left(\frac{\pi}{2} - \theta\right)$$

or $$x(t) = b\cos\left(\frac{\pi}{2} - \frac{2\pi}{30}t\right)$$

or $$x(t) = b\cos\left(\frac{2\pi}{30}t - \frac{\pi}{2}\right) \quad [\because \cos(-\theta) = \cos\theta]$$

This equation represents SHM of amplitude b period 30 s and an initial phase of $-\frac{\pi}{2}$ rad.

23. A man stands on a weighing machine placed on a horizontal platform. The machine reads 50 kg. By means of a suitable mechanism, the platform is made to execute harmonic vibrations up and down with a frequency of two vibrations per second. What will be the effect on the reading of the weighing machine? The amplitude of vibrations of platform is 5 cm. Take, $g = 10\ \text{ms}^{-2}$.

Sol. Here, $m = 50$ kg, $\nu = 2\ \text{s}^{-1}$, $A = 5\ \text{cm} = 0.05$ m

Maximum acceleration

$$a_{max} = \omega^2 A = (2\pi\nu)^2 A = 4\pi^2\nu^2 A$$

$$= 4 \times \left(\frac{22}{7}\right)^2 \times (2)^2 \times 0.05 = 7.9\ \text{ms}^{-2}$$

$\therefore$ Maximum force felt by the man

$$m(g + a_{max}) = 50(10 + 7.9) = 895.0\ \text{N} = 89.5\ \text{kgf}$$

Minimum force felt by the man

$$= m(g - a_{max}) = 50(10 - 7.9)$$

$$= 105.0\ \text{N} = 10.5\ \text{kgf}$$

Hence, the reading of the weighing machine varies between 10.5 kgf and 89.5 kgf.

24. A particle is executing SHM. If v_1 and v_2 are the speeds of the particle at distance x_1 and x_2 from the equilibrium position, show that the frequency of oscillations is

$$f = \frac{1}{2\pi}\left(\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}\right)^{1/2}$$

Sol. The displacement of a particle executing SHM is given by

$$x = A\cos\omega t$$

$$\frac{dx}{dt} = -\omega A\sin\omega t$$

$\therefore$ Velocity, $v = \frac{dx}{dt}$

or $$v^2 = A^2\omega^2\sin^2\omega t$$

$$= A^2\omega^2(1 - \cos^2\omega t)$$

$$= \omega^2(A^2 - x^2)$$

Hence, $$v_1^2 = \omega^2(A^2 - x_1^2) \quad \text{...(i)}$$

and $$v_2^2 = \omega^2(A^2 - x_2^2) \quad \text{...(ii)}$$

Subtracting the above equations,

$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$$

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$$

$\Rightarrow$ $$\omega = \left(\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}\right)^{1/2}$$

But $$\omega = 2\pi f$$

$$f = \frac{1}{2\pi}\left(\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}\right)^{1/2}$$

LONG ANSWER Type II Questions

25. Which of the following functions of time represent (i) simple harmonic (ii) periodic but not simple harmonic and (iii) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant).

(a) $\sin\omega t - \cos\omega t$ (b) $\sin^3\omega t$

(c) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ (d) $\cos\omega t + \cos 3\omega t + \cos 5\omega t$

(e) $\exp(-\omega^2 t^2)$ (f) $1 + \omega t + \omega^2 t^2$ **[NCERT]**

Any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients.

Sol. (a) Assuming $x(t) = \sin\omega t - \cos\omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

Clearly, the above equation is of the form

$$x = A\sin(\omega t \pm \phi)$$

represents SHM.

Again, if $t = \frac{2\pi}{\omega}$ is the period of SHM, then

$$x\left(t + \frac{2\pi}{\omega}\right) = \sqrt{2}\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) - \frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t + 2\pi - \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) = x(t)$$

Hence, the period $T = \frac{2\pi}{\omega}$

and phase angle $= -\frac{\pi}{4}$ or $\left(2\pi - \frac{\pi}{4}\right)$

(b) Let $x(t)=\sin^3 t \Rightarrow x(t)=\frac{1}{4}(3\sin\omega t-\sin 3\omega t)$

Using, $\sin 3\theta=3\sin\theta-4\sin^3\theta$

The above equations represent two SHMs in combination. The combination is periodic but not SHM.

Period of $\frac{3}{4}\sin\omega t=\frac{2\pi}{\omega}=T$

Period of $\frac{1}{4}\sin 3\omega t=\frac{2\pi}{3\omega}=T'=\frac{T}{3}$

Thus, period of the combination = Minimum time after which the combined function repeats

$$= \text{LCM of } T \text{ and } \frac{T}{3}=T$$

(c) $x(t)=3\cos\left(\frac{\pi}{4}-2\omega t\right)=3\cos\left(2\omega t-\frac{\pi}{4}\right)$

As $\cos(-\theta)=\cos\theta$

The above equation is of the form

$$x(t)=A\cos(\omega t\pm\phi)$$

Hence, SHM with period $T=\frac{2\pi}{2\omega}=\frac{\pi}{\omega}$

(d) $x(t)=\cos\omega t+\cos 3\omega t+\cos 5\omega t$

$\cos\omega t$ represents SHM with period $=\frac{2\pi}{\omega}=T$ (say)

$\cos 3\omega t$ represents SHM with period $=\frac{2\pi}{3\omega}=\frac{T}{3}$

$\cos 5\omega t$ represents SHM with period $=\frac{2\pi}{5\omega}=\frac{T}{5}$

The minimum time after which the combined function repeats its value is T. Hence, the given function represents periodic function but not SHM, with period T.

(e) $x(t)=\exp(-\omega^2t^2)$

The given function is an exponential function. It decreases monotonically $x(t)\to 0$ as $t\to\infty$

There is no repetition of the values. Hence, it represents a non-periodic function.

(f) $x(t)=1+\omega t+\omega^2t^2$

Here, as $t\to\infty$, $x(t)\to\infty$

No repetition of values. Hence, it represents non-periodic function.

26. The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t)=A\cos(\omega t+\phi)$

If the initial ($t=0$) position of the particle is 1 cm and its initial velocity is ω cm/s, then what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM, $x=B\sin(\omega t+\alpha)$, then what are the amplitude and initial phase of the particle with the above initial conditions? **[NCERT]**

Sol. Given, $x(t)=A\cos(\omega t+\phi)$ ...(i)

At $t=0$; $x(0)=1$ cm, velocity $v=\omega$ cm/s

Angular frequency $\omega=\pi s^{-1} \Rightarrow 1=A\cos(\omega t+\phi)$

For $t=0$, $1=A\cos\phi$ ...(i)

Now, $v(t)=\frac{dx(t)}{dt}=\frac{d}{dt}A\cos(\omega t+\phi)$

$=-A\omega\sin(\omega t+\phi)$

Again at $t=0$, $v=\omega$ cm/s

$\Rightarrow \omega=-A\omega\sin\phi \Rightarrow -1=A\sin\phi$ ...(ii)

Squaring and adding Eqs. (i) and (ii),

$$A^2\cos^2\phi+A^2\sin^2\phi=(1)^2+(-1)^2$$

$$A^2=2 \Rightarrow A=\pm\sqrt{2}\text{ cm}$$

Hence, the amplitude $=\sqrt{2}$ cm

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{A\sin\phi}{A\cos\phi}=\frac{-1}{1} \text{ or } \tan\phi=-1$$

$$\Rightarrow \phi=-\frac{\pi}{4} \text{ or } \frac{7\pi}{4}$$

Now, if instead of cosine, we choose the sine function, then $x(t)=B\sin(\omega t+\alpha)$

At $t=0$, $x=1$ cm $\Rightarrow 1=B\sin(0+\alpha)$

or $B\sin\alpha=1$ ...(iii)

Velocity $v(t)=\frac{dx(t)}{dt}=\frac{d}{dt}[B\sin(\omega t+\alpha)]$

$=+B\omega\cos(\omega t+\alpha)$

Again at $t=0$, $v(t)=\omega$ cm/s

$\omega=+B\omega\cos(0+\alpha)$

$B\cos\alpha=+1$ ...(iv)

Squaring and adding Eqs. (iii) and (iv),

$$B^2\sin^2\alpha+B^2\cos^2\alpha=(1)^2+(+1)^2$$

$$\Rightarrow B^2\sin^2\alpha+B^2\cos^2\alpha=2$$

$$B^2(\sin^2\alpha+\cos^2\alpha)=2$$

$$B^2 1=2 \Rightarrow B=\pm\sqrt{2}\text{ cm}$$

Hence, amplitude of motion $=\sqrt{2}$ cm

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{B\sin\alpha}{B\cos\alpha}=\frac{1}{1} \text{ or } \tan\alpha=1$$

$$\therefore \alpha=\frac{\pi}{4}$$

27. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t=0$) position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anti-clockwise in every case (x is in cm and t is in second).

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$

(b) $x = \cos\left(\frac{\pi}{6} - t\right)$

(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

(d) $x = 2\cos \pi t$ **[NCERT]**

The given functions are first converted into cosine functions through manipulation and then compared with standard SHM equation to get the parameter values. The same is depicted graphically.

Sol. (a) Given, $x = -2\sin\left(3t + \frac{\pi}{3}\right)$

$$= 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right) = 2\cos\left(3t + \frac{5\pi}{6}\right)$$

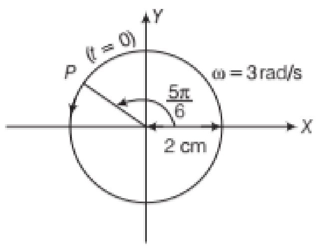
On comparing with standard equation for SHM,

$x = A\cos\left[\frac{2\pi}{T}t + \phi\right]$, we get amplitude $A = 2$ cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3$ rad/s

This corresponding reference circle is plotted as below



(b) $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$ is the standard SHM equation.

The given equation, $x = \cos\left(\frac{\pi}{6} - t\right)$

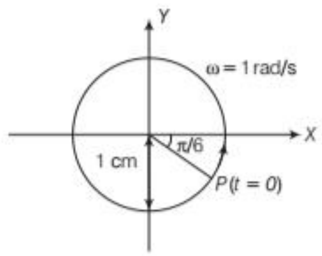
$$= \cos\left(t - \frac{\pi}{6}\right) \quad [\text{as } \cos(-\theta) = \cos\theta]$$

On comparison, amplitude $A = 1$ cm

Phase angle, $\phi = -\frac{\pi}{6} = -30°$

Angular velocity, $\omega = \frac{2\pi}{T} = 1$ rad/s

The reference circle is plotted below.



(c) Given equation, $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right]$$

$$\left[\because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta\right]$$

or $x = 3\cos\left(2\pi t + \frac{3\pi}{4}\right)$

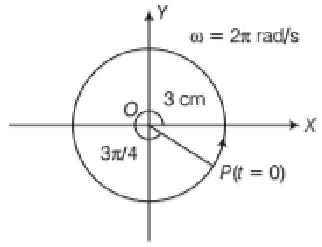
On comparing with standard SHM equation

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$$

We get, $A = 3$ cm

Phase angle, $\phi = \frac{3\pi}{4} = 135°$ and $\omega = \frac{2\pi}{T} = 2\pi$ rad/s

The reference circle of the motion of the particle is plotted below.



(d) Given, $x = 2\cos \pi t$

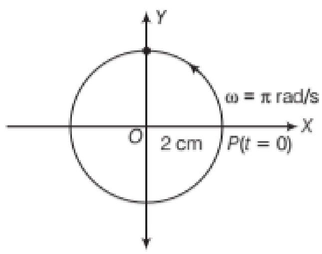
Standard SHM equation, $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$

Then, we get amplitude, $A = 2$ cm

Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi$ rad/s

The reference circle of the motion is plotted below.



ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. The motion of a swing is
(a) periodic but not oscillatory
(b) oscillatory
(c) linear simple harmonic
(d) circular motion.

2. When frequency of oscillations is high then motion is called
(a) periodic (b) non-periodic
(c) vibratory (d) rotatory

3. A particle executing a simple harmonic motion has a period of 6 s. The time taken by the particle to move from the mean position to half the amplitude, starting from the mean position is
(a) $\frac{1}{4}$ s (b) $\frac{3}{4}$ s
(c) $\frac{1}{2}$ s (d) $\frac{3}{2}$ s

4. A particle executing SHM has a maximum speed of 30 cm/s angular frequency 10 rad/s. The amplitude of oscillation is
(a) 3 cm (b) 6 cm
(c) 1 cm (d) 60 cm

5. The relation between acceleration and displacement of four particles are given below **[NCERT Exemplar]**
(a) $a_x = +2x$ (b) $a_x = +2x^2$
(c) $a_x = -2x^2$ (d) $a_x = -2x$
Which, one of the particle is suggesting simple harmonic motion?

Answers

1. (b)	2. (c)	3. (c)	4. (a)	5. (d)

VERY SHORT ANSWER Type Questions

6. What is the phase difference between particle velocity and particle displacement in SHM? [**Ans.** $-\pi/2$]

7. When are two SHMs said to be in anti-phase ?

SHORT ANSWER Type Questions

8. Show that in a simple harmonic motion, particle velocity is ahead in phase by $\frac{\pi}{2}$ rad as compared to its displacement and acceleration is further ahead in phase by $\frac{\pi}{2}$ as compared to velocity.

9. A simple harmonic motion is given by $x = 6.0 \cos\left(100\,t + \frac{\pi}{4}\right)$, where x is in cm and t in second. What is the (i) displacement amplitude, (ii) frequency? [**Ans.** (i) – 6.0 cm, (ii) 16 Hz]

10. The maximum acceleration in a simple harmonic motion is a_m and the maximum velocity is v_0. What is the displacement amplitude of simple harmonic motion? $\left[\textbf{Ans. } \frac{v_0^2}{a_0}\right]$

LONG ANSWER Type I Questions

11. A body oscillates with SHM according to the equation $x(t) = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$, where x is in metres and t is in seconds.
Calculate the following
(a) displacement at time $t = 0$,
(b) angular frequency
(c) magnitude of maximum velocity
[**Ans.** (a) 3.54 m, (b) 6.28 s^{-1}, (c) 31.4 m/s]

12. A body oscillates with SHM according to the equation (in SI units)
$x = 5\cos[2\pi \text{ rad s}^{-1} \cdot t + \pi/4]$
At $t = 1.5$ s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.

LONG ANSWER Type II Questions

13. A particle is subjected to two simple harmonic motions in the same direction having equal magnitude and equal frequency.
If the resultant amplitude is equal to the amplitude of the individual motion. Find the phase difference between two individual motions. [**Ans.** $2\pi/3$]

14. A particle moving with SHM in a straight line has a speed of 6 m/s when 4 m from the centre of oscillation and a speed of 8 m/s when 3 m from the centre of oscillation. Find the amplitude of oscillation and the shortest time taken by the particle in moving from the extreme position to a point mid way between the extreme position and the centre. [**Ans.** ± 5 m and $\pi/6$ s]

15. Derive an expression for the acceleration in the simple harmonic motion and also distinguish the displacement, velocity and acceleration of SHM in a uniform circular motion.

|TOPIC 2|

Force and Energy in SHM and Systems Executing SHM

FORCE LAW FOR SIMPLE HARMONIC MOTION

Whenever a body is displaced a little from its equilibrium position (i.e. mean position), a **restoring force** acts on the body in a direction opposite to its displacement in order to bring the body back to its equilibrium position.

This restoring force is proportional to the displacement, provided the displacement is small. If the body is left there, it returns back to its mean position, under the restoring force.

Then, the body gains some kinetic energy which helps to overshoot its mean position and body goes to other side. Again, a restoring force is set up which slows it down and brings the body back to the mean position and so on.

In this way, the restoring force helps in oscillating the body back and forth about the mean position with a definite period.

Now, from Newton's second law of motion

We know that, $F = ma$

For a body executing SHM, we have studied earlier that $a(t) = -\omega^2 x(t)$

Thus, $F(t) = -m\omega^2 x(t)$

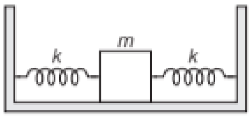
$$\Rightarrow \quad \boxed{\text{Restoring force, } F(t) = -kx(t)} \qquad \ldots\text{(i)}$$

where, $k = m\omega^2$ or $\omega = \sqrt{\dfrac{k}{m}}$

The force law is expressed by Eq. (i) and can be taken as an alternative definition of simple harmonic motion. **It states that simple harmonic motion is the motion executed by a particle subjected to a force, which is proportional to the displacement of the particle and is always directed towards the mean position.**

EXAMPLE |1| Block Fastened With Two Springs

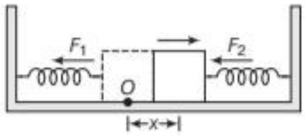
Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in figure. Show that when the mass is displaced from its equilibrium position on either side. It executes a simple harmonic motion. Find the period of oscillations. **[NCERT]**



Let the mass m is displaced by the small length x to the right side of the mean position. Then, the left spring gets elongated by distance x and right spring gets compressed by distance x.

Sol. Force exerted by left spring, trying to pull the mass towards the mean position.

$$F_1 = -kx$$



Similarly, force exerted by the right spring, trying to push the mass towards the mean position,

$$F_2 = -kx$$

The net force acting on the mass due to both the springs is

$$F = F_1 + F_2 = -2kx$$

Thus, the force acting on the mass is proportional to its displacement x and is directed towards its mean position. Hence, the motion of the mass m is simple harmonic.

Comparing the equation,

$$F = -2kx \text{ with } F = -k'x.$$

We have, force constant $k' = 2k$.

$\therefore$ The time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

EXAMPLE |2| A Huge Piston

The vertical motion of a huge piston in a machine is simple harmonic with a frequency of 0.50 s^{-1}. A block of 10 kg is placed on the piston. What is the maximum amplitude of the piston's SHM for the block and the piston to remain together?

Sol. As, $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

$\therefore \quad k = 4\pi^2 m\nu^2$

For maximum displacement $y_{max} = A$

Maximum restoring force,

$$F = -kA = -mg$$

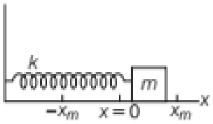
or
$$A = \frac{mg}{k} = \frac{mg}{4\pi^2 m\nu^2} = \frac{g}{4\pi^2\nu^2}$$

$$= \frac{9.8}{4 \times (3.14)^2 \times (0.50)^2}$$

$$= 0.99 \text{ m}$$

EXAMPLE |3| A Block Attached to a Spring

Consider a block of mass 700 g is fastened to a spring having spring constant of 70 N/m. Find out the following parameters if block is pulled a distance of 14 cm from its mean position on a frictionless surface and released from rest at $t = 0$.



(i) The angular frequency, the frequency and the period of the resulting motion.

(ii) The amplitude of the oscillation.

(iii) The maximum speed of the oscillating block.

(iv) The maximum acceleration of the block.

(v) The phase constant and hence the displacement function $x(t)$.

The spring block system forms a linear simple harmonic oscillation with block undergoing SHM. Linear means the restoring force ($F = -kx$) is proportional to x rather than power of x. For a spring, the spring constant being, $k = m\omega^2$.

Sol. (i) The angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{70\,\text{N/m}}{0.700\,\text{kg}}} = 10\ \text{rad/s}$$

Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{10}{2\pi} \simeq 1.59\,\text{Hz}$

The period, $T = \dfrac{1}{f} = \dfrac{1}{1.59} = 0.63 = 630\ \text{ms}$

Since, there is no friction involved, the mechanical energy of the spring block system is conserved. The block is released from rest 14 cm from its equilibrium or mean position. The block will have zero kinetic energy whenever it is again 14 cm from its equilibrium position, which means it will never farther than 14 cm.

(ii) The maximum amplitude of the oscillation

= maximum displacement

$\therefore \quad x_m = 14\,\text{cm} = 0.14\ \text{m}$

(iii) The maximum speed v_m is given by

$$v_m = \omega\, x_m = 10 \times 0.14 = 1.4\ \text{m/s}$$

(iv) The maximum acceleration of the block is given by

$$a_m = \omega^2 x_m = 100 \times 0.14 = 14\ \text{m/s}^2$$

At time $t = 0$, the block is located at position, $x = x_m$

(v) Then, from $x(t) = x_m \cos(\omega t + \phi)$

$$x_m = x_m \cos(0 \times \omega + \phi)$$

$$\cos\phi = 1 \Rightarrow \phi = 0$$

The displacement

$$x(t) = x_m \cos(\omega t + \phi) = 0.14 \times \cos(10t + 0)$$

$$x(t) = 0.14 \cos 10t$$

ENERGY IN SIMPLE HARMONIC MOTION

A particle executing SHM posseses both kinetic energy and potential energy. When a body is displaced from its equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back with a velocity, thus acquiring kinetic energy.

Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values.

Kinetic Energy

At any instant, the displacement of a particle executing SHM is given by

$$x = A\cos(\omega t + \phi)$$

$\therefore$ Velocity, $\quad v = \dfrac{dx}{dt} = -\omega A \sin(\omega t + \phi)$

Hence, kinetic energy of the particle at any displacement x is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

But, $A^2 \sin^2(\omega t + \phi) = A^2[1 - \cos^2(\omega t + \phi)]$

$$= A^2 - A^2\cos^2(\omega t + \phi) = A^2 - x^2$$

Thus, $\quad K = \dfrac{1}{2}kA^2 \sin^2(\omega t + \phi) \quad [\because k = m\omega^2]$

$$= \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

$$\boxed{\text{Kinetic energy, KE} = \frac{1}{2}k(A^2 - x^2)}$$

Hence, kinetic energy is also a periodic function of time, being **zero when the displacement is maximum**.

Kinetic energy is **maximum when the particle is at the mean position**.

Period of kinetic energy is $T/2$.

Potential Energy

When the displacement of a particle from its equilibrium position is x, then restoring force acting on it given as

$$F = -kx$$

If we displace the particle further through a small distance dx, then work done against the restoring force is given by

$$dW = -F dx = -kx\,dx$$

The total work done in moving the particle from mean position ($x = 0$) to displacement x is given by

$$W = \int dW = -\int_0^x kx\,dx = -k\left[\frac{x^2}{2}\right]_0^x = -\frac{1}{2}kx^2$$

This work done against the restoring force is stored as the potential energy of the particle. Hence, potential energy of a particle at displacement x is given by

$$U = -W = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

$$\boxed{\text{Potential energy, } U = \frac{1}{2}kx^2}$$

$$= \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$$

$$U = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

Thus, potential energy of a particle executing simple harmonic motion is also periodic with period $T/2$.

Potential energy is **zero at the mean position and maximum at the extreme position.**

Total Energy

At any displacement x, the total energy of a harmonic oscillation is given by

$$E = K + U = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2$$

$$\boxed{\text{Total energy, } E = \frac{1}{2}kA^2}$$

$$E = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2 \quad [\because \omega = 2\pi\nu]$$

Thus, **the total mechanical energy of a harmonic oscillation is independent of time or displacement, while it depends on the maximum displacement i.e. amplitude.**

Hence, in the absence of any frictional force, the total energy of a harmonic oscillation is **conserved.**

Hence, the total energy of particle in SHM is directly proportional to the mass m of the particle, the square of its frequency ν, and the square of its vibrational amplitude A.

Graphical Representation of Energy in SHM

At mean position, $x = 0$

(i) Kinetic energy, $K = \frac{1}{2}k(A^2 - 0^2) = \frac{1}{2}kA^2$

(ii) Potential energy, $U = \frac{1}{2}k(0^2) = 0$

Hence, at **mean position, the entire mechanical energy is in the form of kinetic energy.**

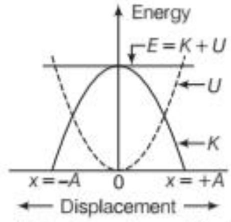
At extreme positions, $x = \pm A$

(i) Kinetic energy, $K = \frac{1}{2}k(A^2 - A^2) = 0$

(ii) Potential energy, $U = \frac{1}{2}kA^2$

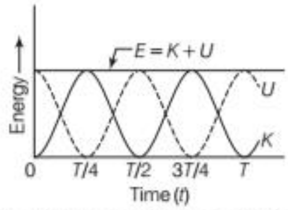
Hence, at **the two extreme positions, the energy is totally potential i.e. kinetic energy is zero.**

From above results, the graph for kinetic energy K, potential energy U and total energy E with displacement x is given below.



K, U and *E* as functions of displacement *x* for a harmonic oscillation

Plot for variation of energies K, U and E of a harmonic oscillator with time t is given below.



K, U and *E* as functions of time *t* for a harmonic oscillation

From the graph, we note that PE or KE completes two vibrations in a time during which SHM completes one vibration, (i.e. PE and KE both repeat after a period $T/2$) **Thus, the frequency of PE or KE is double than that of SHM. But, the total energy remains constant at all t or x.**

EXAMPLE |4| Energy in an SHM

A particle is executing SHM of amplitude A. At what displacement from the mean position is the energy half kinetic and half potential?

Sol. As, $E_k = E_p$

$\therefore \quad \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow \quad A^2 - x^2 = x^2$ or $2x^2 = A^2$

$\Rightarrow \quad x^2 = \frac{A^2}{2}$ or $x = \pm\frac{A}{\sqrt{2}}$

Thus, the energy will be half kinetic and half potential at displacement $\frac{A}{\sqrt{2}}$ on either side of the mean position.

EXAMPLE |5| Energy of a Body Executing SHM

A body of mass 0.2 kg and velocity with 1s after it passes through it, mean position be 6 m/s executes SHM. Find out the total energy and potential energy of the body if time period of the body is 8s during the SHM?

Sol. Given, $m = 0.2\,\text{kg}$, $T = 8\,\text{s}$, $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s

When $t = 1$ s, $v = 6$ m/s

$$v(t) = \omega A \cos \omega t$$

$$6 = \frac{\pi}{4} \times A \cos\left(\frac{\pi}{4} \times 1\right) \Rightarrow 6 = \frac{\pi}{4} \times A \times \frac{1}{\sqrt{2}}$$

$$A = \frac{4\sqrt{2} \times 6}{\pi} = \frac{24\sqrt{2}}{\pi}\,\text{m}$$

The total energy of the body

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \times 0.2 \times \left(\frac{\pi}{4}\right)^2 \times \left(\frac{24\sqrt{2}}{\pi}\right)^2$$

$$= \frac{230.4}{32} = 7.2\,\text{J}$$

Potential energy, $\text{PE} = E - \text{KE} = 7.2 - \frac{1}{2} m v^2$

$$= 7.2 - \frac{1}{2} \times 0.2 \times (6)^2 = 7.2 - 3.6$$

Potential energy = 3.6 J

EXAMPLE |6| Energy of body attached with spring

A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 Nm^{-1}. The block is pulled to a distance $x = 10$ cm from its equilibrium position at $x = 0$ on a frictionless surface from rest at $t = 0$. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position. **[NCERT]**

Sol. Given, $m = 1\,\text{kg}$, $k = 50\,\text{Nm}^{-1}$, $A = 10\,\text{cm} = 0.1\,\text{m}$, $x = 5\,\text{cm} = 0.05\,\text{m}$

Kinetic energy, $K = \frac{1}{2} k(A^2 - x^2)$

$$K = \frac{1}{2} \times 50 \times [(0.1)^2 - (0.05)^2] = 0.1875\,\text{J}$$

Potential energy of a block,

$$U = \frac{1}{2} k x^2 = \frac{1}{2} \times 50 \times (0.05)^2 = 0.0625\,\text{J}$$

Total energy, $E = K + U = 0.1875 + 0.0625$

$E = 0.25$ J

We also know that at maximum displacement, KE is zero and hence the total energy of the system is equal to PE. Therefore, the total energy of the system

$$= \frac{1}{2}(k \times A^2) = \frac{1}{2} \times 50\,\text{Nm}^{-1} \times 0.1 \times 0.1$$

$= 0.25$ J $\quad [\because \text{here}, x = A = 0.1\text{m}]$

Which is same as the sum of two energies at a displacement of 5 cm. This is in confirmity with the principle of conservation of energy.

SOME SYSTEMS EXECUTING SIMPLE HARMONIC MOTION

A pure simple harmonic motion is not possible unless some conditions are not applied on it. Let us consider some of the examples of pure simple harmonic motion under certain conditions.

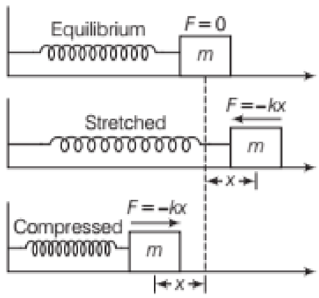
1. OSCILLATION DUE TO A SPRING

To study oscillation due to a spring, let us consider a massless spring lying on a frictionless horizontal table. Its one end is attached to a rigid support and the other end to a block of mass m.

If the block is pulled towards right through a small distance x and released, it starts oscillating back and forth about its equilibrium position under the action of the restoring force.

$$F = -kx$$

where, k is the force constant (restoring force per unit deformation of the spring). The negative sign indicates that the force is directed oppositely to x.



Three different positions of the oscillation due to a spring

If $\frac{d^2x}{dt^2}$ is the acceleration of the body, then

$$m\frac{d^2x}{dt^2} = -kx \qquad \left[\because F = ma = m\frac{d^2x}{dt^2}\right]$$

or $$\frac{d^2x}{dt^2} = \frac{-k}{m} x = -\omega^2 x$$

Thus, **acceleration is proportional to displacement x and acts opposite to it.**

Hence, the block executes simple harmonic motion. Its time period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \quad \text{or} \quad \boxed{\text{Time period, } T = 2\pi\sqrt{\frac{m}{k}}}$$

Frequency of oscillation can also be calculated as

$$\text{Frequency, } \nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

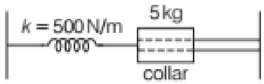
Clearly, the time period T will be small or frequency ν large if the spring is highly stiff (high k) and attached body is light (small m)

EXAMPLE |7| SHM of Body Attach with Spring

A 5 kg collar is attached to a spring of spring constant 500 Nm^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate

(i) the period of oscillation,
(ii) the maximum speed and
(iii) maximum acceleration of the collar. **[NCERT]**

Sol. Draw the figure containing collar of 5 kg attached to a spring of spring constant 500 N /m.



(i) Given, $m = 5\text{kg}$, $k = 500$ N/m, $A = 10\text{cm} = 0.1\text{m}$
The period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \times \sqrt{\frac{5}{500}} = 0.628\,\text{s}$$

(ii) Maximum speed of the collar, $v_{max} = \omega A$

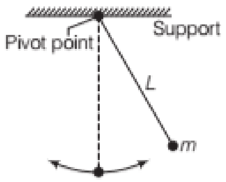
$$v_{max} = \sqrt{\frac{k}{m}} \cdot A = \sqrt{\frac{500}{5}} \times 0.1 = 1\ \text{m/s}$$

(iii) Maximum acceleration of the collar,

$$a_{max} = \omega^2 A = \frac{k}{m} \cdot A = \frac{500}{5} \times 0.1 = 10\,\text{m/s}^2$$

2. SIMPLE PENDULUM

An ideal simple pendulum consists of a point mass suspended by an inextensible and weightless string which is fixed at the other end as shown in figure. But this type of pendulum is not possible to make practically.



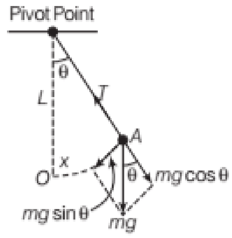
A simple pendulum

So, practically, a simple pendulum is obtained by suspending a small metal bob by a long and fine cotton thread from a support as given in the figure along side.

Expression for Time Period of a Simple Pendulum

Consider a simple pendulum, a small bob of mass m tied to an inextensible massless string of length L.

In the equilibrium position, the bob of a simple pendulum lies vertically below the point of suspension. If the bob is slightly displaced on either side and then released, it begins oscillation about the mean position.



An oscillating pendulum

Let us consider that at any instant during oscillation, the bob lies at position A when its displacement is $OA = X$ and the thread makes an angle θ with the vertical.

The forces acting on the bob are

(i) Weight mg of the bob acting vertically downwards.
(ii) Tension T along the string.

The force mg has two rectangular components

(i) The component $mg \cos\theta$ acting along the thread is balanced by the tension T in the thread and
(ii) The tangential force $mg\sin\theta$ will provide the restoring torque, which tends to bring the bob back to its mean position. Thus, the restoring torque of the force $mg\sin\theta$ about the pivot point is given by

$$\tau = -(mg\sin\theta)L = -mgL\sin\theta \quad \text{...(i)}$$

where, the negative sign shows that the torque acts to reduce θ. L is the length of the simple pendulum. For rotation, the torque can be given as

$$\tau = I\alpha \quad \text{...(ii)}$$

where, I = pendulum's moment of inertia about the pivot point.
α = angular acceleration about the pivot point.

From Eqs. (i) and (ii), we get

$$-mgL\sin\theta = I\alpha$$

$$\Rightarrow \quad \alpha = \frac{-mgL\sin\theta}{I}$$

If θ is in radian, $\sin\theta$ can be expressed as

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

Now, if θ is small, $\sin\theta$ can be approximated by θ

$$\therefore \quad \alpha \approx \frac{-mgL\theta}{I}$$

Therefore, the acceleration of the pendulum is proportional to the angular displacement θ but in opposite sign. Thus, as the pendulum moves to the right, it's pull to the left increases until it stops and begins to return to the left.

Similarly, when it moves towards left, its acceleration to the right tends to return it to the right and so on as it swings to and fro as SHM. Hence, the motion of a simple pendulum swinging through small angles is approximately SHM.

Equation $\alpha = \frac{-mgL}{I}\theta$ is the angular analogue of equation $a = -\omega^2 x$.

On comparing these equations, we have

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{mgL}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{mgL}} \quad \text{...(iii)}$$

where, T = time period of pendulum.

All the masses of a simple pendulum is centered in the mass m of the bob (taken as a point) which is at a distance of L from the pivot point. Therefore, $I = mL^2$. On putting this value in Eq. (iii)

$$T = \frac{1}{2\pi}\sqrt{\frac{mL^2}{mgL}} = \frac{1}{2\pi}\sqrt{\frac{L}{g}}$$

$$\Rightarrow \quad \boxed{\text{Time period of pendulum, } T = 2\pi\sqrt{\frac{L}{g}}}$$

Obviously, the time period of a simple pendulum depends on its length L and acceleration due to gravity g and is independent of the mass m of the bob.

$T = 2\pi\sqrt{\frac{L}{g}}$ is valid only for small length.

If length is large, then $T = 2\pi\sqrt{\frac{R}{\left(1+\frac{R}{L}\right)g}}$

$$\text{as } L \to \infty, T = 2\pi\sqrt{\frac{R}{g}}$$

$$\text{as } L = R, T = 2\pi\sqrt{\frac{R}{2g}}$$

where, R = distance between the pivot point and the centre of mass of the pendulum.

EXAMPLE |8| Second Pendulum

What is the length of a simple pendulum, which ticks seconds?

The time period of simple pendulum which ticks seconds is 2s and called as **second pendulum**.

Sol. Given, $T = 2\text{s}$ and $g = 9.8\ \text{m/s}^2$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L}{9.8}}$$

$$2^2 = 4\pi^2 \times \frac{L}{9.8} \Rightarrow L = \frac{9.8}{\pi^2} = 0.992\ \text{m} \simeq 1.0\ \text{m}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. In simple harmonic motion, the force

(a) is constant in magnitude only.

(b) is constant in direction only.

(c) varies in magnitude as well as in direction

(d) is constant in both magnitude and direction.

Sol. (c) In SHM, force varies in magnitude as well as in direction. As, for the particle executing SHM, the force subjected to it is always proportional to the displacement of the particle and is directed towards the mean position.

2. In SHM,

(a) PE is stored due to elasticity of system

(b) KE is stored due to inertia of system

(c) Both KE and PE are stored by virtue of elasticity of system.

(d) Both (a) and (b)

Sol. (d) In SHM, potential energy depends on its elastic behaviour and kinetic energy on its inertial behaviour. In case of mass m oscillating on spring. KE is due to motion of m and PE is due to stretching of spring.

3. Natural length of the spring is 40 cm and its spring constant is 4000 Nm^{-1}. A mass of 20 kg is hung from it. The extension produced in the spring is ($g = 9.8\ \text{ms}^{-2}$)

(a) 4.9 cm (b) 0.49 cm

(c) 9.4 cm (d) 0.94 cm

Sol. (a) In equilibrium, $kx = mg$

$$\therefore \text{ Extension, } x = \frac{mg}{k}$$

$$x = \frac{20 \times 9.8}{4000}$$

$$x = 0.049\ \text{m}$$

$$x = 4.9\ \text{cm}$$

4. A body of mass 400g connected to a spring with spring constant 10 Nm^{-1}, executes simple harmonic motion, time period of oscillation is

(a) $4\pi \times 10^{-1}$ s (b) $0.3\,\pi$ s

(c) 2 s (d) 5×10^{-1} s

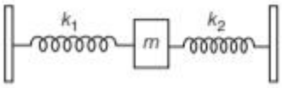
Sol. (a) Here, $m = 400g = 400 \times 10^{-3}$ kg

As, $R = 10\ Nm^{-1}$

$$T = 2\pi\sqrt{\frac{m}{R}}$$

$$= 2\pi\sqrt{\frac{400\times10^{-3}}{10}} = 4\pi\times10^{-1}s$$

5. Two spring of force constants k_1 and k_2 are connected to a mass m as shown in figure. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $f/2$ (b) $f/4$

(c) $4f$ (d) $2f$

Sol. (d) Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

and $$f' = \frac{1}{2\pi}\cdot 2\sqrt{\frac{k_1 + k_2}{m}} = 2f$$

VERY SHORT ANSWER Type Questions

6. What is the force equation of a SHM?

Sol. According to force equation of SHM, $F = -kx$,

where, k is a constant known as force constant.

7. Under what condition is the motion of a simple pendulum be simple harmonic? **[NCERT Exemplar]**

Sol. When the displacement amplitude of the pendulum is extremely small as compared to its length.

8. What is the ratio between the distance travelled by the oscillator in one time period and amplitude? **[NCERT Exemplar]**

Sol. Total distance travelled by an oscillator in one time period, from its mean position to one extreme position, then to other extreme position and finally back to mean position is 4 A, where A is the amplitude of oscillation.

Hence, the ratio $= \frac{4A}{A} = 4$

9. A simple pendulum is transferred from earth to the surface of moon. How will its time period be affected?

Sol. As value of g on moon is less than that on earth, in accordance with the relation $T = 2\pi\sqrt{l/g}$, the time period of oscillations of a simple pendulum on moon will be greater.

10. If the length of a simple pendulum is increased by 25%, then what is the change in its time period?

Sol. $\because$ Time period, $T = 2\pi\sqrt{\frac{l}{g}}$

or $T \propto \sqrt{l}$

$\therefore$ % increase in time period

$$\frac{\Delta T}{T}\times 100 = \frac{1}{2}\cdot\frac{\Delta l}{l}\times 100$$

$$= \frac{1}{2}\times 25 = 12.5\%$$

11. How much is KE for displacement equal to half the amplitude?

Sol. $\because\ x = A/2$, So

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2[A^2 - (A/2)^2]$$

$$= \frac{1}{2}\times\frac{3}{4}[m\omega^2A^2] = \frac{3}{4}(KE)_{max}$$

It is 3/4 th of maximum KE.

SHORT ANSWER Type Questions

12. In case of an oscillating simple pendulum what will be the direction of acceleration of the bob at (a) the mean position, (b) the end points?

Sol. The direction of acceleration of the bob at its mean position is radial i.e. towards the point of suspension.

At extreme points, however, the acceleration is tangential towards the mean position.

13. Justify the following statements

(i) The motion of an artificial satellite around the earth cannot be taken as SHM.

(ii) The time period of a simple pendulum will get doubled if its length is increased four times.

Sol. (i) The motion of an artificial satellite around the earth is periodic as it repeats after a regular interval of time. But it cannot be taken as SHM because it is not a to and fro motion about any fixed point that is, mean position.

(ii) Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}},$$

i.e. $T \propto \sqrt{l}$

Clearly, if the length is increased four times, the time period gets doubles.

14. A body of mass 12 kg is suspended by coil spring of natural length 50 cm and force constant $2.0 \times 10^3\ \text{Nm}^{-1}$. What is the stretched length of the spring? If the body is pulled down further stretching the spring to a length of 5.9 cm and then released, then what is the frequency of oscillation of the suspended mass? (Neglect the mass of the spring)

Sol. Given, $m = 12$ kg, original length $l = 50$ cm,

$$k = 2.0 \times 10^3\ \text{Nm}^{-1}$$

As, $F = ky$

$$\therefore \quad y = \frac{F}{k} = \frac{mg}{k} = \frac{12 \times 9.8}{2 \times 10^3} = 5.9 \times 10^{-2}\ \text{m} = 5.9\ \text{cm}$$

$\therefore$ Stretched length of the spring $= l + y = 50 + 55.9$ cm $= 105.9$cm

Frequency of oscillations, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$= \frac{1}{2 \times 3.14}\sqrt{\frac{2 \times 10^3}{12}} = 2.06\ \text{s}^{-1}$$

15. A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg placed on it. Deduce

(i) the force constant of the spring.

(ii) the depression of the spring under the weight of the body (take $g = 10$ N/kg)

(iii) the period of oscillation, the body is distributed and

(iv) frequency of oscillation

Sol. Here, $F = 10$ N, $\Delta l = 0.1$ m, $m = 4$ kg

(i) $k = \frac{F}{\Delta l} = \frac{10}{0.1} = 100\ \text{Nm}^{-1}$

(ii) $y = \frac{mg}{k} = \frac{4 \times 10}{100} = 0.4$ m

(iii) $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7}\sqrt{\frac{4}{100}} = 1.26$ s

(iv) Frequency, $\nu = \frac{1}{T} = \frac{1}{1.26} = 0.8$Hz

16. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation, $J = -\alpha\theta$, where J is the restoring couple and θ the angle of twist.) **[NCERT]**

This is a question based on torsion pendulum for which $T = 2\pi\sqrt{\frac{I}{\alpha}}$

where, I = moment of inertia of the disc about axis of rotation, α = torsion constant which is restoring couple per unit twist.

Sol. Given, Mass of the disc, $m = 10$ kg

Radius of the disc, $r = 15$ cm $= 0.15$ m

$$T = 1.5\ \text{s}$$

I is the moment of inertia of the disc about the axis of rotation which is perpendicular to the plane of the disc and passing through its centre.

$$\therefore \quad I = \frac{1}{2}mr^2 = \frac{1}{2} \times (10) \times (0.15)^2$$

$$= 0.1125\ \text{kg-m}^2$$

Time period, $T = 2\pi\sqrt{\frac{I}{\alpha}}$

$$\alpha = \frac{4\pi^2 I}{T^2} = \frac{4 \times (3.14)^2 \times 0.1125}{(1.5)^2}$$

$$= 1.972\ \text{N-m/rad}$$

17. Define the restoring force and it characterstic in case of an oscillating body.

Sol. A force which takes the body back towards the mean position in oscillation is called restoring force.

Characteristic of Restoring force

The restoring force is always directed towards the mean position and its magnitude of any instant is directly proportional to the displacement of the particle from its mean position of that instance.

18. A particle executes SHM of period 8 s. After what time of its passing through the mean position, will be energy be half kinetic and half potential?

Sol. Given, PE = KE

i.e. $\frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 (A^2 - x^2)$

$$x^2 = A^2 - x^2 \Rightarrow x = \frac{A}{\sqrt{2}}$$

Now, $\quad x = A\sin\omega t = A\sin\left(\frac{2\pi}{T}\right)t$

So, $\quad \frac{A}{\sqrt{2}} = A\sin 2\pi\frac{t}{8}$

or $\quad \sin\frac{\pi t}{4} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

or $\quad \frac{\pi t}{4} = \frac{\pi}{4}$ or $t = 1$ s

LONG ANSWER Type I Questions

19. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body? **[NCERT]**

Sol. The length of the scale 20 cm reads upto 50 kg.

So, $F = mg = 50 \times 9.8$ N and $y = 20$ cm $= 0.20$ m

Now, force constant, $k = \frac{F}{y} = \frac{50 \times 9.8}{0.20} = 2450\,\text{Nm}^{-1}$

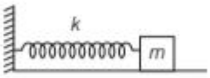
Suppose the spring oscillates with time period of 0.60 s when loaded with a mass of M kg. Then,

$$T = 2\pi\sqrt{\frac{M}{k}} \quad \text{or} \quad T^2 = 4\pi^2 \frac{M}{k}$$

$$M = \frac{T^2 k}{4\pi^2} = \frac{(0.60)^2 \times 2450}{4 \times (3.14)^2} = 22.36\,\text{kg}$$

$\therefore$ Weight $= Mg = 22.36 \times 9.8 = 219.13$ N

20. A spring of force constant $1200\,\text{Nm}^{-1}$ is mounted on a horizontal table. A mass of 3.0 kg is attached to the free end of the spring, pulled sideways to a distance of 2.0 cm and then released.



(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?

[NCERT]

Sol. Here, $k = 1200\,\text{Nm}^{-1}$, $m = 3.0$ kg

and $A = 2.0\,\text{cm} = 2.0 \times 10^{-2}$ m

(i) Frequency of oscillation of the mass,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3.0}} = \frac{1}{2 \times 3.14} \times 20$$

$$= 3.18\,\text{s}^{-1} = 3.2\,\text{s}^{-1}$$

(ii) Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20\,\text{s}^{-1}.$$

$\therefore$ Maximum acceleration of the mass

$$= \omega^2 A = (20)^2 \times 2.0 \times 10^{-2} = 8.0\,\text{ms}^{-2}.$$

(iii) Maximum speed of the mass

$$= \omega \cdot A = 20 \times 2.0 \times 10^{-2} = 0.40\,\text{ms}^{-1}.$$

21. The acceleration due to gravity on the surface of moon is $1.7\,\text{m/s}^2$. What is the time period of a simple pendulum on the surface of moon, if its time period on the surface of earth is 3.5 s? (g on the surface of earth is $9.8\,\text{m/s}^2$.) **[NCERT]**

Consider the two time periods separately. Then, get the ratio and put the given values to get the result.

Sol. Given, acceleration due to gravity on the moon $(g_m) = 1.7\,\text{m/s}^2$

Acceleration due to gravity on the earth, $(g_e) = 9.8\,\text{m/s}^2$

Time period on the earth, $T_e = 3.5$ s

Time period on the moon, $T_m = ?$

On the surface of the earth, time period $= T_e$

$$\therefore \quad T_e = 2\pi\sqrt{\frac{l}{g_e}} \quad \text{...(i)}$$

On the surface of the moon, time period $= T_m$

$$\therefore \quad T_m = 2\pi\sqrt{\frac{1}{g_m}} \quad \text{...(ii)}$$

Suppose g_e, g_m are accelerations due to gravity on the earth and the moon surface, respectively.

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_e}{T_m} = \frac{2\pi}{2\pi}\sqrt{\frac{l}{l} \times \frac{g_m}{g_e}}$$

$$\Rightarrow \quad \frac{T_e}{T_m} = \sqrt{\frac{g_m}{g_e}}$$

$$\Rightarrow \quad T_m = \left(\sqrt{\frac{g_e}{g_m}}\right) T_e$$

Putting the values, we get

$$T_m = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4\,\text{s}$$

22. A mass attached to a spring is free to oscillate, with angular velocity ω in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [**Hint** Start with the equation $x = a\cos(\omega t + \theta)$ and note that the initial velocity is negative.] **[NCERT]**

If in a spring mass system, the mass is displaced and given a velocity, also, it will perform SHM but with an amplitude more than the amount of extension.

Sol. We have for SHM

$$x = A\cos(\omega t + \theta) \quad \text{...(i)}$$

where, x = displacement, A = amplitude

θ = phase constant, we get

On differentiating with respect to t,

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \theta)$$

$$\Rightarrow \quad v = -A\omega\sin(\omega t + \theta) \quad \text{...(ii)}$$

where, v = instantaneous velocity of the particle at t.

Now, at $t = 0$, $x = x_0$

From Eq. (i),

$$\Rightarrow \quad x_0 = A\cos\theta \quad \text{...(iii)}$$

Again at $t = 0$, $v = -v_0$

From Eq. (ii),

$$-v_0 = -A\omega\sin\theta$$

or

$$\frac{v_0}{\omega} = A\sin\theta \quad \text{...(iv)}$$

On squaring and adding Eqs. (iii) and (iv), we get

$$A^2(\cos^2\theta + \sin^2\theta) = x_0^2 + \left(\frac{v_0}{\omega}\right)^2$$

$$\Rightarrow \quad A^2 = x_0^2 + \left(\frac{v_0}{\omega}\right)^2$$

or

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

23. The length of a second pendulum on the surface of earth is 1 m. What will be the length of a second pendulum on the moon? **[NCERT Exemplar]**

Sol. A second pendulum means a simple pendulum having time period, $T = 2$ s

For a simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

where, l = length of the pendulum and g = acceleration due to gravity on surface of the earth.

$$T_s = 2\pi\sqrt{\frac{l_e}{g_e}} \quad \text{...(i)}$$

On the surface of the moon,

$$T_m = 2\pi\sqrt{\frac{l_m}{g_m}} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\therefore \quad \frac{T_s}{T_m} = \frac{2\pi}{2\pi}\sqrt{\frac{l_e}{g_e}} \times \sqrt{\frac{g_m}{l_m}}$$

$T_s = T_m$ to maintain the second pendulum time period.

$$\therefore \quad 1 = \sqrt{\frac{l_e}{l_m} \times \frac{g_m}{g_e}} \quad \text{...(iii)}$$

But the acceleration due to gravity at moon is 1/6 of the acceleration due to gravity at earth, i.e. $g_m = \frac{g_e}{6}$

Squaring Eq. (iii) and putting this value,

$$1 = \frac{l_e}{l_m} \times \frac{g_e/6}{g_e} = \frac{l_e}{l_m} \times \frac{1}{6}$$

$$\Rightarrow \quad \frac{l_e}{6l_m} = 1$$

or

$$l_m = \frac{1}{6}l_e = \frac{1}{6} \times 1 = \frac{1}{6} \text{ m}$$

24. Answer the following questions.

(i) Time period of a particle in SHM depends on the force constant k and mass m of the particle $T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. Then, why is the time period of a pendulum independent of the mass of the pendulum?

(ii) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(iii) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(iv) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity? **[NCERT]**

Sol. (i) For a simple pendulum, k is proportional to m the mass of the particle, hence $\frac{m}{k}$ becomes constant and does not affect the time period.

(ii) If we replace $\sin\theta = \theta$ for large angles, then actually $\sin\theta < \theta$. Now, since this factor is multiplied to the restoring force $mg\sin\theta$ is replaced by $mg\theta$ which means an effective reduction in g for large angles. Hence, there is an increase in time period T over that given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

as compared to the case which it is assumed $\sin\theta = \theta$.

(iii) Yes, since the motion of hands of a wristwatch to indicate time depends on action of the spring and has nothing to do with acceleration due to gravity.

(iv) In a free fall the effective $g = 0$, i.e. gravity disappears (also called **weightlessness**)

$$\therefore \quad \text{Time period } T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{0}} = \infty$$

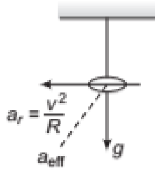
Frequency, $\nu = \frac{1}{T} = 0$

i.e. frequency of oscillation is zero.

25. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, then what will be its time period? **[NCERT]**

For a simple pendulum $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$; so you should always find the g_{eff} in the situation; the pendulum is oscillating and then calculate the time period.

Sol. The bob is subjected to two simultaneous, accelerations perpendicular to each other *viz,* acceleration due to gravity g and radial acceleration, $a_R = \frac{v^2}{R}$ towards the centre of the circular path.



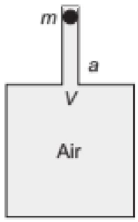
$\therefore$ Effective acceleration, $g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$

$\therefore$ Time period of the simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\left(g^2 + \left(\frac{v^2}{R}\right)^2\right)^{1/2}}}$$

$$= 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}}$$

26. An air chamber of volume V has a neck area of cross-section a into which a ball of mass m just fits and can move up and down without any friction (see figure). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal. **[NCERT]**



This is a system executing SHM. So, proceed in the order finding restoring force, proving SHM and find force constant to determine time period.

Sol. Before pressing the ball, the pressure inside the chamber = pressure outside the chamber = atmospheric pressure.

Let now the ball is depressed by x units. As a result, the volume will decrease and this would increase the pressure inside.

Decrease in the volume of air, $\Delta V = ax$

where, a = cross-sectional area of the neck

Volume strain $= \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V} = \frac{ax}{V}$

Volume stress = Bulk stress = p

As, $B = \frac{-p}{\left(\frac{ax}{V}\right)}$ where, negative sign shows the decrease in volume.

$\therefore$ Increases in the pressure, $p = -\frac{Bax}{V}$

The restoring force on the ball

$$F = pa = -\frac{Bax}{V}\cdot a = \frac{-Ba^2x}{V} \quad \left[\because \frac{Ba^2}{V} = \text{constant}\right]$$

$F \propto -$ displacement (x)

Hence, motion is SHM.

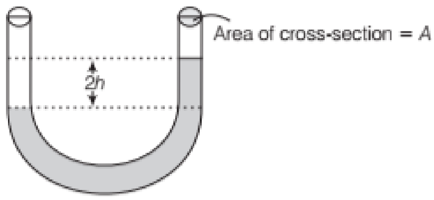
$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mV}{Ba^2}}$$

27. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion. **[NCERT]**

Start from the beginning, find the restoration force and then apply the rule to determine time period in typical SHM.

Sol. Let area of cross-section of each arm of the U-tube be A. When a small pressure difference be maintained between two columns then liquid column falls through height h in one arm.

Now, difference in liquid column in two arms = $2h$



Density of mercury column = ρ

Acceleration due to gravity = g

Restoring force

$F = -$ Weight of mercury column in excess of one arm

$= -$ (Volume $\times$ Density $\times$ g)

$= -(A \times 2h \times \rho \times g)$

$= -2A\rho gh = -k \times$ Displacement in one arm (h)

Clearly, $2A\rho g$ = constant = k (say) [as $F = -kx$]

$F \propto -h$

Hence, motion is SHM, $k = 2A\rho g$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

where, m = mass of the mercury column of length l

If ρ_{Hg} is density of mercury,

Then, $m = Al\rho$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

28. In the previous question, let us take the position of mass when the spring is unstreched as $x = 0$ and the direction from left to right as the positive direction of x-axis. Given x as a function of time t for the oscillating mass, if at the moment we start the stopwatch ($t = 0$), the mass is

(i) at the mean position,
(ii) at the maximum stretched position and
(iii) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase? **[NCERT]**

Standard equation for SHM should be used with different initial phases.

Sol. Assuming the standard equation

$$x(t) = A\sin(\omega t + \phi)$$

(i) When $t = 0$, $x = 0$ [mean position]

$\Rightarrow \quad 0 = A\sin(\omega \times 0 + \phi)$

$A\sin\phi = 0$ [as $A \neq 0$]

or $\sin\phi = 0 \quad \therefore \quad \phi = 0$

$\therefore$ Required function is

$$x(t) = A\sin(\omega t + 0) \text{ or } x(t) = A\sin\omega t$$

where, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20$ rad/s

$\therefore \quad x(t) = A\sin 20t$ or $x(t) = 2\sin 20t$

(ii) When $t = 0$, $x = +A$ (maximum stretched position)

$x(t) = A\sin(\omega t + \phi)$ at $t = 0$ and $x = +A$

$$+A = A\sin(\omega \times 0 + \phi) \text{ or } 1 = \sin\phi \Rightarrow \phi = \frac{\pi}{2}$$

$$\therefore \quad x(t) = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t = A\cos 20t = 2\cos 20t$$

(iii) When the spring is at maximum compressed position.

At $t = 0$, $x(t) = -A$

$$\Rightarrow \quad -A = A\sin(\omega \times 0 + \phi) \text{ or } -1 = \sin\phi \text{ or } \phi = \frac{3\pi}{2}$$

$$\therefore \quad x(t) = A\sin\left(\omega t + \frac{3\pi}{2}\right) = -A\cos\omega t = -2\cos 20t$$

So, the equations only differ in initial phase and in no other factors.

29. A body weighing 10 g has a velocity of 6 cm s^{-1} after one second of its starting from mean position. If the time period is 6 s, then find the kinetic energy, potential energy and the total energy.

Sol. Here, $m = 10$ g, $T = 6$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ rad s}^{-1}$$

When $t = 1$ s, $v = 6$ cm s^{-1}

As $v = A\omega\cos\omega t$

$$6 = A \times \frac{\pi}{3}\cos\frac{\pi}{3} \times 1 = A \times \frac{\pi}{3}\cos 60°$$

$$= A \times \frac{\pi}{3} \times \frac{1}{2} = \frac{\pi A}{6} \quad \text{or } A = \frac{36}{\pi}\text{ cm}$$

Total energy, $E = \frac{1}{2}mA^2\omega^2$

$$= \frac{1}{2} \times 10 \times \left(\frac{36}{\pi}\right)^2 \times \left(\frac{\pi}{3}\right)^2 = 720 \text{ erg}$$

Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times 6^2 = 180$ erg

$\therefore$ Potential energy $=$ Total energy $-$ Kinetic energy

$= 720 - 180 = 540$ erg

30. Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equals the average potential energy over the same period. **[NCERT]**

Sol. Suppose a particle of mass m executes SHM of period T. The displacement of the particle at any instant t is given by

$$y = A\sin\omega t$$

$\therefore$ Velocity, $v = \frac{dy}{dt} = \omega A\cos\omega t$

Kinetic energy, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

Potential energy, $E_p = \frac{1}{2}m\omega^2y^2 = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\therefore$ Average KE over a period of oscillation,

$$E_{k_{av}} = \frac{1}{T}\int_0^T E_k\,dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2A^2\cos^2\omega t\,dt$$

$$= \frac{1}{2T}m\omega^2A^2\int_0^T \frac{(1+\cos 2\omega t)}{2}dt$$

$$= \frac{1}{4T}m\omega^2A^2\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{4T}m\omega^2A^2(T) = \frac{1}{4}m\omega^2A^2 \quad [\because \sin 2\omega T = 0] \quad \text{...(i)}$$

Average PE over a period of oscillation,

$$E_{p_{av}} = \frac{1}{T}\int_0^T E_p\,dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2A^2\sin^2\omega t\,dt$$

$$= \frac{1}{2T}m\omega^2A^2\int_0^T \frac{(1-\cos 2\omega t)}{2}dt$$

$$= \frac{1}{4T}m\omega^2A^2\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{4T} m\omega^2 A^2 (T) = \frac{1}{4} m\omega^2 A^2 \quad \text{...(ii)}$$

Clearly, from Eqs. (i) and (ii),

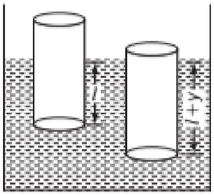
$$E_{k_{av}} = E_{p_{av}}$$

LONG ANSWER Type II Questions

31. A cylindrical piece of cork of base area A, density ρ and height L floats in a liquid of density ρ_L. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically and find its time period of oscillations. **[NCERT]**

Sol. Consider a cylinder of mass m, length L, density of material ρ and uniform area of cross-section A. Therefore,

$$m = AL\rho \quad \text{...(i)}$$



Let the cylinder be floated in the liquid of density ρ_L.

In equilibrium, let l be the length of cylinder dipping in liquid.

In equilibrium, weight of cylinder = Weight of liquid displaced

$$mg = Al\rho_L g$$

$$m = Al\rho_L \quad \text{...(ii)}$$

Let the cylinder be pushed down by y, then

Total upward thrust, $F_2 = A(l + y)\rho_L g$

Restoring force, $F = -(F_2 - mg)$

$$F = -[A(l + y)\rho_L g - Al\rho_L g] = -A\rho_L gy \quad \text{...(iii)}$$

In SHM, $F \propto -y$

$$F = -ky \quad \text{...(iv)}$$

From Eq. (iii) and Eq. (iv),

Spring factor, $k = A\rho_L g$

Inertia factor, $m = AL\rho$

Time period, $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

$$T = 2\pi\sqrt{\frac{AL\rho}{A\rho_L g}} = 2\pi\sqrt{\frac{L\rho}{\rho_L g}} \quad \text{...(v)}$$

Using, $m = Al\rho_L = AL\rho$

So, $l\rho_L = L\rho$

So, another form of time period

$$T = 2\pi\sqrt{\frac{l\rho_L}{g\rho_L}} = 2\pi\sqrt{\frac{l}{g}}$$

32. A person normally weighing 50 kg stands on a massless platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. A weighing machine on the platform gives the persons weight against time.

(a) Will there be any change in weight of the body, during the oscillation?

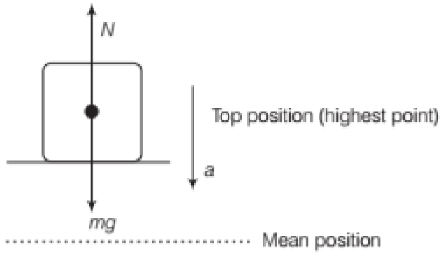
(b) If answer to part (a) is yes, then what will be the maximum and minimum reading in the machine and at which position?

[NCERT Exemplar]

Sol. (a) This is a case of variable acceleration. In accelerated motion, weight of body depends on the magnitude and direction of acceleration for upward or downward motion.

Hence, the weight of body changes.

(b) Considering the situation in two extreme positions, as their acceleration is maximum in magnitude.



We have, $mg - N = ma$

Note, at the highest point, the platform is accelerating downward.

$\Rightarrow$ $N = mg - ma$

But $a = \omega^2 A$ (in magnitude)

$\therefore$ $N = mg - m\omega^2 A$

where, A = amplitude of motion.

Given, $m = 50$ kg, frequency $\nu = 2\ s^{-1}$

$\therefore$ $\omega = 2\pi\nu = 4\pi$ rad/s

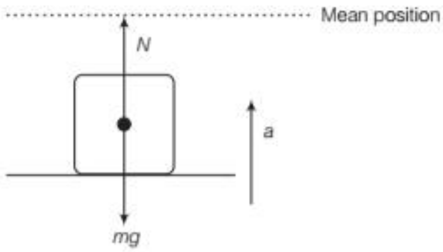
$A = 5\text{ cm} = 5 \times 10^{-2}$ m

$\therefore$ $N = 50 \times 9.8 - 50 \times (4\pi)^2 \times 5 \times 10^{-2}$

$= 50[9.8 - 16\pi^2 \times 5 \times 10^{-2}]$

$= 50[9.8 - 7.89] = 50 \times 1.91 = 95.5$ N

When the platform is at the lowest position of its oscillation,



It is accelerating towards mean position that is vertically upwards.

Writing equation of motion

$$N - mg = ma = m\omega^2 A$$

or

$$N = mg + m\omega^2 A$$

$$= m[g + \omega^2 A]$$

Putting the data, $N = 50[9.8 + (4\pi)^2 \times 5 \times 10^{-2}]$

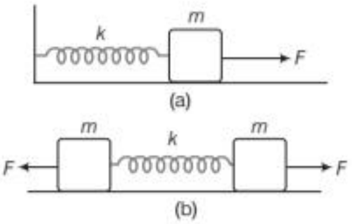
$$= 50[9.8 + (12.56)^2 \times 5 \times 10^{-2}]$$
$$= 50[9.8 + 7.88] = 50 \times 17.68$$
$$= 884.4 \text{ N}$$

Now, the machine reads the normal reaction. It is clear that

maximum weight = 884.4 N (at lowest point)

minimum weight = 95.5 N (at top point)

33. Fig. (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Fig. (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.

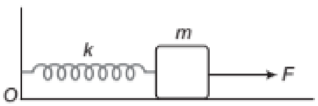


(i) What is the maximum extension of the spring in both the cases?

(ii) If the mass in Fig. (a) and the two masses in Fig. (b) are released, then what is the period of oscillation in each case? **[NCERT]**

Draw the free body diagram to find the extension in the spring in two cases.
In both cases, the system would perform SHM. In the first, the mean position will be the position of mass for unstretched case of the spring whereas in second case it will be centre of mass of the spring block system which will be the mean position for oscillation.

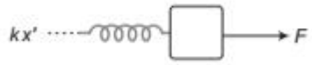
Sol. (i) For **Case** (a), as we know that, $F = -kx \Rightarrow |F| = kx$



$\therefore$ So, $x = \dfrac{F}{k}$

Case (b)

If x' is the extension in the spring, then drawing free body diagram of either mass (as the system under applied force is under equilibrium).



$$kx' = F$$

$\therefore$ $x' = \dfrac{F}{k}$

In both the cases, extension is the same $\left(\dfrac{F}{k}\right)$.

(ii) The period of oscillation in case (a),

As, $F = -kx$

where, x = given extension

But $F = ma$

$\therefore$ $ma = -kx$

$\Rightarrow$ $a = -\left(\dfrac{k}{m}\right)x$ ...(i)

$a \propto -x$ [as, $\dfrac{k}{m}$ is a constant]

On comparing Eq. (i) with $a = -\omega^2 x$, we get

$\therefore$ $\omega = \sqrt{\dfrac{k}{m}}$

Period of oscillation, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

Case (b)

The system is divided into two similar systems with spring divided in two equal halves. $k' = 2k$

Hence, $F = -k'x$

$k' = 2k$ (on cutting a spring in two halves, its k doubles)

$\Rightarrow$ $F = -2kx$

But $F = ma$

$\therefore$ $ma = -2kx$

$\Rightarrow$ $a = -\left(\dfrac{2k}{m}\right)x$ ...(ii)

$\Rightarrow$ $a \propto -$ displacement [as $\dfrac{2k}{m}$ is a constant]

On comparing Eq. (ii) with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{2k}{m}}$$

Period of oscillation,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. For simple harmonic motion of an object of mass m,

(a) $\mathbf{F} = -m\omega^2 x$

(b) $\mathbf{F} = -m\omega x$

(c) Force always acts in the opposite direction of displacement

(d) Both (a) and (c)

2. The expression for displacement of an object in SHM is $x = A\cos(\omega t)$. The potential energy at $t = T/4$ is

(a) $\frac{1}{2}kA^2$ (b) $\frac{1}{8}kA^2$ (c) $\frac{1}{4}kA^2$ (d) zero

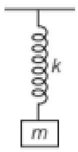
3. A simple pendulum suspended from the roof of a lift oscillates with frequency ν when the lift is at rest. If the lift falls freely under gravity, its frequency of oscillations becomes

(a) zero (b) ν (c) 2ν (d) infinite

4. If we do an experiment by swinging a small ball by a thread of length 100 cm, what will be the approximate time for complete to and fro periodic motion?

(a) 4 s (b) 2 s (c) 6 s (d) 1 s

5. A block is left in the equilibrium position as shown in the figure. If now it is stretched by $\frac{mg}{k}$, the net stretch of the spring is



(a) $\frac{mg}{k}$ (b) $\frac{mg}{2k}$ (c) $\frac{2mg}{k}$ (d) $\frac{mg}{4k}$

Answers

1. (d) | 2. (d) | 3. (a) | 4. (b) | 5. (c)

VERY SHORT ANSWER Type Questions

|1 Mark|

6. Two simple pendulums of equal length cross each other at mean position. What is their phase difference? [**Ans.** π rad, i.e. 180°]

7. A body executes SHM with a period of 11/7 s and an amplitude of 0.025 m. What is the maximum value of acceleration? [**Ans.** $-$ 0.4 m/s^2]

SHORT ANSWER Type Questions

8. When a particle oscillates simple harmonically, its potential energy varies periodically. If ν is the frequency of oscillation of the particle, then what is the frequency of variation of potential energy? [**Ans.** 2ν]

LONG ANSWER Type I Questions

9. Derive the expression for total energy in oscillations.

10. When the mass is displaced a little to one side, one spring gets compressed and another is elongated. Due to which the combination of springs works as parallel combination of springs. Here, effective spring factor k will be given by

$k = k_1 + k_2 = 600 + 600 = 1200\ \text{Nm}^{-1}$

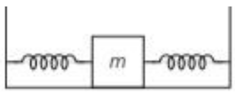
[**Ans.** $T = 0.314$ s, $v_{max} = 1\ \text{ms}^{-1}$, $E = 1.5$ J]

LONG ANSWER Type II Questions

11. A point particle of mass 0.1 kg is executing SHM with amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} joule. Obtain the equation of motion of this particle, if the initial phase of oscillation is 45°.

12. A mass of 1 kg is executing SHM which is given by $x = 6.0\cos\left(100\,t + \frac{\pi}{4}\right)$ cm. What is the maximum kinetic energy? [**Ans.** $-$ 18 J]

13. In the figure shown below, two identical springs of spring constant 7580 N/m are attached to a block of mass 0.245 kg. What is the frequency of oscillation on the frictionless floor? [**Ans.** 39.6 Hz]



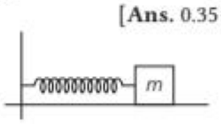
14. A spring of force constant 1200 Nm^{-1} is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring, pulled sideways to a distance of 2.0 cm and released. What is

(i) the speed of the mass, if the spring is compressed?

(ii) the potential energy of the mass, when the mass comes to rest?

(iii) total energy of the mass during oscillation?

[**Ans.** 0.35 ms^{-1}, 0.24 J, 0.24 J]



SUMMARY

- **Periodic motion** A motion which repeats itself after a regular interval of time is called a **periodic motion**.
- **Oscillatory motion** A motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position) is called **oscillatory** or **vibratory motion**.
- **Simple harmonic motion** It is a special type of periodic motion in which the body moves to and fro repeatedly about a mean position under the influence of a restoring force is known as simple harmonic motion (SHM). The restoring force is directly proportional to its displacement from the mean position.

$$F \propto x \text{ or } F = -kx$$

where, k is the force constant or spring constant.

- **Displacement in SHM** The displacement of a particle executing SHM is expressed as, $y = A\sin(\omega t + \phi)$ where, A is amplitude of SHM, ω is the angular frequency $\left(\text{where}, \omega = \frac{2\pi}{T} = 2\pi\nu\right)$ and ϕ is the initial phase of SHM.
- **Time period** It is the time taken by a particle to complete one oscillation about its mean position. It is denoted by T.
- **Frequency** It is the number of oscillations completed per second by a particle about its mean position. It is denoted by ν and is equal to the reciprocal of time period. Thus, $\nu = \frac{1}{T}$. Frequency is measured in hertz.

$$1 \text{ hertz} = 1\,\text{Hz} = 1 \text{ oscillation per second} = 1\,\text{s}^{-1}.$$

- **Angular frequency** It is the quantity obtained by multiplying frequency ν by a factor of 2π. It is denoted by ω. Thus, $\omega = 2\pi\nu = \frac{2\pi}{T}$. SI unit of $\omega = \text{rad s}^{-1}$.
- **Phase** The phase of vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant. It is denoted by ϕ.
- **Velocity in SHM** The velocity of particle executing SHM is defined as the time rate of change of its displacement at particular instant. Velocity, $v = \omega\sqrt{A^2 - y^2}$
  At mean position ($y = 0$) during its motion, $v = A\omega = v_{max}$ and at extreme positions ($y = \pm A$), $v = 0$.
  where, v_{max} velocity amplitude $= A\omega$ or Velocity, $v(t) = -\omega A\sin(\omega t + \phi)$
- **Acceleration in SHM** The acceleration of a SHM at an instant is defined as the time rate of change of velocity at that instant. Acceleration, $a = -\omega^2 y$
- The acceleration is also variable. At mean position ($y = 0$), acceleration, $a = 0$ and at extreme position ($y = \pm A$), acceleration is $a_{max} = \mp A\omega^2$.
  where, a_{max}, Acceleration amplitude $= A\omega^2$ or Acceleration, $a(t) = \omega^2 A\cos(\omega t + \phi)$

  Time Period in SHM Time period of SHM, $T = \frac{2\pi}{\omega} 2\pi\sqrt{\frac{|y|}{|a|}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

- **Energy in SHM** If a particle of mass m executes SHM, then at a displacement x from mean position, the particle possesses potential and kinetic energy.
  At any displacement x,
  (i) Potential energy, $U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}kx^2$
  (ii) Kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$
  (iii) Total energy, $E = U + K = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2$
- **Simple pendulum** A simple pendulum, in practice, consist of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string. The motion of a simple pendulum is simple harmonic whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and } \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Choose the correct option.
   (a) Every periodic motion is oscillatory
   (b) Every oscillatory motion is periodic
   (c) Both (a) and (b)
   (d) Neither (a) nor (b)

2. The function $\log_a(\omega t)$
   (a) is a periodic function
   (b) is a non-periodic function
   (c) could represents oscillatory motion
   (d) can represent circular motion.

3. Which of the following expression does not represent simple harmonic motion?
   (a) $x = A\cos\omega t + B\cos\omega t$
   (b) $x = A\cos(\omega t + \alpha)$
   (c) $x = B\sin(\omega t + \beta)$
   (d) $x = A\sin\omega t\cos^2\omega t$

4. A body performing simple harmonic motion is expressed by the displacement equation $y = 4\sin 2t$. The magnitude of maximum acceleration of the body is
   (a) 12 (b) 8
   (c) 16 (d) 20

5. Two simple harmonic motions of angular frequency 100 $rads^{-1}$ and 1000 $rads^{-1}$ have the same displacement amplitude. The ratio of their maximum acceleration is
   (a) $1:10$ (b) $1:10^2$
   (c) $1:10^3$ (d) $1:10^4$

6. A particle executing simple harmonic motion with an amplitude A and angular frequency ω. The ratio of maximum acceleration to the maximum velocity of the particle is
   (a) ωA (b) $\omega^2 A$
   (c) ω (d) $\frac{\omega^2}{A}$

7. For a SHM, if the maximum potential energy become double, choose the correct option.
   (a) Maximum kinetic energy will become double
   (b) The total mechanical energy will become double
   (c) Both (a) and (b)
   (d) Neither (a) nor (b)

8. The ratio of frequencies of two pendulums oscillating are 2 : 3, then their lengths are in ratio
   (a) $\sqrt{2/3}$ (b) $\sqrt{3/2}$
   (c) 4/9 (d) 9/4

9. The acceleration due to gravity on the surface of the moon is $1.7 ms^{-2}$. The time period of a simple pendulum on the moon, if its time period on the earth is 3.5 s is
   (a) 2.2 s (b) 4.4 s
   (c) 8.4 s (d) 16.8 s

10. A simple pendulum is mounted in a satellite revolving around the earth. Then, choose the correct one.
   (a) Frequency of oscillation is zero.
   (b) Gravitational acceleration is absent inside the satellite.
   (c) Both (a) and (b)
   (d) Neither (a) nor (b)

ASSERTION AND REASON

Directions (Q.Nos. 11-15) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

11. **Assertion** Vibrations and oscillations are two different types of motion.

    Reason For vibration frequency is more and for oscillation the frequency is less.

12. **Assertion** $x(t) = A\sin(\omega t)$ is periodic but cannot represent an oscillatory motion.

    Reason $\sin\theta$ is a sinusoidal periodic function.

13. **Assertion** $x = A\cos(\omega t)$ and $x = A\sin(\omega t)$ can represent same motion depending on initial position of particle.

Reason Phase constant depends on position of particle at $t = 0$.

14. **Assertion** In $X = A\cos(\omega t)$, ω represents the angular velocity of a particle moving on a circular path.

Reason The uniform circular motion and simple harmonic motion can be correlated.

15. **Assertion** For force (**F**) that represents spring force, $\mathbf{F} = -m\omega^2\mathbf{x}$.

Reason Magnitude of spring force is directly proportional to net stretch in the spring.

CASE BASED QUESTIONS

Directions (Q. Nos. 16-17) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

16. **Energy in SHM**

A particle executing SHM posseses both kinetic energy and potential energy. When a body is displaced from its equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back with a velocity, thus acquiring kinetic energy.

Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values

(i) In SHM,

(a) PE is stored due to elasticity of system
(b) KE is stored due to inertia of system
(c) Both KE and PE are stored by virtue of elasticity of system.
(d) Both (a) and (b)

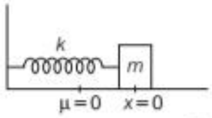
(ii) The expression for displacement of an object in SHM is $x = A\cos(\omega t)$. The potential energy at $t = T/4$ is

(a) $\frac{1}{2}kA^2$ (b) $\frac{1}{8}kA^2$
(c) $\frac{1}{4}kA^2$ (d) zero

(iii) For a SHM, if the maximum potential energy become double, choose the correct option.

(a) Maximum kinetic energy will become double
(b) The total mechanical energy will become double
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(iv) A block is in simple harmonic motion as shown in the figure on a frictionless surface. i.e. $\mu = 0$.
Choose the correct option.



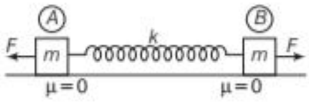
(a) The kinetic energy varies between a maximum value and zero.
(b) The potential energy varies between a maximum value and zero.
(c) Total energy remains constant.
(d) All of the above

(v) In simple harmonic motion, let the time period of variation of potential energy is T_1 and time period of variation of position is T_2, then relation between T_1 and T_2 is

(a) $T_1 = T_2$ (b) $T_1 = 2T_2$
(c) $2T_1 = T_2$ (d) None of these

17. **Spring-Mass System**

A system of two blocks is attached with a spring and stretched by two equal and opposite forces as shown in the figure. Let the maximum stretch of spring towards each block is $\frac{X}{2}$.



(i) The magnitude of spring force on block B is

(a) kx (b) $\frac{kx}{2}$
(c) $2kx$ (d) None of these

(ii) The equation of SHM for block B when F from both blocks is removed, is

(a) $F_{SHM} = -m\omega^2 x$ (b) $F_{SHM} = -\frac{m\omega^2 x}{2}$
(c) $F_{SHM} = -kx$ (d) None of these

(iii) The time period of the two blocks system is

(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$
(c) $2\pi\sqrt{\frac{m}{2k}}$ (d) $2\pi\sqrt{\frac{m}{4k}}$

(iv) In SHM,

(a) PE is stored due to elasticity of system
(b) KE is stored due to inertia of system
(c) Both KE and PE are stored by virtue of elasticity of system.
(d) Both (a) and (b)

(v) I. Time period of a spring-mass system depends on its amplitude.

II. Time period of a spring-mass system depends on its mass.

III. Time period of a spring-mass system depends on spring constant.

Choose the correct option regarding the above statements.

(a) I and II (b) I and III

(c) II and III (d) All of these

Answers

1. (b)	2. (b)	3. (d)	4. (c)	5. (b)
6. (c)	7. (c)	8. (d)	9. (c)	10. (c)
11. (d)	12. (d)	13. (a)	14. (a)	15. (b)
16. (i) (d)	(ii) (d)	(iii) (c)	(iv) (d)	(v) (c)
17. (i) (a)	(ii) (b)	(iii) (c)	(iv) (d)	(v) (c)

VERY SHORT ANSWER Type Questions

18. What is the period of each of the function, $\sec\omega t$ and $\operatorname{cosec}\omega t$?

19. Name the quantity which is conserved during the collision.

20. Two identical springs of force constant k each are connected in parallel. What will be the equivalent spring constant?

21. A pendulum clock is thrown out of an aeroplane. What will be its time period and how it will be effected in free fall?

22. How will a simple pendulum behave if it is taken to the moon?

23. What would be the effect on the time period if the amplitude of a simple pendulum increases?

24. A simple pendulum moves from one end to the other in $\frac{1}{4}$ second. What is its frequency?

25. Write the relation between time period T, displacement x and acceleration a of a particle in SHM.

26. Write the relation between acceleration, displacement and frequency of a particle executing SHM.

SHORT ANSWER Type Questions

27. What is meant by phase of an oscillating particle?

28. On what factors does the energy of a harmonic oscillator depend?

29. What would be the time period of a simple pendulum at the centre of the earth?

30. A simple harmonic motion of acceleration a and displacement x is represented by $a + 4\pi^2 x = 0$.

What is the time period of SHM?

LONG ANSWER Type I Questions

31. Write down the differential equation for SHM. Give its solution. Hence, obtain expression for the time period of SHM.

32. Obtain an expression for the velocity of a particle executing SHM, when is this velocity (i) maximum and (ii) minimum.

33. The relation between the acceleration a and displacement x of a particle executing SHM is

$a = -\left(\frac{p}{q}\right)y$, where p and q are constants.

LONG ANSWER Type II Questions

34. Draw the graphical representation of simple harmonic motion showing the
(i) displacement-time curve
(ii) velocity-time curve and
(iii) acceleration-time curve

35. Explain the relation in phase between displacement, velocity and acceleration in SHM, graphically as well as theoretically.

36. Define the terms harmonic oscillator, displacement, amplitude, cycle, time period, frequency, angular frequency and phase with reference to an oscillatory system.

37. Show that simple harmonic motion may be regarded as the projection of uniform circular motion along a diameter of the circle. Hence, derive an expression for the displacement of a particle in SHM.