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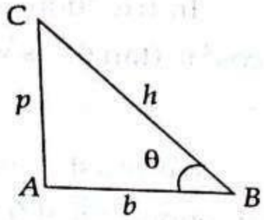
Trigonometric Ratio of Specific Angles

1. **Trigonometric Ratio :**

To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with $\angle A = 90°$.



We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively $\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB}$ and $\frac{BC}{AC}$. If $\angle B = \theta$ then these ratio are respectively called $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$ and $\text{cosec}\,\theta$. Clearly for the given angle θ, AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are as follows (see the given figure)

$$\sin\theta = \frac{p}{h} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}} = \frac{AC}{BC}$$

$$\cos\theta = \frac{b}{h} = \frac{\text{base}}{\text{hypotenuse}} = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}} = \frac{AB}{BC}$$

$$\tan\theta = \frac{p}{b} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta} = \frac{AC}{AB}$$

$$\cot\theta = \frac{b}{p} = \frac{\text{base}}{\text{perpendicular}} = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta} = \frac{AB}{AC}$$

$$\sec\theta = \frac{h}{b} = \frac{\text{hypotenuse}}{\text{base}} = \frac{\text{hypotenuse}}{\text{side adjacent to angle }\theta} = \frac{BC}{AB}$$

$$\text{cosec}\,\theta = \frac{h}{p} = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\text{hypotenuse}}{\text{side opposite to angle }\theta} = \frac{BC}{AC}$$

Clearly $\sin\theta$ and $\text{cosec}\,\theta$ are reciprocals to each other. Similarly $\cos\theta$ and $\sec\theta$ are reciprocals to each other while $\tan\theta$ and $\cot\theta$ are reciprocals to each other.

$\therefore$ $\sin\theta \cdot \text{cosec}\,\theta = 1$

$\sec\theta \cdot \cos\theta = 1$

$\tan\theta \cdot \cot\theta = 1$

Since $p^2 + b^2 = h^2$ i.e. $(perpendicular)^2 + (base)^2 = (hypotenuse)^2$, therefore we can find the other trigonometric ratio when any one of them is known.

Following relations among natural number will help in solving the problems on trigonometric ratio angle.

$3^2 + 4^2 = 5^2$, $\quad 6^2 + 8^2 = 10^2$

$5^2 + 12^2 = 13^2$, $\quad 10^2 + 24^2 = 26^2$

$8^2 + 15^2 = 17^2$, $\quad 16^2 + 30^2 = 34^2$

$20^2 + 21^2 = 29^2$ $\quad 9^2 + 40^2 = 41^2$ etc.

2. Meaning of $\sin^2\theta$, $\cos^2\theta$:

In trigonometric ratio $(\sin\theta)^2$ is written as $\sin^2\theta$, $(\cos\theta)^2$ is written as $\cos^2\theta$, $(\tan\theta)^3$ is written as $\tan^3\theta$ etc.

3. Value of some specific angle of trigonometrical (t)-ratio function :

We must learn the following table to solve the questions based on trigonometrical (t)-ratio angle 0°, 30°, 45°, 60°, 90°.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefine
$\cot\theta$	undefine	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefine
$\text{cosec}\,\theta$	undefine	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Tricks to learn the table are as follows :

3.1 Values of $\sin\theta$ are respectively $\sqrt{\frac{0}{4}}, \sqrt{\frac{1}{4}}, \sqrt{\frac{2}{4}}, \sqrt{\frac{3}{4}}, \sqrt{\frac{4}{4}}$

3.2 Value of $\cos\theta$ are in reverse order to that of $\sin\theta$.

Thus values of $\cos\theta$ are respectively $\sqrt{\frac{4}{4}}, \sqrt{\frac{3}{4}}, \sqrt{\frac{2}{4}}, \sqrt{\frac{1}{4}}, \sqrt{\frac{0}{4}}$

3.3 Divide $\sin\theta$ by $\cos\theta$ to get the values of $\tan\theta$.

3.4 Divide $\cos\theta$ by $\sin\theta$ to get the values of $\cot\theta$.

3.5 Values of $\sec\theta$ are reciprocals to the values of $\cos\theta$ i.e., values of $\sec\theta$ are respectively $\sqrt{\frac{4}{0}}, \sqrt{\frac{4}{1}}, \sqrt{\frac{4}{2}}, \sqrt{\frac{4}{3}}, \sqrt{\frac{4}{4}}$ here $\sqrt{\frac{4}{0}}$ is undefined.

3.6 Values of $\text{cosec}\,\theta$ are reciprocals to the values of $\sin\theta$.

We must note that values of $\sin\theta$, $\tan\theta$ and $\sec\theta$ are increasing in 0° to 90°. While values of '*c*' functions i.e., $\cos\theta$, $\cot\theta$ and $\text{cosec}\,\theta$ are decreasing in 0° to 90°.

4. Minimum and Maximum value of t-ratio functions when $0 \le \theta \le 90°$.

From t-ratio table it is clear that for $0 \le \theta \le 90°$.

4.1 Maximum value of each of $\sin\theta$ and $\cos\theta$ is 1.

4.2 Minimum value of each of $\sin\theta$ and $\cos\theta$ is 0.

4.3 Maximum value of $\tan\theta$ and $\cot\theta$ are not defined.

4.4 Minimum value of $\tan\theta$ and $\cot\theta$ are 0.

4.5 Maximum value of $\sec\theta$ and $\text{cosec}\,\theta$ are undefined.

4.6 Minimum value of each of $\sec\theta$ and $\text{cosec}\,\theta$ is 1.

5. Some important tricks for Maximum and Minimum values :

5.1 Maximum and Minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$, where θ be any angle.

5.2 If $a > b$ then maximum and minimum value of $a\cos^2\theta + b\sin^2\theta$ are respectively a and b. If $a < b$ then maximum and minimum value of $a\cos^2\theta + b\sin^2\theta$ are respectively b and a.

[i.e., which ever is greater between a and b is the greater value and smaller one is the least value].

5.3 Minimum value of $a\tan^2\theta + b\cot^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.4 Minimum value of $a\sec^2\theta + b\cos^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.5 Minimum and Maximum value of $a\,\text{cosec}^2\theta + b\sin^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.6 If m and n are positive integers then $(\sin\theta)^n \le \sin\theta \le 1$ and $(\cos\theta)^m \le \cos\theta \le 1$

[**Explanation :** $\because 0 \le \sin^2\theta \le 1$ i.e., $\sin^2\theta$ is a proper fraction, value of $(\sin^{2n}\theta)$ decreases as its power increases; e.g. when $\theta = 30°$

$\sin\theta = \frac{1}{2}, (\sin\theta)^2 = \frac{1}{4}, (\sin\theta)^3 = \frac{1}{8}\ldots\ldots$

Clearly $\sin^3\theta < \sin^2\theta < \sin\theta$]

5.7 $\because$ From above mentioned point we can say that $(\sin^2\theta)^n \le \sin^2\theta$ and $(\cos^2\theta)^m \le \cos^2\theta$,

adding, $\sin^{2n}\theta + \cos^{2m}\theta \le \sin^2\theta + \cos^2\theta$

or, $\boxed{\sin^{2n}\theta + \cos^{2m}\theta \le 1}$

e.g., $\sin^4\theta + \cos^4\theta \le 1$,

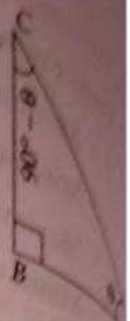
$\sin^6\theta + \cos^6\theta \le 1$,

$\sin^8\theta + \cos^{10}\theta \le 1$ etc.

6. Complementary Angle.

For a given angle θ its complementary angle is $(90^\circ - \theta)$.

From definition,



$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\text{and } \cos(90^\circ - \theta) = \frac{\text{side along with angle } (90^\circ - \theta)}{\text{hypotenuse}} = \frac{BC}{AC}$$

$\therefore$ $\boxed{\sin\theta = \cos(90^\circ - \theta)}$

Similarly, we can prove that

$\therefore$ $\boxed{\cos\theta = \sin(90^\circ - \theta)}$, $\tan(90^\circ - \theta) = \cot\theta$, $\cot(90^\circ - \theta) = \tan\theta$,

$\sec(90^\circ - \theta) = \text{cosec}\,\theta$, $\text{cosec}\,(90^\circ - \theta) = \sec\theta$

7. Trigonometric Inequalities :

If $0 < \theta < 90^\circ$ then

7.1 Value of $\sin\theta$ increase as θ increase.

Value of $\tan\theta$ increase as θ increase.

Value of $\sec\theta$ increase as θ increase.

Thus, $\sin\theta$, $\tan\theta$ and $\sec\theta$ follow the same sign of inequality i.e.,

$$\sin\theta_1 > \sin\theta_2 \Rightarrow \theta_1 > \theta_2$$
$$\tan\theta_1 > \tan\theta_2 \Rightarrow \theta_1 > \theta_2$$
$$\sec\theta_1 > \sec\theta_2 \Rightarrow \theta_1 > \theta_2$$

7.2 Value of $\cos\theta$ decreases as θ increases.

Value of $\cot\theta$ decreases as θ increases.

Value of $\text{cosec}\,\theta$ decreases as θ increases.

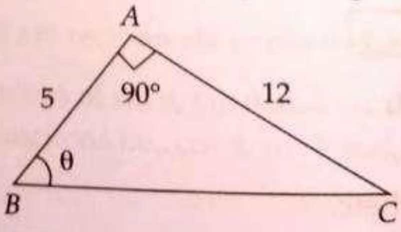
Thus $\cos\theta$, $\cot\theta$ and $\text{cosec}\,\theta$ follow the opposite sign of inequality i.e.

$$\cos\theta_1 > \cos\theta_2 \Rightarrow \theta_1 < \theta_2$$
$$\cot\theta_1 > \cot\theta_2 \Rightarrow \theta_1 < \theta_2$$
$$\text{cosec}\,\theta_1 > \text{cosec}\,\theta_2 \Rightarrow \theta_1 < \theta_2$$

Trick : Sign of inequality reverse in '*c*' function i.e., in $\cos\theta$, $\cot\theta$ and cosecθ

Solved Example

1. Write all the six t-ratios value in the given figure :



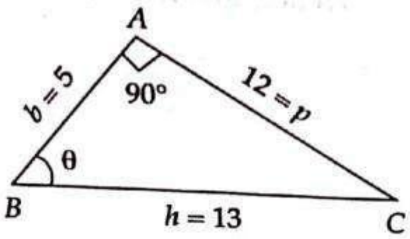
...on : Given ΔABC is, a right angle triangle with $\angle A = 90°$,

Let $AC = 12 = p$ and $AB = 5 = b$

Then from Pythagoras theorem,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13$$



Here side opposite to θ is AC which is p.

Side adjacent to θ is AB, which is b.

Side opposite to right angle is BC, which is hypotenuse h.

$\therefore \sin\theta = \frac{p}{h} = \frac{12}{13}$ $\qquad$ $\text{cosec}\,\theta = \frac{h}{p} = \frac{13}{12}$

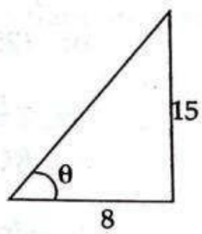
$\cos\theta = \frac{b}{h} = \frac{5}{13}$ $\qquad$ $\sec\theta = \frac{h}{b} = \frac{13}{5}$

$\tan\theta = \frac{p}{b} = \frac{12}{5}$ $\qquad$ $\cot\theta = \frac{b}{p} = \frac{5}{12}$

If $15\cot\theta = 8$ then calculate the remaining trigonometric ratio.

...tion : Given, $\cot\theta = \frac{8}{15} = \frac{b}{p}$

Let $b = 8k$ and $p = 15\,k$



From Pythagoras Theorem, $h^2 = p^2 + b^2 = (15\,k)^2 + (8\,k)^2$

or, $h^2 = 225\,k^2 + 64k^2 = 289\,k^2$

or, $h = \sqrt{289k^2} = 17\,k$

Hence, $\sin\theta = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$ $\qquad$ $\cos\theta = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$

$\tan\theta = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$ $\qquad$ $\sec\theta = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$

$\text{cosec}\,\theta = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$

If $\tan\theta = \frac{q}{p}$ then find the value of $\frac{p\sin\theta + q\cos\theta}{p\cos\theta + q\sin\theta}$

...lution : $\tan\theta = \frac{q}{p} = \frac{\text{perpendicular}}{\text{base}}$

Let perpendicular $= qk$ and base $= pk$ then

$$\text{hypotenuse} = \sqrt{(\text{perpendicular})^2 + (\text{base})^2} = \sqrt{q^2k^2 + p^2k^2} = \sqrt{q^2 + p^2}\,k$$

$$\therefore \sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{qk}{\sqrt{q^2+p^2}\,k} = \frac{q}{\sqrt{q^2+p^2}}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{pk}{k\sqrt{q^2+p^2}} = \frac{p}{\sqrt{q^2+p^2}}$$

Hence, given expression $= \dfrac{p\sin\theta + q\cos\theta}{p\cos\theta + q\sin\theta}$

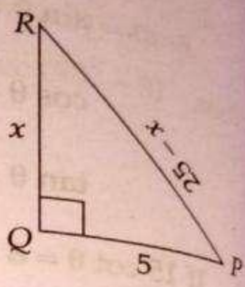
$$= \frac{\dfrac{pq}{\sqrt{q^2+p^2}} + \dfrac{qp}{\sqrt{q^2+p^2}}}{p\cdot\dfrac{p}{\sqrt{q^2+p^2}} + q\cdot\dfrac{q}{\sqrt{q^2+p^2}}}$$

$$= \frac{pq+qp}{p^2+q^2} = \frac{2pq}{p^2+q^2}$$

4. In ΔPQR, with $\angle Q$ at right angle, given that $PR + QR = 25$ cm and $PQ = 5$ cm. Find the value of :

(i) $\sin P$, $\cos P$ and $\tan P$.

(ii) $\sec R - \cot P$.



Solution : Given, $\angle Q = 90°$

$PQ = 5$ cm, $PR + QR = 25$ cm

Let $QR = x$ cm then $PR = (25 - x)$ cm

From Pythagoras theorem, $RP^2 = RQ^2 + QP^2$

or, $(25 - x)^2 = x^2 + 5^2$ or, $625 - 50x + x^2 = x^2 + 25$

or, $-50x = 25 - 625 = -600$ or, $x = \dfrac{-600}{-50} = 12$

$\therefore$ $RQ = 12$ cm and $RP = (25 - 12)$ cm $= 13$ cm

$$\therefore \quad \sin P = \frac{\text{side opposite to } \angle P}{\text{hypotenuse}} = \frac{RQ}{RP} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{RP} = \frac{5}{13}$$

$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side associated with } \angle P} = \frac{RQ}{PQ} = \frac{12}{5}$$

$$\cot P = \frac{\text{side associated with } \angle P}{\text{side opposite to } \angle P} = \frac{PQ}{RQ} = \frac{5}{12}$$

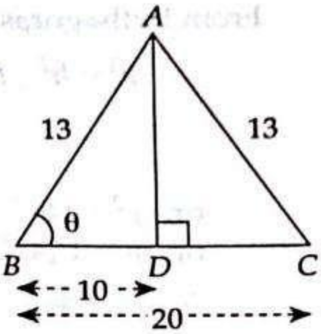
$$\sec R = \frac{\text{hypotenuse}}{\text{side associated to } \angle R} = \frac{RP}{RQ} = \frac{13}{12}$$

$$\therefore \quad \sec R - \cot P = \frac{13}{12} - \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$$

5. ABC is an isosceles triangle with $AB = AC = 13$ cm and $BC = 20$ cm. If $\angle ABC = \theta$ then find the value of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$ and cosecθ.

Solution : In ΔABC given that, $AB = AC = 13$, $BC = 20$ and $\angle ABD = \theta$ (see the figure)

Draw $AD \perp$r BC.



Then $\angle ADB = \angle ADC = 90°$

and $BD = DC = 10$ cm

Clearly $\angle ABC = \angle ABD = \theta$

Using Pythagoras theorem in ΔABD.

$$AD^2 + BD^2 = AB^2$$

or, $AD^2 + 10^2 = 13^2$

or, $AD^2 = 13^2 - 10^2 = 169 - 100 = 69$

or, $AD = \sqrt{69}$

Now, In ΔABD,

$$\sin\theta = \frac{AD}{AB} = \frac{\sqrt{69}}{13} \qquad \cos\theta = \frac{BD}{AB} = \frac{10}{13}$$

$$\tan\theta = \frac{AD}{BD} = \frac{\sqrt{69}}{10} \qquad \cot\theta = \frac{BD}{AD} = \frac{10}{\sqrt{69}}$$

$$\sec\theta = \frac{AB}{BD} = \frac{13}{10} \qquad \operatorname{cosec}\theta = \frac{AB}{AD} = \frac{13}{\sqrt{69}}$$

6. $PQRS$ is a rhombus whose diagonals are $PR = 6$ cm and $QS = 8$ cm. If O is the point of intersection of diagonals and $\angle PQO = \theta$ then find the value of $\sin\theta$, $\tan\theta$ and $\sec\theta$.

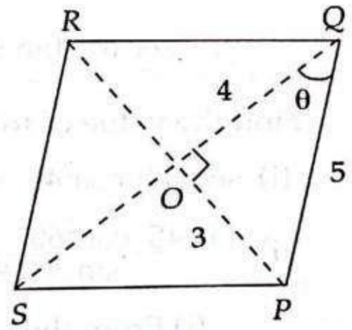
Solution : We know that diagonals of a rhombus intersect at right angle.

Hence in ΔPOQ (see the figure)

$$\angle POQ = 90°,\ OQ = \frac{QS}{2} = 4 \text{ cm and } OP = \frac{PR}{2} = 3 \text{ cm}$$

Using Pythagoras Theorem,

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{3^2 + 4^2} = 5$$



In triangle POQ,

$$\sin\theta = \frac{OP}{PQ} = \frac{3}{5}$$

$$\tan\theta = \frac{OP}{OQ} = \frac{3}{4}$$

$$\sec\theta = \frac{PQ}{OQ} = \frac{5}{4}$$

7. If $\sec\theta = x + \frac{1}{4x}$ then prove that

$$\sec\theta + \tan\theta = 2x \text{ or } \frac{1}{2x}$$

Solution : Given, $\sec\theta = x + \frac{1}{4x} = \frac{4x^2 + 1}{4x} = \frac{h}{b}$

Let $h = (4x^2 + 1)k$ and $b = (4x)k$

From Pythagoras theorem,

$$p^2 = h^2 - b^2 = (4x^2+1)^2 k^2 - (16x^2)k^2$$
$$= (16x^4 + 8x^2 + 1)k^2 - (16x^2)k^2$$
$$= (16x^4 - 8x^2 + 1)k^2$$

or, $p^2 = (4x^2-1)^2k^2$

or, $p = \pm(4x^2-1)k$ or, $(1-4x^2)k$

$\therefore$ $p = (4x^2-1)k$

[It must be noted that if $x^2 > \frac{1}{4}$ then $p = (4x^2-1)k$ and if $x^2 < \frac{1}{4}$ then $p = (1-4x^2)k$ as p is always a positive quantity]

If $p = (4x^2-1)k$ then

$$\sec\theta + \tan\theta = \frac{h}{b} + \frac{p}{b} = \frac{h+p}{b}$$
$$= \frac{(4x^2+1)k + (4x^2-1)k}{(4x)k}$$
$$= \frac{(4x^2+1+4x^2-1)k}{(4x)k} = \frac{8x^2}{4x} = 2x$$

If $p = (1-4x^2)k$ then

$$\sec\theta + \tan\theta = \frac{h}{b} + \frac{p}{b} = \frac{h+p}{b}$$
$$= \frac{(4x^2+1)k - (4x^2-1)k}{4xk}$$
$$= \frac{(4x^2+1-4x^2+1)k}{4xk} = \frac{2}{4x} = \frac{1}{2x}$$

$\therefore$ $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$; **Proved.**

8. Find the value of following expressions :

(i) $\sec^2 60° \cos^2 45° - \operatorname{cosec}^2 30°$

(ii) $\dfrac{\tan 45° \cot^2 60° + \tan^2 30° \cot 45°}{\sin 30° + \cos 60°}$

Solution : (i) From the t-ratio table,

$$\sec 60° = 2, \cos 45° = \frac{1}{\sqrt{2}}, \operatorname{cosec} 30° = 2$$

$$\therefore \sec^2 60° \cos^2 45° - \operatorname{cosec}^2 30° = 2^2\left(\frac{1}{\sqrt{2}}\right)^2 - 2^2 = \frac{4}{2} - 4 = -2$$

(ii) We know that

$\tan 45° = 1$, $\cot 60° = \frac{1}{\sqrt{3}}$, $\tan 30° = \frac{1}{\sqrt{3}}$, $\cot 45° = 1$, $\sin 30° = \frac{1}{2}$ and $\cos 60° = \frac{1}{2}$

$\therefore \dfrac{\tan 45° \cot^2 60° + \tan^2 30° \cot 45°}{\sin 30° + \cos 60°}$

$$= \frac{1\cdot\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\cdot 1}{\frac{1}{2}+\frac{1}{2}} = \frac{\frac{1}{3}+\frac{1}{3}}{1} = \frac{2}{3}$$

9. Find the acute angle θ if $4\sin^2\theta = 3$.

Solution : Given that $4\sin^2\theta = 3$

or, $\sin^2\theta = \frac{3}{4}$

or, $\sin\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ $\therefore \theta = 60°$

10. Find the maximum value of $\frac{1}{\operatorname{cosec}\theta}$

Solution : $\because \frac{1}{\operatorname{cosec}\theta} = \sin\theta$ and maximum value of $\sin\theta$ is 1.

$\therefore$ Maximum value of $\frac{1}{\operatorname{cosec}\theta}$ is 1.

11. Prove that minimum value of $a\tan^2\theta + b\cot^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities. Find the minimum value of $16\tan^2\theta + 9\cot^2\theta$.

Solution : $a\tan^2\theta + b\cot^2\theta = (\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2 + 2\sqrt{a}\sqrt{b}\tan\theta\cot\theta$

$= (\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2 + 2\sqrt{ab}$ $(\because \tan\theta\cdot\cot\theta = 1)$

But $(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2$ is either 0 or greater than zero.

$\therefore$ $a\tan^2\theta + b\cot^2\theta \geq 0 + 2\sqrt{ab}$

or, $a\tan^2\theta + b\cot^2\theta \geq 2\sqrt{ab}$

Since value of $a\tan^2\theta + b\cot^2\theta$ is greater than or equal to $2\sqrt{ab}$, its minimum value is $2\sqrt{ab}$.

[A special comment : Do not write the given expression as $(\sqrt{a}\tan\theta + \sqrt{b}\cot\theta)^2 - 2\sqrt{ab}\tan\theta\cot\theta$.

In this situation minimum value of, $\sqrt{a}\tan\theta + \sqrt{b}\cot\theta$ cannot be zero.

Second part : Minimum value of $16\tan^2\theta + 9\cot^2\theta$

$= 2\sqrt{16\times 9} = 2\sqrt{144} = 2\times 12 = 24$

12. Find the difference between square of greatest and least value of $15\cos\theta - 8\sin\theta + 5$.

Solution : We know that maximum and minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$.

Here, $a = 15$, $b = 8$

$\therefore \quad \sqrt{a^2+b^2} = \sqrt{15^2+8^2} = 17$

Hence, maximum value = 17 + 5 = 22

and minimum value = – 17 + 5 = – 12

∴ Difference between maximum and minimum value

$= (22)^2 - (-12)^2 = 22^2 - 12^2 = (22 + 12)(22 - 12) = 34 \times 10 = 340$

13. If $\tan 3\theta = \cot(75° - 2\theta)$ then find the value of $\sin 4\theta$.

Solution : $\tan 3\theta = \cot(75° - 2\theta) = \tan(90° - (75° - 2\theta))$

or, $3\theta = 15° + 2\theta$ or, $\theta = 15°$

$\therefore \quad \sin 4\theta = \sin 60° = \frac{\sqrt{3}}{2}$

14. Choose the correct statements among following :

(a) $\cos 40° > \cos 70°$

(b) $\sin 35° > \sin 65°$

(c) $\tan 45° < \tan 46°$

(d) $\cot 40° < \cot 39°$

(e) $\sec 20° > \sec 40°$

(f) $\text{cosec}\,20° < \text{cosec}\,30°$

Solution : When $0 < \theta < 90°$,

(a) Value of $\cos\theta$ decreases as θ increases, hence, $40° < 70° \Rightarrow \cos 40° > \cos 70°$. The statement is true.

(b) As θ increases, value of $\sin\theta$ also increase, hence $65° > 35°$ $\Rightarrow \sin 65° > \sin 35°$. Hence, the statement is false.

(c) As θ increases value of $\tan\theta$ increases, hence $45° < 46°$ $\Rightarrow \tan 45° < \tan 46°$. Hence, given statement is true.

(d) As θ increases, value of $\cot\theta$ decreases, hence $40° > 39°$ $\Rightarrow \cot 40° < \cot 39°$. Hence, the statement is true.

(e) As θ increases, value of $\sec\theta$ increases hence $20° < 40°$ $\Rightarrow \sec 20° < \sec 40°$. Hence, the statement is false.

(f) As θ increases, value of $\text{cosec}\,\theta$ decreases, Hence $20° < 30°$ $\Rightarrow \text{cosec}\,20° > \text{cosec}\,30°$. Hence, given statement is false.

Exercise–10A

1. If $\tan x = \frac{3}{4}, 0 < x < 90°$ then what is the value of $\sin x \cos x$?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

2. Which of the following is true ?

(a) $\tan x > 1; 45° < x < 90°$

(b) $\sin x > \frac{1}{2}; 0° < x < 30°$

(c) $\cos x > \frac{1}{2}; 60° < x < 90°$

(d) For some value of x in $30° < x < 45°$, $\sin x = \cos x$

3. **Statement (A)** : $\sin 1° < \cos 1°$

Reason (R) : $\sin\theta < \cos\theta$ When $0° < \theta < 90°$ then

(a) Both A and R are correct and R is the correct explanation of A.

(b) Both A and R are correct but R is not a correct explanation of A.

(c) A is correct, R is incorrect.

(d) A is incorrect, R is correct.

4. If $\cos A = \frac{5}{13}$ then what is the value of $\frac{\sin A - \cot A}{2\tan A}$?

(a) $\frac{395}{3644}$ (b) $\frac{395}{3844}$ (c) $\frac{395}{3744}$ (d) $\frac{385}{3744}$

5. If $\sin x = \cos y$ where x and y are acute angle then what is the relation between x and y ?

(a) $x - y = \frac{\pi}{2}$ (b) $x + y = \frac{3\pi}{2}$ (c) $x + y = \frac{\pi}{2}$ (d) $x + y = \frac{\pi}{4}$

6. In a right angled triangle base $BC = 15$ cm and $\sin B = \frac{4}{5}$, then what is the length of hypotenuse AB ?

(a) 25 cm (b) 20 cm (c) 5 cm (d) 4 cm

7. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then value of $\tan\theta$ is

(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{2mn}{m^2+n^2}$ (c) $\frac{m^2-n^2}{2mn}$ (d) $\frac{m^2+n^2}{2mn}$

8. If $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$, then value of x is

(a) 15° (b) 30° (c) 45° (d) 60°

9. If $1 + \tan\theta = \sqrt{2}$, then value of $\cot\theta - 1$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$

10. If $\sin(x + 54°) = \cos x$, where $0 < x$, $x + 54° < 90°$ then what is the value of x?

(a) 54° (b) 36° (c) 27° (d) 18°

11. If $x\cos 60° + y\cos 0° = 3$ and $4x\sin 30° - y\cot 45° = 2$, then what is the value of x ?

(a) –1 (b) 0 (c) 1 (d) 2

12. If $\cos x + \cos^2 x = 1$ then what is the value of $\sin^2 x + \sin^4 x$?

(a) 0 (b) 1 (c) 2 (d) 4

13. If $x + y = 90°$ and $\sin x : \sin y = \sqrt{3} : 1$ then $x : y$ equals,

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 3 : 2

14. If $0 \le x \le \frac{\pi}{2}$ then which one of the following is always true ?

(a) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x > \frac{1}{2}$ (b) $\sin^2 x > \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$

(c) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$

(d) At least one of $\sin^2 x$, $\cos^2 x$ is less than 1.

15. If $p = \tan^2 x + \cot^2 x$ then which one is true ?

(a) $p \le 2$ (b) $p \ge 2$ (c) $p < 2$ (d) $p > 2$

16. Value of $\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}$ is

(a) -1 (b) 0 (c) 1 (d) 2

17. If $0 < x < 45^\circ$ and $45^\circ < y < 90^\circ$, then which of the following is true.

(a) $\sin x = \sin y$ (b) $\sin x < \sin y$ (c) $\sin x > \sin y$ (d) $\sin x \le \sin y$

18. What is the value of $\sin^3 60^\circ \cot 30^\circ - 2\sec^2 45^\circ + 3\cos 60^\circ \tan 45^\circ - \tan^2 60^\circ$?

(a) $\frac{35}{8}$ (b) $\frac{-35}{8}$ (c) $\frac{-11}{8}$ (d) $\frac{11}{8}$

19. If $\tan\theta = \frac{p}{q}$, then what is the value of $\frac{p\sec\theta - q\,\text{cosec}\,\theta}{p\sec\theta + q\,\text{cosec}\,\theta}$?

(a) $\frac{p-q}{p+q}$ (b) $\frac{q^2-p^2}{q^2+p^2}$ (c) $\frac{p^2-q^2}{q^2+p^2}$ (d) 1

20. Value of $\text{cosec}^2\theta - 2 + \sin^2\theta$ is always

(a) less than zero (b) non negative

(c) zero (d) 1

21. If $\cot\theta = \frac{2xy}{x^2-y^2}$, then which one is equal to $\cos\theta$?

(a) $\frac{x^2-y^2}{x^2+y^2}$ (b) $\frac{x^2+y^2}{x^2-y^2}$ (c) $\frac{2xy}{x^2+y^2}$ (d) $\frac{2xy}{\sqrt{x^2+y^2}}$

22. For what value of θ $(\sin\theta + \text{cosec}\,\theta) = 2.5$, where $0 < \theta \le 90^\circ$?

(a) 30° (b) 45° (c) 60° (d) 90°

23. If $0 < \theta < \phi < 90^\circ$, then which one of the following is true ?

(a) $(\sin\theta + \cos\theta)^2 > 2$ (b) $(\sin^2\theta + \cos^2\phi) \le 2$

(c) $(\sin^2\theta + \cos^2\phi) < 2$ (d) $(\sin^2\theta + \cos^2\phi) > 2$

24. If $\tan\theta = \frac{2t}{1-t^2}$ then $\sin\theta + \cos\theta$ equals

(a) $\frac{t^2-1}{t^2+1}$ (b) $\frac{t^2-2t-1}{t^2+1}$ (c) $\frac{(t-1)^2}{t^2+1}$ (d) $\frac{1+2t-t^2}{1+t^2}$

25. If $0 \le \theta < \frac{\pi}{2}$ and $p = \sec^2\theta$ then which one of the following is true ?

(a) $p < 1$ (b) $p = 1$ (c) $p > 1$ (d) $p \ge 1$

26. In a ΔABC, $\angle ABC = 90^\circ$, $\angle ACB = 30^\circ$, $AB = 5$ cm. What is the length of AC?

(a) 10 cm (b) 5 cm (c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm

If $0 \le \theta \le \frac{\pi}{2}$ and $\cos\theta + \sqrt{3}\sin\theta = 2$, then what is the value of θ ?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Suppose ABC is a right angled triangle with right angled at C. If length of sides opposite to angle A, B, C are respectively u, v and w then $\tan A + \tan B$ equals

(a) $\frac{u^2}{vw}$ (b) 1 (c) $u + v$ (d) $\frac{w^2}{uv}$

ABC is a right angled triangle with right angle at A. If $\tan B = \frac{1}{\sqrt{3}}$, and which one is the form of a hypotenuse for real k ?

(a) $3k$ (b) $2k$ (c) $5k$ (d) $9k$

If α is an acute angle and $\sin\alpha = \sqrt{\frac{x-1}{2x}}$ then $\tan\alpha$ is equal to which of the following ?

(a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$

(c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$

If θ is in first quadrant and $\cos\theta \ge \frac{1}{2}$ then which one of the following is true ?

(a) $\theta \le \frac{\pi}{3}$ (b) $\theta \ge \frac{\pi}{3}$ (c) $\theta \le \frac{\pi}{6}$ (d) $\theta \ge \frac{\pi}{6}$

What is the value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ$?

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

If $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$, then among the following which is/are true ?

I. $\sin A + \sin B = \cos A + \cos B$

II. $\tan A + \tan B = \cot A + \cot B$

Use the alternative given below to get the correct answer

(a) only I (b) only II

(c) both I and II (d) Neither I nor II

If $\sin 17^\circ = \frac{a}{b}$ then value of $\sec 17^\circ - \sin 73^\circ$ is

(a) $\frac{a}{b\sqrt{a^2+b^2}}$ (b) $\frac{b^2}{a\sqrt{b^2-a^2}}$

(c) $\frac{a^2}{b\sqrt{b^2-a^2}}$ (d) 0

5. Consider the Earth as a sphere of radius R, radius of circle at latitude 40°S is

(a) $R\cos 40^\circ$ (b) $R\sin 80^\circ$ (c) $R\sin 40^\circ$ (d) $R\tan 40^\circ$

36. If $\frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3}$ and $0° < \theta < 90°$ then what is the value of $\tan\theta$?

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

37. Choose the correct statement :

(a) $\sin 40° > \sin 20°$ (b) $\sin 40° > \sin 50°$

(c) $\cos 20° > \cos 10°$ (d) Both (a) and (c) are correct

38. Given below are respectively base and hypotenuse of four right angle triangles :

1 and $\sqrt{5}$, 2 and $\sqrt{13}$, 3 and 5, 4 and $\sqrt{41}$

$\theta_1, \theta_2, \theta_3, \theta_4$ are respectively angle included between them. What are the increasing order of these values.

1. $\sin\theta_1$ 2. $\tan\theta_2$ 3. $\cos\theta_3$ 4. $\sec\theta_4$

Choose the correct code among following :

Code :

(a) 4 – 1 – 2 – 3 (b) 1 – 4 – 3 – 2 (c) 3 – 1 – 2 – 4 (d) 3 – 1 – 4 – 2

39. Consider the following statements about the expression

$\sin^3\theta + 2\sin^2\theta + 3\sin\theta$

1. For any $\theta \in R$, maximum value of this expression is 6.
2. For any $\theta \in R$, value of this expression cannot be zero.

Among above statement which is/are true ?

(a) only 1 (b) only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

40. Consider right angled ΔABC with $\angle B = 90°$. If $\angle ACB = 60°$, then $AB : BC : CA$ equals

(a) $\sqrt{3}:1:2$ (b) $1:\sqrt{3}:2$ (c) $1:1:\sqrt{2}$ (d) $\sqrt{2}:1:\sqrt{3}$

41. In ΔABC, $\angle ABC = 60°$ and AD is perpendicular from A to BC. If $AB = x$ and $AC = \frac{3x}{2}$ the CD is equal to which of the following ?

(a) $\frac{x}{2}$ (b) $\sqrt{\frac{3}{2}}\,x$ (c) $\frac{3x}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2x}$

42. If $0° \le \theta \le 90°$ then for any value of θ which one is correct ?

(a) $\sin\theta = \sqrt{2}$ (b) $\sin\theta + \cos\theta = 2$

(c) $\sin\theta + \cos\theta = 0$ (d) $\sin\theta - \cos\theta = 1$

43. If θ is acute then value of $\sin\theta + \cos\theta$ will be

(a) less than 1 (b) equal to 2

(c) more than 1 (d) more than one and less than 2

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44. If $\sin\theta + \operatorname{cosec}\theta = 2$ then value of $\sin^4\theta + \cos^4\theta$?

(a) 2 (b) 2^2 (c) 2^3 (d) 1

If $\sec\theta = \frac{13}{5}$, then what is the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$?

(a) 1 (b) 2 (c) 3 (d) 4

If $0° \le x \le 90°$ and $\sin x + \sqrt{3}\cos x = 1$, then what is the value of x ?

(a) 30° (b) 45° (c) 60° (d) 90°

In a right angled ΔABC if $\angle B = 90°$, $AC = 2\sqrt{5}$ and $AB - BC = 2$ then what is the value of $\cos^2 A - \cos^2 C$?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{1}{\sqrt{5}}$

Statement (A) : If $\tan\theta + \cot\theta = 2$, then for all $n \in N$, $\tan^n\theta + \cot^n\theta = 2$

Reason (R) : For all $n \in N$, $\tan\theta + \cot\theta = \tan^n\theta + \cot^n\theta$

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not a correct explanation of A.
(c) A is correct, R is incorrect.
(d) A is incorrect, R is correct.

49. What is the value of expression $\cos^2\frac{\pi}{8} + 4\cos^2\frac{\pi}{4} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3} + \sin^2\frac{\pi}{8}$?

(a) 8 (b) 10 (c) 16 (d) 18

50. If $q\,\text{cosec}\,\theta = p$ and θ is acute then value of $\left(\sqrt{p^2 - q^2}\right)\tan\theta$ is

(a) p (b) q (c) pq (d) $\sqrt{p^2 + q^2}$

51. If $2x^2\cos 60° - 4\cot^2 45° - 2\tan 60° = 0$, then what is the value of x ?

(a) 2 (b) 3 (c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

52. If $13\cos\theta = 12k - 5$ where $0 \le \theta \le 90°$ and k is an integer then number of possible value of k is

(a) 0 (b) 1 (c) 2 (d) more than 2

53. **Statement (A) :** $\tan 50° > 1$.

Reason (R) : For $0° < \theta < 90°$, $\tan\theta > 1$

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not a correct explanation of A.
(c) A is correct, R is incorrect.
(d) A is incorrect, R is correct.

54. Which one of the following is true ?

(a) $\sin 35° > \cos 55°$ (b) $\cos 61° > \frac{1}{2}$

(c) $\sin 32° > \frac{1}{2}$ (d) $\tan 44° > 1$

55. What is the value of x in the expression

$$\frac{x\,\text{cosec}^2 30° \sec^2 45°}{8\cos^2 45° \sin^2 60°} = \tan^2 60° - \tan^2 30° \text{ ?}$$

(a) $x = 1$ (b) $x = 2$ (c) $x = \frac{1}{2}$ (d) $x = \frac{3}{2}$

56. If $\sin\alpha + \cos\beta = 2 (0^\circ \le \beta < \alpha \le 90^\circ)$, then $\sin\left(\frac{2\alpha+\beta}{3}\right) =$

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$ (c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

57. Value of $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cdot \cot 80^\circ$ is

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

58. If $\tan\theta = 1$ then what is the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$?

(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) $\frac{4}{5}$

59. What of the value of $\tan\frac{\pi}{8} \cdot \tan\frac{\pi}{12} \cdot \tan\frac{\pi}{4} \cdot \tan\frac{3\pi}{8} \tan\frac{5\pi}{12}$?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

60. If $\tan 15^\circ = 2 - \sqrt{3}$ then value of $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$ is

(a) 14 (b) 12 (c) 10 (d) 8

Answer–10A

1. (c)	2. (a)	3. (c)	4. (c)	5. (c)	6. (a)	7. (c)	8. (c)
9. (b)	10. (d)	11. (d)	12. (b)	13. (c)	14. (d)	15. (b)	16. (b)
17. (b)	18. (b)	19. (c)	20. (b)	21. (c)	22. (a)	23. (c)	24. (d)
25. (d)	26. (a)	27. (a)	28. (d)	29. (b)	30. (a)	31. (a)	32. (b)
33. (c)	34. (c)	35. (a)	36. (b)	37. (a)	38. (c)	39. (a)	40. (a)
41. (b)	42. (d)	43. (d)	44. (d)	45. (c)	46. (d)	47. (a)	48. (a)
49. (c)	50. (b)	51. (d)	52. (c)	53. (c)	54. (c)	55. (a)	56. (b)
57. (d)	58. (a)	59. (b)	60. (a)				

Explanation

1. (c) $\because \tan x = \frac{3}{4} = \frac{p}{b}$

$\therefore \quad h = \sqrt{p^2 + b^2} = \sqrt{3^2 + 4^2} = 5$

$\therefore \quad \sin x \cdot \cos x = \frac{p}{h} \cdot \frac{b}{h} = \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$

2. (a) $\tan 45^\circ = 1$ and $\tan\theta > 1$. When $45^\circ < \theta < 90^\circ$.

3. (c) A is true R is false.

4. (c) Given,

$\cos A = \frac{5}{13} = \frac{b}{h}$

$\therefore \quad p = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12$

$\therefore \quad \frac{\sin A - \cot A}{2\tan A} = \frac{\frac{12}{13} - \frac{5}{12}}{2 \times \frac{12}{5}} = \frac{144 - 65}{13 \times 12 \times 2 \times \frac{12}{5}} = \frac{395}{3744}$

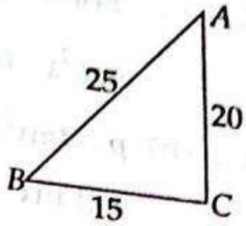
(c) $\sin x = \cos y \Rightarrow \sin x = \sin\left(\frac{\pi}{2} - y\right)$

$x = \frac{\pi}{2} - y \quad \Rightarrow x + y = \frac{\pi}{2}$

(a) Given, $\sin B = \frac{4}{5} \Rightarrow \cos B = \frac{3}{5} = \frac{BC}{AB}$

$BC = 15$ cm

$\therefore AB = BC \times \frac{5}{3} = 15 \times \frac{5}{3} = 25$ cm.



(c) $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2} = \frac{p}{h}$

$\therefore \quad b = \sqrt{h^2 - p^2}$

$= \sqrt{m^4 + n^4 + 2m^2n^2 - (m^4 + n^4 - 2m^2n^2)} = \sqrt{4m^2n^2} = 2mn$

$\therefore \tan\theta = \frac{p}{b} = \frac{m^2 - n^2}{2mn}$

(c) Given, $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$

$\Rightarrow x - y = 30°$ and $x + y = 60°$ solving

$x = 45°$ and $y = 15°$

(b) Given, $1 + \tan\theta = \sqrt{2} \Rightarrow \tan\theta = \sqrt{2} - 1$

$\therefore \cot\theta - 1 = \frac{1}{\sqrt{2} - 1} - 1 = \frac{\sqrt{2} + 1}{1} - 1 = \sqrt{2}$

10. (d) Given, $\sin(x + 54°) = \cos x = \sin(90° - x)$

$\Rightarrow x + 54° = 90° - x$

$\Rightarrow 2x = 36° \quad \Rightarrow \quad x = 18°$

11. (d) Given, $x\cos 60° + y\cos 0° = 3$

$\Rightarrow \frac{x}{2} + y = 3$

$\Rightarrow x + 2y = 6 \qquad \ldots$ (i)

and $4x \sin 30° - y\cot 45° = 2$

$\Rightarrow 4x \times \frac{1}{2} - y \cdot 1 = 2$

$\Rightarrow 2x - y = 2 \qquad \ldots$ (ii)

Solving equation (i) & (ii), $\quad x = y = 2$

12. (b) Given, $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$

Squaring both sides,

$\cos^2 x = \sin^4 x \Rightarrow 1 - \sin^2 x = \sin^4 x$

$\Rightarrow 1 = \sin^2 x + \sin^4 x$

13. (c) $\sin x : \sin y = \sqrt{3} : 1 = \frac{\sqrt{3}}{2} : \frac{1}{2} = \sin 60° : \sin 30°$

$\therefore \quad x : y = 60 : 30 \quad \Rightarrow x : y = 2 : 1$

14. (d) For $0 \le x \le \frac{\pi}{2}$,

$\cos^2 x$ and $\sin^2 x$, lie between 0 and 1. Hence option (d) is correct.

15. (b) $p = \tan^2 x + \cot^2 x = (\tan x - \cot x)^2 + 2\tan x \cot x$

$= (\tan x - \cot x)^2 + 2 \ge 0 + 2 = 2$

16. (b) $\frac{5\sin 75° \sin 77° + 2\cos 13° \cos 15°}{\cos 15° \sin 77°} - \frac{7\sin 81°}{\cos 9°}$

$= \frac{5\cos 15° \sin 77° + 2\sin 77° \cos 15°}{\cos 15° \sin 77°} - \frac{7\cos 9°}{\cos 9°} = 7 - 7 = 0$

17. (b) We know that value of $\sin x$, 0 increases as θ increases from 0° to 90°

$\therefore \quad \sin y > \sin x$

18. (b) $\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan 45° - \tan^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^3 (\sqrt{3}) - 2(\sqrt{2})^2 + 3\left(\frac{1}{2}\right)(1) - (\sqrt{3})^2$

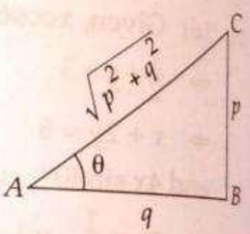
$= \frac{9}{8} - 4 + \frac{3}{2} - 3 = -\frac{35}{8}$

19. (c) Given, $\tan\theta = \frac{p}{q}$

$\Rightarrow \quad \sec\theta = \frac{\sqrt{p^2+q^2}}{q}$ and $\operatorname{cosec}\theta = \frac{\sqrt{p^2+q^2}}{p}$

$\therefore \quad \frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$

$= \frac{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) - q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) + q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}$



$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{p}{q} + \frac{q}{p}} = \frac{p^2 - q^2}{p^2 + q^2}$

20. (b) $\operatorname{cosec}^2\theta - 2 + \sin^2\theta = (\sin\theta - \operatorname{cosec}\theta)^2$

Hence it is always non negative.

21. (c) Given, $\cot\theta = \frac{2xy}{x^2 - y^2} = \frac{b}{p}$

In ΔABC,

$\therefore \quad h^2 = (x^2 - y^2)^2 + (2xy)^2$

$\Rightarrow \quad h^2 = (x^2 + y^2)^2$

$\Rightarrow \quad h = x^2 + y^2$

$\therefore \quad \cos\theta = \frac{b}{h} = \frac{2xy}{x^2 + y^2}$

22. (a) Given, $(\sin\theta + \text{cosec}\theta) = 2.5$

$\Rightarrow \left(\sin\theta + \frac{1}{\sin\theta}\right) = \frac{5}{2}$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = \frac{1}{2} \qquad (\because \sin\theta \neq 2)$

$\Rightarrow \theta = 30°$

23. (c) When $0 < \theta < \phi < 90°$

$0 < \sin^2\theta, \cos^2\theta < 1$

$\therefore \sin^2\theta + \cos^2\phi < 2$

24. (d) $\tan\theta = \frac{2t}{1-t^2} = \frac{p}{b}$

$\therefore h = \sqrt{p^2 + b^2} = \sqrt{(2t)^2 + (1-t^2)^2} = \sqrt{4t^2 + 1 + t^4 - 2t^2} = 1 + t^2$

Hence, $\sin\theta + \cos\theta = \frac{p}{h} + \frac{b}{h} = \frac{2t + 1 - t^2}{1 + t^2}$

25. (d) We know that when $\theta \in \left[0, \frac{\pi}{2}\right]$, value of $\sec^2\theta$, increases from 1 to ∞.

$\therefore p \geq 1$

26. (a) In ΔBAC, $\cos 60° = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{5}{AC} \quad \therefore \quad AC = 10$ cm

27. (a) Given, $\cos\theta + \sqrt{3}\sin\theta = 2$

$\Rightarrow \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1$

$\Rightarrow \sin 30° \cos\theta + \cos 30° \sin\theta = 1$

$\Rightarrow \sin(30° + \theta) = \sin 90°$

$\Rightarrow 30° + \theta = 90°$

$\Rightarrow \theta = 60° = \frac{\pi}{3}$

28. (d) In ΔABC, $\tan A = \frac{BC}{AC} = \frac{u}{v}$, $\tan B = \frac{v}{u}$

Also, $u^2 + v^2 = w^2$ (Pythagoras theorem)

$\therefore \quad \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2 + v^2}{vu} = \frac{w^2}{uv}$

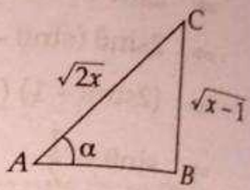
29. (b) Given, $\tan B = \frac{k}{\sqrt{3}k}$

From Pythagoras theorem, $AB^2 + AC^2 = BC^2$

$\Rightarrow \quad (\sqrt{3}k)^2 + (1k)^2 = BC^2 \quad \Rightarrow \quad BC^2 = 4K^2 \quad \Rightarrow \quad BC = 2k$

30. (a) Given, $\sin\alpha = \sqrt{\frac{x-1}{2x}}$

Applying Pythagoras theorem in ΔABC,



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad 2x = AB^2 + (x-1)$

$\Rightarrow \quad AB^2 = x + 1$

$\Rightarrow \quad AB = \sqrt{x+1} \qquad \therefore \quad \tan\alpha = \frac{BC}{AB} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

31. (a) We know that as θ increases value of $\cos\theta$ decreases,

$\therefore \quad \cos\theta \geq \frac{1}{2}$

$\Rightarrow \quad \cos\theta \geq \cos\frac{\pi}{3} \quad \Rightarrow \quad \theta \leq \frac{\pi}{3}$

32. (b) $\because \quad \cos 90^\circ = 0$

$\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots\ldots \cos 90^\circ = 0$

33. (c) If $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$

I. $\quad \sin A + \sin B = \sin\frac{\pi}{6} + \sin\frac{\pi}{3}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

and $\cos A + \cos B = \cos\frac{\pi}{6} + \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}$

$\Rightarrow \quad \sin A + \sin B = \cos A + \cos B$

II. $\quad \tan A + \tan B = \tan\frac{\pi}{6} + \tan\frac{\pi}{3}$

$= \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

and $\cot A + \cot B = \cot\frac{\pi}{6} + \cot\frac{\pi}{3} = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \quad \tan A + \tan B = \cot A + \cot B$

Alternate method, $A + B = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$ ... (i)

I. $\sin A + \sin B = \sin\left(\frac{\pi}{2} - B\right) + \sin\left(\frac{\pi}{2} - A\right) = \cos B + \cos A = \cos A + \cos B$

33. $\tan A + \tan B = \tan\left(\frac{\pi}{2} - B\right) + \tan\left(\frac{\pi}{2} - A\right)$

$$= \cot B + \cot A = \cot A + \cot B$$

Hence both statements are true.

34. (c) $\sin 17^\circ = \frac{a}{b} = \frac{\text{perpendicular}}{\text{hypotenuse}} \Rightarrow \text{base} = \sqrt{b^2 - a^2}$

$\therefore \quad \sec 17^\circ = \frac{\text{hypotenuse}}{\text{base}} = \frac{b}{\sqrt{b^2 - a^2}}$

and $\sin 73^\circ = \sin(90^\circ - 17^\circ) = \cos 17^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Hence, $\sec 17^\circ - \sin 73^\circ = \frac{b}{\sqrt{b^2 - a^2}} - \frac{b^2 - a^2}{b} = \frac{b^2 - b^2 + a^2}{b\sqrt{b^2 - a^2}} = \frac{a^2}{b\sqrt{b^2 - a^2}}$

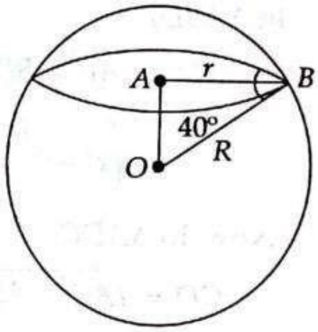
35. (a) See the figure,

In ΔOAB,

$\cos 40^\circ = \frac{AB}{OB}$

$\Rightarrow \cos 40^\circ = \frac{r}{R}$

$\therefore \; r = R \cos 40^\circ$



36. (b) $\because \quad \frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3} \Rightarrow \frac{1 + \sin^2\theta}{1 - \sin^2\theta} = \frac{5}{3}$

$\Rightarrow 3 + 3\sin^2\theta = 5 - 5\sin^2\theta$

$\Rightarrow 8\sin^2\theta = 2 \quad \Rightarrow \quad \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30^\circ$

$\therefore \; \tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$

37. (a) In first quadrant, value of $\sin\theta$ increases as θ increases and value of $\cos\theta$ decreases as θ increases. Thus $40^\circ > 20^\circ \Rightarrow \sin 40^\circ > \sin 20^\circ$.

38. (c) $\because \quad \sin\theta_1 = \frac{2}{\sqrt{5}} = 0.894$

$\tan\theta_2 = \frac{3}{2} = 1.5$

$\cos\theta_3 = \frac{3}{5} = 0.6$

and $\sec\theta_4 = \frac{\sqrt{41}}{4} = 1.6$

$\therefore$ Correct order is $3 - 1 - 2 - 4$.

39. (a) We know the maximum value of $\sin\theta = 1$

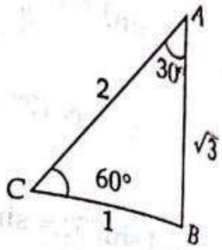
$\therefore$ Maximum value of $\sin^3\theta + 2\sin^2\theta + 3\sin\theta = 1 + 2 + 3 = 6$

at $\theta = 0°$, $\sin\theta = 0$ $\therefore \sin^3\theta + 2\sin^2\theta + 3\sin\theta = 0$

Hence only statement (1) is correct.

40. (a) $\because$ $\angle C = 60°$ and $\angle B = 90°$

$\therefore$ $\angle A = 180° - (90° + 60°)$
$= 180° - 150° = 30°$



$\therefore$ $\sin C = \sin 60° = \frac{\sqrt{3}}{2}$

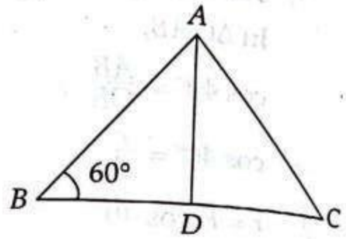
and $\sin A = \sin 30° = \frac{1}{2}$

From figure it is clear that $AB : BC : CA = \sqrt{3} : 1 : 2$

41. (b) In ΔABD,

$\cos 60° = \frac{BD}{AB}$ $\Rightarrow$ $\frac{1}{2} = \frac{BD}{x}$ $\Rightarrow$ $BD = \frac{x}{2}$

In ΔABD,



$AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{x^2 - \frac{x^2}{4}} = \frac{\sqrt{3}x}{2}$

Now, In ΔADC,

$CD = \sqrt{AC^2 - AD^2}$

$= \sqrt{\frac{9x^2}{4} - \frac{3x^2}{4}} = \sqrt{\frac{6x^2}{4}} = \sqrt{\frac{3}{2}}x$

42. (d) (a) $\sin\theta = \sqrt{2}$ is not possible as $\sin\theta \le 1$

(b) Maximum value of $\sin\theta$ is 1 and Maximum value of $\cos\theta$ is also 1 but both cannot be attained simultaneously. Hence $\sin\theta + \cos\theta = 2$ is not possible.

(c) $\sin\theta + \cos\theta = 0$
$\Rightarrow$ $\sin\theta - \cos\theta \Rightarrow \tan\theta = -1$, but when $0° < \theta < 90°$ $\tan\theta$ is positive. So, option (c) is incorrect.

(d) $\sin\theta - \cos\theta = 1$ is true when $\theta = 90°$.

Hence option (d) is correct.

43. (d) Since maximum value of each of $\sin\theta$ and $\cos\theta$ is 1, but they cannot be 1 simultaneously.

Hence $\sin\theta + \cos\theta \ne 2$. Again when $\theta = 45°$.

$\sin\theta + \cos\theta = \cos 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$, which indicates that (d) is correct.

44. (d) $\sin\theta + \text{cosce}\theta = 2$

$\Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\Rightarrow \sin\theta = \sin 90^\circ \Rightarrow \theta = 90^\circ$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 90^\circ + \cos^4 90^\circ = 1 + 0 = 1$

45. (c) $\because \sec\theta = \dfrac{13}{5} = \dfrac{h}{b}$

$\therefore p = \sqrt{h^2 - p^2} = \sqrt{13^2 - 5^2} = 12$

Required expression $= \dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \dfrac{2\cdot\dfrac{p}{h} - \dfrac{3\cdot b}{h}}{4\cdot\dfrac{p}{h} - 9\cdot\dfrac{b}{h}}$

$= \dfrac{2p - 3b}{4p - 9b} = \dfrac{2\times 12 - 3\times 5}{4\times 12 - 9\times 5} = 3$

46. (d) $\because \sin x + \sqrt{3}\cos x = 1$

$\Rightarrow \dfrac{1}{2}\sin x + \dfrac{\sqrt{3}}{2}\cos x = \dfrac{1}{2}$

$\Rightarrow \sin x \sin 30^\circ + \cos x \cos 30^\circ = \dfrac{1}{2}$

$\Rightarrow \cos(x - 30^\circ) = \cos 60^\circ$

$\Rightarrow x - 30^\circ = 60^\circ \quad \therefore x = 90^\circ$

47. (a) $\because AC^2 = AB^2 + BC^2$

$\therefore (2\sqrt{5})^2 = (BC + 2)^2 + (BC)^2$

$\Rightarrow 20 = 2BC^2 + 4BC + 4$

$\Rightarrow 2BC^2 + 4BC - 16 = 0$

$\Rightarrow 2(BC^2 + 2BC - 8) = 0$

$\Rightarrow (BC + 4)(BC - 2) = 0$

$\Rightarrow BC = 2$

($\because BC$ cannot be negative)

Hence, $AB - BC = 2 \Rightarrow AB = 2 + BC = 2 + 2 = 4$

In ΔABC, $\cos^2 A - \cos^2 C = \left(\dfrac{4}{2\sqrt{5}}\right)^2 - \left(\dfrac{2}{2\sqrt{5}}\right)^2 = \dfrac{12}{20} = \dfrac{3}{5}$

48. (a) $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0 \Rightarrow \tan\theta = 1$

$\therefore \tan^n\theta + \cot^n\theta = 1 + 1 = 2$

Hence (A) and (R) is correct and R is correct statement of A.

49. (c) $\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)+4\cos^2 45^\circ-\sec 60^\circ+5\tan^2 60^\circ$

$$=1+4\left(\frac{1}{\sqrt{2}}\right)^2-2+5(\sqrt{3})^2=1+2-2+15=16$$

50. (b) Given, $\operatorname{cosec}\theta=\frac{p}{q}=\frac{\text{hypotenuse}}{\text{perpendicular}}$

Now, $\tan\theta=\frac{\text{perpendicular}}{\text{base}}$ $\therefore$ base $=\sqrt{(\text{hypotenuse})^2-(\text{perpendicular})^2}$

$$=\sqrt{p^2-q^2}$$

$\therefore\quad \tan\theta=\frac{q}{\sqrt{p^2-q^2}}$

Now, $\sqrt{p^2-q^2}.\tan\theta=\left(\sqrt{p^2-q^2}\right)\frac{\text{perpendicular}}{\text{base}}$

$$=\sqrt{p^2-q^2}\times\frac{q}{\sqrt{p^2-q^2}}=q$$

51. (d) $2x^2\times\frac{1}{2}-4(1)^2-2\sqrt{3}=0$

$x^2-4-2\sqrt{3}=0$

$x^2=4+2\sqrt{3}=(\sqrt{3}+1)^2\quad\therefore\quad x=\sqrt{3}+1$

52. (c) $\cos\theta=\frac{12k-5}{13}$

$\because\quad 0\le\cos\theta\le 1$

$\therefore\quad 0\le\frac{12k-5}{13}\le 1$

or, $0\le 12k-5\le 13$ or, $5\le 12k\le 18$

or, $\frac{5}{12}\le k\le\frac{18}{12}$ or, $\frac{5}{12}\le k\le\frac{3}{2}$

Clearly it is true for integral value of $k=0,1$. Hence k has two value.

53. (c) We know that as θ increases from 0° to 90°, value of $\tan\theta$ increases,

$\therefore\quad \tan 50^\circ>\tan 45^\circ=1$

54. (c) $\sin 32^\circ>\sin 30^\circ\Rightarrow\sin 32^\circ>\frac{1}{2}$

55. (a) $\frac{x\times(2)^2\times(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2}=(\sqrt{3})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

or, $\frac{x\times 4\times 2}{8\times\frac{1}{2}\times\frac{3}{4}}=3-\frac{1}{3}$ or, $\frac{8x}{3}=\frac{8}{3}\quad\therefore\quad x=1$

56. (b) ∵ $\sin\alpha \le 1; \cos\beta \le 1$

∴ $\sin\alpha + \cos\beta = 2$ is possible only when $\sin\alpha = 1$ and $\cos\beta = 1$

⇒ $\alpha = 90^\circ; \beta = 0^\circ$

∴ $\sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180^\circ}{3}\right) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos\frac{\alpha}{3} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

57. (d) ∵ $\cot 10^\circ \cdot \cot 80^\circ \cdot \cot 20^\circ \cdot \cot 70^\circ \cdot \cot 60^\circ$

$= \cot 10^\circ \cdot \tan 10^\circ \cdot \cot 20^\circ \cdot \tan 20^\circ \cdot \cot 60^\circ$ $\left[\begin{array}{l} \because \tan(90^\circ - \theta) = \cot\theta \\ \tan\theta \cdot \cot\theta = 1 \end{array}\right]$

$= 1 \cdot 1 \cdot \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

58. (a) ∵ $\tan\theta = 1 = \tan 45^\circ$

∴ $\theta = 45^\circ$

∴ $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta} = \frac{8\sin 45^\circ + 5\cos 45^\circ}{\sin^3 45^\circ - 2\cos^3 45^\circ + 7\cos 45^\circ}$

$= \frac{8\times\frac{1}{\sqrt{2}} + 5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\times\left(\frac{1}{\sqrt{2}}\right)^3 + 7\times\frac{1}{\sqrt{2}}} = \frac{\frac{1}{\sqrt{2}}(8+5)}{\frac{1}{\sqrt{2}}\left(\frac{1}{2} - 1 + 7\right)} = \frac{13}{\left(\frac{13}{2}\right)} = 2$

59. (b) $\frac{\pi}{8} = \frac{180^\circ}{8} = 22\frac{1^\circ}{2}, \frac{\pi}{12} = \frac{180^\circ}{12} = 15^\circ, \frac{\pi}{4} = 45^\circ$

$\tan\frac{3\pi}{8} = \tan\frac{3\times 180^\circ}{8} = \tan 67\frac{1^\circ}{2} = \cot 22\frac{1^\circ}{2}$

$\tan\frac{5\pi}{12} = \tan(5\times 15^\circ) = \tan 75^\circ = \cot 15^\circ$

∴ Required value $= \tan 22\frac{1^\circ}{2} \tan 15^\circ \tan 45^\circ \cot 22\frac{1^\circ}{2} \cot 15 = 1$

($\because \tan\theta \cdot \cot\theta = 1$)

60. (a) $\tan 15^\circ \cdot \cot 75^\circ + \tan 75^\circ \cdot \cot 15^\circ$

$= \tan 15^\circ \cdot \cot(90^\circ - 15^\circ) + \tan(90^\circ - 15^\circ) \cdot \cot 15^\circ$

$= \tan^2 15^\circ + \cot^2 15^\circ$ ... (i) $\left[\begin{array}{l} \because \tan(90^\circ - \theta) = \cot\theta \\ \cot(90^\circ - \theta) = \tan\theta \end{array}\right]$

∵ $\tan 15^\circ = 2 - \sqrt{3}$

∴ $\cot 15^\circ = \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = 2 + \sqrt{3}$

∴ $\tan^2 15^\circ + \cot^2 15^\circ$

$= (2-\sqrt{3})^2 + (2+\sqrt{3})^2 = 2(4+3) = 14$

Exercise–10B

1. If $\sin(x+y) = \cos[3(x+y)]$ then what is the value of $\tan[2(x+y)]$?
(a) $\sqrt{3}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{3}}$
[SSC Tier-I 2012]

2. If $\sin 2\theta = \frac{1}{2}$ then what is the value of $\cos(75^\circ - \theta)$?
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$
[SSC Tier-I 2012]

3. In a right angled triangle ABC, $AB = 2.5$ cm, $\cos B = 0.5$, $\angle ACB = 90^\circ$. Length of side AC in cm is
(a) $5\sqrt{3}$ (b) $\frac{5}{2}\sqrt{3}$ (c) $\frac{5}{4}\sqrt{3}$ (d) $\frac{5}{16}\sqrt{3}$
[SSC Tier-I 2012]

4. If $\cos\theta = \frac{4}{5}$, then value of $\frac{\operatorname{cosec}\theta}{1+\cot\theta}$ is
(a) $\frac{7}{5}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{4}{7}$
[SSC Tier-I 2012]

5. Maximum value of $24\sin\theta + 7\cos\theta$ is
(a) 7 (b) 17 (c) 24 (d) 25
[SSC Tier-I 2012]

6. In ΔABC, $\angle A$ is right angle and AD is perpendicular to BC. If $AD = 4$ cm, $BC = 12$ cm, then value of $(\cot B + \cot C)$ is
(a) 6 (b) 3 (c) 4 (d) $\frac{3}{2}$
[SSC Tier-I 2012]

7. If $5\tan\theta = 4$ then value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right)$ is
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{2}{5}$
[SSC Tier-I 2012]

8. What is the minimum value of $4\operatorname{cosec}^2\alpha + 9\sin^2\alpha$?
(a) 10 (b) 11 (c) 12 (d) 14
[SSC Tier-I 2012]

9. In a right angled triangle $\angle B$ is right angle and $AC = 2\sqrt{5}$ cm. If $AB - BC = 2$ cm then what is the value of $(\cos^2 A - \cos^2 C)$?
(a) $\frac{3}{5}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$ (d) $\frac{2}{5}$
[SSC Tier-I 2012]

10. If $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$, where A and B are positive acute angle and $A \geq B$ then A and B are
(a) $A = 60^\circ, B = 30^\circ$ (b) $A = 45^\circ, B = 45^\circ$
(c) $A = 75^\circ, B = 15^\circ$ (d) None of these
[SSC Tier-I 2012]

Answer-10B

1. (b) 2. (b) 3. (c) 4. (c) 5. (a) 6. (b) 7. (a) 8. (c)
9. (a) 10. (a)

Explanation

1. (b) $\sin(x+y) = \cos(3(x+y)) = \sin\left(\frac{\pi}{2} - 3(x+y)\right)$

$\therefore\ (x+y) = 90° - 3(x+y)$

or, $4(x+y) = 90°$

or, $2(x+y) = 45°$

$\therefore\ \tan(2(x+y)) = \tan 45° = 1$

2. (b) $\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = 30° \Rightarrow \theta = 15°$

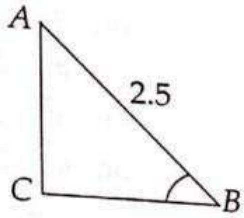
$\therefore\ \cos(75° - \theta) = \cos(75° - 15°) = \cos 60° = \frac{1}{2}$

3. (c) $\cos B = 0{\cdot}5 = \frac{1}{2} \Rightarrow B = 60°$

Now, $\sin B = \frac{AC}{AB}$

$\Rightarrow\ AC = AB \sin B$

$= (2{\cdot}5)\sin 60° = \frac{5}{2}\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$



4. (c) $\cos\theta = \frac{4}{5} = \frac{b}{h}$

$\therefore\ p = \sqrt{h^2 - b^2} = \sqrt{25 - 16} = 3$

Now, $\frac{\operatorname{cosec}\theta}{1+\cot\theta} = \frac{h/p}{1+\frac{b}{p}} = \frac{h}{p+b} = \frac{5}{3+4} = \frac{5}{7}$

5. (d) Recall that maximum and minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$.

Hence maximum value of $24\sin\theta + 7\cos\theta = \sqrt{24^2+7^2}$

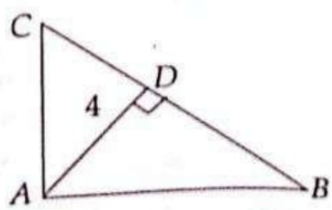
$= \sqrt{576+49} = \sqrt{625} = 25$

6. (b) In right angle ΔABD, $\cot B = \frac{BD}{4}$

In right angle ΔACD, $\cot C = \frac{CD}{4}$

$\therefore\ \cot B + \cot C = \frac{BD+CD}{4}$

$= \frac{BC}{4} = \frac{12}{4} = 3$ cm



7. (a) (tricky solution), $5\tan\theta = 4$

$\Rightarrow \frac{5\sin\theta}{\cos\theta} = 4 \Rightarrow \frac{5\sin\theta}{3\cos\theta} = \frac{4}{3}$

by componendo-dividendo

$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{4-3}{4+3} = \frac{1}{7}$

8. (c) $4\,\text{cosec}^2\alpha + 9\sin^2\alpha$

$= (2\text{cosec}\alpha - 3\sin\alpha)^2 + 2\cdot 2\text{cosec}\alpha\cdot 3\sin\alpha$

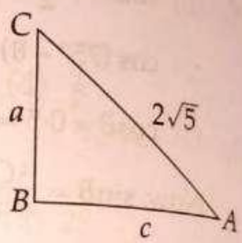
$= (2\text{cosec}\alpha - 3\sin\alpha)^2 + 12$

$\because\ 2\text{cosec}\alpha - 3\sin\alpha \geq 0$

$\therefore$ Required value ≥ 12

9. (a) Given $AB - BC = 2$ i.e., $c - a = 2$

and $c^2 + a^2 = 20$ *(Pythagoras Theorem)*



putting $c = a + 2$ in $c^2 + a^2 = 20$

$(a+2)^2 + a^2 = 20$

or, $2a^2 + 4a + 4 = 20$

or, $a^2 + 2a + 2 = 10$

or, $a^2 + 2a - 8 = 0$

or, $a = 2, -4$

$\therefore\ c = 4$

Now, $\cos^2 A - \cos^2 C = \left(\frac{c}{2\sqrt{5}}\right)^2 - \left(\frac{a}{2\sqrt{5}}\right)^2 = \frac{c^2 - a^2}{20} = \frac{16-4}{20} = \frac{3}{5}$

10. (a) $\sin(A - B) = 1 \quad \Rightarrow A + B = 90°$

$\cos(A - B) = \frac{\sqrt{3}}{2} \quad \Rightarrow A - B = 30°$

Adding, $2A = 120° \Rightarrow A = 60°$

$\therefore\ B = 30°$

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