CHAPTER 6

Solution of Linear Equations in Two Variables

Let's consider the following situation :

Ram's age is 3 years more than double the age of his son Abhimanyu.

i.e. $y = 2x + 3$ ......(i)

Where x = the age of Abhimanyu and y = the age of Ram.

Here, equation (i) represents a linear equation in two variables x and y.

Linear Equation in Two Variables : An equation of the form $ax + by + c = 0$ or $ax + by = c$, where a, b, c are real numbers and $a \neq 0$, $b \neq 0$, is called a linear equation in two variables x and y.

Examples : Each of the following equations is a linear equation :

(i) $2x + 3y = 6$

(ii) $x - 3y = 5$

(iii) $\sqrt{5}x - \sqrt{2}y = 0$

The condition $a \neq 0$, $b \neq 0$, is often denoted by $a^2 + b^2 \neq 0$

Solution of a Linear Equation : Any pair of values of x and y is said to be the solution of a linear equation $ax + by + c = 0$, where a, b, c are real numbers and $a \neq 0$, $b \neq 0$ if it satisfies the equation.

i.e.,. $x = \alpha$ (alpha) and $y = \beta$ (beta) is an solution of $ax + by + c = 0$ if $a\alpha + b\beta + c = 0$

Example : Show that $x = 1$ and $y = 2$ is a solution of $3x + y = 5$

Sol. : Substituting $x = 1$ and $y = 2$ in the given equation, we get,

LHS = $3 \times 1 + 2 = 5$ = RHS

$\therefore$ $x = 1$ and $y = 2$ is a solution of $3x + y = 5$.

Example : Show that $x = -3$ and $y = 5$ is not a solution of $5x - 2y = 8$

Sol. : Substituting $x = -3$ and $y = 5$ in the equation, we get,

LHS = $5 \times (-3) - 2 \times 5 = -15 - 10 = -25 \neq$ RHS

$\therefore$ $x = 3$ and $y = 5$ is not a solution of $5x - 2y = 8$

Simultaneous Linear Equations in Two Variables

A pair of linear equations in two variables is said to form a system of simultaneous linear equation.

Thus, a pair of linear equations in x and y. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$

Solution of System of Simultaneous Linear Equations in Two Variables

A pair of values of the variables x and y satisfying each of the equations in a given system of two linear equations in x and y is called a solution of the system.

Example : Show that $x = 2$, $y = 1$ is a solution of the system of linear equations $3x + 2y = 8$ and $5x - y = 9$

Sol. :

$3x + 2y = 8$ ...(i)

$5x - y = 9$ ...(ii)

Substituting $x = 2$, $y = 1$ in equation (i), we get,

LHS = $3 \times 2 + 2 \times 1 = 6 + 2 = 8$ = RHS

Substituting $x = 2$ and $y = 1$ in equation (ii), we get,

LHS = $5 \times 2 - 1 = 9$ = RHS

Hence, $x = 2$ and $y = 1$ is a solution of the given system of linear equations.

Consistency and Inconsistency

- A system of a pair of linear equations in two variables is said to be consistent if it has at least one solution.
- A system of a pair of linear equations in two variables is said to be inconsistent if it has no solution.
- The system of a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has :

(i) a unique solution (i.e. consistent) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

The graph of the linear equations intersect at only one point.

(ii) no solution (i.e. inconsistent) if $\frac{a_1}{b_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The graph of the two linear equations are parallel to each other i.e.e the lines do not intersect.

(iii) an infinite number of solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The graph of the linear equations are coincident.

- Homogeneous equation of the form $ax + by = 0$ is a line passing through the origin. Therefore, this system is always consistent.

Algebraic Method of Solving a Pair of Linear Equations in Two Variables

The most commonly used methods are :

(i) Substitution Method

(ii) Elimination Method

(iii) Cross-mutliplication Method

Algebraic Solution by Substitution Method

Working Rule :

Step I : Write the given equations

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

Step II : Choose one of the two equations and express y in terms of x (or x in terms of y) i.e. express one variable in terms of the other.

Step III : Substitute this value of y obtained in step II, in the other equation to get a linear equation in x.

Step IV : Solve the linear equation obtained in step III and find the value of x.

Step V : Substitute this value of x in the relation obtained in step II and find the value of y.

Example : Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Sol. :

$2x + 3y = 11$ ...(i)

$2x - 4y = -24$ ...(ii)

From equation (i), $y = \frac{11-2x}{3}$ ... (iii)

Substituting $y = \frac{11-2x}{3}$ in equation (ii),

we get, $2x - 4\left(\frac{11-2x}{3}\right) = -24$

$\Rightarrow \frac{6x - 44 + 8x}{3} = -24$

$\Rightarrow 14x - 44 = -72$

$\Rightarrow 14x = -28 \Rightarrow x = -2$

Substituting $x = -2$ in equation (iii)

We get, $y = \frac{11 - 2\times(-2)}{3} = 5$

Now substituting $x = -2$ and $y = 5$ in $y = mx + 3$ we get,

$5 = m \times (-2) + 3$

$\Rightarrow 5 = -2m + 3$

$\Rightarrow m = -1$

Algebraic Solution by Elimination Method

Working Rule :

Step I : Write the given equation as $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

Step II : Multiply the given equations by suitable numbers so that the coefficient of one of variables is numerically equal.

Step III : If the numerically equal coefficient are opposite in sign, then add the new equations otherwise subtract.

Step IV : Solve the linear equations in one variable obtained in step III and get the value of one variable.

Step V : Substitute the value of the variable so obtained in step IV in any of the two equations and find the value of the other variable.

Example : $4x + \frac{y}{3} = \frac{8}{3}$

and $\frac{x}{2} + \frac{3y}{4} = -\frac{5}{2}$

Sol. : We have, $4x + \frac{y}{3} = \frac{8}{3}$

$\Rightarrow 12x + y = 8$ ...(i)

and, $\frac{x}{2} + \frac{3y}{4} = -\frac{5}{2}$

$\Rightarrow 2x + 3y = -10$ ...(ii)

Multiplying equation (ii) by 6 and subtracting from equation (i)

$$\begin{array}{rcl} 12x + \;\; y &=& 8 \\ 12x + 18y &=& -60 \\ - \quad - & & + \\ \hline -17y &=& 68 \end{array}$$

$\Rightarrow y = -4$

Putting $y = -4$ in equation (ii),

$2x + 3 \times (-4) = -10$

$\Rightarrow 2x - 12 = -10$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

Algebraic Solution by Cross-multiplication

System of two linear equations $a_1x + b_1y + G = 0$ and $a_2x + b_2y + c_2 = 0$ where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ has a unique solution given by

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

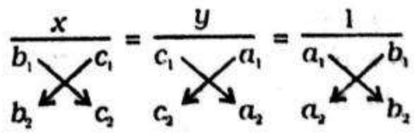
and $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Remarks

(i) The above solution can also be written as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{y}{a_1b_2 - a_2b_1}$$

(ii) The following diagram helps in remembering the above solution :



SOLVED OBJECTIVE QUESTIONS

1. If $13x - 15 = -2x + 105$ then $x = ?$

(1) 8 (2) 6
(3) 3 (4) 4

2. If $\frac{3}{x-1} + \frac{1}{x-3} = \frac{4}{x-2}$ then $x = ?$

(1) –4 (2) 4
(3) 2 (4) –2

3. If $\frac{x+1}{x} + \frac{x}{x+1} = 2\frac{1}{6}$, then the value of x is :

(1) 2 (2) –3
(3) 2 or –3 (4) 2 or 4

4. If $\frac{x+1}{x-1} + \frac{x+2}{x-2} = \frac{22x+30}{11x-18}$ then the value of x is :

(1) –16 (2) –6
(3) 6 (4) 8

5. If $\frac{x-7}{x-3} + \frac{x-2}{x-9} = 2$, then the value of $x = ?$

(1) –5 (2) 5
(3) 4 (4) –4

6. If $\frac{(x+1)(x+2)}{(x+3)(x+4)} = \frac{x+3}{x+7}$, then $x = ?$

(1) $-2\frac{1}{5}$ (2) $2\frac{1}{5}$

(3) $3\frac{1}{5}$ (4) $-3\frac{1}{5}$

7. Solve the following system of equations :

(i) $7x - 2y = 3$ (ii) $11x - \frac{3}{2}y = 8$

(1) $x = 1, y = 2$ (2) $x = 2, y = 1$
(3) $x = 2, y = 3$ (4) $x = 3, y = -2$

8. Solve the following system of equations :

$4x - 3y = 8$ $6x - y = \frac{29}{3}$

(1) $x = \frac{3}{2}, y = \frac{2}{3}$ (2) $x = \frac{3}{2}, y = \frac{-2}{3}$

(3) $x = 2, y = 3$ (4) $x = 3, y = -2$

9. Solve the following system of equations :

$x + y = a + b$
$ax - by = a^2 - b^2$

(1) $x = 2a, y = 2b$ (2) $x = 2a, y = b$
(3) $x = a, y = b$ (4) $x = -a, y = -b$

10. Solve the following system of equations :

$a(x + y) + b(x - y) = a^2 - ab + b^2$
$a(x + y) - b(x - y) = a^2 + ab + b^2$

(1) $x = \frac{b^2}{2a}, y = \frac{2a^2 + b^2}{2a}$

(2) $x = \frac{a^2}{2a}, y = \frac{b^2}{2a}$

(3) $x = a, y = b$
(4) $x = 2a, y = 2b$

11. The value of k for which the system of equations $x + 2y = 5$, $3x + ky + 15 = 0$ has no solution, is

(1) 6 (2) –6
(3) 2 (4) –2

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

12. The area bounded by the lines $x = 0$, $y = 0$, $x + y = 1$, $2x + 3y = 6$ (in square units) is

(1) 2 (2) $2\frac{1}{3}$

(3) $2\frac{1}{2}$ (4) 3

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (Ist Sitting)]

ANSWERS

1. (1)	**2.** (2)	**3.** (3)	**4.** (2)	**5.** (1)
6. (1)	**7.** (1)	**8.** (2)	**9.** (3)	**10.** (1)
11. (1)	**12.** (3)			

EXPLANATIONS

1. (1) $13x - 15 = -2x + 105$

$\Rightarrow 13x + 2x = 105 + 15$

$\Rightarrow 15x = 120$

$\therefore x = \frac{120}{15} = 8$

2. (2) $\frac{3}{x-1} + \frac{1}{x-3} = \frac{4}{x-2}$

$\Rightarrow \frac{3}{x-1} + \frac{1}{x-3} = \frac{3}{x-2} + \frac{1}{x-2}$

$\Rightarrow \frac{3}{x-1} - \frac{3}{x-2} = \frac{1}{x-2} + \frac{1}{x-3}$

$\Rightarrow \frac{3(x-2) - 3(x-1)}{(x-1)(x-2)} = \frac{(x-3) - (x-2)}{(x-2)(x-3)}$

$$\Rightarrow \frac{3x-6-3x+3}{x-1} = \frac{x-3-x+2}{x-2}$$

$$\Rightarrow \frac{-3}{x-1} = \frac{-1}{x-3}$$

$$\Rightarrow \frac{3}{x-1} = \frac{1}{x-3}$$

$\Rightarrow 3x-9 = x-1 \Rightarrow 2x = 8 \Rightarrow x = 4$

3. (3) $\frac{x+1}{x} + \frac{x}{x+1} = 2\frac{1}{6}$

Subtracting 2 from the both sides,

$$\Rightarrow \left(\frac{x+1}{x}-1\right)+\left(\frac{x}{x+1}-1\right) = 2\frac{1}{6}-2$$

$$\Rightarrow \frac{x+1-x}{x} + \frac{x-x-1}{x+1} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$$

$$\Rightarrow \frac{x+1-x}{x(x+1)} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{x^2+x} = \frac{1}{6}$$

$\Rightarrow x^2+x = 6$ [By cross multiplication]

$\Rightarrow x^2+x-6 = 0$

$\Rightarrow x^2+3x-2x-6 = 0$

$\Rightarrow x(x+3)-2(x+3) = 0$

$\Rightarrow (x+3)(x-2) = 0$

If $\left.\begin{array}{l} x+3=0 \text{ then, } x=-3 \\ x-2=0 \text{ then, } x=\ \ 2 \end{array}\right\}$

4. (2) $\frac{x+1}{x-1} + \frac{x+2}{x-2} = \frac{22x+30}{11x-18}$

$$\Rightarrow \frac{x+1}{x-1} - 1 + \frac{x+2}{x-2} - 1 = \frac{22x+30}{11x-18} - 2$$

$$\Rightarrow \frac{x+1-x+1}{x-1} + \frac{x+2-x+2}{x-2}$$

$$= \frac{22x+30-22x+36}{11x-18}$$

$$\Rightarrow \frac{2}{x-1} + \frac{4}{x-2} = \frac{66}{11x-18}$$

$$\Rightarrow \frac{2(x-2)+4(x-1)}{(x-1)(x-2)} = \frac{66}{11x-18}$$

$$\Rightarrow \frac{2x-4+4x-4}{x^2-3x+2} = \frac{66}{11x-18}$$

$$\Rightarrow \frac{6x-8}{x^2-3x+2} = \frac{66}{11x-18}$$

$\Rightarrow (6x-8)(11x-18) = 66(x^2-3x+2)$

$\Rightarrow 66x^2-196x+144 = 66x^2-198x+132$

$\Rightarrow 198x-196x = 132-144$

$\Rightarrow 2x = -12$

$\therefore x = -6$

5. (1) $\frac{x-7}{x-3} + \frac{x-2}{x-9} - 2 = 0$

$$\Rightarrow \frac{x-7}{x-3} - 1 + \frac{x-2}{x-9} - 1 = 0$$

$$\Rightarrow \frac{x-7-x+3}{x-3} + \frac{x-2-x+9}{x-9} = 0$$

$$\Rightarrow \frac{-4}{x-3} + \frac{7}{x-9} = 0$$

$$\Rightarrow \frac{7}{x-9} = \frac{4}{x-3}$$

$\Rightarrow 7x-21 = 4x-36$ (By cross-multiplication)

$\Rightarrow 7x-4x = 21-36$

$\Rightarrow 3x = -15$

$\Rightarrow x = -5$

6. (1) $\frac{(x+1)(x+2)}{(x+3)(x+4)} = \frac{x+3}{x+7}$

$$\Rightarrow \frac{x^2+3x+2}{x^2+7x+12} = \frac{x+3}{x+7}$$

$$\Rightarrow \frac{x^2+3x+2}{x+3} = \frac{x^2+7x+12}{x+7}$$

$$\Rightarrow \frac{x(x+3)+2}{x+3} = \frac{x(x+7)+12}{x+7}$$

$$\Rightarrow x + \frac{2}{x+3} = x + \frac{12}{x+7} \quad \left[\because \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right]$$

$$\Rightarrow \frac{2}{x+3} = \frac{12}{x+7}$$

$\Rightarrow 12(x+3) = 2(x+7)$

$\Rightarrow 12x+36 = 2x+14$

$\Rightarrow 12x-2x = -36+14$

$\Rightarrow 10x = -22$

$$\therefore x = \frac{-22}{10} = \frac{-11}{5} = -2\frac{1}{5}$$

7. (1) $7x-2y = 3$

$11x - \frac{3}{2}y = 8$

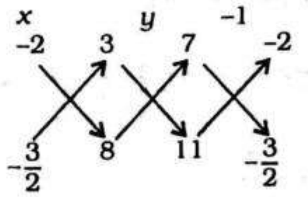
Here, $a_1 = 7$, $b_1 = -2$, $c_1 = 3$

$a_2 = 11$, $b_2 = -\frac{3}{2}$, $c_2 = 8$

$\therefore \frac{a_1}{a_2} = \frac{7}{11}, \frac{b_1}{b_2} = \frac{-2}{-\frac{3}{2}} = \frac{4}{3} \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ The given system of equations has a unique solution.

We write the co-efficients in the following way for an easy solution.



$$\frac{x}{(-2)(8)-(3)\left(-\frac{3}{2}\right)}$$

$$= \frac{y}{(3)(11)-(7)(8)} = \frac{-1}{(7)\left(-\frac{3}{2}\right)-(11)(-2)}$$

$$\Rightarrow \frac{x}{-16+\frac{9}{2}} = \frac{y}{33-56} = \frac{-1}{-\frac{21}{2}+22}$$

$$\Rightarrow \frac{x}{-\frac{23}{2}} = \frac{y}{-23} = \frac{-2}{23} \Rightarrow x = \frac{-2}{23} \times \frac{-23}{2} = 1$$

and $y = \frac{-2}{23} \times -23 = 2$

Hence, $x = 1$, $y = 2$

8. (2) $4x - 3y = 8$ ...(*i*)

$6x - y = \frac{29}{3}$ ...(*ii*)

By equation (i) × 3 – equation (ii) × 2, we have,

$12x - 9y = 24$

$12x - 2y = \frac{58}{3}$

$-\quad + \quad -$

$-7y = 24 - \frac{58}{3}$

$\Rightarrow -7y = \frac{14}{3} \Rightarrow y = \frac{14}{3} \times -\frac{1}{7} = -\frac{2}{3}$

Substituting the value of y in equationl (i),

$4x - 3\left(-\frac{2}{3}\right) = 8$

$\Rightarrow 4x + 2 = 8 \Rightarrow 4x = 8 - 2 = 6$

$\therefore x = \frac{6}{4} = \frac{3}{2}$

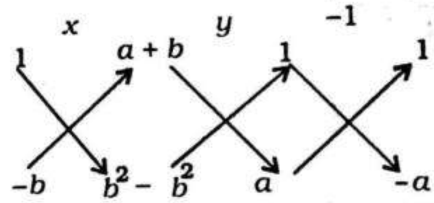
Hence, $x = \frac{3}{2}, y = \frac{-2}{3}$

9. (3) Here, $a_1 = 1$, $b_1 = 1$, $c_1 = a + b$
$a_2 = a$, $b_2 = -b$, $c_2 = a^2 - b^2$

Here, $\frac{a_1}{a_2} = \frac{1}{a}, \frac{b_1}{b_2} = \frac{1}{-b}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The system of equations has a unique solution.



$$\frac{x}{(a^2-b^2)+b(a+b)} = \frac{y}{a(a+b)-1(a^2-b^2)} = \frac{-1}{(1)(-b)-(a)(1)}$$

$$\Rightarrow \frac{x}{a^2-b^2+ab+b^2} = \frac{y}{a^2+ab-a^2+b^2} = \frac{-1}{-b-a}$$

$$\Rightarrow \frac{x}{a^2+ab} = \frac{y}{b^2+ab} = \frac{1}{a+b}$$

$$\Rightarrow x = \frac{a^2+ab}{a+b} = \frac{a(a+b)}{a+b} = a$$

$$y = \frac{b^2+ab}{a+b} = \frac{b(b+a)}{a+b} = b$$

Hence the solution is : $x = a$, $y = b$

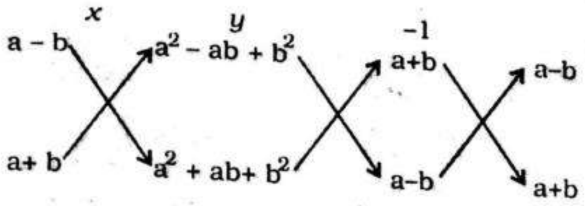
10. (1) $(a + b)x + (a - b)y = a^2 - ab + b^2$
$(a - b)x + (a + b)y = a^2 + ab + b^2$

Here, $\frac{a_1}{a_2} = \frac{a+b}{a-b}$

and $\frac{b_1}{b_2} = \frac{a-b}{a+b}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ The system of equations has a unique solution.



$$\Rightarrow \frac{x}{(a-b)\left(a^2+ab+b^2\right)-(a+b)\left(a^2-ab+b^2\right)}$$

$$=\frac{y}{(a-b)\left(a^2-ab+b^2\right)-(a+b)\left(a^2+ab+b^2\right)}$$

$$=\frac{-1}{(a+b)(a+b)-(a-b)(a-b)}$$

$$\Rightarrow \frac{x}{a^3-b^3+a^3-b^3}=\frac{y}{-2b\left(2a^2+b^2\right)}=\frac{-1}{4ab}$$

$$\Rightarrow \frac{x}{-2b^3}=\frac{y}{-2b(2a^2+b^2)}=-\frac{1}{4ab}$$

$$\Rightarrow x=-\frac{1}{4ab}\times -2b^3=\frac{b^2}{2a}$$

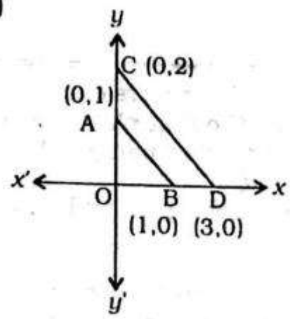
$$y=-\frac{1}{4ab}\times -2b\left(2a^2+b^2\right)=\frac{2a^2+b^2}{2a}$$

The solution is : $x=\frac{b^2}{2a}, y=\frac{2a^2+b^2}{2a}$

11. (1) $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ will have no solution if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}\Rightarrow\frac{1}{3}=\frac{2}{k}\qquad\Rightarrow k=6$$

12. (3)



$x=0$ is the equation of y-axis.
$y=0$ is the equation of x-axis.
Putting $x=0$ in $x+y=1$, $y=1$
Putting $y=0$ in $x+y=1$, $x=1$
Putting $x=0$ in $2x+3y=6$
$3y=6\Rightarrow y=2$
Putting $y=0$ in $2x+3y=6$
$2x=6\Rightarrow x=3$
$\therefore$ OB = 1; OA = 1
OD = 3; OC = 2
$\therefore$ Required area = ΔOCD − ΔOAB

$$=\frac{1}{2}\times3\times2-\frac{1}{2}\times1\times1$$

$$=3-\frac{1}{2}=2\frac{1}{2}\text{ sq. units}$$

□□□