
Sample Paper-05
Mathematics
Class – XI

Time allowed: 3 hours

M. M: 100

General Instructions:

- (i) All questions are compulsory.
 - (ii) This question paper contains 29 questions.
 - (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
 - (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
 - (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
 - (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.
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Section A

- 1. Identify a function $f(x)$ so that $f(x).f(y) = f(x+y)$
- 2. If $A = \{(x, y) : y = a^x, x \in R\}$ and $B = \{(x, y) : y = a^{-x}, x \in R\}$ then what is $(A \cap B)$
- 3. If R is a relation from a set A containing p elements to a set B containing q elements the find the number of subsets of $A \times B$
- 4. Check whether the given lines are parallel or perpendicular.

$$ax - by + c = 0 \quad \text{and} \quad \frac{ax}{2} - \frac{by}{2} + d = 0$$

Section B

- 5. Find the area of the triangle whose vertices are $(2,0), (5,3), (2,6)$
- 6. Write the equation of a circle with center $(0,0)$ and radius 5
- 7. Prove that $f'(a+b) = f'(a) + f'(b)$ when $f(x) = x^2$ and when $f(x) = x^3$
- 8. If α, β are the roots of the equation $x^2 + px + q = 0$ Find $\alpha^3 + \beta^3$.
- 9. A positive 3 digit number has its units digit zero. Find the probability that the number is divisible by 4.
- 10. Prove that $\tan(45 + x) = \sec 2x + \tan 2x$
- 11. Prove by mathematical induction that $n(n+1)$ is even
- 12. Find $n[(A \cup B \cup C)]$ if $n(A) = 4000$ $n(B) = 2000$ $n(C) = 1000$ and

$$n(A \cap B) = n(B \cap C) = n(A \cap C) = 400, n(A \cap B \cap C) = 200$$

Section C

- 13. Find the latus rectum, eccentricity and coordinates of the foci of the ellipse $x^2 + 3y^2 = k^2$
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14. Find the area of the circle passing through the points $(-8, 0), (0, 8), (12, 0)$
15. If S_1, S_2, S_3 are the sums of $n, 2n, 3n$ terms respectively of an AP prove that $S_3 = 3(S_2 - S_1)$
16. Find the least value of $f(x)$ if $f(x) = 3x^2 - 6x - 11$
17. Find $f(x) + f(1-x)$ if $f(x) = \frac{a^x}{a^x + \sqrt{a}}$
18. Prove that $\frac{\tan 2x \tan x}{\tan 2x - \tan x} = \sin 2x$
19. Find the limit $\lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$
20. Evaluate $x^3 + x^2 - 4x + 13$ when $x = 1+i$ and when $x = 1-i$
21. Prove that the roots of the equation $(x-\alpha)(x-\beta) = k^2$ is always real
22. If the roots of the equation $lx^2 + nx + n = 0$ are in the ratio $p:q$ then prove that
- $$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$$
23. Find $\lim_{x \rightarrow \pi} (\pi - x) \tan \frac{x}{2}$

Section D

24. If a, b, c are 3 consecutive integers prove that $(a-i)(a+i)(c+i)(c-i) = b^4 + 1$
25. Prove that $\frac{(1+i)^n}{(1-i)^{n-2}} = 2i^{n-1}$
26. Determine the coordinates of a point which is equidistant from the point $(1, 2)$ and $(3, 4)$ and the shortest distance from the line joining the points $(1, 2)$ and $(3, 4)$ to the required point is $\sqrt{2}$
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Answers

Section A

1. Solution :

$$f(x) = a^x$$

$$f(y) = a^y$$

$$f(x).f(y) = a^x.a^y = a^{x+y} = f(x).f(y)$$

2. Solution :

When $x = 0, y = 1$ in both cases. Hence

$$(A \cap B) = \{0, 1\}$$

3. Solution : 2^{pq}

4. Solution

They are parallel since

$$\begin{vmatrix} a & -b \\ \frac{a}{2} & \frac{-b}{2} \end{vmatrix} = 0$$

Section B

5. Solution

Area of a triangle

$$\frac{1}{2} \begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} = 9$$

6. Solution

$$x^2 + y^2 = 25$$

7. Solution

When $f(x) = x^2$

$$f(x) = x^2$$

$$f'(x) = 2x$$

$$f'(a+b) = 2(a+b)$$

$$f'(a) = 2a$$

$$f'(b) = 2b$$

$$f'(a) + f'(b) = 2(a + b)$$

$$= f'(a + b)$$

When $f(x) = x^3$

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f'(a + b) = 3(a + b)^2$$

$$f'(a) = 3a^2$$

$$f'(b) = 3b^2$$

$$f'(a) + f'(b) = 3(a^2 + b^2)$$

$$\neq f'(a + b)$$

8. Solution

$$\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] = -p[p^2 - 3q]$$

9. Solution

Total number of 3 digit numbers with 0 in units place = 90

The digits that can go into tens place for the number to be divisible by 4 = 0, 2, 4, 6, 8

100th place can be formed with any of the 9 digits excepting 0

Hence total number of 3 digits number divisible by 4 is $9 \times 5 = 45$

$$\text{Probability} = \frac{45}{90} = \frac{1}{2}$$

10. Solution

$$\begin{aligned} \tan(45 + x) &= \frac{1 + \tan x}{1 - \tan x} \\ &= \frac{\cos x + \sin x}{\cos x - \sin x} \\ &= \frac{\cos^2 x + \sin^2 x + 2 \sin x \cos x}{\cos^2 x - \sin^2 x} \\ &= \frac{1 + \sin 2x}{\cos 2x} = \sec 2x + \tan 2x \end{aligned}$$

11. Solution

$$P(n) = n(n+1)$$

$$P(1) = 2, \text{ even}$$

$$P(k) = k(k+1) \text{ let this be true}$$

$$P(k+1) = (k+1)(k+2)$$

$$= k^2 + 3k + 2$$

$$= k^2 + k + 2k + 2$$

$$= k(k+1) + 2(k+1) \text{ True}$$

12. Solution

$$n[(A \cup B \cup C)] = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n[(A \cup B \cup C)] = 4000 + 2000 + 1000 - 400 - 400 - 400 + 200$$

$$n[(A \cup B \cup C)] = 6000$$

Section C

13. Solution.

$$\frac{X^2}{k^2} + \frac{y^2}{\frac{k^2}{3}} = 1$$

$$\text{Latus rectum is} = \frac{\left(\frac{2k^2}{3}\right)}{k}$$

$$= \frac{2k}{3}$$

$$e = \sqrt{\frac{k^2 - \frac{k^2}{3}}{k^2}}$$

$$= \sqrt{\frac{2}{3}}$$

$$= \frac{\sqrt{6}}{3}$$

Coordinates of foci are $(ae, 0)$ and $(-ae, 0)$

$$\text{Coordinates are} = \left(\frac{\sqrt{6}}{3}k, 0\right) \text{ and } \left(-\frac{\sqrt{6}}{3}k, 0\right)$$

14. Solution

Slope of line AB joining the points $(-8, 0)$ and $(12, 0) = 0$

Its midpoint = $(2, 0)$

Equation to the line perpendicular to AB and passing through $(2, 0)$ is $x = 2$

Slope of line AC joining the points $(-8, 0)$ and $(0, 8) = 1$

Its midpoint = $(-4, 4)$

Equation to the line perpendicular to AC and passing through $(-4, 4)$ is $y = -x$

So the center of the circle will be the point of intersection of line AB and line AC . Center of circle at point $(2, -2)$

$$\text{Radius} = \sqrt{(2-0)^2 + (-2-8)^2} = \sqrt{104}$$

$$\text{Area} = 104\pi$$

15. Solution

$$2S_1 = n[2a + (n-1)d]$$

$$2S_2 = 2n[2a + (2n-1)d]$$

$$2S_3 = 3n[2a + (3n-1)d]$$

$$\frac{2S_1}{n} = 2a + (n-1)d$$

$$\frac{2S_3}{3n} = 2a + (n-1)d$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + d(n-1+3n-1)$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 4a + 2(2n-1)d$$

$$\frac{2S_1}{n} + \frac{2S_3}{3n} = 2 \cdot \frac{2S_2}{2n}$$

$$\frac{2S_3}{3n} = \frac{2S_2}{2n} - \frac{2S_1}{n}$$

$$\frac{2S_3}{3n} = \frac{4S_2}{2n} - \frac{4S_1}{2n}$$

$$S_3 = 3(S_2 - S_1)$$

16. Solution

$$f(x) = 3x^2 - 6x - 11$$

$$f(x) = 3(x^2 - 2x - \frac{11}{3})$$

$$f(x) = 3(x^2 - 2x + 1 - 1 - \frac{11}{3})$$

$$f(x) = 3[(x-1)^2 - \frac{11}{3} - 1]$$

$$f(x) = 3[(x-1)^2 - \frac{14}{3}]$$

$$f(x) = 3(x-1)^2 - 14$$

$f(x)$ will attain minimum when $x = 1$

Minimum value of $f(x) = -14$

17. Solution

$$f(x) = \frac{a^x}{a^x + \sqrt{a}}$$

$$f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}}$$

$$f(x) + f(1-x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{a^{1-x}}{a^{1-x} + \sqrt{a}} = 1$$

18. Solution

$$\begin{aligned} \frac{\tan 2x \tan x}{\tan 2x - \tan x} &= \frac{\frac{2 \tan x}{1 - \tan^2 x} \tan x}{\frac{2 \tan x}{1 - \tan^2 x} - \tan x} \\ &= \frac{2 \tan^2 x}{\tan x + \tan^2 x} \\ &= \frac{2 \tan x}{1 + \tan^2 x} \\ &= \sin 2x \end{aligned}$$

19. Solution

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!} &= \lim_{n \rightarrow \infty} \frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)!} \\ &= \lim_{n \rightarrow \infty} \frac{(n+3)}{(n+1)} \\ &= \lim_{n \rightarrow \infty} \frac{1 + \frac{3}{n}}{1 + \frac{1}{n}} \\ &= 1 \end{aligned}$$

20. Solution

Form a quadratic equation whose roots are

$$1+i \quad \text{and} \quad 1-i$$

The equation is

$$x^2 - 2x + 2 = 0$$

The given expression

$$x^3 + x^2 - 4x + 13 = x(x^2 - 2x + 2) + 3(x^2 - 2x + 2) + 7$$

$$x^3 + x^2 - 4x + 13 = x(0) + (0) + 7$$

$$x^3 + x^2 - 4x + 13 = 7$$

21. Solution

$$x^2 - (\alpha + \beta)x + \alpha\beta - k^2 = 0$$

Discriminant of the above quadratic is

$\{(\alpha + \beta)\}^2 - 4(\alpha\beta - k^2) = (\alpha - \beta)^2 + k^2$ is always positive and hence the roots are real.

22. Solution

Let the roots be

$p\alpha$ and $q\beta$

Then

$$p\alpha + q\alpha = -\frac{n}{l} \dots (1)$$

$$pq\alpha^2 = \frac{n}{l}$$

$$\alpha = \frac{\sqrt{n}}{\sqrt{l}} \times \frac{1}{\sqrt{pq}} \dots (2)$$

Hence substituting equation 2 in equation 1

$$(p+q) \frac{\sqrt{n}}{\sqrt{l}} \times \frac{1}{\sqrt{pq}} + \frac{n}{l} = 0$$

On simplifying ,

$$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$$

23. Solution

$$\lim_{x \rightarrow \pi} (\pi - x) \tan \frac{x}{2} = \lim_{x \rightarrow \pi} \frac{2(\pi - x)}{2} \cot \frac{\pi - x}{2}$$

$$= \lim_{x \rightarrow \pi} \frac{2(\pi - x)}{2} \frac{\cos \frac{\pi - x}{2}}{\sin \frac{\pi - x}{2}}$$

$$= \lim_{x \rightarrow \pi} 2 \frac{\cos \frac{\pi - x}{2}}{\frac{\sin \frac{\pi - x}{2}}{\frac{\pi - x}{2}}}$$

$$= \lim_{\frac{x - \pi}{2} \rightarrow 0} 2 \frac{\cos \frac{\pi - x}{2}}{\frac{\sin \frac{\pi - x}{2}}{\frac{\pi - x}{2}}}$$

$$= \lim_{\frac{x-\pi}{2} \rightarrow 0} 2 \frac{\cos \frac{x-\pi}{2}}{\sin \frac{x-\pi}{2}} = 2 \quad \text{since the limit of} \quad \frac{\sin \frac{x-\pi}{2}}{\frac{x-\pi}{2}} = 1$$

Section D

24. Solution

Let

$$a = x - 1$$

$$b = x$$

$$c = x + 1$$

Then

$$\begin{aligned} (x-1-i)((x-1+i)(x+1+i)(x+1-i)) &= \{(x-1)^2 - i^2\} \{(x+1)^2 - i^2\} \\ &= \{(x-1)^2 + 1\} \{(x+1)^2 + 1\} \\ &= \{(x-1)(x+1)\}^2 + (x-1)^2 + (x+1)^2 + 1 \\ &= (x^2 - 1)^2 + (x-1)^2 + (x+1)^2 + 1 \\ &= x^4 + 1 \\ &= b^4 + 1 \end{aligned}$$

25. Solution

Multiply both Numerator and denominator with $(1-i)^2$ Then

$$\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^n (1-i)^2}{(1-i)^n}$$

multiplying both Numerator & denominator with $(1+i)^n$

$$= \frac{(1+i)^n (-2i) (1+i)^n}{(1-i)^n (1+i)^n}$$

Simplifying

$$= \frac{\{(1+i)^2\}^n (-2i)}{(1-i^2)^n}$$

On expanding and simplifying

$$\begin{aligned}
 &= \frac{2^n i^n (-2)i}{2^n} \\
 &= -2i^{n+1} \\
 &= \frac{2(i)^{n+1}}{i^2} = 2i^{n-1}
 \end{aligned}$$

26. Solution

Let the point be $A(1,2)$ and $B(3,4)$

The mid-point of the line joining A and B is $C(2,3)$

$$\text{Slope of line AB} = \frac{4-2}{3-1} = 1$$

Let the required point be $D(\alpha, \beta)$

Then D must be a point on the line perpendicular to the line AB and passing through point C

$$\therefore \text{Slope of } CD = -1$$

Equation of CD

$$y - 3 = -1(x - 2)$$

$$x + y = 5$$

Equation of AB

$$y - 2 = 1(x - 1)$$

$$x - y + 1 = 0$$

The point $D(\alpha, \beta)$ must satisfy the equation

$$x + y = 5$$

$$\therefore \alpha + \beta = 5 \dots (1)$$

The perpendicular distance from (α, β) to AB is

$$\frac{\alpha - \beta + 1}{\sqrt{2}} = \sqrt{2}$$

$$\alpha - \beta = 1 \dots (2)$$

Solving equations 1 and 2

$$\alpha = 3, \beta = 2$$
