Sample Paper 11

Class- X Exam - 2022-23

Mathematics - Basic

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section - A consists of 20 questions of 1 mark each.)

1. The common zero of the polynomial $x^3 + 1$, $x^2 - 1$ and $x^2 + 2x + 1$ is:

(a) 1 (b) –1

(c) 3 (d) –2 1

2. If p, $2p - 1$, $2p + 1$ are three consecutive terms of an A.P., then the value of 'p' is:

(a) 3 (b) 4

(c) 5 (d) 6 1

3. In which quadrant does the point (–1, –2) lie ?

(a) I (b) II

(c) III (d) IV 1

4. If radii of two concentric circles are 4 cm and 5 cm, then the length of the chord to one circle which is tangent to the other circle is:

(a) 6 cm (b) 5 cm

(c) 8 cm (d) 9 cm 1

5. The tangent of a circle makes angle with radius at point of contact is:

(a) 60° (b) 30°

(c) 90° (d) none of these 1

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is:

(a) 10 units (b) 14 units

(c) 15 units (d) 12 units 1

7. What is the area of a circle inscribed in a square of side 'a' units ?

(a) $\frac{\pi a^2}{4}$ (b) πa^2

(c) $\frac{\pi}{a}$ (d) $\frac{\pi a}{2}$ 1

8. In the distribution given below, the modal class is:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
Frequency	3	12	27	57	75	80

(a) 50–60 (b) 10–20

(c) 40–50 (d) 30–40 1

9. A number from 11 to 30 was chosen at random. The probability of this chosen number being a multiple of 2 is:

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{2}$ 1

10. A car travels 0.99 km in which each wheel makes 450 complete revolutions. The radius of the wheel is:

(a) 30 cm (b) 32 cm

(c) 35 cm (d) 40 cm 1

11. If α and β are the zeros of the polynomial $p(x) = x^2 - px + q$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is:

(a) p (b) $\frac{-p}{q}$

(c) pq (d) $\frac{p}{q}$ 1

12. The sum of first 20 even numbers is:

(a) 420 (b) 320

(c) 140 (d) 450 1

13. The discriminant of the quadratic equation $(p + 3)x^2 - (5 - p)x + 1 = 0$ is:

(a) $(p + 13)$ (b) $(p + 1)$

(c) $(p + 2)$ (d) $(p - 13)(p - 1)$ 1

14. In a ΔABC, DE || BC with D on AB and E on AC. If $\frac{AD}{DB} = \frac{2}{3}$, then $\frac{BC}{DE}$ is:

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$ 1

15. The median of the following data is:

5, 2, 7, 9, 3, 2, 4, 8.

(a) 5.5 (b) 4

(c) 4.5 (d) 5 1

16. The discriminant of the equation $3x^2 + 2x = 0$ is:

(a) 4 (b) 2

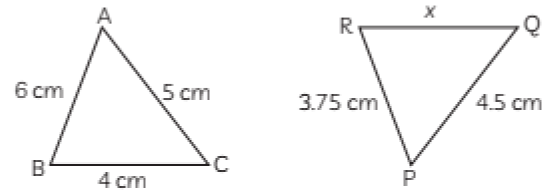
(c) 1 (d) –5 1

17. The 8th term from the end of A.P.: – 12, – 7, – 2, ..., 68 is:

(a) 33 (b) 35

(c) 30 (d) 36 1

18. In the given figure, $\Delta ABC \sim \Delta PQR$. The value of x is:



(a) 4 cm (b) 6 cm

(c) 3 cm (d) 5 cm 1

Direction for questions 19 and 20: In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : The length of the tangent will be 7 cm in a circle with a radius of 3 cm and a point's distance from the centre of the circle is 5 cm.

Reason (R) : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$ 1

20. Assertion (A) : If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, then median is 21.

Reason (R) : Median $\left(\frac{n+1}{2}\right)^{th}$ value, if n is odd. 1

SECTION - B

10 marks

(Section - B consists of 5 questions of 2 marks each.)

21. Show that $3 + \sqrt{5}$ is an irrational number, assume that $\sqrt{5}$ is an irrational number.

OR

Solve algebraically: $4x + 3y = 14$ and $3x - 4y = 23$. 2

22. Evaluate: $(\sin^4 60° + \sec^4 30°) - 2(\cos^2 45° - \sin^2 90°)$

OR

Prove that $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ 2

23. A dice is thrown twice. Find the probability that 5 will not come up either time. 2

24. Find a point on x-axis which is equidistant from A (–3, 4) and B (7, 6). 2

25. If the circumfrence of a circle increases from 4π to 8π, then find the percentage increase in the area of the circle. 2

SECTION - C

18 marks

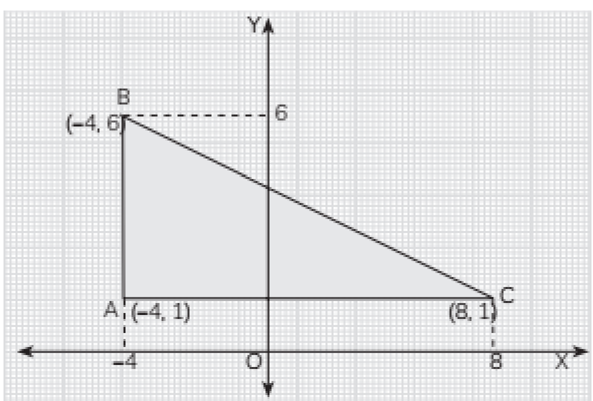
(Section - C consists of 6 questions of 3 marks each.)

26. The sum of the squares of two consecutive multiples of 7 is 637. Find the two multiples.

OR

Find the last term of an AP having 9 terms whose last term is 28 and sum of all the terms is 144. 3

27. The figure drawn on the graph paper shows a ΔABC with vertices A (–4, 1), B (–4, 6) and C (8, 1).



(A) Find the length of BC;

(B) Find sin ∠ABC;

(C) Find cos ∠BCA. 3

28. Prove that the length of two tangents drawn from an external point to a circle are equal. 3

29. The minute hand of a clock is 2 cm long. Find the area of the face of the clock described by the minute hand between 7 am and 7 : 15 am.

OR

A cone of height 24 cm and radius of base 6 cm is made up from modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere. 3

30. Determine the median for the following frequency distribution:

Class	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

3

31. A bag contains 15 white balls and some black balls. If the probability of drawing a black ball from the bag is thrice that of a white ball, find the number of black balls in the bag. 3

SECTION - D
20 marks

(Section - D consists of 4 questions of 5 marks each.)

32. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(A) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$

(B) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

OR

Two men on either side of a 75 m high building and in line with base of the building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men (use $\sqrt{3}$ = 1.73). 5

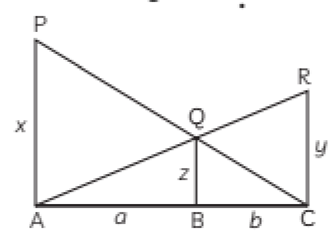
33. Solve for x and y graphically:

$x - y + 1 = 0;\quad 3x + 2y - 12 = 0$

OR

The shadow of a vertical tower on level ground increases by 16 m, when the altitude of the Sun changes from angles of elevation 60° to 45°. Find the height of the tower, correct to one place of decimal. (Use $\sqrt{3}$ = 1.73). 5

34. In the given figure, PA, QB and RC are perpendiculars to AC such that PA = x, RC = y, QB = z, AB = a and BC = b. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



5

35. If $a + b = c + d$ where a, b, c, d are rational numbers, them prove that either $a = c$ and $b = d$ or b and d are squares of rational numbers. 5

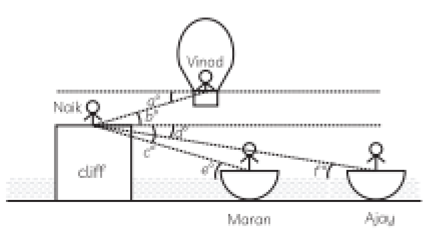
SECTION - E
12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *Mr. Naik is a paramilitary Intelligence Corps officer who is tasked with planning a coup on the enemy at a certain date. Currently he is inspecting the area standing on top of the cliff. Agent Vinod is on a hot air balloon in the sky. When Mr. Naik looks down below the cliff towards the sea, he has Ajay and Maran in boats positioned to get a good vantage point.*

The main goal is to scope out the range and angles at which they should train their soldiers.



On the basis of the above information, answer the following questions:

(A) Write a pair of 'angles of elevation' and 'angle of depression'. 1

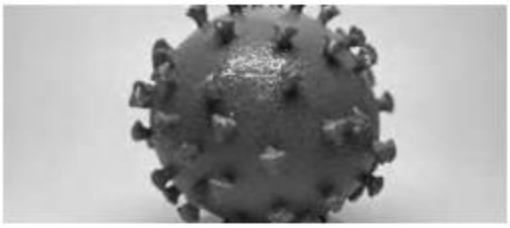
(B) If the vertical height of the balloon from the top of the cliff is 12 m and ∠b = 30°, then find the distance between the Naik and vinod. 1

(C) Ajay's boat is 25 m away from the base of the cliff. If ∠d = 30° . What is the height of the cliff? (use $\sqrt{3}$ = 1.73).

OR

If the height of the cliff is 30 m , ∠c = 45° and ∠d = 30°, then find the horizontal distance between the two boats (use $\sqrt{3}$ = 1.73). 2

37. *Nimmi, a 10th class student makes a project on corona virus in Science for an exhibition in her school. For this project, she picks a sphere which has volume 38808 cu. cm and 11 cylindrical shapes rods each of volume 1540 cu. cm and length 10 cm.*



Based on the above, answer the following questions :

(A) **What is the diameter of the base of a cylindrical rod?**

OR

What is the diameter of the sphere? 2

(B) **What is the curved surface area of a cylindrical rod?** 1

(C) **How much curved surface area of the sphere is covered by 11 cylindrical rods ?** 1

38. *Prime Minister's National Relief Fund (PMNRF) was established to help the families of earthquake affected village. The allotment officer is trying to come up with a method to calculate fair division of funds across various affected families so that the fund amount and amount received per family can be easily adjusted based on daily revised numbers.*

The total fund allotted is formulated by the officer as:

$$x^3 - 5x^2 - 2x - 6$$

The officer has also divided the fund equally among families of the village and each family receives an amount of $x^2 + 2x + 1$. After distribution, an amount of $11x + 1$ should be left to have some buffer for future disbursements.

On the basis of the above information, answer the following questions:

(A) **If an amount of ₹ 540 is left after distribution, what is value of x?** 1

(B) **How much amount (In rupees) does each family receive?** 1

(C) **What is the amount of fund (In rupees) allocated ?**

OR

Find the value of k, if $x - 1$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$. 2

SOLUTION

SECTION - A

1. *(b) –1*

Explanation: Here

$$x^3 + 1 = (x + 1)(x^2 - x + 1)$$
$$x^2 + 1 = (x + 1)(x - 1)$$
$$x^2 + 2x + 1 = (x + 1)^2$$

The common factor is $(x + 1)$, *i.e.* the common zero is –1.

2. *(a) 3*

Explanation: Since p, $2p - 1$ and $2p + 1$ are consecutive terms of an A.P.,

$$2(2p - 1) = p + (2p + 1)$$

$\Rightarrow \quad 4p - 2 = 3p + 1$

$\Rightarrow \quad p = 3$

3. *(c) III*

Explanation: Third quadrant since, both '*x*' and '*y*' are negative.

⚠ **Caution**

➥ *The points given in a ordered pair represent the first point as x-coordinate and the second point as y-coordinate.*

4. *(a) 6 cm*

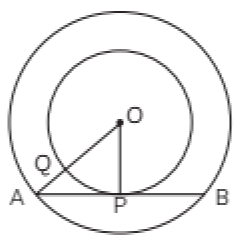
Explanation: Here, OP ⊥ AB and

$\therefore \quad AP = PB = \frac{1}{2} AB$

We know tangent is perpendicular to radious at the point of contact.

$\therefore \quad OP \perp AB$

Also, AB is a chord to smaller circle and perpendicular from centre to the chord bisects it.



In right ΔOPA, $AP = \sqrt{OA^2 - OP^2}$

$= \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16} = \sqrt{9}$

$= 3$

Thus, $AB = 2 \times AP$

$= 6$ cm

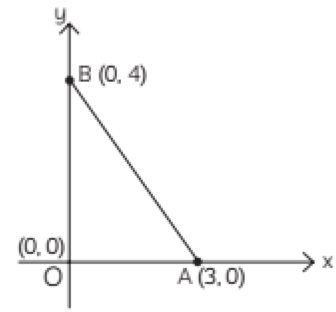
5. *(c) 90°*

Explanation: Tangent at any point of a circle is perpendicular to the radius at the point of contact. So, the tangent makes a right angle with the radius at the point of contact.

6. *(d) 12 units*

Explanation: Perimeter of ΔABC.

$= OA + AB + OB$



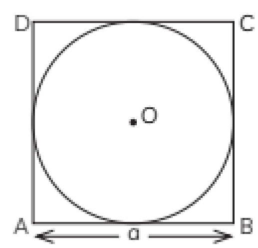
$= 3 + \sqrt{(3-0)^2 + (0-4)^2} + 4$

$= 3 + 5 + 4$

$= 12$ units

7. *(a)* $\frac{\pi a^2}{4}$

Explanation: The radius of the inscribed circle is $\frac{a}{2}$



So, its area is $\pi\left(\frac{a}{2}\right)^2$, *i.e.* $\frac{\pi a^2}{4}$.

8. *(d) 30–40*

Explanation:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	3	9	15	30	18	5

Here, maximum frequency is 30, which belongs to class 30 – 40.

∴ Modal class = 30 – 40.

⚠ **Caution**

➥ *Find the modal class by the value of the maximum frequency.*

9. *(a)* $\frac{1}{2}$

Explanation: Total numbers from 11 to 30 = 20

$\therefore$ Total number of outcomes = 20

Multiples of 2 from 11 to 30 = {12, 14, 16, ... 28, 30}

$\Rightarrow$ Number of favourable outcomes = 10

$\therefore$ P (Multiple of 2) = $\frac{10}{20} = \frac{1}{2}$

10. *(c) 35 cm*

Explanation: The wheel covers a distance of $2\pi r$, in one revolutions.

So, $450 \times 2\pi r = 0.99 \times 1000 \times 100$

$$450 \times 2 \times \frac{22}{7} \times r = 99 \times 1000$$

$$r = 35$$

Thus, the radius of the wheel is 35 cm.

11. *(d)* $\frac{p}{q}$

Explanation: Since α and β are the zeros of $p(x) = x^2 - px + q$,

$\therefore \quad \alpha + \beta = p$ and $\alpha\beta = q$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{p}{q}$

12. *(a) 420*

Explanation: The list of first 20 even numbers is: 2, 4, 6, 8, ..., 40

It is an A.P. with $a = 2$, $d = 2$

So, the required sum = $\frac{20}{2}[2 \times 2 + (20 - 1)(2)]$

$= 10(4 + 38)$

$= 420$

Caution

- *Use the appropriate formula for finding the sum of first 'n' terms, as per the values mentioned in the question.*

13. *(d) (p – 13) (p – 1)*

Explanation:

Discriminant = $b^2 - 4ac$

$= [-(5 - p)]^2 - 4(p + 3)(1)$

$= 25 + p^2 - 10p - 4p - 12$

$= p^2 - 14p + 13$

$= p^2 - 13p - p + 13$

$p(p - 13) - 1(p - 13)$

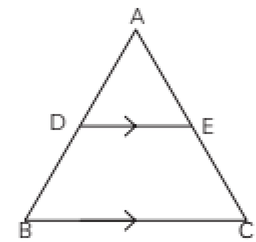
$= (p - 13)(p - 1)$

14. *(a)* $\frac{5}{2}$

Explanation: Since DE is parallel to BC,

$\therefore \quad \Delta ADE \sim \Delta ABC$

$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$ ...(i)



Given $\frac{AD}{DB} = \frac{2}{3}$,

we have

$\Rightarrow \quad \frac{BD}{AD} = \frac{3}{2}$

$\Rightarrow \quad \frac{BD}{AD} + 1 = \frac{3}{2} + 1$

$\Rightarrow \quad \frac{BD + AD}{AD} = \frac{3 + 2}{2}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{5}{2}$

$\frac{AD}{AB} = \frac{2}{5}$

So, by eqn. (i)

$\frac{BC}{DE} = \frac{5}{2}$

15. *(c) 4.5*

Explanation: Arranging the data in ascending order,

2, 2, 3, 4, 5, 7, 8, 9

Here, number of terms (n) = 8 i.e., even

$\therefore$ median =

$$\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2} + 1\right)^{th} \text{term}\right]$$

$= \frac{1}{2}$ [4^{th} term + 5^{th} term]

$= \frac{1}{2}$ [4 + 5]

$= 4.5$

16. *(b) 2*

Explanation: Here, equation is $3x^2 + 2x = 0$

Then comparing it with $ax^2 + bx + c = 0$, we get

$a = 3, b = 2, c = 0$

Then, discriminant,

$$D = \sqrt{b^2 - 4ac}$$
$$= \sqrt{(2)^2 - 4 \times 3 \times 0}$$
$$= \sqrt{4} = 2$$

17. *(a) 33*

Explanation: Write the given A.P in reverse order, then series is,

68, 63, ... – 2, – 7, – 12

Then, $a = -68, d = -5$

$a_8 = a + (8 - 1)\, d = 68 + 7\,(-5) = 33$

18. *(c) 3 cm*

Explanation: Since, Δ ABC ~ Δ PQR

Then, $\frac{AB}{PQ} = \frac{BC}{RQ} = \frac{AC}{PR}$

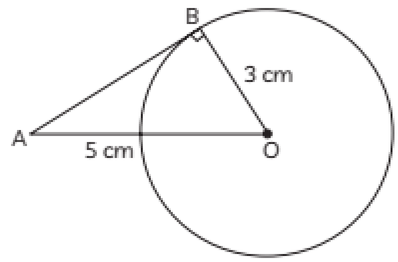
$\Rightarrow \quad \frac{6}{4.5} = \frac{4}{x} = \frac{5}{3.75}$

$\Rightarrow \quad x = \frac{4 \times 4.5}{6}$

$= 3$ cm

19. *(d) Assertion (A) is false but reason (R) is true.*

Explanation:



A tangent at any point of a circle is perpendicular to the radius through the point of contact.

Therefore, □OBA = 90° and ΔOBA is a right-angled triangle.

By Pythagoras theorem,

$$OA^2 = OB^2 + AB^2$$
$$5^2 = 3^2 + AB^2$$
$$AB^2 = 25 - 9$$
$$AB^2 = 16$$
$$AB = \pm 4$$

20. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: Arranging the terms in ascending order, 0, 5, 11, 19, 21, 30, 42, 50, 52.

Median = Value of $(n+1)/2^{th}$ observation

Median score = Value of $(9 + 1)/2^{th}$ term

= Value of 5^{th} term

= 21

SECTION - B

21. If possible, let us assume that $3 + \sqrt{5}$ be a rational number. So, there exists positive integers a and b such that, $3 + \sqrt{5} = \frac{a}{b}$, where a and b are integers having no common factor other than 1 and b ≠ 0.

$\Rightarrow \quad \sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow \quad \sqrt{5} = \frac{a - 3b}{b}$

Since, $\frac{a - 3b}{b}$ is a rational number, $\sqrt{5}$ is a rational number which is a contradiction to the fact that "$\sqrt{5}$ is irrational".

Hence, $3 + \sqrt{5}$ is irrational.

OR

Given, $\quad 4x + 3y = 14 \quad$ ...(i)

and $\quad 3x - 4y = 23 \quad$ ...(ii)

Adding (i) and (ii), we get

$$7x - y = 37$$

□ $\quad y = 7x - 37 \quad$ ...(iii)

Substituting the value of y in equation (i), we get

$4x + 3\,(7x - 37) = 14$

$\Rightarrow \quad 4x + 21x - 111 = 14$

$$\Rightarrow \quad 25x = 125$$
$$\Rightarrow \quad x = 5$$

Putting $x = 5$ in eq. (iii), we get

$$y = 7(5) - 37$$
$$= 35 - 37$$
$$= -2$$

$\therefore$ The solution is $x = 5$ and $y = -2$.

22. $(\sin^4 60° + \sec^4 30°) - 2(\cos^2 45° - \sin^2 90°)$

$$= \left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{2}{\sqrt{3}}\right)^4 - 2\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= \left(\frac{9}{16} + \frac{16}{9}\right) - 2\left(\frac{1}{2} - 1\right)$$

$$= \frac{337}{144} + 1 = \frac{481}{144}$$

Caution

➥ *Learn the table of trigonometric ratios for specific angles properly for solving such types of problems.*

OR

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$[\because 1 = \sin^2\theta + \cos^2\theta]$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta[2\cos^2\theta - (\sin^2\theta + \cos^2\theta)]}$$

$$= \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{R.H.S.}$$

23. Total number of possible outcomes = 36

Number of favourable outcomes = 36 – 11 = 25

[Excluding (1, 5), (5, 1), (2, 5), (5, 2), (3, 5), (5, 3), (4, 5), (5, 4), (5, 5), (5, 6), (6, 5)]

So, required probability = $\frac{25}{36}$

24. Let P $(x, 0)$ be a point on x-axis, equidistant from A (–3, 4) and B (7, 6), *i.e.,*

$$AP = BP \quad \text{or} \quad AP^2 = BP^2$$

i.e. $(-3 - x)^2 + (4 - 0)^2 = (7 - x)^2 + (6 - 0)^2$

i.e. $9 + x^2 + 6x + 16 = 49 + x^2 - 14x + 36$

$$\Rightarrow \quad 6x + 25 = -14x + 85$$
$$\Rightarrow \quad 20x = 60$$
$$\Rightarrow \quad x = 3$$

Thus, the required point is (3, 0).

Caution

➥ *If distances between two lines are equal, then the square of these distances are also equal.*

25. Let r and R be the radii of the initial circle and the increased circle, respectively. Then,

$$2\pi r = 4\pi \text{ and } 2\pi R = 8\pi$$
$$\Rightarrow \quad r = 2 \text{ and } R = 4$$

$\therefore$ Area of initial circle

$$= \pi(2)^2 \textit{ i.e. } 4\pi$$

and Area of increased circle

$$= \pi(4)^2 \textit{ i.e. } 16\pi$$

Thus, % increase = $\frac{16\pi - 4\pi}{4\pi} \times 100$

$$= \frac{12\pi}{4\pi} \times 100$$

$$= 300\%$$

SECTION - C

26. Let $7x$ and $7(x + 1)$ be two consecutive multiples of 7.

Then,

$$(7x)^2 + [7(x + 1)]^2 = 637$$
$$\Rightarrow 49x^2 + 49(x^2 + 2x + 1) = 637$$
$$\Rightarrow 98x^2 + 98x + 49 = 637$$
$$\Rightarrow 98x^2 + 98x - 588 = 0$$
$$\Rightarrow x^2 + x - 6 = 0$$
$$\Rightarrow (x + 3)(x - 2) = 0$$
$$\Rightarrow x + 3 = 0 \text{ or } x - 2 = 0$$
$$\Rightarrow x = -3 \text{ or } x = 2$$

Thus, two consecutive multiples of 7 are 14 and 21, or – 21 and – 14.

OR

Let 'a' be the first term of A.P. and 'd' be the common difference.

Here, total number of terms of A.P. is 9, *i.e.* $n = 9$

n^{th} term = last term = $a_n = a + 8d = 28$ ...(i)

Also, $S_n = S_9 = \frac{9}{2}[2a + (9-1)d] = 144$

$\Rightarrow \quad 9(a + 4d) = 144$

or $\quad 9a + 36d = 144$ ...(ii)

Solving (i) and (ii) simultaneously, we get

$$a = 4; \quad d = 3$$

Thus, the required first term is 4.

27. (A) Length of BC = $\sqrt{AB^2 + AC^2}$

$$= \sqrt{(1-6)^2 + (8-(4))^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169}, \text{ i.e. } 13$$

(B) Here, AB = 5 cm, AC = 12 cm and BC = 13 cm

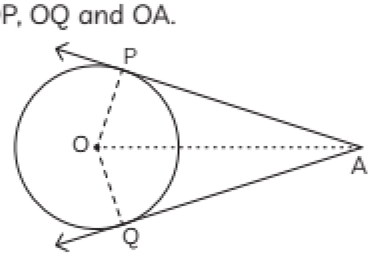
So, $\sin \angle ABC = \frac{AC}{BC} = \frac{12}{13}$

(C) $\cos \angle BCA = \frac{AC}{BC} = \frac{12}{13}$

28. Here, AP and AQ are two tangents drawn from an external point A, to the circle with centre O.

We need to prove that, AP = AQ

Join OP, OQ and OA.



Proof: Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ OQ $\perp$ QA and OP $\perp$ PA

So, $\angle OQA = 90° = \angle OPA$

Now, in two Δs OQA and OPA, we have:

(i) OQ = OP(Radii of the same circle)

(ii) $\angle OPA = \angle OQA$ (each 90°)

(iii) OA = OA (common)

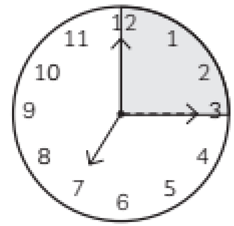
So, by RHS congruence criterion,

$$\Delta OPA \cong \Delta OQA$$

This concludes that, AP = AQ

Thus, the lengths of tangents drawn from an external point to a circle are equal.

29. We know that a minute hand sweeps by 6° angle in one minute.



So, it will sweep by an angle of 90° in 15 minutes.

So, area swept by the minute-hand in 15 minutes

$$= \frac{90}{360} \times \pi(2)^2 \text{ sq cm}$$

$$= \pi \text{ sq cm, or } \frac{22}{7} \text{ sq cm.}$$

OR

Height of the cone, h = 24 cm

Radius of base of the cone, r = 6 cm

Let, the radius of the sphere formed by reshaping cone be R.

As the cone is reshaped into sphere:

$\therefore$ Volume of cone = Volume of sphere

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$r^2 h = 4R^3$$

$$6 \times 6 \times 24 = 4 \times R^3$$

$$R^3 = 6 \times 6 \times 6$$

$$R = 6 \text{ cm}$$

$\therefore$ Radius (R) of the sphere formed = 6 cm

Hence, diameter of sphere = 6 × 2 = 12 cm

30.

Class	Frequency	Cum. frequency
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50

Here, N = 50, $\frac{N}{2} = 25$

Cumulative frequency just greater than 25 is 26, which belongs to class 120 – 140.

Hence, the median class is 120-140

For this class,

$l = 120,\ f = 14,\ cf = 12,\ h = 20$

Thus Median $= l + \dfrac{\frac{N}{2} - cf}{f} \times h$

$= 120 + \dfrac{25 - 12}{14} \times 20$

$= 120 + 18.57$

$= 138.57$

Thus, median of the given data is 138.57.

31. Let the bag contains '*x*' number of black balls. Then, the total number of balls in the bag is (*x* + 15).

It is given that,

P(a black ball) = 3 × P(a white ball)

$\Rightarrow \quad \dfrac{x}{x+15} = 3 \times \dfrac{15}{x+15}$

$\Rightarrow \quad x = 45$

Thus, there are 45 black balls in the bag.

SECTION - D

32. (A) L.H.S. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\dfrac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta}\times\dfrac{\sin\theta}{\sin\theta-\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\dfrac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)]$

$= \dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta+\cos^2\theta = 1]$

$= \dfrac{1}{\sin\theta\cos\theta}+1$

$= 1+\dfrac{1}{\sin\theta\cos\theta}$

$= 1 + \sec\theta \operatorname{cosec}\theta$ = R.H.S.

(B) L. H. S. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \left(\sin A+\dfrac{1}{\sin A}\right)^2+\left(\cos A+\dfrac{1}{\cos A}\right)^2$

$= \sin^2 A + \dfrac{1}{\sin^2 A} + 2.\sin A.\dfrac{1}{\sin A} + \cos^2 A$

$+ \dfrac{1}{\cos^2 A} + 2.\cos A.\dfrac{1}{\cos A}$

$= 2 + 2 + \sin^2 A + \cos^2 A + \dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 4 + 1 + \dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 5 + \operatorname{cosec}^2 A + \sec^2 A$

$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$

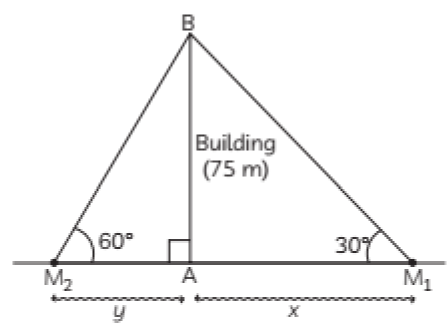
$[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta,\ \sec^2\theta = 1 + \tan^2\theta]$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

OR

Let AB be the building and, M_1 and M_2 be the positions of two men.



∴ Given, AB = 75 m,

$\angle AM_1B = 30°$

and $\angle AM_2B = 60°$

Let $AM_1 = x$ and $AM_2 = y$

From right triangle, M_1 AB,

$$\frac{AB}{M_1A} = \tan 30°$$

$$\Rightarrow \quad \frac{75}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = 75\sqrt{3}$$

From right triangle, M_2 AB,

$$\frac{AB}{M_2A} = \tan 60°$$

$$\Rightarrow \quad \frac{75}{y} = \sqrt{3}$$

$$\Rightarrow \quad y = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad y = 25\sqrt{3}$$

Thus, $M_1 M_2 = x + y$

$$= (1\sqrt{3} + 2\sqrt{3})$$

$$= 100\sqrt{3}$$

$$= 100 \times 1.73$$

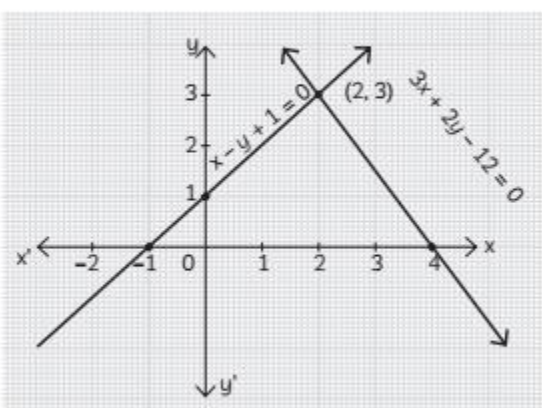
= 173 metres.

33. Table for the value of $x - y + 1 = 0$

x	0	1	2
y	1	2	3

Table for the value of $3x + 2y - 12 = 0$

x	2	3	4
y	3	1.5	0



From the graph, the point of intersection of the two lines is (2, 3).

So, the required solution is $x = 2$, $y = 3$.

OR

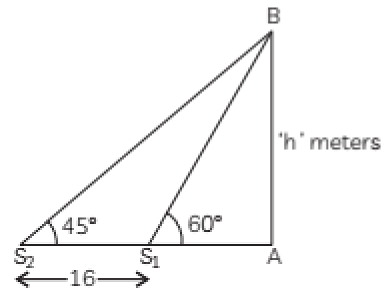
Let AB represent the tower of height 'h' metres.

AS_1, AS_2 be, is shadows when sun's attitude is 60° and 45° respectively.

$$\therefore \quad S_1S_2 = 16 \text{ m}$$

From S_1AB,

$$\frac{AB}{S_1A} = \tan 60°$$



$$\Rightarrow \quad \frac{h}{S_1A} = \sqrt{3} \Rightarrow S_1A = \frac{h}{\sqrt{3}} \quad \text{...(i)}$$

From S_2AB,

$$\frac{AB}{S_2A} = \tan 45°$$

$$\Rightarrow \quad \frac{h}{16 + S_1A} = 1$$

$$\Rightarrow \quad 16 + S_1A = h \quad \text{...(ii)}$$

Subtracting eqn. (i) from (ii), we have

$$16 = h - \frac{h}{\sqrt{3}}$$

$$= h\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$= h\left(\frac{\sqrt{3} - 1}{\sqrt{3}}\right)$$

or

$$h = \frac{16\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{16\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{16\sqrt{3}\,(\sqrt{3}+1)}{2}$$

$$= 8\sqrt{3}(\sqrt{3} + 1)$$

= 37.8 metres.

34. Given: PA, QB and RC are perpendiculars to AC.

To Prove: $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Proof: Since PA, QB and RC are perpendiculars, drawn on same line segment AC,

$$\therefore \quad PA \parallel QB \parallel RC$$

$\therefore$ In ΔARC, $QB \parallel RC$.

$\therefore$ By Basic proportional Theorem,

$$\frac{BQ}{CR} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{z}{y} = \frac{a}{a+b} \quad ...(i)$$

Similarly, In ΔCAP, $QB \parallel PA$.

$$\therefore \quad \frac{QB}{PA} = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{z}{x} = \frac{b}{a+b} \quad ...(ii)$$

Adding (i) and (ii), we get

$$\frac{z}{y} + \frac{z}{x} = \frac{a}{a+b} + \frac{b}{a+b}$$

$$\Rightarrow \quad z\left(\frac{1}{y} + \frac{1}{x}\right) = \frac{a+b}{a+b}$$

$$\Rightarrow \quad z\left(\frac{1}{y} + \frac{1}{x}\right) = 1$$

$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = \frac{1}{z}$$

$$\text{or} \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

35. Let $a = c$, then

$$a + \sqrt{b} = c + \sqrt{d}$$

$$\Rightarrow \quad \sqrt{b} = \sqrt{d}$$

$$\Rightarrow \quad b = d$$

So let $a \neq c$, then, there exist a postive rational number x, such that $a = c + x$.

Now, $a + \sqrt{b} = c + \sqrt{d}$

$$\Rightarrow \quad c + x + \sqrt{b} = c + \sqrt{d} \quad [\because a = c + x]$$

$$\Rightarrow \quad x + \sqrt{b} = \sqrt{d} \quad ...(i)$$

$$\Rightarrow \quad (x + \sqrt{b})^2 = (\sqrt{d})^2$$

$$\Rightarrow \quad x^2 + 2\sqrt{b}\,x + b = d$$

$$\Rightarrow \quad \sqrt{b} = \frac{d - x^2 - b}{2x}$$

$\Rightarrow$ $\sqrt{b}$ is rational [$\therefore$ d, x, b are rationals,

$\therefore \frac{d - x^2 - b}{2x}$ is rational]

$\Rightarrow$ b is a square of a rational number

From (i), we have

$$\sqrt{d} = x + \sqrt{b}$$

$\Rightarrow$ $\sqrt{d}$ is rational

$\Rightarrow$ d is the square of a rational number

Hence, either $a = c$ and $b = d$ or b and d are the squares of rationals number.

SECTION - E

36. (A) A pair of 'angle of elevation' is $\angle b°$, $\angle e$ and one pair of angle of depression is $\angle c°$ and $\angle d°$

(B) Then, sin 30°

$$= \frac{\text{Vertical height}}{\text{Distance between Naik and Vinod}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{12}{D_{N and V}}$$

$\Rightarrow$ Distance = 24 m

(C) Here, $\angle d° = \angle f° = 30°$

Then, $\dfrac{\text{Height of cliff.}}{\text{Distance of Ajay s boat from the base of cliff}} = \tan 30°$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{25}$$

$$\Rightarrow \quad h = \frac{25}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{25}{3} \times \sqrt{3}$$

$$= 14.45 \text{ m}$$

OR

Here, height of cliff = 30 m

Then, $\angle c = \angle e = 45°$

$$\therefore \tan 45° = \frac{\text{height of cliff}}{\text{Distance of Maran's boat}}$$

$$\Rightarrow \quad 1 = \frac{30}{\text{Distance of Maran's boat}}$$

$\Rightarrow$ Distance of maran's boat = 30 m

And $\angle d = \angle f = 30°$

$$\therefore \tan 30° = \frac{\text{height of cliff}}{\text{Distance of Ajay's boat}}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30}{D_A}$

$\Rightarrow \quad D_A = 30\sqrt{3}$

$\therefore$ Distance between boats

$= 30\sqrt{3} - 30$

$= 30(\sqrt{3}-1)$

$= 30(1.73-1)$

$= 30 \times 0.73$

$= 21.9$ m

37. (A) Volume of cylinderical shape

= 1540 cu. Cm

Height of cylinderical shape h = 10 cm

Let r be the radius of the cylinderical shape

$\therefore \quad \pi r^2 h = V$

$\Rightarrow \frac{22}{7} \times r^2 \times 10 = 1540$

$\Rightarrow \quad r^2 = \frac{1540 \times 7}{10 \times 22}$

$= 49$

$\Rightarrow \quad r = 7$ cm

$\therefore \quad$ Diameter $= 2r = 14$ cm

OR

Volume of sphere = 38808 cm^3

Let R be the radius of the sphere

$\therefore \quad \frac{4}{3}\pi R^3 = 38808$

$\Rightarrow \quad R^3 = \frac{38808 \times 3}{4\pi}$

$= \frac{38808 \times 3}{4 \times 22} \times 7$

$= 9261$

$\Rightarrow \quad R = 21$ cm

$\therefore \quad$ Diameter = 2R = 42 cm

(B) For cylinderical rod,

CSA $= 2\pi rh$

$= 72 \times \frac{22}{7} \times 7 \times 7$

= 440 sq cm

(C) Base Area of each cylindrical rod $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

= 154 sq. cm

Total area covered = 11 × 154

= 1694 sq. cm

38. (A) Amount lift = $11x + 1$

$\therefore \quad 11x + 1 = 540$

$x = \frac{539}{11} = 49$

(B) Since, $x = 49$

$\therefore$ Amount received by each family is

$x^2 + 2x + 1 = (49)^2 + 2(49) + 1$

$= 2401 + 98 + 1$

$= 2500$

(C) Since, $x = 49$

$\therefore$ Fund alloted is—

$x^3 - 5x^2 - 2x - 6$

$= (49)^3 - 5(49)^2 - 2(49) - 6$

$= 117649 - 12005 - 98 - 6$

$= 1,05,540$

OR

Let $g(x) = x - 1$

$g(x) = 0 = x - 1 \Rightarrow x = 1$

so, $\quad p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$= 2 + k + \sqrt{2} \quad$ ...(i)

As $(x - 1)$ is a factor of $p(x)$

$\therefore \quad p(x) = 0$

$2 + k + \sqrt{2} = 0$

$k = -[2 + \sqrt{2}]$