BCECE
ENGINEERING ENTRANCE EXAM
SOLVED PAPER 2005

Physics

1. Consider the following statements :
The total energy of a particle executing simple harmonic motion depends on its :
(I) amplitude
(II) period
(III) displacement
Of these statements :
(a) I and II are correct
(b) II and III are correct
(c) I and III are correct
(d) I, II and III are correct

2. The maximum velocity of a simple harmonic motion represented by

$$y = 3\sin\left(100\,t + \frac{\pi}{6}\right)\text{m}$$

is given by :
(a) 300 m/s (b) $\frac{3\pi}{6}$ m/s
(c) 100 m/s (d) $\frac{\pi}{6}$ m/s

3. When earth moves round the sun, the quantity which remains constant is :
(a) angular velocity (b) kinetic energy
(c) potential energy (d) areal velocity

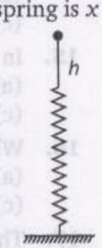
4. 1 kg body explodes into three fragments. The ratio of their masses is 1:1:3. The fragments of same mass move perpendicular to each other with speeds 30 m/s, while the heavier part remains in the initial direction. The speed of heavier part is :
(a) $\frac{10}{\sqrt{2}}$ m/s (b) $10\sqrt{2}$ m/s
(c) $20\sqrt{2}$ m/s (d) $30\sqrt{2}$ m/s

5. The dimensional formula for the gravitational constant is :
(a) $[M^{-1}L^3T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[ML^2T^{-2}]$ (d) $[M^{-1}L^3T]$

6. A particle is moving in a circle of radius R with constant speed v. If radius is doubled, then its centripetal force to keep the same speed gets :
(a) twice as great as before
(b) half
(c) one-fourth
(d) remains constant

7. If linear density of a rod of length 3 m varies as $\lambda = 2 + x$, then the position of the centre of gravity of the rod is :
(a) $\frac{7}{3}$ m (b) $\frac{12}{7}$ m
(c) $\frac{10}{7}$ m (d) $\frac{9}{7}$ m

8. A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. The maximum compression in the spring is x then :



(a) $mgh = \frac{1}{2}k\,x^2$
(b) $mg(h+x) = \frac{1}{2}k\,x^2$
(c) $mgh = \frac{1}{2}k(x+h)^2$
(d) $mg(h+x) = \frac{1}{2}k(x+h)^2$

9. A ball of mass m moves with speed v and it strikes normally with a wall and reflected back normally. If its time of contact with wall is t, then find force exerted by ball on the wall.
(a) $\frac{2mv}{t}$ (b) $\frac{mv}{t}$
(c) mvt (d) $\frac{mv}{2t}$

10. A wheel of radius 1m rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is :
(a) 2π (b) $\sqrt{2}\,\pi$
(c) $\sqrt{\pi^2 + 4}$ (d) π

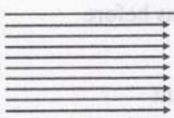
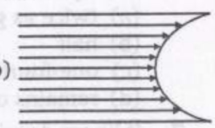
11. Water is flowing in a pipe of diameter 4 cm with a velocity 3 m/s. The water then enters into a pipe of diameter 2 cm. The velocity of water in the other pipe is :

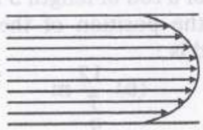
(a) 3 m/s (b) 6 m/s
(c) 12 m/s (d) 8 m/s

12. The surface tension of a liquid is 5 N/m. If a film is held on a ring of area 0.02 m^2, its total surface energy is about :

(a) 2×10^{-2} J (b) 2.5×10^{-2} J
(c) 2×10^{-1} J (d) 3×10^{-1} J

13. A viscous fluid is flowing through a cylindrical tube. The velocity distribution of the fluid is best represented by the diagram :

(a)  (b) 

(c)  (d) none of these

14. A wire is stretched by 1 mm by a force of 1 kN. The work done in stretching the wire is :

(a) 5 erg (b) 5 J
(c) 0.5 erg (d) 0.5 J

15. In isochoric process :

(a) $\Delta U = \Delta Q$ (b) $\Delta Q = \Delta W$
(c) $\Delta U = \Delta W$ (d) none of these

16. Which is not a path function ?

(a) ΔQ (b) $\Delta Q + \Delta W$
(c) ΔW (d) $\Delta Q - \Delta W$

17. The work done, W during an isothermal process in which 1 mole of the gas expands from an initial volume V_1 to a final volume V_2 is given by : (R=gas constant, T= temperature)

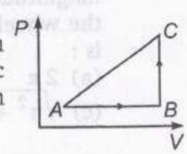
(a) $R(V_2 - V_1)\log_e\left(\frac{T_1}{T_2}\right)$

(b) $R(T_2 - T_1)\log_e\left(\frac{V_2}{V_1}\right)$

(c) $RT\log_e\left(\frac{V_2}{V_1}\right)$

(d) $2RT\log_e\left(\frac{V_1}{V_2}\right)$

18. The P-V diagram of a system undergoing thermodynamic transformation is shown in figure. The work done by the system in going from $A \to B \to C$ is 30 J, and 40 J heat is given to the system. The change in internal energy between A and C is :



(a) 10 J (b) 70 J
(c) 84 J (d) 134 J

19. The degrees of freedom of a molecule of a triatomic gas are :

(a) 2 (b) 4
(c) 6 (d) 8

20. Ozone layer blocks the radiations of wavelength :

(a) less than 3×10^{-7} m
(b) equal to 3×10^{-7} m
(c) more than 3×10^{-7} m
(d) all of the above

21. Two simple harmonic waves of the same amplitude and frequency differ by a phase $\pi/2$. When they are fed simultaneously to the X and Y-plates of a CRO, the screen would display the trace of :

(a) a circle (b) an ellipse
(c) a straight line (d) a square

22. The speed of a wave is 360 m/s and frequency is 500 Hz. Phase difference between two consecutive particles is 60°, then path difference between them will be :

(a) 0.72 cm (b) 120 cm
(c) 12 cm (d) 7.2 cm

23. A string of length 2 m is fixed at both ends. If this string vibrates in its fourth normal mode with a frequency of 500 Hz, then the waves would travel on it with a velocity of :

(a) 125 m/s (b) 250 m/s
(c) 500 m/s (d) 1000 m/s

24. The fundamental frequency of a sonometer wire is n. If its radius is doubled and its tension becomes half, the material of the wire remains same, the new fundamental frequency will be :

(a) n (b) $\frac{n}{\sqrt{2}}$
(c) $\frac{n}{2}$ (d) $\frac{n}{2\sqrt{2}}$

25. In open organ pipe, if fundamental frequency is n, then the other frequencies are :

(a) $n, 2n, 3n, 4n$ (b) $n, 3n, 5n$
(c) $n, 2n, 4n, 8n$ (d) none of these

26. A sound wave of frequency f propagating through air with a velocity c, is reflected from a surface which is moving away from the source with a constant speed v. The frequency of the reflected wave, measured by the observer at the position of the source, is :

(a) $\frac{f(c-v)}{c+v}$ (b) $\frac{f(c+v)}{c-v}$

(c) $\frac{f(c+2v)}{c+v}$ (d) $\frac{f(c-v)}{c-2v}$

27. A convex lens of focal length 10 cm and image formed by it, is at least distance of distinct vision then the magnifying power is :

(a) 3.5 (b) 2.5

(c) 1.5 (d) 1.4

28. If two +5 D, lenses are mounted at some distance apart, the equivalent power will always be negative, if the distance is :

(a) greater than 40 cm

(b) equal to 40 cm

(c) equal to 10 cm

(d) less than 10 cm

29. A Galilean telescope has an objective of focal length 100 cm and magnifying power 50. The distance between the two lenses in normal adjustment will be :

(a) 98 cm (b) 100 cm

(c) 150 cm (d) 200 cm

30. A film projector magnifies a 100 cm^2 film strip on a screen. If the linear magnification is 4, the area of the magnified film on the screen is :

(a) 1600 cm^2 (b) 400 cm^2

(c) 800 cm^2 (d) 200 cm^2

31. The exposure time of a camera lens at $f/2.8$ setting is 1/200 s. The correct exposure time at $f/5.6$ setting is :

(a) 0.02 s (b) 0.04 s

(c) 0.20 s (d) 0.40 s

32. In Young's double slit experiment, the slit width and the distance of slits from the screen both are doubled. The fringe width :

(a) increases

(b) decreases

(c) remains unchanged

(d) none of the above

33. In Young's double slit experiment, the aperture screen distance is 2 m. The slit width is 1 mm. Light of 600 nm is used. If a thin plate of glass ($\mu = 1.5$) of thickness 0.06 mm is placed over one of the slits, then there will be a lateral displacement of the fringes by :

(a) zero (b) 6 cm

(c) 10 cm (d) 15 cm

34. A single slit is used to observe diffraction pattern with red light. On replacing the red light with violet light the diffraction pattern would :

(a) remain unchanged

(b) become narrower

(c) become broader

(d) disappear

35. The resistance of a discharge tube is :

(a) zero (b) ohmic

(c) non-ohmic (d) infinity

36. Which one of the following processes depends on gravity ?

(a) Conduction

(b) Convection

(c) Radiation

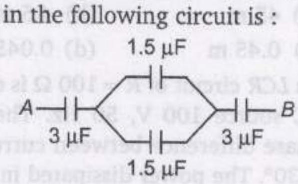
(d) None of the above

37. The radius of solid metallic non-conducting sphere is 60 cm and charge on the sphere is 500 μC. The electric field at a distance 10 cm from centre of sphere is :

(a) 2×10^6 N/C (b) 2×10^8 N/C

(c) 5×10^6 N/C (d) 5×10^8 N/C

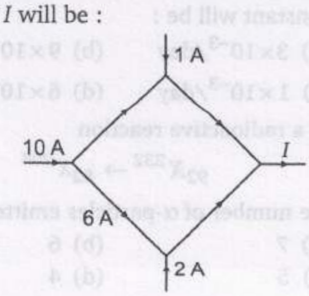
38. The equivalent capacitance between the points A and B in the following circuit is :



(a) 1 μF (b) 2μF

(c) 4 μF (d) 8 μF

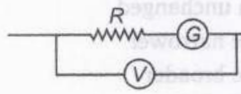
39. The figure shows a network of currents. The magnitude of current is shown here. The current I will be :



(a) 3 A (b) 9 A

(c) 13 A (d) 19 A

40. If resistance of voltmeter is 10000 Ω and resistance of galvanometer is 2 Ω, then find R when voltmeter reads 12 V and galvanometer reads 0.1 A.



(a) 118 Ω (b) 120 Ω
(c) 124 Ω (d) 114 Ω

41. Two bulbs 25 W, 220 V and 100 W, 220 V are given. Which has higher resistance ?

(a) 25 W bulb
(b) 100 W bulb
(c) Both bulbs will have equal resistance
(d) Resistance of bulbs cannot be compared

42. An electron (mass $= 9.1 \times 10^{-31}$ kg, charge $= 1.6 \times 10^{-19}$ C) experiences no deflection, if subjected to an electric field of 3.2×10^5 V/m, and a magnetic field of 2.0×10^{-3} Wb/m^2. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius :

(a) 45 m (b) 4.5 m
(c) 0.45 m (d) 0.045 m

43. An LCR circuit of $R = 100\ \Omega$ is connected to an AC source 100 V, 50 Hz. The magnitude of phase difference between current and voltage is 30°. The power dissipated in the LCR circuit is :

(a) 50 W (b) 86.6 W
(c) 100 W (d) 200 W

44. If half-life of radium is 77 days, its decay constant will be :

(a) 3×10^{-3}/day (b) 9×10^{-3}/day
(c) 1×10^{-3}/day (d) 6×10^{-3}/day

45. In a radioactive reaction

$$_{92}X^{232} \rightarrow {}_{82}X^{204}$$

the number of α-particles emitted is :

(a) 7 (b) 6
(c) 5 (d) 4

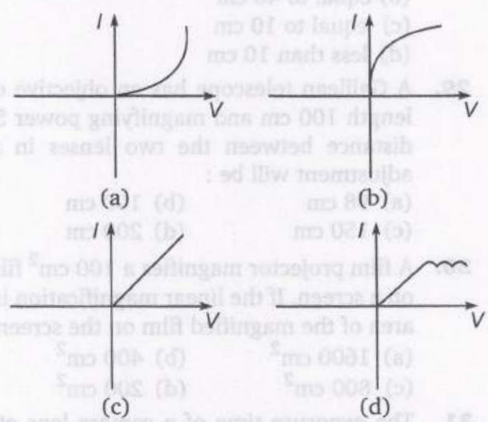
46. According to Hubble's law, the red-shift (Z) of a receding galaxy and its distance r from earth are related as :

(a) $Z \propto r$ (b) $Z \propto \frac{1}{r}$
(c) $Z \propto \frac{1}{r^2}$ (d) $Z \propto r^{3/2}$

47. The de-Broglie wavelength of a body of mass m and kinetic energy E is given by :

(a) $\lambda = \frac{h}{mE}$ (b) $\lambda = \frac{\sqrt{2mE}}{h}$
(c) $\lambda = \frac{h}{2mE}$ d) $\lambda = \frac{h}{\sqrt{2mE}}$

48. Different voltages are applied across a p-n junction and the currents are measured from each value. Which of the following graphs is obtained between voltage and current ?



49. A logic gate having two inputs A and B and output C has the following truth table

A	B	C
1	1	0
1	0	1
0	1	1
0	0	1

It is :

(a) an OR gate (b) an AND gate
(c) a NOR gate (d) a NAND gate

50. For sky wave propagation of 10 MHz signal, what should be the minimum electron density in ionosphere ?

(a) $\sim 1.2 \times 10^{12}\ m^{-3}$ (b) $\sim 10^6\ m^{-3}$
(c) $\sim 10^{14}\ m^{-3}$ (d) $\sim 10^{22}\ m^{-3}$

Chemistry

1. The number of molecules of CO_2 present in 44 g of CO_2 is :
(a) 6.0×10^{23} (b) 3×10^{23}
(c) 12×10^{23} (d) 3×10^{10}

2. Magnitude of kinetic energy in an orbit is equal to :
(a) half of the potential energy
(b) twice of the potential energy
(c) one fourth of the potential energy
(d) none of the above

3. A *p*-orbital in a given shell can accommodate upto :
(a) four electrons
(b) two electrons with parallel spin
(c) six electrons
(d) two electrons with opposite spin

4. Which of the following is Heisenberg uncertainty principle ?
(a) $\Delta x\cdot\Delta p\geq\frac{h}{4\pi}$ (b) $\Delta x\cdot\Delta p=\frac{h}{4\pi}$
(c) $\Delta x\cdot\Delta p\leq\frac{h}{4\pi}$ (d) $\Delta x\cdot\Delta p<\frac{h}{4\pi}$

5. Chromium is represented by the electronic configuration :
(a) $[Ne]\,3s^2\,3P^6\,3d^1\,4s^2$
(b) $[Ne]\,3s^2 3p^6 3d^2 4s^1$
(c) $[Ne]\,3s^2\,3p^6\,3d^5\,4s^1$
(d) $[Ne]\,3s^2\,3p^6\,4s^2\,3d^4$

6. Number of neutron in C^{12} is :
(a) 6 (b) 7
(c) 8 (d) 9

7. H-bond is not present in :
(a) water
(b) glycerol
(c) hydrogen fluoride
(d) hydrogen sulphide

8. Oxidation number of S in SO_4^{2-} :
(a) +6 (b) +3
(c) +2 (d) −2

9. 2 g of O_2 at 27°C and 760 mm of Hg pressure has volume :
(a) 1.4 L (b) 2.8 L
(c) 11.2 L (d) 22.4 L

10. If pressure increases then its effect on given equilibrium $2NO(g)\rightleftharpoons N_2(g)+O_2(g)$ is shift in :
(a) forward direction
(b) backward direction
(c) no effect
(d) none of the above

11. Which is Lewis base : $I_2+I^-\longrightarrow I_3^-$?
(a) I_2 (b) I_3
(c) I^- (d) None of these

12. The heat of neutralisation of any strong acid and a strong base is nearly equal to :
(a) −75.3 kJ (b) + 57.3 kJ
(c) − 57.3 kJ (d) + 75.3 kJ

13. pH of 10^{-8} M HCl solution is :
(a) 8 (b) between 7 and 8
(c) between 6 and 7 (d) between 8 and 9

14. A reaction $A\longrightarrow B$ follows a second order kinetic. Doubling the concentration of *A* will increase the rate of formation of *B* by a factor of :
(a) 1/4 (b) 4
(c) 1/2 (d) 2

15. Which of the following is always negative for exothermic reaction ?
(a) ΔH (b) ΔS
(c) ΔG (d) None of these

16. Unit of equivalent conductance is :
(a) $ohm^{-1}\,cm^2\,(g\text{-}eq)^{-1}$
(b) ohm cm(g - eq)
(c) $ohm\,cm^2\,(g\text{-}eq)^{-1}$
(d) $ohm^{-1}\,cm(g\text{-}eq)^{-1}$

17. For cell reaction
$$Zn+Cu^{2+}\rightarrow Zn^{2+}+Cu$$
cell representation is:
(a) $Zn|Zn^{2+}||Cu^{2+}|Cu$
(b) $Cu|Cu^{2+}||Zn^{2+}|Zn$
(c) $Cu|Zn^{2+}||Zn|Cu^{2+}$
(d) $Cu^{2+}|Zn||Zn^{2+}|Cu$

18. Molal solution means 1 mole of solute present in :
(a) 1000g of solvent (b) 1 L of solvent
(c) 1 L of solution (d) 1000g of solution

19. Which of the following shows maximum depression in freezing point ?
(a) K_2SO_4 (b) NaCl
(c) Urea (d) Glucose

20. An emulsion is a colloidal dispersion of :
(a) a liquid in a gas
(b) a liquid in a liquid
(c) a solid in a liquid
(d) a gas in a solid

21. Size of colloidal particle is :
(a) 1 nm (b) 1-100 nm
(c) < 100 nm (d) > 1000 nm

22. Ether show isomerism with :
(a) alcohol (b) ethane
(c) halide (d) aldehyde

23. Which of the following shows geometrical isomerism ?
(a) C_2H_5Br (b) $(CH_2)(COOH)_2$
(c) $(CH)_2(COOH)_2$ (d) C_2H_6

24. Which of the following is present in natural gas ?
(a) *n*-butane (b) Ethane
(c) Methane (d) Propane

25. Which of the following have delocalised electron ?
(a) Benzene (b) Cyclohexane
(c) CH_4 (d) C_2H_6

26. Nitration of benzene is :
(a) electrophilic substitution
(b) electrophilic addition
(c) nucleophilic substitution
(d) nucleophilic addition

27. Alkyl halide can be converted into alkene by :
(a) nucleophilic substitution reaction
(b) elimination reaction
(c) both nucleophilic substitution and elimination reaction
(d) rearrangement

28. Which of the following compounds is most acidic ?
(a) CH_4 (b) C_2H_6
(c) $CH \equiv CH$ (d) C_2H_5OH

29. Which of the following reacts with $KMnO_4$ but does not react with $AgNO_3$?
(a) C_2H_6 (b) CH_4
(c) C_2H_4 (d) C_2H_2

30. Which of the following compound does not show anti-Markownikoff's addition ?
(a) Propene (b) 1-butene
(c) 2-butene (d) 2-pentene

31. Methyl ketone is identified by :
(a) Iodoform test (b) Fehling solution
(c) Tollen's reagent (d) Schiff's reagent

32. Which of the following does not give Fehling solution test ?
(a) Acetone (b) Propanal
(c) Ethanal (d) Butanal

33. Cyanide group on hydrolysis gives :
(a) acid (b) acetamide
(c) amine (d) hydrate

34. In alkyl cyanide alkyl group attached with :
(a) C of CN group
(b) N of CN group
(c) either C or N of CN group
(d) both C and N of CN group

35. Fat on hydrolysis gives which alcohol ?
(a) Glycerol (b) Propanol
(c) Butanol (d) Ethanol

36. Which of the following is natural polymer ?
(a) PVC (b) Nylon 66
(c) Teflon (d) Cellulose

37. Condensation product of caprolactum is :
(a) Nylon 6 (b) Nylon 66
(c) Nylon 60 (d) Nylon 6, 10

38. Which inert gas have highest boiling point ?
(a) Xe (b) Ar
(a) Kr (d) He

39. In Ostwald process of manufacturing of HNO_3, catalyst used is :
(a) Mo (b) Fe
(c) Mn (d) Pt

40. Important ore of Mg is :
(a) Gypsum (b) Carnallite
(c) Magnatide (d) Carnolite

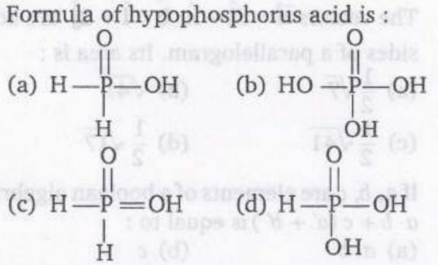
41. Purification of Al by electrolysis method is called :
(a) Hall's process (b) Baeyer process
(c) Ostwald process (d) Hoope's process

42. Decreasing size of ions is :
(a) $I > I^- > I^+$ (b) $I^- > I > I^+$
(c) $I^+ > I^- > I$ (d) $I > I^+ > I^-$

43. Which of the following is kept in water ?
(a) White phosphorus
(b) Sodium
(c) Potassium
(d) Calcium

44. When heated NH_3 is passed over CuO gas evolved is :
(a) N_2 (b) N_2O
(c) HNO_3 (d) NO_2

45. Formula of hypophosphorus acid is :



46. Most powerful reducing agent is :
(a) Li (b) Na (c) Ca (d) Mg

47. When calomel reacts with NH_4OH solution the compound formed is :
(a) $NH_2—Hg—Cl$ (b) $Hg_2Cl_2NH_3$
(c) $Hg(NH_3)_2Cl_2$ (d) $HgCl_2NH_3$

48. Which of the following is coloured compound ?
(a) CuF_2 (b) CuI
(c) NaCl (d) $MgCl_2$

49. H_2O_2 is formed by which of the following compounds ?
(a) Na_2O_2 (b) NaOH
(c) Na_2O (d) KO_2

50. Copper sulphate solution reacts with KCN and gives :
(a) $K_3[Cu(CN)_4]$
(b) CuCN
(c) $Cu(CN)_2$
(d) $K_2[Cu(CN)_4]$

Mathematics

1. If $A \subseteq B$, then $B \cup A$ is equal to :
(a) $B \cap A$ (b) A
(c) B (d) none of these

2. The real value of α for which the expression $\frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely real, is :
(a) $(2n+1)\frac{\pi}{2}$ (b) $(n+1)\frac{\pi}{2}$
(c) $n\pi$ (d) none of these

3. The medians AD and BE of the triangle with vertices $A(0, b), B(0, 0)$ and $C(a, 0)$ are mutually perpendicular, if :
(a) $b = a$ (b) $b = -2\sqrt{a}$
(c) $a = \pm\sqrt{2}b$ (d) $b = \sqrt{2}a$

4. The equation of line parallel to the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) and passing through origin , is :
(a) $xy_1 + x_1 y = 0$ (b) $x x_1 - y y_1 = 0$
(c) $x x_1 + y y_1 = 0$ (d) $x y - x_1 y = 0$

5. $\sin 200° + \cos 200°$ is :
(a) positive (b) negative
(c) zero (d) zero or positive

6. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is :
(a) $\frac{4\pi}{3}$ (b) $\frac{5\pi}{3}$
(c) $-\frac{2\pi}{3}$ (d) $-\frac{\pi}{3}$

7. If $xy + yz + zx = 1$, then $\sum \frac{x+y}{1-xy}$ is equal to :
(a) $\frac{4}{xyz}$ (b) $\frac{1}{xyz}$
(c) xyz (d) none of these

8. A vertical pole (more than 100 m high) consists of two portions, the lower being one-third of the whole. If the upper portion subtends an angle $\tan^{-1}\frac{1}{2}$ at a point in a horizontal plane through the foot of the pole and distance 40 ft from it, then the height of the pole is :
(a) 100 ft (b) 120 ft
(c) 150 ft (d) none of these

9. When the three coins are tossed simultaneously, then the probability of getting one head will be :
(a) $\frac{3}{7}$ (b) $\frac{1}{7}$ (c) $\frac{3}{8}$ (d) $\frac{7}{8}$

10. The cosine of the angle between any two diagonals of a cube is :
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

11. If f is any function, then $\frac{1}{2}[f(x) + f(-x)]$ is always :
(a) odd
(b) even
(c) neither even nor odd
(d) one-one

12. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $f(g(x))$ is equal to :
(a) $f(3x)$ (b) $(f(x))^3$
(c) $3f(x)$ (d) $-f(x)$

13. $\lim_{x\to 0}[\cos x]$ is equal to :
(a) -1 (b) 1
(c) 0 (d) none of these

14. $\frac{d}{dx}(\sin^{-1} 2x\sqrt{1-x^2})$ is equal to :
(a) $-\frac{2}{\sqrt{1-x^2}}$ (b) $\frac{2}{\sqrt{1-x^2}}$
(c) $\cos 2x$ (d) none of these

15. $\int_0^{\pi/2}|\sin x-\cos x|\,dx$ is equal to :
(a) $2(\sqrt{2}+1)$ (b) $\sqrt{2}-1$
(c) $2(\sqrt{2}-1)$ (d) 0

16. If p and q are the roots of the equation $x^2+pq=(p+1)x$, then q is equal to :
(a) -1 (b) 2
(c) 1 (d) -2

17. If a line lies in the octant $OXYZ$ and it makes equal angle with the axes, then :
(a) $l=m=n=\frac{1}{\sqrt{3}}$ (b) $l=m=n=\pm\frac{1}{\sqrt{3}}$
(c) $l=m=n=-\frac{1}{\sqrt{3}}$ (d) $l=m=n=\pm\frac{1}{\sqrt{2}}$

18. If a, b, c are in AP, a, mb, c are in GP, then a, m^2b, c are in :
(a) HP (b) GP
(c) AP (d) none of these

19. If $p\Rightarrow(q\vee r)$ is false, then the truth values of p, q, r are respectively :
(a) T, F, F
(b) F, F, F
(c) F, T, T
(d) T, T, F

20. In the expansion of $(3x+2)^4$ the coefficient of middle term is :
(a) 36 (b) 216
(c) 54 (d) 81

21. For any 2×2 matrix A, if $A\,(\text{adj}\,A)=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$, then $|A|$ *i.e.*, det A is equal to :
(a) 20 (b) 100
(c) 10 (d) 0

22. The vectors $\vec{a}=3\hat{i}-\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}$ are adjacent sides of a parallelogram. Its area is :
(a) $\frac{1}{2}\sqrt{7}$ (b) $\sqrt{41}$
(c) $\frac{1}{2}\sqrt{41}$ (d) $\frac{1}{2}\sqrt{17}$

23. If a, b, c are elements of a boolean algebra, then $a\cdot b+c\,(a'+b')$ is equal to :
(a) $a\cdot b$ (b) c
(c) $a\cdot b+c$ (d) none of these

24. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$ implies that :
(a) $\vec{a}=-\vec{b}$ (b) $\vec{a}\neq\vec{b}$
(c) $|\vec{a}|=|\vec{b}|$ (d) $\vec{a}=\vec{b}$

25. 3 persons have 4 coats, 5 waist coats and 6 hats. The number of ways in which they put on the clothes are :
(a) $4^3\times 5^3\times 6^3$ (b) $4\times 5\times 6$
(c) $4!\,5!\,6!$ (d) none of these

26. If an integer p is chosen at random in the interval $0\le p\le 5$, the probability that the roots of the equation $x^2+px+\frac{p}{4}+\frac{1}{2}=0$ are real, is :
(a) $\frac{4}{5}$ (b) $\frac{2}{3}$
(c) $\frac{3}{5}$ (d) none of these

27. $\int_0^1 x(1-x)^4\,dx$ is equal to :
(a) 1 (b) 0 (c) $\frac{1}{30}$ (d) $\frac{1}{5}$

28. The function $f(x)=2-3x$ is :
(a) increasing
(b) decreasing
(c) neither decreasing nor increasing
(d) none of the above

29. The value of $\int_0^{\pi/2}\frac{\frac{\pi}{4}-x}{\sqrt{\sin x+\cos x}}\,dx$ is :
(a) $\frac{\pi\sqrt{3}}{4}$ (b) $\frac{\pi}{4\sqrt{2}}$
(c) 0 (d) none of these

30. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to :
(a) $\frac{1}{2y+1}$ (b) $\frac{1}{2y-1}$
(c) $\frac{1}{xy}$ (d) 1

31. If $f : A \to B$ is a bijection, then :
(a) $n(A) = n(B)$ (b) $n(A) \leq n(B)$
(c) $n(A) \geq n(s)$ (d) none of these

32. The equation of a straight line passing through (–3, 2) and cutting an intercept equal in magnitude but opposite in sign from axis, is given by :
(a) $x - y + 5 = 0$
(b) $x + y - 5 = 0$
(c) $x - y - 5 = 0$
(d) $x + y + 5 = 0$

33. If $\frac{2z_1}{3z_2}$ is a purely imaginary number, then $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$ is equal to :
(a) $\frac{3}{2}$ (b) 1
(c) $\frac{2}{3}$ (d) $\frac{4}{9}$

34. If pth, qth, rth, sth terms of an arithmetic progression are in geometric progression, then $p - q$, $q - r$ and $r - s$ are in :
(a) HP (b) GP
(c) AP (d) no particular order

35. Two bodies are projected from the same point with the same velocity but in different directions. If the range in each case be R and times of flight be t_1 and t_2, then R is equal to :
(a) $\frac{1}{2} g t_1 t_2$ (b) $g t_1 t_2$
(c) $\frac{1}{4} g t_1 t_2$ (d) $2g t_1 t_2$

36. The reciprocal of the eccentricity of rectangular hyperbola is :
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) 2

37. If $y = \frac{a^{\cos^{-1} x}}{1 + a^{\cos^{-1} x}}$, $z = a^{\cos^{-1} x}$, then $\frac{dy}{dx}$ is equal to :
(a) $\frac{1}{1 + a^{\cos^{-1} x}}$ (b) $\frac{1}{(1 + a^{\cos^{-1} x})^2}$
(c) $\frac{-1}{1 + a^{\cos^{-1} x}}$ (d) none of these

38. The value of
$\sin 600° \cos 330° + \cos 120° \sin 150°$ is :
(a) 1 (b) –1
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

39. The area of circle whose centre is (h, k) and radius a is :
(a) $\pi a^2 hk$ sq unit
(b) πa^2 sq unit
(c) $\pi (h^2 + k^2 - a^2)$ sq unit
(d) none of the above

40. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then X is equal to :
(a) $\begin{bmatrix} -3 & 4 \\ -4 & 13 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$
(c) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$

41. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, then $[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}]$ is equal to :
(a) $2|\vec{a}\ \vec{b}\ \vec{c}|$ (b) $|\vec{a}\ \vec{b}\ \vec{c}|$
(c) $3|\vec{a}\ \vec{b}\ \vec{c}|$ (d) 0

42. If the resultant of two unlike parallel forces of magnitudes 10 N and 16 N act along a line at a distance of 24 cm from the line of action of the smaller force is 8 N, then the distance between the lines of action of the force is :
(a) 12 cm (b) 8 cm
(c) 10.66 cm (d) 18 cm

43. The resultant of two forces $\vec{P}$ and $\vec{Q}$ is of magnitude P. If the force $\vec{P}$ is doubled, $\vec{Q}$ remaining unaltered, then the new resultant will be :
(a) along $\vec{P}$ (b) along $\vec{Q}$
(c) at 60° to $\vec{Q}$ (d) at right angle to $\vec{Q}$

44. If the ratio of the sum of m and n terms of an AP is $m^2 : n^2$, then the ratio of its mth and nth terms is :
(a) $m - 1 : n - 1$
(b) $2m + 1 : 2n + 1$
(c) $2m - 1 : 2n - 1$
(d) none of these

45. A body dropped from a height h at time $t = 0$ reaches the ground at time t_0. It would have reached a height $h/2$ at time :
(a) $\frac{t_0}{2}$ (b) $\frac{t_0}{\sqrt{2}}$
(c) t_0^2 (d) $\frac{1}{t_0^2}$

46. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is :

(a) $\frac{\pi^3}{4}$ sq unit (b) $\frac{\pi^3 - 16}{4}$ sq unit

(c) $\frac{\pi^3 - 8}{2}$ sq unit (d) $\frac{\pi^3 - 8}{4}$ sq unit

47. In triangle ABC, if $3a = b + c$, then $\cot \frac{B}{2} \cot \frac{C}{2}$ is equal to :

(a) $\sqrt{3}$ (b) 1 (c) 2 (d) 3

48. If $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1 + x^2} - x)$, then $f(x)$:

(a) increases in $(-\infty, \infty)$
(b) decreases in $(0, \infty)$
(c) neither increases nor decreases in $(0, \infty)$
(d) sometimes increases and sometimes decreases

49. The series $(1 + 3) \log_e 3 + \frac{1 + 3^2}{2!} (\log_e 3)^2 + \frac{1 + 3^2}{3!} (\log_e 3)^2 + \ldots$ is equal to :

(a) 28
(b) 30
(c) 25
(d) 0

50. $\int \frac{\cos 4x - 1}{\cot x - \tan x} dx$ is equal to :

(a) $-\frac{1}{2} \cos 4x + c$
(b) $-\frac{1}{4} \cos 4x + c$
(c) $-\frac{1}{2} \sin 2x + c$
(d) none of these

ANSWERS

PHYSICS

1. (a)	2. (a)	3. (d)	4. (b)	5. (a)	6. (b)	7. (b)	8. (a)	9. (a)	10. (c)
11. (c)	12. (c)	13. (c)	14. (d)	15. (a)	16. (d)	17. (c)	18. (a)	19. (c)	20. (a)
21. (a)	22. (c)	23. (c)	24. (d)	25. (a)	26. (c)	27. (a)	28. (a)	29. (a)	30. (a)
31. (a)	32. (c)	33. (b)	34. (b)	35. (c)	36. (b)	37. (a)	38. (a)	39. (c)	40. (a)
41. (a)	42. (c)	43. (b)	44. (b)	45. (a)	46. (a)	47. (d)	48. (a)	49. (d)	50. (a)

CHEMISTRY

1. (a)	2. (a)	3. (c)	4. (a)	5. (c)	6. (a)	7. (d)	8. (a)	9. (a)	10. (c)
11. (c)	12. (c)	13. (c)	14. (b)	15. (a)	16. (a)	17. (a)	18. (c)	19. (a)	20. (b)
21. (b)	22. (a)	23. (c)	24. (c)	25. (a)	26. (a)	27. (b)	28. (d)	29. (c)	30. (c)
31. (a)	32. (a)	33. (a)	34. (a)	35. (a)	36. (d)	37. (a)	38. (a)	39. (d)	40. (b)
41. (d)	42. (b)	43. (a)	44. (a)	45. (a)	46. (a)	47. (a)	48. (a)	49. (a)	50. (a)

MATHEMATICS

1. (c)	2. (c)	3. (c)	4. (c)	5. (b)	6. (d)	7. (b)	8. (b)	9. (c)	10. (a)
11. (b)	12. (c)	13. (c)	14. (b)	15. (c)	16. (c)	17. (a)	18. (a)	19. (a)	20. (b)
21. (c)	22. (b)	23. (c)	24. (c)	25. (a)	26. (b)	27. (c)	28. (b)	29. (c)	30. (b)
31. (a)	32. (a)	33. (b)	34. (b)	35. (a)	36. (a)	37. (d)	38. (b)	39. (b)	40. (c)
41. (d)	42. (c)	43. (d)	44. (c)	45. (b)	46. (d)	47. (c)	48. (a)	49. (a)	50. (d)

HINTS & SOLUTIONS

Physics

1. Key Idea : *Total energy of a particle executing simple harmonic motion is obtained by summing its potential and kinetic energies.*

Potential energy of particle in SHM

$$U = \frac{1}{2} m\omega^2 x^2$$

or $$U = \frac{1}{2} m(2\pi f)^2 x^2$$

or $$U = 2\pi^2 m f^2 x^2 \quad \text{...(i)}$$

Kinetic energy of particle in SHM

$$K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

or $$K = 2\pi^2 m f^2 (A^2 - x^2) \quad \text{...(ii)}$$

Hence, total energy

$$E = K + U$$
$$= 2\pi^2 m f^2 x^2 + 2\pi^2 m f^2 (A^2 - x^2)$$
$$= 2\pi^2 m f^2 A^2 = \frac{2\pi^2 m A^2}{T^2}$$
$$\left(\because T = \frac{1}{f}\right)$$

Thus, it is obvious that total energy of particle executing simple harmonic motion depends on amplitude (A) and period (T).

2. Key Idea : *Equating the given equation with general equation of SHM.*

The given equation is written as

$$y = 3\sin\left(100t + \frac{\pi}{6}\right) \quad \text{...(i)}$$

The general equation of simple harmonic motion is written as

$$y = a\sin(\omega t + \phi) \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$a = 3, \quad \omega = 100$$

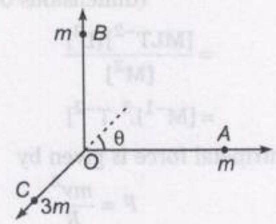
Maximum velocity, $v = a\omega$

$$= 3 \times 100$$
$$= 300 \text{ m/s}$$

3. When earth moves round the sun then according to Kepler's second law, the radius vector drawn from the sun to earth, sweeps out equal areas in equal time, *i. e.*, its areal velocity (or the area swept out by it per unit time) is constant. While in such motion, angular velocity, kinetic energy and potential energy change.

4. Key Idea : *Equate the momenta of the system along two perpendicular axes.*

Let u be the velocity and θ the direction of the third piece as shown.



Equating the momenta of the system along OA and OB to zero, we get

$$m \times 30 - 3m \times v\cos\theta = 0 \quad \text{...(i)}$$

and $$m \times 30 - 3m \times v\sin\theta = 0 \quad \text{...(ii)}$$

These give $3mv\cos\theta = 3mv\sin\theta$

or $$\cos\theta = \sin\theta$$

$\therefore$ $$\theta = 45°$$

Thus, $\angle AOC = \angle BOC = 180° - 45° = 135°$

Putting the value of θ in Eq. (i), we get

$$30m = 3mv\cos 45° = \frac{3mv}{\sqrt{2}}$$

$\therefore$ $$v = 10\sqrt{2} \text{ m/s}$$

The third piece will move with a velocity of $10\sqrt{2}$ m/s in a direction making an angle of 135° with either piece.

Alternative : Key Idea : *The square of momentum of third piece is equal to sum of squares of momentum of first and second piece.*

As from key idea,

$$p_3^2 = p_1^2 + p_2^2$$

or $$p_3 = \sqrt{p_1^2 + p_2^2}$$

or $$3mv_3 = \sqrt{(m \times 30)^2 + (m \times 30)^2}$$

or $$v_3 = \frac{30\sqrt{2}}{3}$$
$$= 10\sqrt{2} \text{ m/s}$$

5. Gravitational constant is equal in magnitude to that force of attraction which acts between two particles each of unit mass separated by a unit distance apart.

$$\therefore \quad G = \frac{Fr^2}{m_1 m_2}$$

(Newton's law of gravitation)

where m_1 and m_2 are masses, r is the distance between them, and F is force.

$\therefore$ Dimensions of gravitational constant

$$= \frac{\text{dimensions of force} \times (\text{length})^2}{(\text{dimensions of mass})^2}$$

$$= \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{M}^2]}$$

$$= [\text{M}^{-1}\text{L}^3\,\text{T}^{-2}]$$

6. Centripetal force is given by

$$F = \frac{mv^2}{R}$$

where m is mass of particle, v is speed, and R is radius of circular path.

$$\Rightarrow \quad F \propto \frac{1}{R}$$

or $\quad \dfrac{F_2}{F_1} = \dfrac{R_1}{R_2}$

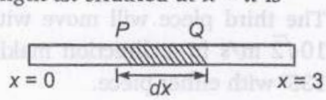
Given, $\quad R_2 = 2R_1$

$\therefore \quad \dfrac{F_2}{F_1} = \dfrac{R_1}{2R_1} = \dfrac{1}{2}$

or $\quad F_2 = \dfrac{F_1}{2}$

Therefore, centripetal force will become half.

7. Let rod is placed along x - axis. Mass of element PQ of length dx situated at $x = x$ is



$$dm = \lambda dx = (2 + x)\,dx$$

The COM of the element has coordinates $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod will be

$$x_{\text{COM}} = \frac{\int_0^3 x\,dm}{\int_0^3 dm}$$

$$= \frac{\int_0^3 x(2+x)\,dx}{\int_0^3 (2+x)\,dx}$$

$$= \frac{\int_0^3 (2x + x^2)\,dx}{\int_0^3 (2+x)\,dx}$$

$$= \frac{\left[\dfrac{2x^2}{2} + \dfrac{x^3}{3}\right]_0^3}{\left[2x + \dfrac{x^2}{2}\right]_0^3}$$

$$= \frac{\left[(3)^2 + \dfrac{(3)^3}{3}\right]}{\left[2 \times 3 + \dfrac{(3)^2}{2}\right]}$$

$$= \frac{9+9}{6+9/2} = \frac{18 \times 2}{21}$$

$$= \frac{12}{7}\ \text{m}$$

8. **Key Idea :** *Potential energy of block at height h is converted into potential energy of spring.*

When block is at height h, it has potential energy mgh, which is converted into potential energy of spring to conserve energy

i.e., $\quad mgh = \dfrac{1}{2} k x^2$

where k is force constant of spring.

9. Initial velocity of ball $= v$

When it strikes the wall normally and reflected back, then final velocity $= -v$

Change in velocity $= v - (-v) = 2v$

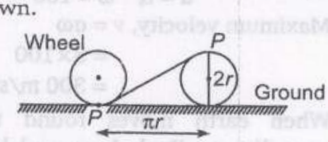
Force exerted by the ball on the wall is given by Newton's second law, i.e.,

$$F = ma$$

$$= \frac{m\Delta v}{\Delta t}$$

$$= \frac{m(2v)}{t} = \frac{2mv}{t}$$

10. When wheel rolls half a revolution, the point (P) of the wheel which is in contact with the ground initially, moves at the top of the wheel as shown.



Horizontal displacement of wheel,

$x = \pi r$, where r being the radius.

Vertical displacement of point P,

$$y = 2r$$

Net displacement $= \sqrt{x^2 + y^2}$

$$= \sqrt{(\pi r)^2 + (2r)^2}$$

$$= r\sqrt{\pi^2 + 4}$$

$$= \sqrt{\pi^2 + 4} \qquad (\because r = 1 \text{ m})$$

11. According to principle of continuity, when an incompressible and non-viscous liquid flows in a streamlined motion through a tube of non-uniform cross-section, then the product of the area of cross-section and the velocity of flow is same at every point in the tube.

Thus, $Av = \text{constant}$

or $A_1 v_1 = A_2 v_2$

or $\pi r_1^2 v_1 = \pi r_2^2 v_2$

Given, $r_1 = \frac{4}{2} \text{ cm} = 0.02 \text{ m}$,

$r_2 = \frac{2}{2} \text{ cm} = 0.01 \text{ m}$,

$v_1 = 3 \text{ m/s}$

$\therefore \quad \pi (0.02)^2 \times 3 = \pi (0.01)^2 v_2$

or $\quad v_2 = \left(\frac{0.02}{0.01}\right)^2 \times 3 = 12 \text{ m/s}$

12. Surface energy is related to the surface tension by the relation

$$U = Td\,A$$

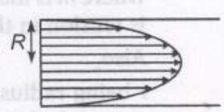
Given, $T = 5 \text{ N/m}$

$d\,A = 2A = 2 \times 0.02 = 0.04 \text{ m}^2$

$\therefore \quad U = 5 \times 0.04 = 0.20 \text{ J}$

$= 2 \times 10^{-1} \text{ J}$

13. Figure shows the flow speed profile for laminar flow of a viscous fluid in a long cylindrical pipe.



The speed is greatest along the axis and zero at the pipe walls.

14. Work done by force F in stretching the wire through distance x is,

$$W = \frac{1}{2} Fx$$

Given, $F = 1 \text{ kN} = 1000 \text{ N}$,

$x = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

$\therefore \quad W = \frac{1}{2} \times 1000 \times 1 \times 10^{-3} = 0.5 \text{ J}$

15. An isochoric process is a constant volume process. In an isochoric process

$V = \text{constant} \quad \text{or} \quad \Delta V = 0$

So, work done

$$\Delta W = P\Delta V = 0$$

From first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$\Rightarrow \quad \Delta Q = \Delta U$

Note: P-V diagram of isochoric process is a straight line parallel to P-axis.

16. In thermodynamics there is a function of the thermodynamic coordinates (P, V and T) whose final value minus its initial value equals the change $\Delta Q - \Delta W$ in the process. We call this function the internal energy ΔU function which depends only on the initial and final positions, so it is not a path function.

17. Work done during isothermal process in expanding volume of gas from V_1 to V_2 is given by

$$W = \int_{V_1}^{V_2} P dV$$

$$= \int_{V_1}^{V_2} \left(\frac{nRT}{V}\right) dV \qquad \left(\text{as } P = \frac{nRT}{V}\right)$$

$$= nRT \int_{V_1}^{V_2} \frac{dV}{V} \qquad (\text{as } T = \text{constant})$$

$$= nRT \log_e \left(\frac{V_2}{V_1}\right)$$

For expansion of 1 mole of gas, *i.e.*, $n = 1$

$$W = RT \log_e \left(\frac{V_2}{V_1}\right)$$

Note: For a process to be isothermal, any heat flow into or out of the system must occur slowly enough, so that thermal equilibrium is maintained.

18. Since, work is done by the system, so it is positive. Therefore,

$$\Delta W = 30 \text{ J}$$

Heat given to the system,

$$\Delta Q = 40 \text{ J}$$

According to first law of thermodynamics, change in internal energy is given by

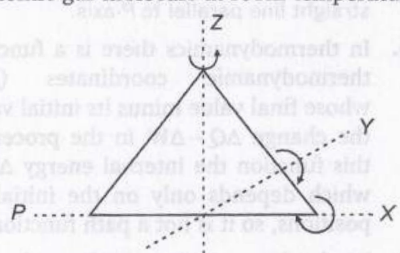
$$\Delta U = \Delta Q - \Delta W$$

$$= 40 - 30$$

$$= 10 \text{ J}$$

19. **Key Idea** : *A triatomic molecule can rotate about any of three coordinate axes.*

The molecule of a triatomic gas has a tendency of rotating about any of three coordinate axes. So, it has 6 degrees of freedom; 3 translational and 3 rotational. At high enough temperature a triatomic molecule has 2 vibrational degrees of freedom. But as temperature requirement is not given, so we answer simply by assuming triatomic gas molecule at room temperature.



Thus,

$$f = 6$$

(3 translational + 3 rotational) at room temperature.

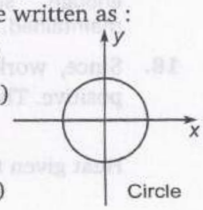
20. Ozone layer extends from 30 km to nearly 50 km above the earth's surface in ozone sphere. This layer absorbs the major part of ultraviolet radiations coming from the sun and does not allow them to reach the earth's surface.

The range of ultraviolet radiations is 100 Å to 4000 Å. Thus, it blocks the radiations of wavelength less than 3×10^{-7} m (or 3000 Å).

21. Two simple harmonic waves of same amplitude and frequency with phase difference $\frac{\pi}{2}$ in x and y-directions respectively are written as :

$$x = a \sin \omega t \quad \text{...(i)}$$

$$y = a \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{...(ii)}$$



From Eqs. (i) and (ii),

$$\sin \omega t = \frac{x}{a} \quad \text{...(iii)}$$

$$\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t = \frac{y}{a} \quad \text{...(iv)}$$

Squaring and adding Eqs. (iii) and (iv), we have

$$x^2 + y^2 = a^2$$

This is an equation of a circle.

22. The relation between velocity, frequency and wavelength is given by

$$v = n\lambda$$

or $$\lambda = \frac{v}{n}$$

Given, $v = 360$ m/s, $n = 500$ Hz

$\therefore$ $$\lambda = \frac{360}{500} = 0.72 \text{ m}$$

Path difference $= \frac{\lambda}{2\pi} \times$ phase difference

i. e., $$\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$$

or $$\Delta x = \frac{0.72}{2\pi} \times \frac{\pi}{3} \quad \left(\because \Delta\phi = 60° = \frac{\pi}{3}\right)$$

$$= 0.12 \text{ m} = 12 \text{ cm}$$

23. A normal mode of an oscillating system is a motion in which all particles of the system move sinusoidally with the same frequency.

In general, pth mode of a string fixed at ends has frequency

$$n = \frac{pv}{2l} \qquad p = 1, 2, 3 \ldots$$

where v is velocity of wave and l is length of string. In fourth normal mode, $p = 4$

$\therefore$ $$n = \frac{4v}{2l}$$

Given, $n = 500$ Hz, $l = 2$ m

Hence, $$500 = \frac{4v}{2 \times 2}$$

or $$v = \frac{500 \times 4}{4} = 500 \text{ m/s}$$

24. Frequency of sonometer wire is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where m is mass of string per unit length, and T is tension in the string.

Also, $$m = \pi r^2 d$$

r being radius of string and d is the density of material of string.

So, $$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}}$$

or $$n \propto \frac{\sqrt{T}}{r}$$

or $$\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} \times \left(\frac{r_2}{r_1}\right)$$

Given, $r_2 = 2r_1$, $T_2 = \frac{T_1}{2}$, $n_1 = n$

Hence, $$\frac{n}{n_2} = \sqrt{2} \times 2 \text{ or } n_2 = \frac{n}{2\sqrt{2}}$$

25. In an open organ pipe, both ends of the pipe are open. There are pressure nodes (or displacement antinodes) at both ends. In open pipe at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency, *i.e.*, all even and odd harmonics are present. Therefore, if fundamental frequency is n, then other frequencies are $n, 2n, 3n, 4n, \ldots$.

26. **Key Idea** : *The frequency perceived by the observer depends upon the relative motion between source and observer.*

In our case both source and observer are moving, so perceived frequency

$$f' = \frac{f(c - v_o)}{(c - v_s)}$$

where v_o is the velocity of observer, v_s is the velocity of source, and c is velocity of sound.

Given, $v_o = -2v, \quad v_s = -v$

$$\therefore \quad f' = \frac{f(c + 2v)}{(c + v)}$$

27. The converging lens used for magnification is called simple microscope or a magnifier.

When image is formed at D, the least distance of distinct vision, then magnifying power

$$M = 1 + \frac{D}{f}$$

Given, $D = 25$ cm, $f = 10$ cm

$$\therefore \quad M = 1 + \frac{25}{10}$$

$$= 1 + 2.5 = 3.5$$

Note : When image is formed at infinity, $M = \frac{D}{f}$. This relation is called magnifying power for normal adjustment.

28. When two lenses are separated by some distance x, then equivalent power

$$P = P_1 + P_2 - x P_1 P_2$$

$$\therefore \quad P = 5 + 5 - x \times 5 \times 5$$

or $\quad P = 10 - 25x$

Power P will be negative, if $10 - 25x$ will be negative

i.e., $\quad 25x > 10$

or $\quad x > \frac{10}{25}$ m

or $\quad x > \frac{10}{25} \times 100$ cm

or $\quad x > 40$ cm

29. In Galilean telescope a convergent lens is used as the objective and a divergent lens as the eye piece. Magnifying power and length of telescope are written as :

$$M = \frac{f_o}{u_e} \text{ and } L = f_o - u_e$$

In normal adjustment, *i.e.*, in relaxed eye state

$$u_e = f_e$$

So, $M_\infty = \frac{f_o}{f_e} = 50$ or $f_e = \frac{f_o}{50} = \frac{100}{50} = 2$ cm

and $\quad L_\infty = f_o - f_e$

$\therefore \quad L_\infty = 100 - 2 = 98$ cm

30. Linear magnification $= 4$

Areal magnification $=$ (linear magnification)^2

$$= (4)^2 = 16$$

Therefore, area of magnified film on the screen

$=$ areal magnification $\times$ area

$= 16 \times 100 = 1600 \text{ cm}^2$

31. The exposure time of camera lens is given by

$$\text{Time of exposure} \propto \frac{1}{(\text{Aperture})^2}$$

Also, $\quad f\text{ - number} = \frac{\text{Focal length } (f)}{\text{Aperture } (A)}$

or $\quad \text{Aperture } (A) = \frac{\text{Focal length } (f)}{f \text{ number}}$

Therefore, $\quad \frac{T_1}{T_2} = \frac{A_2}{A_1}$

Given, $T_1 = \frac{1}{200}, A_1 = \frac{f}{2.8}, A_2 = \frac{f}{5.6}$

$$\therefore \quad \frac{1/200}{T_2} = \left(\frac{f/5.6}{f/2.8}\right)^2$$

or $\quad \frac{1}{200\,T_2} = \left(\frac{2.8}{5.6}\right)^2$

or $\quad T_2 = \left(\frac{5.6}{2.8}\right)^2 \times \frac{1}{200} = 0.02$ s

Note : Smaller the f-number larger will be the aperture and lesser will be the time of exposure and faster will be the camera. This is why movie cameras have very low f-numbers such as $f/1.5$.

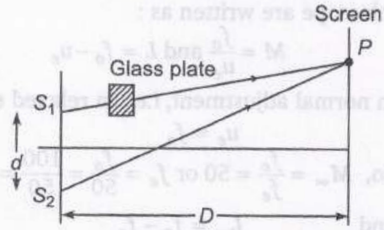
32. Distance between two adjacent bright (or dark) fringes is called the fringe width. It is denoted by β, thus,

$$\beta = \frac{D\lambda}{d}$$

where D is the distance between slit source and screen and d is separation of slits.

Since, D and d are increased to same extent, so fringe width (w) will remain unchanged.

33. When a thin glass plate of thickness t is placed over one of the slits, then lateral displacement is given by



$$x = \frac{(\mu - 1)tD}{d}$$

Given, $\mu = 1.5, t = 0.06 \text{ mm} = 6 \times 10^{-5} \text{ m}$

$D = 2 \text{ m}, \quad d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Putting the values in the above relation, we get

$$x = \frac{(1.5-1) \times 6 \times 10^{-5} \times 2}{1 \times 10^{-3}}$$

$$= 0.5 \times 12 \times 10^{-2}$$

$$= 0.06 \text{ m} = 6 \text{ cm}$$

Note : Lateral displacement is independent of λ *i.e.*, if white light is used, then shift of red colour fringe = shift of violet colour fringe

34. In diffraction pattern, fringe width is proportional to λ. We know that wavelength of violet light is less than that of red light, so on replacing red light with violet light, diffraction pattern would become narrower.

35. In discharge tube, the current is due to flow of positions ions and electrons. Moreover secondary emission of electrons is also possible. So, *V-I* curve is non-linear, hence its resistance is non-ohmic.

36. (a) Conduction is the process of transmission of heat in a body from the hotter part to the colder part without any bodily movement of constituent atoms or molecules of the body.

(b) In convection, the heated lighter particles move upwards and colder heavier particles move downward to take their place. This depends on weight and hence, on gravity.

(c) Radiation is the process of transmission of heat from one body to another body through electromagnetic waves even through vacuum, irrespective of their temperatures.

Hence, choice (b) is correct.

37. The point 10 cm from centre of sphere will be inside the sphere. Hence,

$$E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}$$

Here, $q = 500 \times 10^{-6}$ C, $r = 10$ cm $= 0.1$ m

$R = 60$ cm $= 0.60$ m

$$\therefore E_{\text{inside}} = (9.0 \times 10^9) \times \frac{500 \times 10^{-6} \times 0.1}{(0.60)^3}$$

$$= 2 \times 10^6 \text{ N/C}$$

38. The two capacitors each of value 1.5μF are in parallel, so their equivalent capacitance.

$$C_1 = 1.5 + 1.5 = 3 \mu\text{F}$$

A —| 3 μF |— 3 μF |— 3 μF |— B

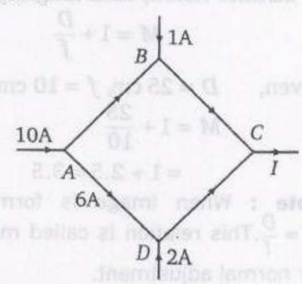
Now, three capacitors each of value 3μF are in series. Hence, their equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

or $\frac{1}{C} = \frac{3}{3}$

or $C = 1 \mu$ F

39. Regarding Kirchhoff's junction rule, the circuit can be redrawn as



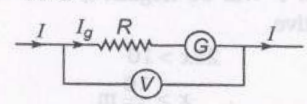
Current in arm, $AB = 10 - 6 = 4$A

Current in arm, $DC = 6 + 2 = 8$A

Current in arm, $BC = 4 + 1 = 5$A

Hence, $I = 5 + 8 = 13$A

40. The circuit diagram with current variation can be drawn as :



Let current I_g flows through R and G.

For a voltmeter with full scale reading, we have

$$V = I_g (G + R)$$

or $R = \frac{V}{I_g} - G$

Given, $G = 2\,\Omega$, $V = 12$ volt, $I_g = 0.1$ A

$\therefore$ $R = \frac{12}{0.1} - 2$

$= 120 - 2$

$= 118\,\Omega$

41. Power in electric bulb

$$P = \frac{V^2}{R}$$

So, resistance of electric bulb

$$R = \frac{V^2}{P}$$

Given, $P_1 = 25$ W, $P_2 = 100$ W,

$V_1 = V_2 = 220$ volt

Therefore, for same potential difference V

$$R \propto \frac{1}{P}$$

Thus, we observe that for minimum power, resistance will be maximum and *vice-versa*. Hence, resistance of 25 W bulb is maximum and 100 W bulb is minimum.

42. Since, electron has no deflection in electric and magnetic field, so

magnetic force on electron=electric force on electron

i.e., $Bev = eE$

or $v = \frac{E}{B}$

Given, $E = 3.2 \times 10^5$ V/m,

$B = 2.0 \times 10^{-3}$ Wb/m^2

$\therefore$ $v = \frac{3.2 \times 10^5}{2.0 \times 10^{-3}}$

$= 1.6 \times 10^8$ m/s

When electric field is switched off, then electron will move on circular path of radius

$$r = \frac{mv}{eB}$$

$$= \frac{9.1 \times 10^{-31} \times 1.6 \times 10^8}{1.6 \times 10^{-19} \times 2 \times 10^{-3}}$$

$= 0.45$ m

43. Average power dissipated in an AC circuit

$$P_{av} = V_{rms} I_{rms} \cos\phi \quad \text{...(i)}$$

where the term $\cos\phi$ is known as power factor.

Given, $V_{rms} = 100$ volt, $R = 100\,\Omega$, $\phi = 30°$

$\therefore$ $I_{rms} = \frac{V_{rms}}{R} = \frac{100}{100} = 1$ A

Putting the values in Eq. (i), we get

$P_{av} = 100 \times 1 \times \cos 30°$

$= 100 \frac{\sqrt{3}}{2}$

$= 50\sqrt{3}$

$= 86.6$ W

Note : The product of V_{rms} and I_{rms} gives the apparent power, while the true power is obtained by multiplying the apparent power by the power factor $\cos\phi$.

44. The time required for the number of parent nuclei to fall to 50% is called half-life $T_{1/2}$ and may be related to λ as follows.

Since, $0.5 N_0 = N_0 e^{-T_{1/2}}$

we have, $\lambda T_{1/2} = \ln(2) = 0.693$

or $T_{1/2} = \frac{0.693}{\lambda}$

or $\lambda = \frac{0.693}{T_{1/2}}$

Given, $T_{1/2} = 77$ days

$\therefore$ $\lambda = \frac{0.693}{77}$

$= 9 \times 10^{-3}$/ days

45. Key Idea : *α-particle is a nucleus of helium atom and its symbol is* $_2He^4$.

When an α-particle is emitted from a nucleus, the resultant nucleus reduces in mass number by 4 unit and in atomic number by 2 unit.

Loss in mass number $= 232 - 204 = 28$

Therefore, number of α-particles emitted

$= \frac{28}{4}$

$= 7$

46. Hubble's law is the statement in physical cosmology that the red-shift (Z) in light coming from different galaxies is proportional to their distance (r).

i.e., $Z \propto r$

It is considered the first observational basis for the expanding space and today serves as one of the most often cited pieces of evidence in support of the Big bang.

47. de-Broglie wavelength of a body moving with velocity v is given by

$$\lambda = \frac{h}{mv} \quad \text{...(i)}$$

where h is Planck's constant.

The kinetic energy of the body moving with velocity v is

$$E = \frac{1}{2}mv^2$$

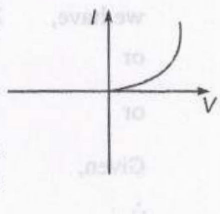
or $$mv = \sqrt{2Em} \quad \text{...(ii)}$$

Combining Eqs. (i) and (ii), we have

$$\lambda = \frac{h}{\sqrt{2mE}}$$

48. When the junction diode is forward biased, *i.e.*, p-side is kept at higher potential, the current in the diode changes with the voltage applied across the diode. The current increases very slowly till the voltage across the diode crosses a certain value.

After this voltage, the diode current increases rapidly even for very small increase in the diode voltage. The current-voltage relation for diode is shown.



Note : The value of the cut-in voltage is about 0.2 V for a germanium diode and 0.7 V for a silicon diode.

49. The given truth table follows a 'NAND' gate whose output is 1 only if at least one of its inputs is zero. Its Boolean expression is

$$Y = \overline{A \cdot B}$$

so that
$$\overline{1 \cdot 1} = \overline{1} = 0$$
$$\overline{1 \cdot 0} = \overline{0} = 1$$
$$\overline{0 \cdot 1} = \overline{0} = 1$$
$$\overline{0 \cdot 0} = \overline{0} = 1$$

Note: NAND gate is called universal or digital building block because by the repeated use of NAND gates we can perform all the logic functions like OR, AND, etc.

50. The critical frequency of sky wave undergoing reflection from a layer of ionosphere is

$$f_c = 9\sqrt{N_{max}}$$

where N is electron density per m^3.

$$N_{max} = \frac{f_c^2}{81} = \frac{(10 \times 10^6)^2}{81} = 1.2 \times 10^{12}\ m^{-3}$$

Chemistry

1. Key Idea : *1 mole = molecular mass in gram* $= 6.02 \times 10^{23}$ *molecules*

Given mass of $CO_2 = 44$ g

Molecular mass of $CO_2 = 12 + 16 \times 2 = 44$

$\therefore$ No. of molecules in 44 g of $CO_2 = 6.02 \times 10^{23}$

2. Key Idea : *Kinetic energy in an orbit*

$$= \frac{Ze^2}{8\pi E^\circ r} \quad \text{...(i)}$$

Potential energy in an orbit $= \frac{Ze^2}{4\pi E^\circ r}$ ...(ii)

Comparing Eqs. (1) and (2).

$$KE = \frac{1}{2} PE$$

3. A p-orbital has 3 dumbles and each dumble can accommodate maximum of 2 electrons. So maximum number of electrons in p orbital is 6.

4. Key Idea : *It is impossible to determine simultaneously the exact position and momentum of moving particle like electron, proton, neutron.*

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

Where Δx = uncertainty in position.

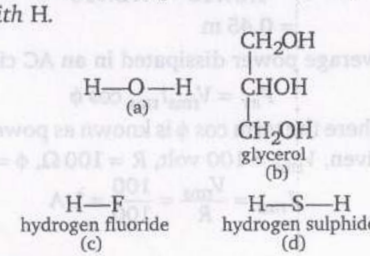
Δp = uncertainty in momentum.

5. Key Idea : *Atomic number of* Cr = 24 *and its configuration is* $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1, 3d^5$ *or* [Ne] $3s^2, 3p^6, 3d^5, 4s^1$ *because exactly full filled orbitals are more stable than nearly fulfilled orbitals.*

6. Key Idea : *No. of neutron = Atomic mass − atomic number.*

For C^{12} No. of neutron $= 12 - 6 = 6$

7. Key Idea : *Hydrogen bond is formed between molecules of compounds having* O, F *and* N *with* H.



$\because$ H_2S does not have O, F or N

$\therefore$ It does not form hydrogen bond.

8. Key Idea : *The sum of oxidation states of all elements in an ion is equal to charge on it.*

Let the oxidation state of S in $SO_4^{2-} = x$

$\therefore$ $x + (-2 \times 4) = -2$

or $x = +6$

9. Key Idea : *Molecular mass in gram* = 22.4 L *at* STP.

Given mass of $O_2 = 2$ g at 27° C and 760 mm Hg

32 g of O_2 = 22.4 L at STP

$\therefore$ 2 g of $O_2 = \frac{22.4}{32} \times 2 = 1.4$ L

10. Key Idea : *According to Le-Chatelier principle the reactions in which number of moles of reactants is equal to number of moles of products, is not effected by change in pressure.*

$$2NO(g) \rightleftharpoons N_2(g) + O_2(g)$$

Moles of reactants = 2

Moles of products = 2

$\because$ There is no change in number of moles of reactants and products.

$\therefore$ The reaction is not effected by change in pressure.

11. Key Idea : *Electron donors are Lewis base.*

In the reaction $I_2 + I^- \longrightarrow I_3^-$, I^- donates a pair of electron and I_2 accepts it to form I^{3-}

$\therefore$ I^- is Lewis base.

12. Key Idea : *The heat of neutralization of strong acid and strong base is constant because it is infact heat of formation of water by* H^+ *and* OH^-. Its value is almost equal to – 57.3 kJ.

13. Key Idea : *In a dilute solution, the* H^+ *furnished by water cannot be neglected*

Given $HCl = 10^{-8}$ M

$$\underset{10^{-8}}{HCl} \longrightarrow \underset{10^{-8}}{H^+} + \underset{10^{-8}}{Cl^-}$$

$$H_2O \rightleftharpoons \underset{10^{-7}}{H^+} + \underset{10^{-7}}{OH^-}$$

$\therefore$ Total $[H^+] = 10^{-7} + 10^{-8}$

$= 10^{-7}(1 + 10^{-1})$

$pH = -\log[H^+]$

$= -\log 10^{-7}(1 + 10^{-1})$

$= 6.98$

$\therefore$ The pH is between 6 and 7.

14. Key Idea : *For second order reaction*

$r = k[A]^2$ ...(i)

When concentration of *A* (reactant) is doubled

$r' = k[2A]^2$

$r' = 4k[A]^2$ ...(ii)

Comparing Eqs. (i) and (ii)

$r' = 4r$

15. Key Idea : *The reaction in which heat is evolved is called exothermic reaction. For exothermic reaction*

$H_R > H_P$ (Where H_R = Heat content of reactant H_p = Heat content of product.

$\therefore$ ΔH is always negative.

16. Key Idea : *Equivalent conductance is conductance of all ions produced by one gram equivalent of electrolyte dissolved in V* cm^3 *of solution.*

$$\Lambda_{eq} = \kappa_c \times \frac{1000}{N} = \kappa V$$

units of equivalent conductance Λ_{eq} are ohm^{-1} cm^2 $(g\ eq)^{-1}$ or S cm^2 eq^{-1}.

17. Key Idea : *Cell representation is done as follows*

Anode | Anodicelectrolyte | | cathodic electrolyte | cathode

(i) *Oxidation is loss of electron and it takes place at anode. Reduction is gain of electron and it takes place at cathode.*

$\therefore$ For cell reaction

$$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$$

Zn is anode and Cu is cathode.

$\therefore$ cell representation is

$$Zn | Zn^{2+} || Cu^{2+} | Cu$$

18. Key Idea : *Molarity* $= \frac{\text{moles of solute}}{\text{V of solution in litre}}$

$\therefore$ Molar solution means 1 mole of solute is present 1 L of solution.

19. Key Idea : *Depression in freezing point is a colligative property. It depends on no. of particles. More the number of particles, more will be depression in freezing point.*

(a) $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

It gives 3 particles

(b) $NaCl \longrightarrow Na^+ + Cl^-$

It gives 2 particles

(c) Urea $\longrightarrow$ No dissociation.

(d) Glucose $\longrightarrow$ No dissociation.

∵ K_2SO_4 produces maximum number of particles

∴ K_2SO_4 has maximum depression in freezing point.

20. Emulsions are colloidal system in which dispersion medium and dispersed phase both are liquids. So emulsion is dispersion of liquid in liquid.

21. The particle size of colloids is 1 – 100 nm or 50° Å to 2000 Å.

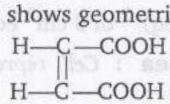
22. Ether are functional isomers of alcohols. Ethane, aldehyde and halide cannot be isomer of ether because they have different molecular formula.

23. **Key Idea :** *The compounds must fulfill two conditions to show geometrical isomerism.*

(i) *The compound should have at least one* C = C.

(ii) *The two groups attached to same carbon atom must be different.*

Out of given choices only (c) fulfills both conditions and shows geometrical isomerism.



(c)
2–butene1,4–dioic acid

24. Natural gas is a mixture of gaseous hydrocarbons. Methane (about 85%) is its main constituent.

25. **Key Idea :** *Aromatic compounds have delocalised π electrons.*

Out of given choices cyclohexane, CH_4, C_2H_6 and benzene, only benzene is aromatic compound. Benzene has 6 delocalised π electrons.

26. **Key Idea :** *During nitration benzene ring is attacked by* NO_2^+ *and hydrogen of benzene ring is replaced by* NO_2 *group.*

∴ Nitration of benzene is electrophilic substitution because NO_2^+ is an electrophile.

27. **Key Idea :**

$$R—CH_2—CH_2X + KOH(alc) \longrightarrow R—CH = CH_2 + KX + H_2O$$

Alkyl halide undergo β-elimination to form alkene

∴ Elimination reaction is correct choice.

(i) Nucleophilic substitution reaction is used to convert alcohol into alkyl halide.

(ii) In rearrangement reaction one molecule changes into other *via* internal changes.

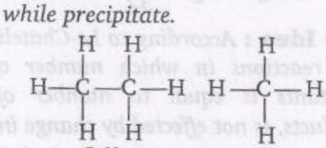
28. **Key Idea :** *Proton donors are acids.*

Among given choices C_2H_5OH can give proton (H^+) most easily.

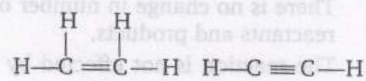
∴ C_2H_5OH is most acidic among C_6H_6, CH_4, $CH \equiv CH$ and C_2H_5OH

29. **Key Idea :** (i) *Alkene and alkynes both react with* $KMnO_4$ *and decolorise it.*

(ii) *Only alkynes react with* $AgNO_3$ *to give while precipitate.*



C_2H_6 (a) alkane CH_4 (b) alkane



C_2H_4 (c) alkene C_2H_2 (d) alkyne

∴ C_2H_4 (an alkene) reacts with $KMnO_4$ and decolorises it and does not react with $AgNO_3$, C_2H_6 and CH_4 are alkane they do not react with $KMnO_4$ and $AgNO_3$.

30. **Key Idea :** *Anti Markownikoff's addition is shown when unsymmetrical reagent reacts with unsymmetrical double or triple bond containing compound.*

$CH_3—CH = CH_2$ propene (a) $CH_3—CH_2—CH = CH_2$ butene-1 (b)

$CH_3—CH = CH—CH_3$ butene-2 (c)

$CH_3—CH_2—CH = CH—CH_3$ pentene-2

∵ Choice (c) butene-2 is symmetrical alkene

∴ It does not show Anti Markownikoff addition.

31. (a) Iodoform test is done to detect presence of CH_3CO group in organic compounds.

(b) Fehling solution identifies aldehydes

(c) Tollen's reagent identifies aldehydes

(d) Schiff's reagent identifies aldehydes

Methyl ketone is $CH_3-\overset{O}{\overset{\|}{C}}-R$.

$\because$ It has $CH_3-\overset{O}{\overset{\|}{C}}$ group. It is tested by using iodoform test. The compound having CH_3CO group give yellow ppt. on reaction with I_2 and aqueous alkali.

32. Key Idea : *Fehling solution is cupric ion complex with tartarate anion. Aldehydes reduce it to red precipitate. The red precipitate is chemically* Cu_2O

$CH_3-\overset{O}{\overset{\|}{C}}-CH_3$ acetone (a) $\quad CH_3CH_2CHO$ propanal (b)

CH_3COH ethanal (c) $\quad CH_3CH_2CH_2CHO$ butanal (d)

$\therefore$ Only acetone which is ketone not an aldehyde does not give iodoform test.

33. $\underset{\text{alkyl cyanide}}{RCN} \xrightarrow[H^+/OH^-]{H_2O} \underset{\text{amide}}{RCONH_2} \xrightarrow[H^+/OH^-]{H_2O} \underset{\text{acid}}{RCOOH}$

$\therefore$ Alkyl cyanide on hydrolysis give acid.

34. Key Idea : *Draw structure of CN group.*

$$R-C\equiv N$$

$\therefore$ Alkyl group is attached to carbon.

35. Key Idea : *Write hydrolysis reaction for fats*

$$\text{fat} + NaOH \text{ or } KOH \longrightarrow \underset{\text{glycerol}}{\begin{matrix} CH_2OH \\ | \\ CHOH \\ | \\ CH_2OH \end{matrix}} +$$

Sodium or potassium salt of fatty acid.

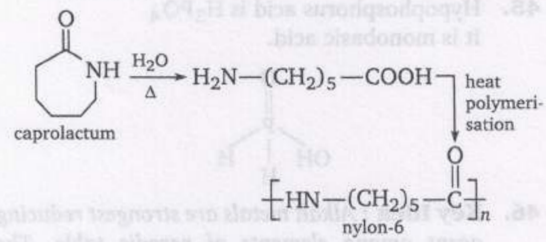
$\therefore$ Glycerol is alcohol, formed by hydrolysis of fats.

36. Key Idea : *Natural polymers are found in nature.*

(i) Cellulose is a natural polymer. It is found in cell walls.

(ii) Nylon 66, teflon and PVC are man made polymers.

37. Key Idea : *Caprolactum condenses to form nylon-6.*



38. Key Idea : *The boiling point of inert gases increases with increase in molecular weight due to increase in van der Waals' forces.*

$\because$ Xe has largest size, among inert gases.

$\therefore$ Xe has highest boiling point.

39. Key Idea : *Write the reaction to find catalyst of Ostwald process.*

$$4NH_3 + SO_2 \xrightarrow{Pt} 4NO + 6H_2O + \text{Heat}$$

$$2NO + O_2 \xrightarrow{50°\,C} 2NO_2(g)$$

$$3NO_2 + H_2O \longrightarrow 2HNO_3 + NO$$

$\therefore$ Pt is catalyst in Ostwald process.

40. Gypsum is $CaSO_4 \cdot 2H_2O$

Carnallite is $KCl \cdot MgCl_2 \cdot 6H_2O$

$\therefore$ Carnallite is ore of Mg.

41. (i) **Ostwald process :** It is used to manufacture HNO_3

(ii) **Hoope's process :** It is method to purify aluminium. Pure Al makes anode and impure aluminium makes cathode in this reaction.

(iii) **Hall's process** It is used to purify bauxite having nospecific impurity.

(iv) **Baeyer's process :** It is used to purify bauxite having chief impurity of iron.

$\therefore$ Hoope's process is correct answer.

42. Key Idea : *The size of cation is always smaller than corresponding neutral atom and anion is always larger in size the corresponding neutral atom.*

$\therefore$ $I^- > I > I^+$ is correct order.

43. ∵ White phosphorus is most reactive and most important allotrop of phosphorus. It is insoluble in water.

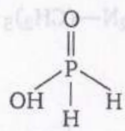
∴ It is kept in water to prevent it from catching fire.

44. $3CaO + 2NH_3 \longrightarrow 3Ca + N_2 + 3H_2O$

∴ N_2 gas is evolved when CaO reacts with NH_3

45. Hypophosphorus acid is H_3PO_4

It is monobasic acid.



46. Key Idea : *Alkali metals are strongest reducing agent among elements of perodic table. The reducing character decreases down the group.*

∴ Li is strongest reducing agent among Li, Na, Mg and Ca.

47. $HgCl_2 + 2NH_3 \xrightarrow{H_2O} \underset{\text{mercuric amino chloride}}{Hg + NH_2HgCl} + NH_4Cl$

∴ $HgCl_2$ on reaction with NH_4OH (or $NH_3 + H_2O$) forms mercuric amino chloride.

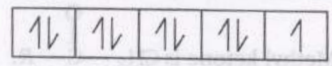
48. Key Idea : *The d block elements form coloured compounds. These compounds have ions with unpaired electron in d- subshell.*

(i) Na and Mg belong to s-block, so NaCl and $MgCl_2$ are colourless compounds.

(ii) CuF_2

oxidation state of Cu in Cu F_2 is + 2

$Cu^{2+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^9$



∴ CuF_2 in which Cu has one unpaired electron is coloured.

(iii) CuI

Oxidation state of Cu in CuI = +1

$Cu^+ - 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^{10}$

It has no unpaired electron. So CuI is colourless.

∴ Only CuF_2 is coloured among given choices.

49. $Na_2O_2 + H_2SO_4 \xrightarrow{\text{ice cold}} Na_2SO_4 + H_2O_2$

∴ H_2O_2 is formed by reaction Na_2O_2 (oxone) on dil H_2SO_4.

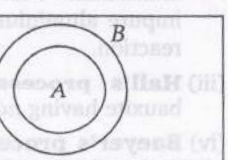
50. $CuSO_4 + 2KCN \longrightarrow K_2SO_4 + \underset{\substack{\text{cupric cyanide} \\ \text{(unstable)}}}{Cu(CN)_2}$

$$2Cu(CN)_2 \longrightarrow \underset{\text{white ppt}}{2CuCN} + (CN)_2$$

$$3KCN + CuCN \longrightarrow \underset{\substack{\text{potassium cuprocyanide} \\ \text{(soluble complex)}}}{K_3[Cu(CN)_4]}$$

Mathematics

1. Since, it is given that $A \subseteq B$. It is clear from the figure that $A \cup B = B$.



$A \subseteq B$

2. Key Idea : *If z is purely real, then the coefficient of imaginary part will be zero.*

Let $z = \dfrac{1 - i \sin\alpha}{1 + 2i \sin\alpha}$

$$= \frac{1 - i \sin\alpha}{1 + 2i \sin\alpha} \times \frac{(1 - 2i \sin\alpha)}{(1 - 2i \sin\alpha)}$$

$$= \frac{1 - 2\sin^2\alpha - 3i \sin\alpha}{1 + 4\sin^2\alpha}$$

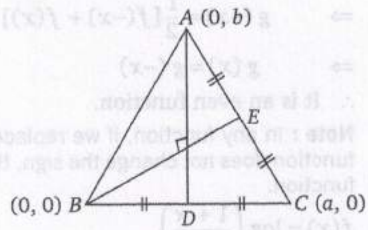
$$= \frac{(1 - 2\sin^2\alpha) - 3i \sin\alpha}{1 + 4\sin^2\alpha}$$

Since, z is real, therefore the coefficient of imaginary part will be zero.

$\Rightarrow 3\sin\alpha = 0$

$\Rightarrow \alpha = n\pi$, where n is integer

3. Here, the vertices of a triangle are $A(0, b)$, $B(0, 0)$ and $C(a, 0)$



Mid point of BC, $D = \left(\frac{a+0}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$

Mid point of AC, $E = \left(\frac{a}{2}, \frac{b}{2}\right)$

Slope of $AD = \frac{b-0}{0-\frac{a}{2}} = -\frac{2b}{a}$

Slope of $BE = \frac{\frac{b}{2}-0}{\frac{a}{2}-0} = \frac{b}{a}$

Since, AD is perpendicular to BE

$\Rightarrow$ slope of $AD \times$ slope of $BE = -1$

$\Rightarrow \quad -\frac{2b}{a} \times \frac{b}{a} = -1$

$\Rightarrow \quad a^2 = 2b^2 \Rightarrow a = \pm\sqrt{2}b$

4. As we know that tangent on point (x_1, y_1) of the circle $x^2 + y^2 = r^2$ is $x x_1 + y y_1 = r^2$ ...(i)

The required line is parallel to the tangent line

i.e., $\quad x x_1 + y y_1 = k$

Since, it is passing through origin

$\Rightarrow \quad k = 0$

$\therefore \quad x x_1 + y y_1 = 0$

5. Let $S = \sin 200° + \cos 200°$

As we know that 200° lies in the IIIrd quadrant. In that quadrant sin and cos is always be negative and sum of negative terms will be negative.

6. **Key Idea :** *The principal value of* $\sin^{-1} x$ *is* $-\frac{\pi}{2}$ *to* $\frac{\pi}{2}$.

We have

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$$

$\Rightarrow \quad \sin\theta = -\frac{\sqrt{3}}{2}$

$= -\sin\frac{\pi}{3}$

$= \sin\left(-\frac{\pi}{3}\right)$

$\therefore$ Principal value is $\left(-\frac{\pi}{3}\right)$.

7. Since, $xy + yz + zx = 1$

Put, $x = \cot A$, $y = \cot B$, $z = \cot C$

$\Rightarrow \cot A \cot B + \cot C [\cot B + \cot A] = 1$ ...(i)

$\Rightarrow \cot C [\cot A + \cot B] = 1 - \cot A \cot B$

$\Rightarrow \quad \cot C = \frac{1 - \cot A \cot B}{\cot A + \cot B}$

$\Rightarrow \quad \frac{\cot A + \cot B}{1 - \cot A \cot B} = \frac{1}{\cot C}$

$\therefore \quad \sum \frac{x+y}{1-xy} = \sum \frac{\cot A + \cot B}{1 - \cot A \cot B}$

$= \sum \frac{1}{\cot C}$

$= \frac{1}{\cot C} + \frac{1}{\cot A} + \frac{1}{\cot B}$

$= \frac{\cot A \cot B + \cot B \cot C + \cot A \cot C}{\cot A \cot B \cot C}$

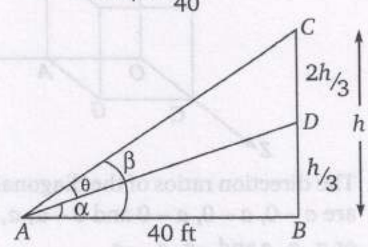
$= \frac{1}{\cot A \cot B \cot C}$ [from (i)]

$= \frac{1}{xyz}$

8. Let the height of the pole BC be h.

In ΔABC,

$$\tan\beta = \frac{h}{40} \quad \text{...(i)}$$



and in ΔABD,

$$\tan\alpha = \frac{h/3}{40} = \frac{h}{120} \quad \text{...(ii)}$$

Now, $\quad \tan\theta = \frac{1}{2}$ (given)

$\Rightarrow \quad \tan(\beta - \alpha) = \frac{1}{2}$

$\Rightarrow \quad \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha} = \frac{1}{2}$

$$\Rightarrow \frac{\frac{3h}{120}-\frac{h}{120}}{1+\frac{3h^2}{14400}}=\frac{1}{2}\Rightarrow h=120,\ 40$$

But h cannot be taken according to the given condition, therefore $h = 120$ ft.

9. Total number of cases $n(S) = 2^3 = 8$

Favourable cases

$$=\{(H,T,T),(T,H,T),(T,T,H)\}$$

$$n(F)=3$$

$\therefore$ Required probability $=\frac{3}{8}$

Alternate Solution :

Probability of getting head in one coin is $p=\frac{1}{2}$

$$\Rightarrow \quad q=\frac{1}{2}$$

$\therefore$ Probability of getting one head in three tosses

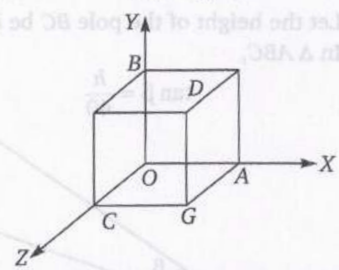
$$={}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$$

$$=3\left(\frac{1}{2}\right)^3=\frac{3}{8}$$

10. Let the sides of a cube be a.

$\therefore$ Coordinate of vertices are

$O\ (0,0,0)$, $D\ (a,a,a)$, $B\ (0,a,0)$ and $G\ (a,0,a)$



The direction ratios of the diagonals OD and BG are $a-0$, $a-0$, $a-0$ and $0-a$, a, $0-a$ or a, a, a and $-a$, a, $-a$.

$$\therefore \quad \cos\theta=\frac{|-a^2+a^2-a^2|}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}$$

$$=\frac{a^2}{\sqrt{3}a\sqrt{3}a}$$

$$=\frac{a^2}{3a^2}=\frac{1}{3}$$

$$\Rightarrow \cos\theta=\frac{1}{3}$$

11. Let $\quad g(x)=\frac{1}{2}[f(x)+f(-x)]$

$$\Rightarrow \quad g(-x)=\frac{1}{2}[f(-x)+f(x)]$$

$$\Rightarrow \quad g(x)=g(-x)$$

$\therefore$ It is an even function.

Note : In any function, if we replace x by $-x$, the function does not change the sign, that is an even function.

12. $f(x)=\log\left(\frac{1+x}{1-x}\right)$

and $\quad g(x)=\frac{3x+x^3}{1+3x^2}$

$$\therefore \quad f(g(x))=\log\left(\frac{1+g(x)}{1-g(x)}\right)$$

$$=\log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right)$$

$$=\log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

$$=\log\left[\frac{(1+x)^3}{(1-x)^3}\right]$$

$$=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

13. Key Idea : *If x is any real number, then* $[x]\le x$.

Now, $\quad \lim_{x\to 0}[\cos x]$

When $x\to 0^-$, $-\frac{\pi}{2}<x<0$

$$0<\cos x<1 \Rightarrow [\cos x]=0$$

$$\therefore \quad \lim_{x\to 0^-}[\cos x]=0$$

when $\quad x\to 0^+, 0<x<\frac{\pi}{2}$

$$\Rightarrow \quad 0<\cos x<1 \Rightarrow [\cos x]=0$$

$$\therefore \quad \lim_{x\to 0^+}[\cos x]=0$$

Hence, $\quad \lim_{x\to 0}[\cos x]=0$

Note : If any function LHL is equal to RHL, then the limit will exist otherwise not.

14. $\frac{d}{dx}(\sin^{-1} 2x\sqrt{1-x^2})$

Put, $x = \sin\theta$

$\therefore \frac{d}{dx}(\sin^{-1} 2\sin\theta\sqrt{1-\sin^2\theta})$

$= \frac{d}{dx}(\sin^{-1} 2\sin\theta\cos\theta)$

$= \frac{d}{dx}(\sin^{-1}\sin 2\theta) = \frac{d}{dx}(2\theta)$

$= \frac{d}{dx}(2\sin^{-1}x) = \frac{2}{\sqrt{1-x^2}}$

15. Let $I = \int_0^{\pi/2} |\sin x - \cos x|\, dx$

$= \int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$

$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$

$= \left[\frac{1}{\sqrt2} + \frac{1}{\sqrt2} - (1)\right] + \left[0 - 1 - \left(-\frac{1}{\sqrt2} - \frac{1}{\sqrt2}\right)\right]$

$= [\sqrt2 - 1 - 1 + \sqrt2] = 2(\sqrt2 - 1)$

Alternate Solution :

Let $I = \int_0^{\pi/2} |\sin x - \cos x|\, dx$

$= \sqrt2 \int_0^{\pi/2} \left|\frac{1}{\sqrt2}\sin x - \frac{1}{\sqrt2}\cos x\right| dx$

$= \sqrt2 \int_0^{\pi/2} \left|\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x\right| dx$

$= \sqrt2 \int_0^{\pi/2} \left|\sin\left(x - \frac{\pi}{4}\right)\right| dx$

Put, $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$= \sqrt2 \int_{-\pi/4}^{\pi/4} |\sin t|\, dt$

$= 2\sqrt2 \int_0^{\pi/4} \sin t\, dt$

$= 2\sqrt2 [-\cos t]_0^{\pi/4}$

$= -2\sqrt2\left[\frac{1}{\sqrt2} - 1\right]$

$= 2(\sqrt2 - 1)$

16. Since, p and q are the roots of the equation $x^2 + pq = (p+1)x$

or $x^2 - (p+1)x + pq = 0$

$\therefore \quad p + q = (p+1)$

$\Rightarrow \quad q = 1$

17. Since, the line makes an equal angle to the coordinate axes *i.e.*, $(\alpha = \beta = \gamma)$

As we know, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \quad \cos\alpha = \pm\frac{1}{\sqrt3}$

Since the line lies in the *OXYZ* octant, so we take +ve sign

$\therefore \quad l = m = n = \frac{1}{\sqrt3}$

18. Since, a, b, c are in AP, then

$2b = a + c$ ...(i)

and a, mb, c are in GP, then

$mb = \sqrt{ac}$

or $m^2b^2 = ac$ ...(ii)

From Eqs. (i) and (ii)

$\frac{m^2b^2}{2b} = \frac{ac}{a+c}$

$\Rightarrow \quad m^2b = \frac{2ac}{a+c}$

$\Rightarrow \quad a, m^2b, c$ are in HP.

19. $p \Rightarrow q$ is false only, when p is true and q is false

$\therefore p \Rightarrow q$ is false when p is true and $q \vee r$ is false and $q \vee r$ is false when both q and r are false.

Hence, truth values of p, q, r are respectively T, F, F.

20. Key Idea : *In the expansion of $(1+x)^n$, the middle term is $\frac{n}{2}$th if n is even and $\left(\frac{n}{2}+1\right)$th if n is odd.*

In the given expansion $(3x+2)^4$

Here, $n = 4$

$\therefore$ Middle term $= \frac{n}{2} + 1, = \frac{4}{2} + 1$

$= 3$rd term

$\therefore \quad T_3 = T_{2+1} = {}^4C_2 (3x)^2 (2)^2$

The coefficient of middle term $= {}^4C_2 3^2 2^2$

$= 6 \times 9 \times 4 = 216$

21. Since, $A\,(\text{adj}\, A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ ...(i)

As we know, $A^{-1} = \frac{1}{|A|}\text{adj}\, A$

$\Rightarrow \quad A\,A^{-1}|A| = A\,\text{adj}\,A$

$= A\,(\text{adj}\,A) = |A|\,I$ ...(ii)

From Eqs. (i) and (ii), we get

$$|A| I = 10\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$\Rightarrow \quad |A| I = 10 I \quad \Rightarrow \quad |A| = 10$

22. Key Idea : *If $\vec{a}$, $\vec{b}$ are the sides of a parallelogram, then area* $= |\vec{a} \times \vec{b}|$.

We have, $\vec{a} = 3\hat{i} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\3 & 0 & -1\\1 & 2 & 0\end{vmatrix}$

$= \hat{i}(0+2) - \hat{j}(0+1) + \hat{k}(6-0)$

$= 2\hat{i} - \hat{j} + 6\hat{k}$

$\therefore$ Area of parallelogram $= |\vec{a} \times \vec{b}|$

$= |2\hat{i} - \hat{j} + 6\hat{k}|$

$= \sqrt{4+1+36}$

$= \sqrt{41}$

Note : If $\vec{a}$ and $\vec{b}$ are the diagonals of a parallelogram, then area $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

23. $a \cdot b + c(a' + b') = a \cdot b + c \cdot (a \cdot b)'$

$= (a \cdot b + c)[ab + (ab)']$

$= (a \cdot b + c) \cdot 1$

$= a \cdot b + c$

24. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 0$

$\Rightarrow \quad |\vec{a}|^2 - |\vec{b}|^2 = 0$

$\Rightarrow \quad |\vec{a}|^2 = |\vec{b}|^2$

$\Rightarrow \quad |\vec{a}| = |\vec{b}|$

25. Total, number of coats = 4, waist coats = 5 and hats = 6

The total number of ways, in which one person put the clothes $= 4 \times 5 \times 6$

The total number of ways, that all three persons put the clothes $= 4^3 \times 5^3 \times 6^3$

26. Key Idea : *If roots of the quadratic equation are real, then discriminant is always be greater than equal to zero.*

Given equation is $x^2 + px + \frac{p}{4} + \frac{1}{2} = 0$

Since, roots are real, therefore discriminant ≥ 0

$\Rightarrow \quad p^2 - 4\left(\frac{p}{4} + \frac{1}{2}\right) \geq 0$

$\Rightarrow \quad p^2 - p - 2 \geq 0$

$\Rightarrow \quad (p-2)(p+1) \geq 0$

$\Rightarrow \quad p \geq 2$ or $p \leq -1$

Since, it is given $0 \leq p \leq 5$, so we neglect $p \leq -1$.

The possible values of p are 2, 3, 4, 5

$\therefore$ Required probability $= \frac{4}{6} = \frac{2}{3}$

27. Let $I = \int_0^1 x(1-x)^4 dx$

$= \int_0^1 (1-x)(1-(1-x))^4 dx$

$= \int_0^1 (1-x) x^4 dx$

$= \int_0^1 (x^4 - x^5) dx$

$= \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1 = \left[\frac{1}{5} - \frac{1}{6}\right]$

$= \frac{1}{30}$

Alternate Solution :

Let $I = \int_0^1 x(1-x)^4 dx$

$= \left[-x\frac{(1-x)^5}{5}\right]_0^1 - \int_0^1 -\frac{(1-x)^5}{5} dx$

$= 0 + \frac{1}{5}\left[-\frac{(1-x)^6}{6}\right]_0^1$

$= \frac{1}{5}\left(0 - \frac{-1}{6}\right) = \frac{1}{30}$

28. Key Idea : If $f(x)$ is a function, it will be increasing or decreasing if $f'(x) > 0$ or $f'(x) < 0$.

We have $f(x) = 2 - 3x$

On differentiating w.r.t. x, we get

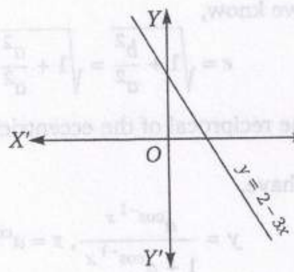
$f'(x) = -3 < 0$

$\therefore$ Function is decreasing for every value of x.

Alternate Solution : Let $y = f(x) = 2 - 3x$

$\Rightarrow \quad y + 3x = 2$

$\Rightarrow \quad \frac{x}{2/3} + \frac{y}{2} = 1$



It is clear from the figure that for increasing the value of x from $-\infty$ to ∞, we will get the decreasing value of y from ∞ to $-\infty$.

$\therefore$ It is decreasing function.

29. Let $I = \int_0^{\pi/2} \frac{\frac{\pi}{4} - x}{\sqrt{\sin x + \cos x}} dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\frac{\pi}{4} - \left(\frac{\pi}{2} - x\right)}{\sqrt{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{x - \frac{\pi}{4}}{\sqrt{\cos x + \sin x}} dx \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{0}{\sqrt{\sin x + \cos x}} dx$$

$$\Rightarrow \quad I = 0$$

30. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$

$$\Rightarrow \quad y = \sqrt{x + y}$$

$$\Rightarrow \quad y^2 = x + y$$

$$\Rightarrow \quad y^2 - y - x = 0$$

On differentiating w.r.t. x, we get

$$2y \frac{dy}{dx} - \frac{dy}{dx} - 1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{(2y - 1)}$$

31. **Key Idea :** *A one-one onto mapping is a bijection mapping.*

Since, $f : A \to B$ is a bijection. It means that mapping A to B is one-one onto

$\Rightarrow$ Both the sets have same number of elements

$$\therefore \quad n(A) = n(B)$$

Note : Any function, which is either increasing or decreasing function that is an bijective mapping.

32. The intercept form of equation of equal magnitude but opposite sign is

$$\frac{x}{a} + \frac{y}{(-a)} = 1$$

$$\Rightarrow \quad x - y = a$$

Since, it is passing through (−3, 2)

$$\Rightarrow \quad -3 - 2 = a$$

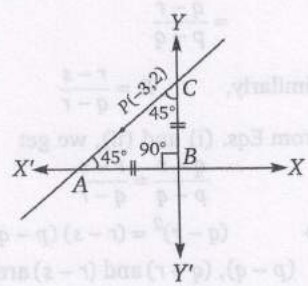
$$\Rightarrow \quad a = -5$$

$$\therefore \quad x - y = -5$$

$$\Rightarrow \quad x - y + 5 = 0$$

Alternate Solution :

Since the required line intercept the coordinate axes in equal magnitude and opposite sign, then the intercepted line makes an angle 45° to the x-axis.



$\therefore$ Equation of straight line is

$$y - 2 = \tan 45° (x + 3)$$

$$\Rightarrow \quad y - 2 = x + 3$$

$$\Rightarrow \quad x - y + 5 = 0$$

33. Since, $\frac{2z_1}{3z_2}$ is a purely imaginary number.

Let $\frac{2z_1}{3z_2} = ib$

$$\Rightarrow \quad \frac{z_1}{z_2} = \frac{3}{2} ib$$

Now, $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \left|\frac{\frac{z_1}{z_2} - 1}{\frac{z_1}{z_2} + 1}\right|$

$$= \left|\frac{i\frac{3}{2}b - 1}{i\frac{3}{2}b + 1}\right|$$

$$= \frac{\sqrt{1^2 + \left(\frac{3}{2}b\right)^2}}{\sqrt{1^2 + \left(\frac{3}{2}b\right)^2}} = 1$$

34. The nth term of AP

$$T_n = a + (n-1)d$$

$\therefore$ $$T_p = a + (p-1)d$$

$$T_q = a + (q-1)d$$

$$T_r = a + (r-1)d$$

and $$T_s = a + (s-1)d$$

Since, T_p, T_q, T_r and T_s are in GP.

$\therefore$ Common ratio, $R = \frac{T_q}{T_p} = \frac{T_r}{T_q}$

$$= \frac{T_q - T_r}{T_p - T_q}$$

$$= \frac{a + (q-1)d - [a + (r-1)d]}{a + (p-1)d - [a + (q-1)d]}$$

$$= \frac{q-r}{p-q} \quad \text{...(i)}$$

Similarly, $$R = \frac{r-s}{q-r} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{q-r}{p-q} = \frac{r-s}{q-r}$$

$\Rightarrow$ $$(q-r)^2 = (r-s)(p-q)$$

$\therefore$ $(p-q)$, $(q-r)$ and $(r-s)$ are in GP.

35. We know that for a given range R there are two directions of projection, *viz* α and $\frac{\pi}{2} - \alpha$. Thus if u is the velocity of projection, then

$$R = \frac{u^2 \sin 2\alpha}{g}, t_1 = \frac{2u}{g} \sin \alpha$$

and $$t_2 = \frac{2u \sin\left(\frac{\pi}{2} - \alpha\right)}{g}$$

$$= \frac{2u \cos \alpha}{g}$$

$\therefore$ $$t_1 t_2 = \frac{4u^2 \sin \alpha \cos \alpha}{g^2}$$

$\Rightarrow$ $$t_1 t_2 = \frac{2u^2 \sin 2\alpha}{g}$$

$\Rightarrow$ $$\frac{1}{2} g\, t_1 t_2 = \frac{u^2 \sin 2\alpha}{g}$$

$\Rightarrow$ $$\frac{1}{2} g\, t_1 t_2 = R$$

36. **Key Idea :** *The eccentricity of rectangular hyperbola is* $\sqrt{2}$.

Let the equation of rectangular hyperbola be $x^2 - y^2 = a^2$.

As we know,

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{a^2}{a^2}} \quad [\because b^2 = a^2]$$

$\therefore$ The reciprocal of the eccentricity is $\frac{1}{\sqrt{2}}$.

37. We have,

$$y = \frac{a^{\cos^{-1} x}}{1 + a^{\cos^{-1} x}}, z = a^{\cos^{-1} x}$$

$\Rightarrow$ $$y = \frac{z}{1+z}$$

On differentiating w.r.t. z, we get

$$\frac{dy}{dz} = \frac{(1+z) - z(1)}{(1+z)^2}$$

$$= \frac{1}{(1+z)^2}$$

$$= \frac{1}{(1 + a^{\cos^{-1} x})^2}$$

Now, $$z = a^{\cos^{-1} x}$$

On differentiating w.r.t. x, we get

$$\frac{dz}{dx} = a^{\cos^{-1} x} \log a \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$\therefore$ $$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{1}{(1 + a^{\cos^{-1} x})^2} \cdot a^{\cos^{-1} x} \left(-\frac{1}{\sqrt{1-x^2}}\right) \log a$$

$$= -\frac{a^{\cos^{-1} x}}{(1 + a^{\cos^{-1} x})^2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot \log a$$

38. $\sin 600^\circ \cos 330^\circ + \cos 120^\circ \sin 150^\circ$

$= \sin(720^\circ - 120^\circ) \cos(180^\circ + 150^\circ)$
$+ \cos 120^\circ \sin 150^\circ$

$= \sin 120^\circ \cos 150^\circ + \cos 120^\circ \sin 150^\circ$

$= \sin(120^\circ + 150^\circ)$

$= \sin 270^\circ = \sin(180^\circ + 90^\circ)$

$= -\sin 90^\circ$

$= -1$

39. As we know that area of circle $= \pi r^2$

Since, radius a is given.

$\therefore$ $$A = \pi a^2 \text{ sq unit}$$

40. We have

$$\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} \quad \text{...(i)}$$

Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$

$\Rightarrow \quad |A| = 3 - 4 = -1$

$\text{adj } A = \begin{bmatrix} 1 & -1 \\ -4 & 3 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{bmatrix} 1 & -1 \\ -4 & 3 \end{bmatrix}}{-1} = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix}$

Now, $AX = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$ [from (i)]

$\Rightarrow \quad X = A^{-1} \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$

41. Key Idea : *Since, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar vectors, then the scalar triple product will be zero.*

$\therefore [\vec{a} + \vec{b} \;\; \vec{b} + \vec{c} \;\; \vec{c} + \vec{a}]$

$= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$

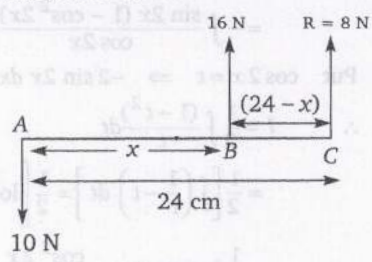
$= (\vec{a} + \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}]$

$= [\vec{a}\;\vec{b}\;\vec{c}] + [\vec{a}\;\vec{b}\;\vec{a}] + [\vec{a}\;\vec{c}\;\vec{a}] + [\vec{b}\;\vec{b}\;\vec{c}] + [\vec{b}\;\vec{b}\;\vec{a}] + [\vec{b}\;\vec{c}\;\vec{a}]$

$= [\vec{a}\;\vec{b}\;\vec{c}] + [\vec{a}\;\vec{b}\;\vec{c}]$

$= 2[\vec{a}\;\vec{b}\;\vec{c}] = 0 \quad [\because \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are coplanar}]$

42. Let the forces of magnitudes 10 N and 16 N be acting at A and B respectively. Suppose the resultant acts at C. Then



Taking moment of forces about B.

$10 \cdot x = 8 \cdot (24 - x)$

$\Rightarrow \quad 10x = 192 - 8x$

$\Rightarrow \quad 18x = 192$

$\Rightarrow \quad x = 10.66$ cm

43. Let α be the angle between the forces $\vec{P}$ and $\vec{Q}$.

It is given that the resultant of $\vec{P}$ and $\vec{Q}$ is of magnitude P.

$\therefore \quad P^2 = P^2 + Q^2 + 2PQ \cos\alpha$

$\Rightarrow \quad Q(Q + 2P\cos\alpha) = 0$

$\Rightarrow \quad Q + 2P\cos\alpha = 0 \quad \text{...(i)}$

Let θ be the angle between the forces $\vec{Q}$ and the new resultant.

$\therefore \quad \tan\theta = \frac{2P\sin\alpha}{Q + 2P\cos\alpha} = \infty$ [from (i)]

$\Rightarrow \quad \theta = \frac{\pi}{2}$

$\therefore$ The new resultant is at right angle to $\vec{Q}$.

44. Let a and d be the first term and common difference respectively.

Given that $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\Rightarrow \quad \frac{m/2[2a + (m-1)d]}{n/2[2a + (n-1)d]} = \frac{m^2}{n^2}$

$\Rightarrow \quad \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$

Replace m by $2m - 1$ and n by $2n - 1$

$\Rightarrow \quad \frac{2a + (2m-2)d}{2a + (2n-2)d} = \frac{2m-1}{2n-1}$

$\frac{a + (m-1)d}{a + (n-1)d} = \frac{2m-1}{2n-1}$

$\Rightarrow$ Ratio of mth and nth terms

$= (2m-1) : (2n-1)$

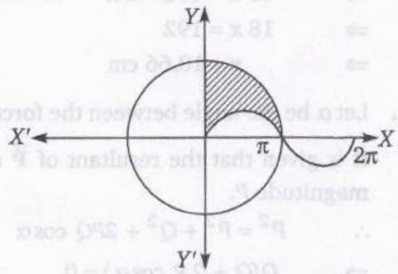
45. We have, $h = \frac{1}{2} g t_0^2 \quad \text{. . . (i)}$

Let t_1 be the time taken by the body to reach a height $\frac{h}{2}$. Then

$\frac{h}{2} = \frac{1}{2} g t_1^2 \Rightarrow \frac{1}{4} g t_0^2 = \frac{1}{2} g t_1^2$ [from (i)]

$\Rightarrow \quad t_1 = \frac{t_0}{\sqrt{2}}$

46. The point of intersection between the curves $x^2 + y^2 = \pi^2$ and $y = \sin x$ in the first quadrant is at $x = \pi$



$\therefore$ Required area $= \int_0^{\pi} (y_2 - y_1)\, dx$

$$= \int_0^{\pi} (\sqrt{\pi^2 - x^2} - \sin x)\, dx$$

$$= \left[\frac{x}{2}\sqrt{\pi^2 - x^2} + \frac{\pi^2}{2}\sin^{-1}\left(\frac{x}{\pi}\right) + \cos x\right]_0^{\pi}$$

$$= 0 + \frac{\pi^3}{4} - 1 - (0 + 1) = \frac{\pi^3 - 8}{4} \text{ sq unit}$$

47. $\because \quad 3a = b + c$

$\Rightarrow \quad \frac{b + c}{a} = 3$

Applying sine rule, we get

$$\frac{\sin B + \sin C}{\sin A} = 3$$

$$\Rightarrow \quad \frac{2\sin\frac{B + C}{2}\cos\frac{B - C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = 3$$

$$\Rightarrow \quad \frac{\cos\frac{A}{2}\cos\frac{B - C}{2}}{\cos\left(\frac{B + C}{2}\right)\cos\frac{A}{2}} = 3$$

$$\Rightarrow \cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2} = 3\left[\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right]$$

$$\Rightarrow \quad 2\cos\frac{B}{2}\cos\frac{C}{2} = 4\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\Rightarrow \quad \cot\frac{B}{2}\cot\frac{C}{2} = 2$$

Alternate Solution :

We have, $\quad 3a = b + c \quad$ ...(i)

Now, $\cot\frac{B}{2}\cot\frac{C}{2}$

$$\Rightarrow \quad \sqrt{\frac{s(s - b)}{(s - a)(s - c)}} \cdot \sqrt{\frac{s(s - c)}{(s - a)(s - b)}}$$

$$= \frac{s}{s - a} = \frac{\frac{a + b + c}{2}}{\frac{a + b + c}{2} - a}$$

$$= \frac{a + b + c}{-a + b + c} = \frac{a + 3a}{-a + 3a} \quad \text{[from (i)]}$$

$$= \frac{4a}{2a} = 2$$

48. We have,

$$f(x) = 2x + \cot^{-1} x + \log(\sqrt{1 + x^2} - x)$$

On differentiating w.r.t. x, we get

$$f'(x) = 2 - \frac{1}{1 + x^2} + \frac{1}{(\sqrt{1 + x^2} - x)} \times \left(\frac{x}{\sqrt{1 + x^2}} - 1\right)$$

$$= 2 - \frac{1}{1 + x^2} - \frac{1}{\sqrt{1 + x^2}}$$

$$= \frac{2 + 2x^2 - 1 - \sqrt{1 + x^2}}{1 + x^2}$$

$$= \frac{x^2 + \sqrt{1 + x^2}\,(\sqrt{1 + x^2} - 1)}{1 + x^2} > 0$$

$\Rightarrow f(x)$ is increasing in $(-\infty, \infty)$.

49. Let $S = \log_e 3 + \frac{(\log_e 3)^2}{2!} + \frac{(\log_e 3)^3}{3!} + \ldots + 3\log_e 3 + \frac{(3\log_e 3)^2}{2!} + \ldots$

$$= (e^{\log_e 3} - 1) + (e^{3\log_e 3} - 1)$$

$$= (3 - 1) + (3^3 - 1)$$

$$= 2 + 26 = 28$$

50. Let $\quad I = \int \frac{\cos 4x - 1}{\cot x - \tan x}\, dx$

$$= \int \frac{2\sin^2 2x}{\frac{\cos^2 x - \sin^2 x}{\cos x \sin x}}\, dx$$

$$= -\int \frac{\sin 2x\,(1 - \cos^2 2x)}{\cos 2x}\, dx$$

Put $\quad \cos 2x = t \quad \Rightarrow \quad -2\sin 2x\, dx = dt$

$$\therefore \quad I = \frac{1}{2}\int \frac{(1 - t^2)}{t} dt$$

$$= \frac{1}{2}\left[\int\left(\frac{1}{t} - t\right) dt\right] = \frac{1}{2}\left[\log t - \frac{t^2}{2}\right] + c$$

$$= \frac{1}{2}\log\cos 2x - \frac{\cos^2 2x}{4} + c$$