16. Right Circular Cone

Let us Work Out 16

1. Question

I have made a closed right circular cone whose length of the base radius is 15cm and slant height is 24cm. Let us calculate the curved surface area and total surface area of that cone.

Answer

Base radius of the cone, r = 15 cm

Slant height of the cone, l = 24 cm

∴ Height of the cone, $h = \sqrt{(l^2 - r^2)} = \sqrt{(24^2 - 15^2)} = \sqrt{(576 - 225)} = \sqrt{351} = 18.73$ cm

∴ Curved surface area of the cone,

$= \pi rl \text{ cm}^2$

$= \frac{22}{7} \times 15 \times 24 \text{ cm}^2$

$= 1131.43 \text{ cm}^2$

∴ Total surface area,

$= \pi r^2 + \pi rl \text{ cm}^2$

$= 22/7 \times 15^2 + 1131.43 \text{ cm}^2$

$= 707.14 + 1131.43 \text{ cm}^2$

$= 1838.57 \text{ cm}^2$

2. Question

Let us determine the volume of the cone when, (i) base area is 1.54 sq. m and height is 2.4m (ii) the length of base diameter is 21m and slant height is 17.5m.

Answer

i) Base area, $\pi r^2 = 1.54$ sq.m.

height, h = 2.4 m

∴ volume of the cone,

$= \pi r^2 \times h/3\ m^3$

$= 1.54 \times 2.4/3\ m^3$

$= 1.232\ m^3$

ii) base diameter = 21 m

∴ base radius, r = 21/2 = 10.5 cm

Slant height, l = 17.5 m

∴ height, $h = \sqrt{(l^2 - r^2)} = \sqrt{(17.5^2 - 10.5^2)} = \sqrt{(306.25 - 110.25)} = \sqrt{196} = 14$ m

∴ volume of the cone,

$= \pi r^2 h/3$

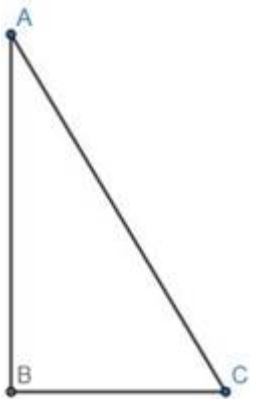
$= 22/7 \times 10.5^2 \times 14/3$

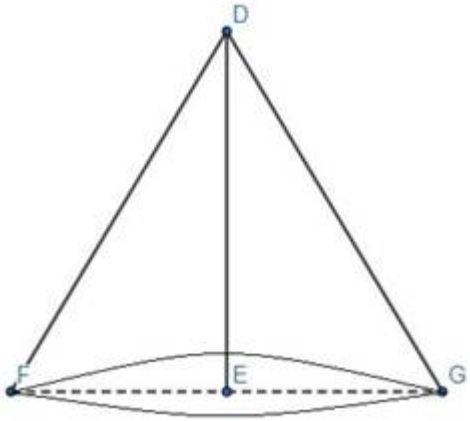
$= 1617\ m^3$

3. Question

Amina has drawn a right-angled triangle whose lengths of two sides adjacent to right angle are 15cm and 20 cm. Let us determine the curved surface area, total surface area and volume of the solid which is formed by taking the side of length 15cm. which is formed by completely revolving the triangle once around the side of the triangle with the length of 15cm, having been taken as an axis.

Answer





The solid formed by taking the side of length 15 cm and completely revolving the triangle once is a right circular cone.

According to problem,

Height of the cone, h = 15 cm

Length of base radius, r = 20 cm

∴ slant height, $l = \sqrt{(h^2 + r^2)}\sqrt{(15^2 + 20^2)} = \sqrt{(225 + 400)} = 25$ cm

∴ curved surface area,

$= \pi rl$

$= 22/7 \times 20 \times 25$

= 1571.43 sq. cm.

∴ Total surface area,

$= \pi r^2 + \pi rl$

$= 22/7 \times 20^2 + 1571.43$

= 1257.14 + 1571.43

= 2828.57 sq cm.

∴ volume,

$= \pi r^2 h/3$

$= 22/7 \times 20^2 \times 15/3$

= 6285.71 cubic cm.

4. Question

If the height and slant height of a cone are6cm.and 10cm. respectively, then let us determine total surface area and volume of the cone.

Answer

Height, h = 6 cm

Slant height, l = 10 cm

∴ radius, $r = \sqrt{(l^2 - h^2)} = \sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = 8$ cm

∴ Total surface area,

$= \pi r^2 + \pi rl$

$= 22/7 \times 8^2 + 22/7 \times 8 \times 10$

$= 201.14 + 251.43$

$= 452.57$ sq. cm.

∴ volume,

$= \pi r^2 h/3$

$= 22/7 \times 8^2 \times 6/3$

$= 402.28$ cubic cm

5. Question

If the volume of a right circular cone is $100\ \pi\ cm^3$ and height is 12cm, then let us write by calculating, the slant height of the cone.

Answer

Let, base radius = r cm and slant height = l cm

Height = 12 cm

According to problem,

$\Rightarrow \pi \times r^2 \times h/3 = 100\pi$

$$\Rightarrow \frac{\pi r^2 \times 12}{3} = 100\pi$$

$\Rightarrow 4r^2 = 100$

$\Rightarrow r = 5$

Base radius = 5 cm

∴ Slant height $= \sqrt{(h^2 + r^2)} = \sqrt{(12^2 + 5^2)} = \sqrt{(144 + 25)} = 13$ cm

6. Question

77sq.m. tripal is required to make a right circular conical tent. If the slant height is 7m. Then let us write by calculating, the base area of the tent.

Answer

Structure of a tent is like a right circular cone.

∴ tripal required to make tent = curved surface area of the cone.

Let, base radius of the tent = r m

Slant height = 7 m

According to problem,

⇒ π × r × 7 = 77

⇒ 22/7 × r = 11

⇒ r = 7/2 = 3.5

∴ base radius of the tent = 3.5 m

∴ Base area of the tent,

= πr^2

= 22/7 × 3.5^2

= 38.5 sq. m

7. Question

The base area of a right circular cone is 21m. and height is 14m. Let us calculate the expenditure to colour the curved surface at the rate of ₹1.50 per sq.m.

Answer

Let, base radius of the cone = r m

According to problem,

⇒ πr^2 = 21

⇒ r^2 = 21 ×7/22

⇒ r = 2.58

∴ radius of the base = 2.58 m

Height of the cone = 14 m

∴ slant height of the cone,

= $\sqrt{(2.58^2 + 14^2)}$

= $\sqrt{(6.66 + 196)}$

= 14.24 cm

∴ curved surface area of the cone,

= πrl

= 22/7 × 2.58 × 14.24

= 115.47 sq. m

∴ Expenditure to colour the curved surface area,

= 115.47 × 1.50

= Rs. 173.2

8. Question

The length of the base diameter of a wooden toy of the conical shape is 10cm. The expenditure for polishing the whole surfaces of the toy at the rate of ₹2.10 per m^2 is ₹429. Let us calculate the height of the toy. Let us also determine the quantity of wood which is required to make the toy.

Answer

Let, whole surface area of the toy = x m^2

According to problem,

⇒ x × 2.1 = 429

⇒ x = 429/2.1

⇒ x = 204.286

Base diameter of the toy = 10 cm

∴ base radius of the toy, r = 10/2 = 5 cm

Let, slant height = l cm

According to problem,

⇒ $\pi r^2 + \pi rl = 204.286$

⇒ $22/7(5^2 + 5l) = 204.286$

⇒ 5l + 25 = 65

⇒ 5l = 40

⇒ l = 8

∴ slant height = 8 cm

∴ height = $\sqrt{(l^2 - r^2)} = \sqrt{(8^2 - 5^2)} = \sqrt{(64 - 25)} = 6.24$ cm

∴ Volume of the toy,

$= \pi r^2 h/3$

$= 22/7 \times 5^2 \times 6.24/3$

$= 163.43\ m^3$

∴ Quantity of wood required to make the toy = $163.43\ m^3$

9. Question

The quantity of iron sheet to make boya of right circular conical shape is $75\frac{3}{7}$ m^2. If the slant height of it is 5m, then let us write, by calculating, the volume of air in the boya and its height. Let us determine the expenditure to colour the whole surface of the boya at the rate of ₹2.80 per m^2. [The width of the iron-sheet not to be considered while calculating.]

Answer

Let, base radius = r m

Slant height, l = 5 m

According to problem,

$$\Rightarrow \pi r^2 + \pi rl = 75\frac{3}{7}$$

$$\Rightarrow \frac{22}{7} \times r^2 + \frac{22}{7} \times r \times 5 = \frac{528}{7}$$

$\Rightarrow r^2 + 5r = 24$

$\Rightarrow r^2 + 8r - 3r - 24 = 0$

$\Rightarrow r(r + 8) - 3(r + 8) = 0$

$\Rightarrow (r + 8)(r - 3) = 0$

$\Rightarrow r = -8$ or 3

∴ base radius = 3 m

∴ height of the boya, $h = \sqrt{(5^2 - 3^2)}$

$= \sqrt{(25 - 9)} = 4$ m

∴ volume of air in the boya,

$= \pi r^2 h/3$

$= 22/7 \times 3^2 \times 4/3$

$\Rightarrow 37.71\ m^3$

10. Question

In a right circular conical tent 11 persons can stay. For each person $4m^2$ space in the base and $20m^3$ air are necessary. Let us determine the height of the tent put up exactly for 11 persons.

Answer

Let, base radius of the tent = r m

Height of the tent = h m

Space in the base in the tent = $\pi r^2\ m^2$

According to problem,

$\Rightarrow \pi r^2 = 11 \times 4$

$\Rightarrow \pi r^2 = 44$ …….. (1)

Volume of air in the tent = $\Rightarrow \pi r^2 h/3$

According to problem,

$\Rightarrow \pi r^2 h/3 = 11 \times 20$

$\Rightarrow 44 \times h = 660$ [putting the value from (1)]

$\Rightarrow h = 15$

$\therefore$ height of the tent = 15 m

11. Question

The external diameter of a conical-coronet made off thermocol is 21cm. in length. To wrap up the outer surface of the coronet with foil, the expenditure will be ₹57.75 at the rate of 10p. per cm^2. Let us write by calculating, the height and slant height of the coronet.

Answer

External diameter of coronet = 21 cm

$\therefore$ external radius of coronet, r = 21/2 = 10.5 cm

Let, slant height = l cm

Height = h cm

According to problem,

$\Rightarrow \pi rl = 57.75/0.1$ [10p. = Rs. 0.1]

$\Rightarrow 22/7 \times 10.5 \times l = 577.5$

$\Rightarrow l = 577.5 \times 7/(22 \times 10.5)$

$\Rightarrow l = 17.5$

$\therefore$ slant height = 17.5 cm

$\therefore$ height = $\sqrt{(17.5^2 - 10.5^2)} = \sqrt{(306.25 - 110.25)} = 14$ cm

12. Question

A heap of wheat is in the shape of a right circular cone, its base diameteris 9m. and height is 3.5m. Let us determine the total volume of wheat. Let us calculate the minimum quantity of plastic sheet to be required to cover up this heap of wheat [suppose $\pi = 3.14$, $\sqrt{130} = 11.1$]

Answer

Base diameter = 9 m

$\therefore$ base radius = 9/2 = 4.5 m

Height, h = 3.5 m

$\therefore$ volume of wheat,

$= \pi r^2 h/3$

$= 3.14 \times 4.5^2 \times 5/3$

$= 105.975\ m^3$

Slant height,

$l = \sqrt{(4.5^2 + 3.5^2)} = \sqrt{(20.25 + 12.25)} = 5.7$ m

$\therefore$ Plastic sheet required to cover the wheat,

$= \pi rl$

$= 3.14 \times 4.5 \times 5.7$

$= 80.541\ m^2$

13 A1. Question

If the slant height of a right circular cone is 15 cm. and the length of the base diameter is 16cm, then the lateral surface area of the cone is

A. 60π

B. $68\pi\ cm^2$

C. $120\pi\ cm^2$

D. $130\pi\ cm^2$

Answer

Slant height, l = 15 cm

Base diameter = 16 cm

∴ base radius, r = 16/2 = 8 cm

∴ lateral surface area of cone,

= πrl

= π × 8 × 15

= $120\pi\ cm^2$

13 A2. Question

If the ratio of the volumes of two right circular cones is 1:4 and the ratio of their radii of their bases is 4:5, then the ratio of the heights is

A. 1:5

B. 5:4

C. 25:16

D. 25:64

Answer

Volume of the first cone = v and second cone = 4v

First cone's base radius = 4r and second cone's base radius = 5r

Let, first cone's height = x and second cone's height = y

According to the problem,

$$\Rightarrow \frac{\frac{\pi\times(4r)^2\times x}{3}}{\frac{\pi\times(5r)^2\times y}{3}} = \frac{v}{4v}$$

$$\Rightarrow \frac{16r^2 \times x}{25r^2 \times y} = \frac{1}{4}$$

⇒ x/y = 25/64

⇒x: y = 25 : 64

13 A3. Question

Keeping the radius of a right circular cone same, if the height of it is increased twice, the volume of it will be

A. 100%

B. 200%

C. 300%

D. 400%

Answer

In 1st case,

Base radius = r

Height = h

∴ Volume, x = $\pi r^2 h/3$

In 2nd case,

Base radius = r

Height = 2h

∴ volume, y = $\pi r^2 \times 2h/3 = 2\pi r^2 h/3$

∴ increase in volume,

$$= \frac{y - x}{x} \times 100\%$$

$$= \frac{\frac{2\pi r^2 h}{3} - \frac{\pi r^2 h}{3}}{\frac{\pi r^2 h}{3}} \times 100\%$$

= 100%

∴ volume will be 200%

13 A4. Question

If each of radius of a cone is increased by twice of its length, then the volume of it will be

A. 3 times

B. 4 times

C. 6 times

D. 8 times of the previous one.

Answer

In 1^{st} case,

Base radius = r

Height = h

∴ Volume, $x = \pi r^2 h/3$

In 2^{nd} case,

Base radius = 2r

Height = h

∴ volume, $y = \pi(2r)^2 \times h/3 = 4\pi r^2 h/3$

$$\therefore \frac{y}{x} = \frac{\frac{\pi r^2 h}{3}}{\frac{4\pi r^2 h}{3}}$$

⇒ y/x = 4

⇒ y = 4x

∴ The volume will be 4 times.

13 A5. Question

If the length of the radius of a cone is $\frac{r}{2}$ unit and slant height of it is 2l unit, then the total surface area is

A. $2\pi r(\ell + r)$ sq.unit

B. $\pi r(\ell + \frac{r}{4})$ sq.unit

C. $\pi r(\ell + r)$ sq. unit

D. $2\pi\ell$ sq.unit

Answer

Radius = r/2 unit

Slant height = 2l unit

∴ Total surface area,

$$= \pi\left(\frac{r}{2}\right)^2 + \pi\left(\frac{r}{2}\right)2l$$

$$= \pi r\left(l + \frac{r}{4}\right) \text{ sq unit}$$

13 B. Question

Let us write whether the following statements are true or false:

(i) If the length of the base radius of a right circular cone is decreased by half and its height is increased by twice of it then the volume remains the same.

(ii) The height, radius and slant height of a right circular cone are always the three sides of a right-angles triangles.

Answer

(i) false

In 1^{st} case,

Base radius = r

Height = h

∴ Volume, $x = \pi r^2 h/3$

In 2^{nd} case,

Base radius = r/2

Height = 2h

∴ volume, $y = \pi(r/2)^2 \times 2h/3 = \pi r^2 h/6$

∴ $x \neq y$

∴ volume will not remain same

(ii) True

The height is equivalent to the perpendicular of a right-angle triangle.

The radius is equivalent to the base and the slant height is equivalent to the hypotenuse.

13 C. Question

Let us fill in the blanks:

(i) AC is the hypotenuse of a right-angle triangle ABC, the radius of the right circular cone formed by revolving the triangle once around the side AB as the

axis is___

(ii) If the volume of a right circular cone is V cubic unit and the base area is A sq. unit, then its height is____

(iii) The lengths of the base radii and the heights of a right circular cylinder and a right circular cone are equal. The ratio of their volume is_____.

Answer

(i) BC

AC is equivalent to the slant height.

AB is equivalent to height.

BC is equivalent to radius.

(ii) We know,

Volume of a right circular cone = base area × height

$\Rightarrow$ V = A × height

$\Rightarrow$height = V/A

(iii) In 1^{st} case,

Base radius = r

Height = h

$\therefore$ Volume, $x = \pi r^2 h/3$

In 2^{nd} case,

Base radius = r

Height = h

$\therefore$ volume, $y = \pi r^2 h/3$

$\therefore x : y = (\pi r^2 h/3) : (\pi r^2 h/3) = 1 : 1$

14 A. Question

The height of a right circular cone is 12cm. and its volume is 100 π cm^3. Let us write the length of the radius of the cone.

Answer

Let, radius = r cm

Height = 12 cm

According to problem,

$\Rightarrow \pi \times r^2 \times 12 = 100\pi$

$\Rightarrow 12r^2 = 100$

$\Rightarrow r^2 = 25/3$

$\Rightarrow r = 2.89$

$\therefore$ Length of radius of the cone = 2.89 cm

14 B. Question

The curved surface area of a right circular cone is $\sqrt{5}$ times of its base area. Let us write the ratio of the height and the length of a right circular cone are always the three sides of a right-angled triangle.

Answer

Given: Curved Surface Area of a right circular cone = $\sqrt{5}$ Base Area of right circular cone

Curved Surface Area of a right circular cone = πrl

Base area of right circular cone, A = πr^2

$$\pi r^2 = \sqrt{5}\ \pi\, r\, l$$

$$r = \sqrt{5}\, l \ldots\ldots\ldots\ldots\ldots\ldots\text{eq(i)}$$

Now from the cone we know that,

$$r^2 + h^2 = l^2 \ldots\ldots\ldots\ldots\ldots\ldots\text{eq(ii)}$$

Where, h is the height of cone

Now from (i) and (ii), we get that

$$5l^2 + h^2 = l^2$$

$$h^2 = -4\, l^2$$

Now, h cannot be negative, hence such cone is not possible.

14 C. Question

If the volume of a right circular cone is V cubic unit, base area A sq. unit and height is H unit, then let us write the value of $\frac{AH}{V}$.

Answer

We know,

Volume of a right circular cone = Base area × Height

⇒ V = A × H

⇒ AH/V = 1

14 D. Question

The numerical values of the volume and the lateral surface area of a right circular cone are equal. If the height and the radius of the cone are h unit and r unit respectively, then let us write the value of $\frac{1}{h^2}+\frac{1}{r^2}$.

Answer

Height = h unit

Radius = r unit

∴ slant height, $l = \sqrt{(h^2 + r^2)}$

According to problem,

$$\Rightarrow \frac{\pi r^2 h}{3} = \pi r\sqrt{h^2 + r^2}$$

$$\Rightarrow \frac{rh}{3} = \sqrt{h^2 + r^2}$$

$$\Rightarrow r^2 + h^2 = r^2h^2/9$$

$$\Rightarrow \frac{1}{h^2}+\frac{1}{r^2} = \frac{1}{9}$$

14 E. Question

The ratio of the lengths of the base radii of a right circular cylinder and a right circular cone is 3:4 and the ratio of their heights is 2:3; let us write the ratio of the volumes of the cylinder and the cone.

Answer

Let, base radius of the cone = 4r

Base radius of the cylinder = 3r

Let, the height of the cone = 3h

The height of the cylinder = 2h

∴ volume of the cone,

$= 1/3 \times \pi \times (4r)^2 \times 3h$

$= 16\pi r^2 h$

$\therefore$ volume of the cylinder,

$= \pi \times (3r)^2 \times 2h$

$\Rightarrow 18\pi r^2 h$

$\therefore$ ratio of the volumes of the cylinder and the cone,

$= (16\pi r^2 h) : (18\pi r^2 h)$

$= 8 : 9$