Vedic Mathematics

5.1 In this chapter, you will be able to learn the multiplication of numbers of two and three digits by the Urdhwtiryak formula, multiplication of three numbers under the Nikhilam base and sub-base formula, and cubage and division by Dhawajank method.

5.1.1. Multiplication Operation (Urdhwtirgbhyaam Formula)

Product can be easily done by any method based on Urdhwtiryak formula. In this word Urdhwtiryak, the meaning of first word Urdhw is the just above i.e. the product of upper and below digits and the meaning of last word Tiryak is skew i.e. product of diagonal digits.

Example 1: Let us multiply 32 and 14.

Step-1: Multiplicand 32 and multiplier 14 can be written for multiplication in this way,

$$\begin{array}{r} 32 \\ \times 14 \\ \hline \end{array}$$

Step-2: Create Group-There will be three groups in multiplication of two digits by two digits which is shown by III, II and I.

III	II	I
3	3 2	2
1	1 4	4

> Note: While multiplying, number of groups can be estimated by the (2n - 1) formula. Where n is the maximum digit number in multiplicand and multiplier. In this 32 and 14, the maximum digit is 2. So 2 × 2-1= 3 groups.

Step-3: Multiplication action-

Step-4: Product $3×1$ / $3×4+1×2$ / $2×4$

Step-5: 3 / $12+2$ / 8

Step-6: 3 / 14 / 8

Step-7: Addition

Line 1 Line 2

$$\begin{array}{ccc} 3 & 4 & 8 \\ & 1 & \\ \hline 4 & 4 & 8 \end{array}$$

or

$$\begin{array}{c} 3 / 4 / 8 \\ +1 \\ \hline 4 \quad 4 \quad 8 \end{array}$$ (3+1=4)

Thus, the exact product of 32 and 14 is 448.

> Note: Each group contains single digit. Group II contains two digit number 14 so tens digit 1 of 14 will be added in

Example 2: Multiplication of 123 and 45 by Urdhwtirgbhyaam method.
Sol:

Step-1: 123 is multiplicand and 45 is multiplier. For making multiplier of three digits it will be written 045.

$$= \begin{array}{r} 123 \\ \times 045 \\ \hline \end{array}$$

Step-2: Number of groups will be 5 in three digit numbers which is shown by V, IV, III, II and I.

V	IV	III	II	I
1	1 2	1 2 3	2 3	3
0	0 4	0 4 5	4 5	5

Step-3: Multiplication action and Product:

Multiplication action $1{\times}0 / 1{\times}4 + 2 \times 0 / 1{\times}5 + 3{\times}0 + 2{\times}4 / 2{\times}5 + 3{\times}4 / 3{\times}5$

Product $0 \; / \; 4+0 \; / \; 5+0+8 \; / \; 10+12 \; / \; 15$

Addition $0 / 4 / 13 / 22 / 15$

Step-4: There are 2 digits in first, second and third part but due to the base 10, only one digit is kept in each part. Therefore, the unit digit of first, second and third part is written in first row and tens digit is written in second row (a one place ahead). After adjusting the each row, sum of these digits are made.

Line 1   4   3   2   5
Line 2   1   2   1   -

Step-5: Addition:

$$\begin{array}{cccc} 5 & 5 & 3 & 5 \end{array}$$

So exact product of $123 \times 45 = 5535$

Example 3: Multiply 57×68.

Number of group = 3

$$\begin{array}{r} 57 \\ 68 \\ \hline \end{array}$$

$$5{\times}6 / 5{\times}8 + 6{\times}7 / 7{\times}8$$

$$30 / 40+42 / 56$$

$$30 / 82 / 56$$

$$\underset{+8 \quad\quad +5}{} $$

3876

(Where in 82+ 5=87, 7 is add on tens and 8 is further add on 30 and resultant 38)

So the exact product is 3876.

Example 4: Multiply 349×986.

Sol: number of group $= 5$

$$
\begin{array}{r}
3\ 4\ 9 \\
\times 9\ 8\ 6 \\
\hline
\end{array}
$$

$$3\times9 / 3\times8 + 4\times9 / 3\times6 + 9\times9 + 4\times8 / 4\times6 + 9\times8 / 9\times6$$
$$27 / 24 + 36 / 18 + 81 + 32 / 24 + 72 / 54$$
$$27 / 60 / 131 / 96 / 54$$

By arranging and adding the numbers in queues.

$$
\begin{array}{cccccc}
① & 7 & 0 & 1 & 6 & 4 \\
2 & 6 & 3 & 9 & 5 & \\
& ① & & & & \\
\hline
3 & 4 & 4 & 1 & 1 & 4 \\
\end{array}
$$

Line-1
Line-2
Line-3

Thus, the exact multiplication of number is $349 \times 986 = 344114$

Do and Learn: ◆

1. Multiplying the following by Urdhwtirgbhyaam method.

     (i) 15×12 (ii) 60×18

     (iii) 71×8 (iv) 122×4

     (v) 706×56 (vi) 497×173

5.1.2 Multiplication by NIkhilam Formula (sub-base)

By Nikhilam formula, large deviation can be obtained of any problem but the multiplication of them is very difficult. Suppose in 32×34, if we take base as a 10 then the product of its deviation 22 and 24 is larger.

Therefore, selection of sub-base is done on the basis of approximate tens so that deviation can be obtained smaller than sub-base. For example, in 68×66 we take sub-base $7 \times 10 = 70$ and in 32×27 we take sub-base $3 \times 10 = 30$ and remaining method is like previous method.

Example 5: find the value of 32×34.   Hint:

Step-1: Number   Deviation   (i) Base $= 10$, Sub base $= 3 \times 10 = 30$

            32       $+2$ (ii) Sub base digit $=$ Sub base $\div$ Base $= 30 \div 10 = 3$

            34       $+4$ (iii) Deviation from sub base $=$ number $-$ base

$$= 32 - 30 = +2$$
$$= 34 - 30 = +4$$

Step-2: Solution will be in two parts

$$
\begin{array}{rl}
32 & +2 \\
34 & +4 \\
\hline
/ & 8 \\
\end{array}
$$

$$\left(\begin{array}{l} \text{Multiplication of deviation} \\ \text{in right side} = 2\times4 = 8 \end{array} \right)$$

Step-3: In left side adding any number and the deviation of remaining numbers and multiplying the sum by subbase digit 3.

$$\begin{array}{r}32 \quad +2 \\ 34 \quad +4 \\ \hline 108 \ / \end{array}$$

(where $32+4=36$ or $34+2=36$ and $3\times36=108$)

Step-4: Consolidating from step 1 to 3 and arranging by operating

$$\begin{array}{r}32 \quad +2 \\ 34 \quad +4 \\ \hline = \quad 108\ /\ 8 \end{array}$$

$= 1088$ (Removing skew line)

Thus the exact product of 32×34 is 1088.

Example 6: Solve 54×57.

Sol: Let us use step - 4 from the previous method

Number   Deviation

$$\begin{array}{cc} 54 & +4 \\ 57 & +7 \\ \hline \end{array}$$

$5\{(54+7) \text{ or } (57+4)\} / 4\times7$

$= 5 \times 61 / 28$

$= 305 \underset{\curvearrowleft 2}{/} 8$

$= 3078$

Hint:
(I) Base $=10$, Sub base $=5\times10=50$
(ii) Sub base digit $=5$
(iii) Deviation from sub base $= +4$ and $+7$
(iv) In base 10, single 0 is there so there will be single digit in right side.
(v) Adding the tens digit in left side $= 305+2=307$

Example 7: Solve 78×76.

Sol: Number   Deviation

$$\begin{array}{cc} 78 & -2 \\ 76 & -4 \\ \hline \end{array}$$

$= 8\times(78-4) / -2\times-4$

$= 8\times74 / 8$

$= 592 / 8$

$= 5928$

Hint:
(I) Base 10 , Sub base $= 8\times10=80$
(ii) Sub base digit $= 8$
(iii) Deviation from sub base $= -2$ and -4

Example 8: Solve 63×58.

Sol:

Number	Solution
63	+3
58	- 2

$= 6 \times (63-2) / +3 \times -2$

$= 6 \times (61) / -6$

$= 366 / -6$

$= 365+1 / -6$

$= 365 / 10-6$

$= 365 / 4$

$= 3654$

Hint:

(i) Base = 10 and Sub base = $6 \times 10 = 60$

(ii) Sub base digit = 6

(iii) Deviation from sub base = (63- 60 = +3 and 58 – 60 = -2) = +3 and -2

(iv) Multiplication of deviation in right side = $3 \times -2 = -6$ which is negative therefore convert it into positive, taking 1 tens = 10 from left side in unit side, we get 10 – 6 = 4.

(v) Thus, product = 3654

Do and Learn: ◆ Find the product of the following:

(i) 11×15 (ii) 12×18 (iii) 19×17

(iv) 28×22 (v) 51×49 (vi) 99×96

5.2 Multiplication of Three Numbers
5.2.1 Formula Nikhilam (Base)

By Nikhilam formula, product of three numbers can be easily calculated in which deviation is same as base 10 or relative the power of 10.
Let us try to understand it.

Example 9: Multiply $12 \times 13 \times 17$.

Step 1: Base of $12 \times 13 \times 17$ = 10 and take deviation +2, +3, +7

Number	Deviation
12	+2
13	+3
17	+7

Step 2: Solution will be in three parts which will be known by right part, middle part and left part respectively. These three deviations are multiplied in right part.

Number	Deviation
12	+2
13	+3
17	+7

$/ 2 \times 3 \times 7$

Step 3: In middle part, products of two deviations and their addition are made. In deviation step, 2×3, 3×7 and 7×2.

$$
\begin{array}{rr}
12 & +2 \\
13 & +3 \\
17 & +7 \\
\hline
\end{array}
$$
$$/\ 2 \times 3 + 3 \times 7 + 2 \times 7\ /$$

Step 4: In left part, any single number and deviation of remaining two numbers i.e., we take any one out of $12+3+7$ or $13+2+7$ or $17+3+2$.

$$
\begin{array}{cc}
\text{Number} & \text{Deviation} \\
12 & +2 \\
13 & +3 \\
17 & +7 \\
\hline
\end{array}
$$

$$12 + 3 + 7 = 22$$
$$\text{or}$$
$$13 + 2 + 7 = 22$$
$$\text{or}$$
$$17 + 3 + 2 = 22$$

Step 5: Consolidate step 1 to 4 and arranging by operation.

$$
\begin{array}{cc}
\text{Number} & \text{Deviation} \\
12 & +2 \\
13 & +3 \\
17 & +7 \\
\hline
\end{array}
$$

(If base is 10 then middle part is multiplied by 1 and left part is multiplied by 1^2.)

$$1^2\,(12+3+7)\ /\ 1\,(2 \times 3 + 3 \times 7 + 2 \times 7)\ /\ 2 \times 3 \times 7$$

$$= 22\ /\ 6 + 21 + 14\ /\ 42$$

$$= 22\ /\ 41\ /\ 42$$

$$= 22\ /\ 1\ /\ 2$$
$$\quad\quad\quad^{+4}\quad^{+4}$$

$$= \quad 2\ \ 6\ \ 5\ \ 2$$

(There is only zero in base 10 therefore there will be single digit in right part and middle part. Second will be carried in next term. For example, in 42, 4 will be in middle and 2 will be right side. 4 will be added in 41 and it will be 45 in which 5 will be in middle and 4 will be added in 22 in left side.)

Thus the exact product of $12 \times 13 \times 17 = 2652$

Example 10: Find out multiplication of 9×8×15 by Nikhilam (base 10) formula.

Sol:

Number	Deviation
9	-1
8	-2
15	+5

Same base = 10
Base = 10, Deviation = -1
Base = 10, Deviation = -2
Base = 10, Deviation = +5

Direct use of previous step-5

Number	Deviation
9	-1
8	-2
15	+5

$$1^2 (15-2-1)/1\{5\times(-2)+(-2)\times(-1)+(5)\times(-1)\}/ (-1)\times(-2)\times(5)$$

$$12/-10+2-5/10$$

$$12/-15+2/10$$

$$10+2/-13/10$$

$$10/20-13/10$$

$$10/7/_1 0$$

1080

(Due to base 10, middle term is multiplied by sub base 1 and left term is multiplied by sub base 1^2).

$(12 = 10 + 2)$

Taking 2 from left side, in middle we get $2 \times 10 = 20$ and write $20 - 13 = 7$.

Example-11: Multiply 22×23×24 by Nikhilam formula (sub base).

22	+2
23	+3
24	+4

$$2^2 (22+3+4)/2\{2\times3+3\times4+4\times2\}/2\times3\times4$$
$$= 2^2 (29)/2(6+12+8)/24$$
$$= 4(29)/2(26)/24$$
$$= 116/_5 2/_2 4$$
$$= \overline{12144}$$

Base = 10, Sub base = $10 \times 2 = 20$, sub base digit = 2
Deviation from sub base = +2, +3, +4
Due to sub base 2 ×10, multiply middle term to sub base digit 2 and multiply left term to sub base digit 2^2.

Thus the exact multiplication of 22×23×24 will be = 12144.

Example 12: Multiply $101 \times 102 \times 103$ by formula Nikhilam.
Sol:

$$
\begin{array}{ll}
101 & +01 \\
102 & +02 \\
103 & +03 \\
\hline
101+2+3 \,/\, 2\times3+1\times2+3\times1 \,/\, 06 \\
= 101+2+3 \,/\, 6+2+3 \,/\, 06 \\
= \quad 106 \,/\, 11 \,/\, 06 \\
= \quad 1061106
\end{array}
$$

Same base = 100,
(Deviation = +01)
(Deviation = +02)
(Deviation = +03)

If base is 100 then there will be two digits in right and middle term because there are two zero in 100 so it will be written right side 06 in place of 6.

Example 13: Multiply $99 \times 98 \times 97$ by formula Nikhilam.
Sol:

Same base = 100,
(Deviation = -01)
(Deviation = -02)
(Deviation = -03)

$$
\begin{array}{ll}
99 & -01 \\
98 & -02 \\
97 & -03 \\
\hline
99-02-03 \,/\, (-02)\times(-03)+(-01)\times(-02)+(-03)\times(-01) \,/\, (-01)\times(-02)\times(-03) \\
= \quad 94 \,/\, 6+2+3 \,/\, \overline{06} \\
= \quad 94 \,/\, 11 \,/\, \overline{06} \\
= \quad 94 \,/\, 10 \,/\, 06 \\
= \quad 94 \,/\, 10 \,/\, 100-6 \\
= \quad 941094
\end{array}
$$

From the middle part, 1 will be taken in right side in form of $1 \times 100 = 100$.

Do and Learn:

Multiply the following three numbers by formula Nikhilam:

(i) $11 \times 12 \times 13$ (ii) $8 \times 9 \times 10$

(iii) $6 \times 7 \times 8$ (iv) $27 \times 28 \times 29$

(v) $98 \times 99 \times 99$ (vi) $51 \times 52 \times 53$

5.3 Cube

1, 8, 27, 64..... are cube numbers which are obtained only multiplying by the three times with the same number. E.g.

$$1 \times 1 \times 1 = 1$$
$$2 \times 2 \times 2 = 8$$
$$3 \times 3 \times 3 = 27$$
$$- - - - - - - -$$

Cube numbers are represented by raising to the power 3. E.g. cube of 2 will be 2^3 and 3 will be 3^3. We have studied the multiplication of three numbers by formula Nikhilam. To find out cube, that process will be repeated. Here all three numbers are same.

Example-14: Multiply 11×11×11 by formula Nikhilam.

Sol:

		Base = 10
11	+1	(Deviation = +1)
11	+1	(Deviation = +1)
11	+1	(Deviation = +1)

$$11+1+1 \,\big/\, (1)\times(1)+(1)\times(1)+(1)\times(1) \,\big/\, (1)\times(1)\times(1)$$

i.e., number + 2 × (deviation) / 3 (deviation)^2 / (deviation)^3 is written in standard form. Now we will solve by using standard form.

Example 15: Find the cube of 15.

Sol: In 15, number is 15 and deviation is 5 then,

$$(15)^3 = 15 + 2 \times 5 \big/ 3 \times 5^2 \big/ 5^3$$

$$= 15 + 10 \big/ 75 \big/ 125$$

$$= 25 \big/ 75 \big/ 125$$

Arranging numbers line wise and adding them,

I	2	5	5	5
II		7	2	-
III		1	-	-
	3	3	7	5

or

$$25 \big/ 75 \big/ 5_{12}$$

$$25 \big/ 87 \big/ 5$$

$$= 33 \big/ 7 \big/ 5$$

$$= 3375$$

Thus the multiplication of number 15×15×15 will be =3375.

Example 16:   Find the cube of 103.
Sol: Base = 100, Deviation = +03

$$103^3 = 103 + 2 \times 3 \big/ 3 \times (03)^2 \big/ (03)^3$$

$$= 103 + 6 \big/ 27 \big/ 27$$

$$= 1092727$$

Example 17:   Find the cubage of 96.
Sol: Base = 100, Deviation = -04

$$96^3 = 96 + 2 \times (-04) \big/ 3 \times (-04)^2 \big/ (-04)^3$$

$$= 96 - 08 \big/ 3 \times 16 \big/ -64$$

$$= 88 \quad 48 \quad -64$$

$$= 88 \big/ 48 \big/ 1\text{-}64 \qquad\qquad \text{(1 is in place of hundreds}$$

$$= 88 \big/ 47 \big/ 100\text{-}64 \qquad\quad \text{so the mean of 1 is 100 unit)}$$

$$= 88 \big/ 47 \big/ 36$$

$$= 884736$$

5.4 Dhwajank Method:

This application is based on formula Urdhwtiryak and formula Dhwajank. By this method, every problem of division operation can be solved easily. Following points are kept in mind while writing the problem before the operation start:

(i)   First the divisor is divided in two parts. Unit part of divisor is called Dhwajank and remaining part is called Cardinal or modified divisor. In dhwajank, there can be so many unit digits.

(ii)   As similar the previous methods of division operation, in this method also, a set location is divided in to three parts.

(a)   In first part, we take both parts of divisor. Cardinal is written below i.e. in place of base and dhwajank is written its above i.e., in place of exponent.

(b)   Digits of dhwajank are same as final digit in third part of dividend and remaining digit will be written in middle part.

(iii)   For dhwajank, $529 \div 23$ can be written as the following:

First part	Middle part	Last part
3	5 2	9
2		
	Quotient	Remainder

Method:
(i)   For middle part, dividing the just left digit of dividend by cardinal a first digit which we get is written below the horizontal line at set place of quotient.
(ii)   Obtained remainder is written from the left side below and before the second digit which is now new dividend.
(iii)   From new dividend we get modified dividend by the following formula:
Modified dividend = New dividend - Quotient digit × Dhwajank
(iv)   Previous actions are repeated if cardinal is divided into modified dividend. Completing the division process, we get remainder and quotient.

This method can be explained by the following examples.

Example 18: Divide 23 in to 552 by Dhwajank method.
Sol:

Hint:

$2 \; {}^{3}5 \; {}_{1}5 \quad {}_{1}2$

$ 2 \quad 4 \quad 0$

(i)   Middle part 5 is divided by 2.
(ii)   First digit of quotient 2 is written below the horizontal line.
(iii)   Remainder = 1 written before the middle part 5 and obtained new dividend = 15.
(iv)   Modified dividend = New dividend - First quotient × Dhwajank
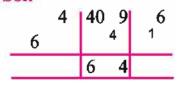$$= 15 - 2 \times 3 = 9$$
(v)   9 divided by cardinal 2. Second digit 4 of quotient is written below the horizontal line.
(vi)   Remainder = 1, written before the third part 2 and obtained new dividend = 12.
(vii)   Modified dividend = New dividend - Second quotient × Dhwajank
$$= 12 - 4 \times 3 = 0$$
(viii)   Thus, quotient = 24, remainder = 0

Example 19: Divide 4096 by 64 (Dhwajank method).

Sol:

```
         4 | 40  9 |  6
   6         4   | 1
   _____
            6   4
```

(i) Dividing 6 in dividend = 40, then quotient = 6 and remainder = 4.

(ii) Modified dividend = $49 - 6 \times 4 = 25$.

(iii) 25 is divided by cardinal 6. Second digit 4 of quotient will be written below the horizontal line.

(iv) Remainder = 1 is written before the third part 6 and get new dividend 16.

(v) Again modified dividend = $16 - 4 \times 4 = 0$

(vi) Thus, quotient = 64, remainder = 0.

Example 20: Divide 87653 by 53 (use Dhwajank method).

Sol:

```
      3 | 8  ₃7  ₄6  ₃5 |  3
   5                    |  5
   _____
          1   6   5   4̶      53-3×3= 44
                      3
```

Hint:

(i) 5 is divided in to middle part 8

(ii) First digit 1 of quotient is written below the horizontal line.

(iii) Remainder 3 is written below and before the middle part 7.

(iv) New dividend = 37

(v) Modified dividend = $37 - 1 \times 3 = 34$

(vi) 34 is divided by cardinal 5. Second digit 6 of quotient will be written below the horizontal line.

(vii) Remainder 4 will be written in middle part before and below 6.

(viii) New dividend = 46

(ix) Modified dividend = $46 - 6 \times 3 = 28$.

(x) 28 is divided by cardinal 5. Third digit 5 of quotient will be written below the horizontal line.

(xi) Remainder 3 will be written in middle part before and below 5.

(xii) New dividend = 35

(xiii) Modified dividend = $35 - 5 \times 3 = 20$.

(xiv) 20 is divided by cardinal 5. Now quotient is 4 and remainder is 0.

(xv) Modified dividend = $3 - 3 \times 4 = -9$. So, not giving the quotient in negative number 3 times quotient is given in place of 4 earlier.

(xvi) Remainder = 05 which is written in last part below 3. Remainder is 0.

(xvii) Remainder = $53 - 3 \times 3 = 44$.

Do and Learn ◆

Division operation using Dhwajank method

   (1) 1737 ÷ 21   (2) 37941 ÷ 47   (3) 23754 ÷ 74

   (4) 3257 ÷ 74   (5) 7453 ÷ 79   (6) 59241 ÷ 82

Exercise 5

1. Multiply by using Urdhwtirgbhyaam formula-
   (i)   101 × 105   (ii) 11 × 15
   (iii) 18 × 81   (iv) 121 × 129

2. Multiply by using Nikhilam formula-
   (i) 48 × 51   (ii) 27 × 29   (iii) 36 × 34
   (iv) 18 × 21   (v) 21 × 22 × 23   (vi) 31 × 28 × 27
   (vii) 96 × 97 × 95   (viii) 18 × 18 × 18   (ix) 99 × 99 × 99

3. Divide by using Dhwajank formula
   (i) 3987 ÷ 28   (ii) 5786 ÷ 78   (iii) 7396 ÷ 82

We Learnt

1. Urdhwtirygbhyaam contains two words Urdhw and Tiryak. Meaning of Urdhw is the product of two just written upper or lower digit and the meaning of Tiryak is the product of two diagonal digits.

2. There are grouping of numbers while multiplying by Urdhwtirygbhyaam formula. When two digits are multiplied by two digit then three groups are made and when three digits are multiplied by three digits then five groups are made.

3. Cubage can be determine in short by Nikhilam formula like this:
   $z^2(x+2y)\big/3y^2z\big/y^3$
   where number x, deviation y and sub base digit is z.

4. To find out the sub base digit, sub base is divided by base.