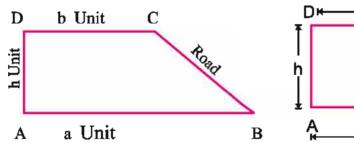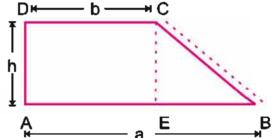# AREA

14.1 In the previous classes we have some method about how to find the area of quadrilateral. Now in this chapter we are going to discuss few more methods to find the area of quadrilateral (like trapezium, parallelogram and simple quadrilateral).

14.2 Trapezium

Let us discuss about the shape of the given plots. What are the techniques you will apply to calculate the area of the given figure. By looking at one pair of parallel side of the figure Aditya consider it as a trapezium. Quadrilateral which has one pair of parallel opposite side is known as trapezium.

He divides the given plot into two parts (figures),one of them is triangle and other is rectangle because he already knows how to find out the area of a rectangle and a triangle.

Area of the plot = area of rectangle AECD + area of triangle ECB

$$= (AE \times AD) + \frac{1}{2}(EB \times EC)$$

$\because$ [EC = AD and AE = DC]

Area of the figure ABCD $= (AE \times AD) + \frac{1}{2}(EB \times AD)$

$$= AD \times [AE + \frac{1}{2} \times EB]$$

$$= AD \times \left(\frac{2AE + EB}{2}\right)$$

$$= AD \times \left(\frac{AE + AE + EB}{2}\right)$$

$$= AD \times \frac{[(AE + EB) + AE]}{2}$$

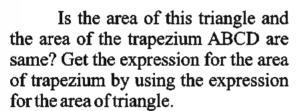$$= AD \times \left(\frac{AB + CD}{2} \right)$$

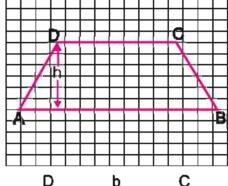$$= h \times \frac{(a+b)}{2}$$

(where a and b are the parallel sides)

Therefore, Area of Trapezium $= \frac{1}{2} \times$ height $\times$ (sum of the parallel sides)
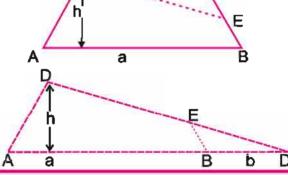
Activity: lets find the area of the trapezium through an activity which has following steps

(i) Draw a trapezium of any size on the graph paper and cut it out.

(ii) Find the mid point E of the side BC and cut the trapezium into two piece, along DE

(iii) Place the triangle DEC in such a way that point C coincides with point D.

(iv) What is the length of the base of the larger triangle?

If height of the triangle is h then its area is given by, $= h \times \frac{(a+b)}{2}$

Is the area of this triangle and the area of the trapezium ABCD are same? Get the expression for the area of trapezium by using the expression for the area of triangle.
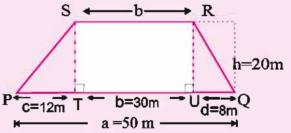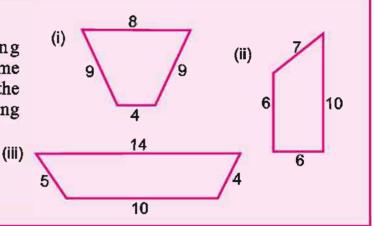

Do and learn ◆

Azhar has a trapezium shaped farm. He divide it into three parts as shown in the figure.
Show that the area of the trapezium PQRS = Area of Δ SPT + area of rectangle STUR + area of ΔRUQ. And compare it with the area of the trapezium $\frac{h \times (a+b)}{2}$
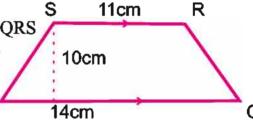
2 . Is trapezium having different parameter has same area? Prove with the help of the data given in the following figures

(i)

8

9   9

4

(ii)

7

6   10

6

(iii)

14

5   4

10

Example 1 Find the area of the trapezium.

S   11cm   R

10cm

P   14cm   Q

Solution ∵ Parallel side of the trapezium PQRS
are PQ = 14 cm and SR = 11 cm
Height, h = 10 cm

Area of the trapezium PQRS

$$= \frac{1}{2} \times height \times (sum\ of\ parallel\ side)$$

$$= \frac{1}{2} \times (14 + 11) \times 10$$

$$= \frac{1}{2} \times 25 \times 10 = 125\ cm$$

Example 2 Two parallel sides of the trapezium is 12cm and 8cm. If area is equal to 60cm^2, then what is the height of the trapezium.

Solution Let the height of the trapezium = h

Area of the trapezium $= \frac{1}{2} \times height \times (sum\ of\ parallel\ side)$

$$60 = \frac{1}{2} \times (12 + 8) \times h$$

$$h = \frac{60 \times 2}{20} = 6\ cm$$
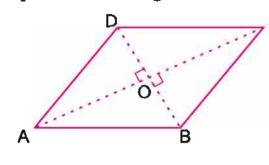
height = 6 cm

8 cm

h
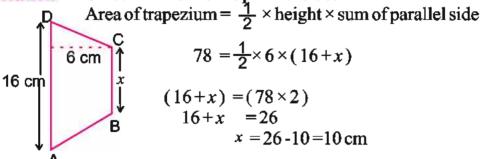
12 cm

Example 3 The sides of the trapezium is given in the figure ABCD. Find the value of x if the area is equal to 78 cm^2.

Solution   Since AD and BC are parallel sides

Area of trapezium $= \frac{1}{2} \times$ height $\times$ sum of parallel side

$$78 = \frac{1}{2} \times 6 \times (16 + x)$$

$$(16 + x) = (78 \times 2)$$
$$16 + x = 26$$
$$x = 26 - 10 = 10 \text{ cm}$$

14.3 Area of Rhombus

We have learnt that a quadrilateral that has equal side is known as Rhombus. the diagonals of the rhombus bisect orthogonally to each other. Let's explain it with the help of an example. ABCD is a rhombus having AC and BD as its diagonals. Let $AC = d_1$ and $BD = d_2$, the diagonals bisect orthogonally each other at point O .i.e the diagonals bisect and make right angle at point O.

Area of the rhombus
= Area of $\triangle$ABC + area of $\triangle$ACD
$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

(OB and OD are the height of $\triangle$ABC and $\triangle$ACD)
$= \frac{1}{2} \times AC [OB + OD]$

$= \frac{1}{2} \times AC \times BD$ $(\because OB + OD = BD)$

$= \frac{1}{2} \times d_1 \times d_2$

> Hence, area of the rhombus $= \frac{1}{2} \times$ Product of the diagonals

In other word the area of the rhombus is equal to the half of the product of its diagonals.

Since rhombus is also a quadrilateral therefore if we know the side and height of the rhombus then

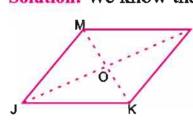> Area of the rhombus = base $\times$ height

Example: 4 The diagonal of the rhombus are respectively 10 cm and 12 cm. What is the area of the rhombus

Solution: Area of the rhombus $= \frac{1}{2} \times$ product of the diagonals

$$= \frac{1}{2} \times 10 \times 12$$

$$= 60 \text{ cm}^2$$

Example: 5 According to given figure JKLM is a rhombus and has area equal to140cm². If the diagonal KM= 14cm then find the value of OL.

Solution: We know that

area of the rhombus $= \frac{1}{2} \times$ product of the diagonals

Given that area of the rhombus=140 cm²
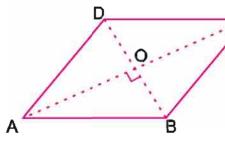
$$140 = \frac{1}{2} \times 14 \times JL$$

$$JL = \frac{140 \times 2}{14} = 20 \text{ cm}$$

Since diagonals of the rhombus bisect each other therefore,

$$OL = \frac{1}{2} \times JL$$

$$= \frac{1}{2} \times 20 = 10 \text{ cm}$$

Example:6 The area of the rhombus shaped play ground is 21600cm². When Rohan walks through the larger diagonal then he has to walk 240m.calculate the total distance covered by Rohan if he walks through the boundary of the given field.

Solution: lets assume that ABCD is a rhombus shaped playing ground.

It is given that area of the ground = 21600m²

And diagonal AC = 240m

Area of the Rhombus $= \frac{1}{2} \times \left(\begin{array}{c}\text{product of the} \\ \text{diagonals}\end{array}\right)$

$$21600 = \frac{1}{2} \times 240 \times BD$$

$$BD = \frac{21600 \times 2}{240} = 180\text{m}$$

Since diagonal of the rhombus perpendicular bisect each other

Therefore, AO $= \frac{1}{2} \times AC = 120$ m and BO $= \frac{1}{2} \times BD = 90$ m

Since $\triangle AOB$, is a triangle .so with the help of pythogorous theorem we have,

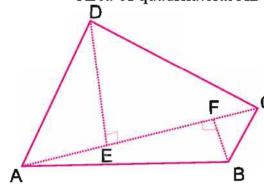$$AB = \sqrt{(AO)^2 + (BO)^2} = \sqrt{(120)^2 + (90)^2} = 150\text{m}$$

Hence distance covered along the boundary of the ground

= perimeter of the ground = 4 ×150 = 600m.

14.4 Area of a Quadrilateral

A general quadrilateral can be split into two triangles by drawing one of its diagonals. After this make perpendicular by each remaining vertex as giving in the figure.

Area of quadrilateral ABCD = (area of $\triangle$ ABC) + (area of $\triangle$ ADC)

$$= \frac{1}{2} \times (AC \times BF) + \frac{1}{2} \times (AC \times DE)$$
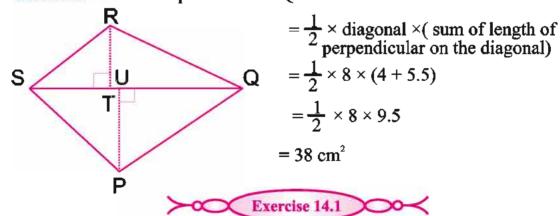
$$= \frac{1}{2} AC \times (BF + DE)$$

$$= \frac{1}{2} d \times (h_1 + h_2).$$

Where , d = length of diagonal AC

h_1 = length of perpendicular BF

h_2 = length of perpendicular DE

> Area of quadrilateral $= \frac{1}{2} \times$ diagonal $\times$ (sum of length of perpendicular on the diagonal)
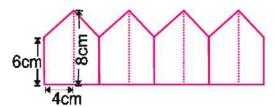
Example 7: The length of the diagonal SQ of the quadrilateral PQRS is 8cm and length of the perpendiculars on the diagonal RT and UP is 4 cm and 5.5cm respectively. Find the area of the quadrilateral.

Solution: Area of the quadrilateral PQRS

$$= \frac{1}{2} \times \text{diagonal} \times (\text{sum of length of perpendicular on the diagonal})$$

$$= \frac{1}{2} \times 8 \times (4 + 5.5)$$

$$= \frac{1}{2} \times 8 \times 9.5$$
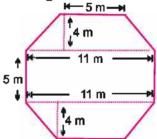
$$= 38 \text{ cm}^2$$

Exercise 14.1

1. The length of two perpendicular sides of a trapezium are 10 cm and 16 cm perpendicular distance between them is 8 cm. find the area of the trapezium.
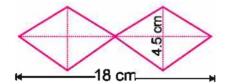2. The roof of the building is in particular shape as shown in the below figure if all the dimension is of equal length then find the area of the whole design.

3. The area and height of the trapezium are 34 cm^2 and 4 cm . One of its parallel side is 10 cm. Find the length of other parallel side.
4. Top surface of a platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.

5. Length between the opposite vertex of the rhombus shaped plot are 12.5 m and 10.4 m .Find the total cost of making this plot as a flat surface if the cost of making a flat surface per square meter is Rs. 180.
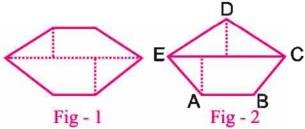6. Find the area of the combined rhombus shaped tiles as given in the figure.

7. The field of the kalyan is in the form of quadrilateral. The diagonal of this field is 220 m and the perpendiculars dropped on it from the remaining opposite vertices are 80 m and 130 m respectively. find the area of the field.
8. Fill in the blanks
    (i) Area of the rhombus is …… product of the diagonals
    (ii) Both the diagonals of the scalene quadrilateral is always …….
    (iii) Area of the………. quadrilateral can be found through the formula height × (sum of parallel sides)/ 2
    (iv) The quadrilateral whose unequal diagonal bisect each other is called…....………
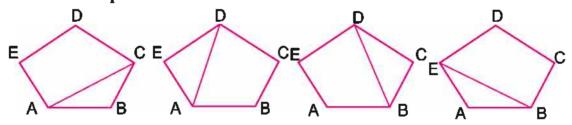
14.5 Area of a Polygon (Field Book)

So far we have learnt how to find out the area of the plane figure like triangles, quadrilaterals, and circles. All the above mentioned figures have some

particular formulas of their areas but if we want to find out the area of the pentagon, hexagon, heptagon,....etc then what will be the processor lets discuss it briefly.
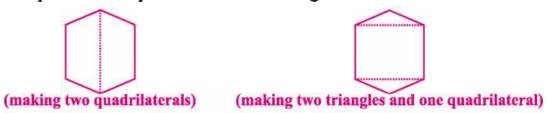
Look at the figures. Can we get some triangle or quadrilateral shaped figure by connecting any two vertices of the given figure. Is sum of the areas of these triangular and quadrilateral shaped figure formed is equal to the area of the given polygon figure?

Fig - 1   Fig - 2

(i) Look at the pentagon ABCDE given in the figure 2. A triangle ECD and a quadrilateral ABCDE is formed if we join the vertices E and C. we can find the area of both the figure i.e. the triangle and the quadrilateral

Area of the polygon ABCDE = area Δ ECD + area of quadrilateral ABCE

(ii) Join different vertices of the pentagon. Calculate the area of the different figure obtained in each case. Is the area calculated in each case equal?
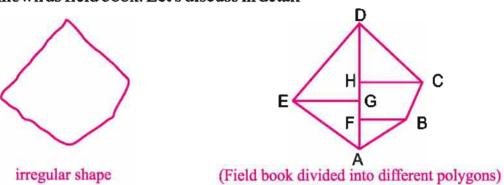

Repeat the same processor with the hexagon.

(making two quadrilaterals)   (making two triangles and one quadrilateral)

14.5.1 Area of Irregular Polygon

You have often seen around you some farm, ground and other land region which are irregular in shape .they do not resemble the shape of any polygon till we have studied.

Generally when patwari and engineer do any survey on the land which are irregular in shape then they divide that land into some small parts. These parts are in the shape of some plane geometric figures and then they can calculate the area all these small parts of the land that they have divided which is equivalent to the area of the whole land. This method of finding the area of irregular shape field is known as field book. Let's discuss in detail

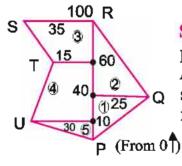irregular shape   (Field book divided into different polygons)

Look at the irregular shaped land given in the figure. It is a tough task to find the area of this land.

For this we draw the straight lines along the length of the figure. Now we drop the perpendicular from the vertices on the straight line drawn. In this way we get different plane figures. From a point A on the straight line drawn we find the distance of points F , G,H,D , In the same way we can find the perpendicular distance of FB, HC, EG . So now we can calculate the area of the given figure.

Area of the whole figure = area of $\triangle$ ABC + area of quadrilateral FBCH + area of $\triangle$ HCD + area of $\triangle$EGD + area of $\triangle$ AHE

Example 8: Find the area of the figure PQRSTU

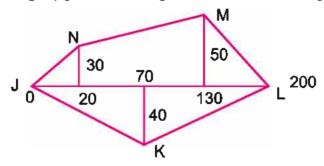**Solution:** By dividing the complete region into parts we get five figures.
Area of PQRSTU = area of first region +area of second region + area of third region +area of fourth region +area of fifth region

$$= [\frac{1}{2}\times 40\times25+ [\frac{1}{2}\times60\times25] + [\frac{1}{2}\times(15+35)\}40] + [\frac{1}{2}\times(30+15)\times50+[\frac{1}{2}\times10\times20]$$
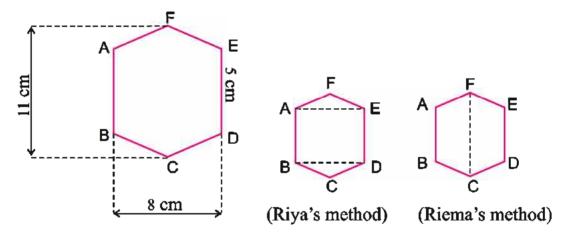
$$= 500+750+1000+1125+150 = 3525 \text{ square unit}$$
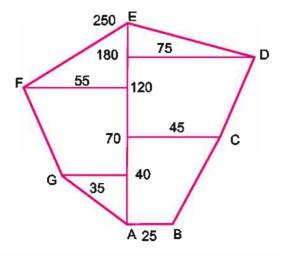
Exercise 14.2

1. Find the area of the polygon according to the measurement given in the figure.

2. Each side of the hexagon ABCDEF has side of length 5cm as given in the figure. Riya and Riema find the area of the region by dividing it into two parts in two different manner. Compare the area in both the case.

(Riya's method)　　　(Riema's method)

3. The measurement of each part of the farm of Ramlal is given in the figure. find the total cost of ploughing at the rate of Rs. 4 per meter .

We Learnt

1. Area of Trapezium = height × $\dfrac{\text{(sum of parallel sides)}}{2}$

2. The diagonals of the trapezium bisect each other

3. Area of Trapezium = $\dfrac{\text{(sum of diagonals)}}{2}$

4. Area of general quadrilateral = $\dfrac{1}{2}$ × diagonal × (sum of length of perpendicular on the diagonal)

5. To find the area of pentagon, hexagon etc. we divide them into triangle or quadrilateral. The total area of the given polygon is equal to sum of all the triangles and quadrilateral that the polygon have been divided into.