Chapter 6
Integer

Introduction to Integers

We already know about natural numbers and whole numbers.

We know for counting number of toffees that we have we can easy do so with the help of whole numbers like, 0, 1, 2, 3.........

Now, what if we want to measure something of the value less than 0 like the loss incurred while selling pen.

Then there was need of integers.

Ethan is going for a picnic with his friends. He wants to carry cupcakes with him, but he has got only 3 cupcakes and there are 4 friends. What is he going to do now?

So, Ethan decided to borrow one cupcake from his sister, which he would return later.

How many cupcakes does he have now?

After borrowing one cupcake from his sister, he has got 4 cupcakes, which he would give to his four friends.

He goes for the picnic, where he gave away the 4 cupcakes to his friends.

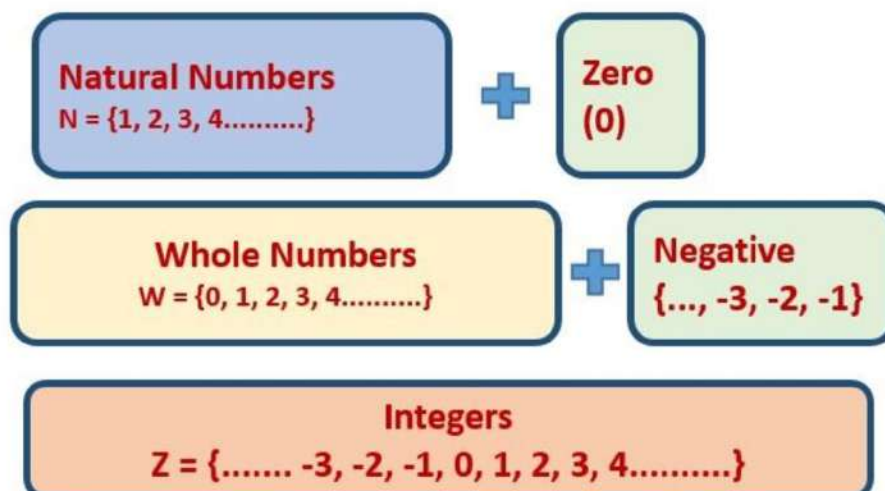Now, how many cupcakes are left with him?

Is your answer zero (0)?

We can say that there are no or 0 cupcakes left with him, but we also have to keep in mind that he has borrowed one cupcake from his sister.

So, in actual Ethan has (-1) cupcake, which means that 1 cupcake is borrowed and did not belong to him.

If he buys 3 more cupcakes next day, he will have to return 1 cupcake to his sister and will be left with 2 cupcakes only.

> **Numbers with a negative sign are less than zero and they are called negative numbers.**

Integers: Integers is a set of whole numbers and negative of all natural numbers.

| Natural Numbers N = {1, 2, 3, 4..........}	+	Zero (0)
Whole Numbers W = {0, 1, 2, 3, 4..........}	+	**Negative** {..., -3, -2, -1}

Integers
Z = {........ -3, -2, -1, 0, 1, 2, 3, 4..........}

Representation of Integers on Number Line

|	
Integers to the right of zero are positive.	
Integers to the left of zero are negative.	
The value of integers increases as we move towards right of 0.	
The value of integers decreases as we move towards left of 0.	

Negative Integers   Positive Integers

-6 -5 -4 -3 -2 -1   0   1   2   3   4   5   6
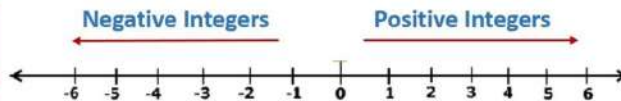
Opposite Integers/ Additive Inverse

The opposite of an integer is at the same distance from zero, but on the opposite sides of the number line. Therefore, one integer will have a positive sign and the other will have a negative sign.

-5 -4 -3 -2 -1   0   1   2   3   4   5
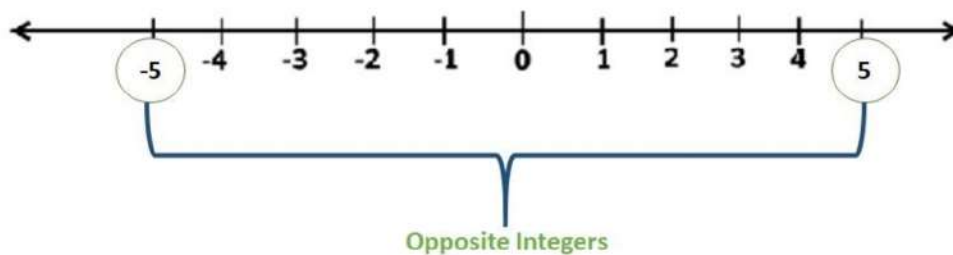
Opposite Integers

So, we can say that 5 and -5 are opposite integers.

Opposite of any integer a is $-a$ and opposite of $-a$ is $-(-a) = a$

Example: Write the opposite of the following integers:
-25, 16, 7, -100

a) -25

The given integer is negative.

The opposite integer of -25 = -(- 25) = 25

b) 16

The given integer is positive.

The opposite integer of 16 = -(16) = -16

c) 7

The given integer is positive.

The opposite integer of 7 = -(7) = -7

d) -100

The given integer is negative.
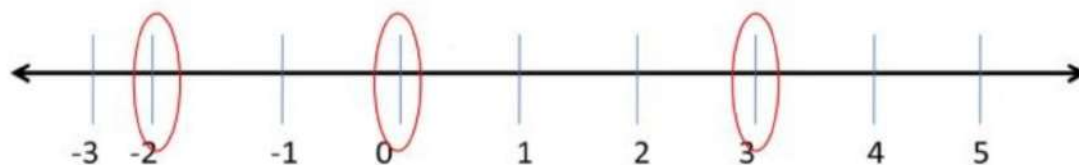
The opposite integer of -100 = - (-100) = 100

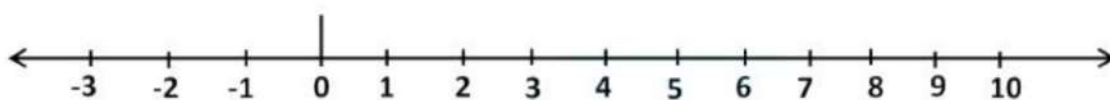Example: Represent following on same the number line:

i) -2
ii) 3
iii) 0

Ordering of Integers

Let us once again observe the integers which are represented on the number line.

We know that 5 > 4 and from the number line shown above, we observe that 5 is to the right of 4.

Similarly, 4 > 0 and 4 is to the right of 0.
Now, since 0 is to the right of –3 so, 0 > – 1.
Again, – 1 is to the right of – 2 so, – 1 > – 2.
Thus, we see that on a number line the number increases as we move to the right and decreases as we move to the left.

Therefore, – 3 < – 2, – 2 < – 1, – 1 < 0, 0 < 1, 1 < 2, 2 < 3 so on.

Hence, the collection of integers can be written as...,−5, −4, − 3, − 2, − 1, 0, 1, 2, 3, 4, 5

Addition of Integers

> **Rule 1: When we add two positive integers, we add their values and the result will take the positive sign (common sign of both the integers)**
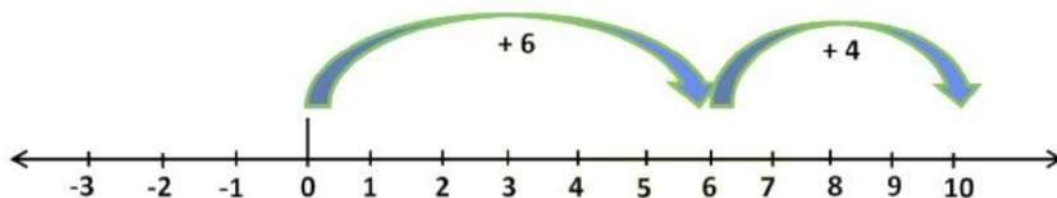
If we add 6 and 4 (both are positive integers), we add their values and the result will also be a positive integer.

So, $6 + 4 = 10$

Let's do this addition on a number line also.

When we add two or more integers on the number line then we move towards the right of any one of the given numbers.

So we start from 0 and jump 6 units towards the right and then again jump 4 units towards right from 6.

We reach 10 on the number line, which is the sum of 6 and 4.

Example: Add 25 and 46

25 and 46 are positive integers. So, we add their values.

$25 + 46 = 71$

The result will be a positive integer. (Common sign of both the integers)

> **Rule 2: When we add a positive and a negative integer, we find the difference of their numerical values, regardless of their signs and give the sign of the integer which is greater.**

Add: 6 + (-9)

Here, one integer is positive and the other integer is negative.
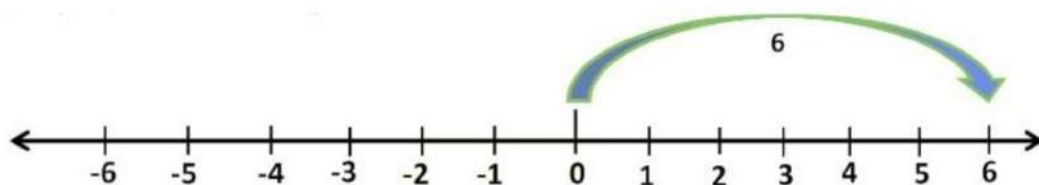
So, we find the difference of the integers, (9 − 6 = 3)

Out of the two integers, which one is greater (do not consider the sign of the integers here)?

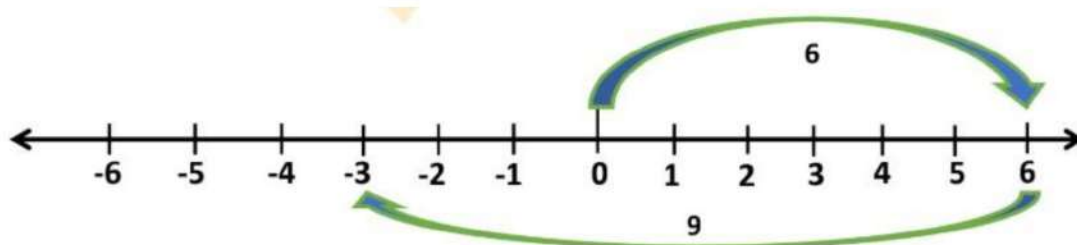Clearly, 9 is the greater integer, but it is a negative integer.

Therefore, the result 3, will have a negative sign.

So, -9 +6 = -3

When we add -9 and 6 on a number line, we first start from 0 and jump 6 places to the right of zero.

We reach 6 on the number line, and then we jump 9 places to the left of 6. (When we add a negative integer on a number line, we move towards left)

We reach -3 on the number line.

So, -9 + 6 = -3

Example: Add -67 and 32

-67 + 32

Here, one integer is positive and the other is negative so we find the difference of their numerical values.

67 – 32 = 35

Now, 67 is the greater integer (signs of the integers are not considered).

As 67 is a negative integer, the result will take the negative sign.

-67 + 32 = -35

Example: Write down a pair of integers whose:
i) Sum is -7
iii) Sum is 0

i) Sum is -7
Consider the pair of integers (-10, 3).
As one of the integers is negative, we find the difference between their numerical values and put the sign of the greater integer.

Sum of -10 and 3 = -10 + 3 = -7

ii) Sum is 0

Consider the pair of integers (-7, 7)

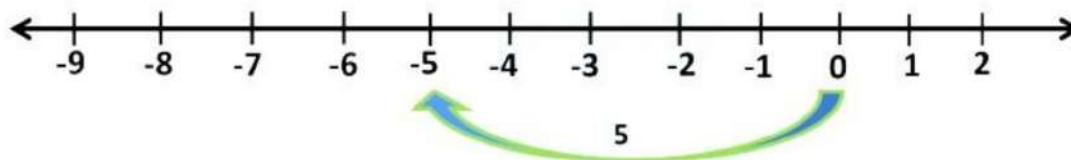One of the integers is negative, we find the difference between the numerical values.

Sum of -7 and 7 = -7 + 7 = 0

> **Rule 3: If two negative integers are added then we add their values and the result will take the negative sign.**
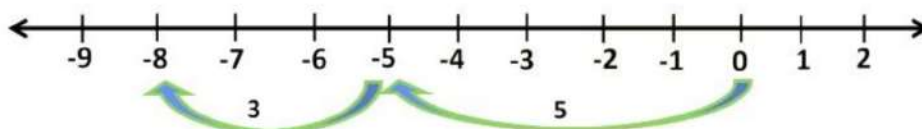
Add: -5 and -3

-5 + (-3) = -5 – 3 = -8

If we do the addition of -5 and -3 on the number line, then we start from 0 and jump 5 places to the left of 0.

We reach -5 on the number line, and then we jump 3 places to the left of -5. (As we are adding a negative integer on a number line, we move towards left)

We reach -8 on the number line.

-5 + (-3) = -8

Example: Add -78 and -36

Both the integers are negative, so we add their values and the result will take the negative sign.

-78 + (-36) = -78 – 36 = -114

Subtraction of Integers on Number Line

When we subtract one integer from the other, we convert the integer to be subtracted to its negative and then add the two integers.

Subtracting an integer from the other is same as adding the additive inverse of the integer.

$$a - b = a + (\text{Opposite of } b) = a + (-b)$$
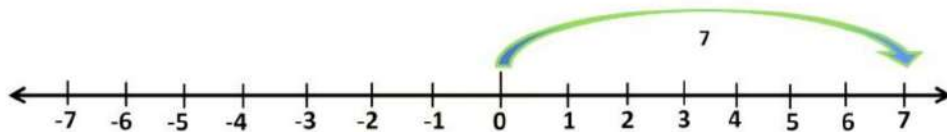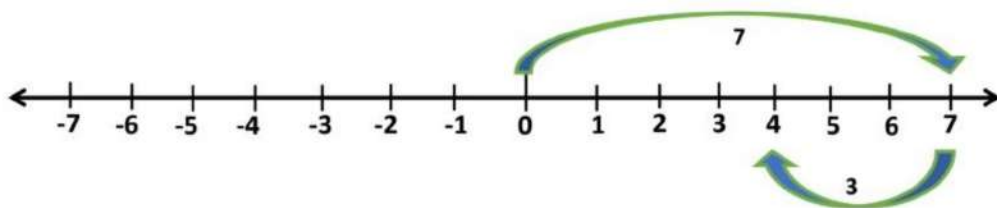
Subtract 3 from 7,

7 – 3

Opposite of 3 is -3

So we add 7 to the additive inverse of 3 that is -3.

7 -3 = 7 + (-3) = 4

On a number line, we jump 7 places to the right of zero

We reach 7 on the number line, and then we jump 3 places to the left of 7. As we are adding a negative integer we move towards left.

We reach 4 on the number line.

So, 7 + (-3) = 4

Example: Subtract 18 from 76

76 – 18

Additive inverse of 18 is -18. So, we add 76 to the additive inverse of 18, which is -18.

= 76 + (-18) = 58

Example: Subtract 45 from 34

34 – 45

Now, we add 34 to the additive inverse of 45, which is -45.

= 34 + (-45) = -11

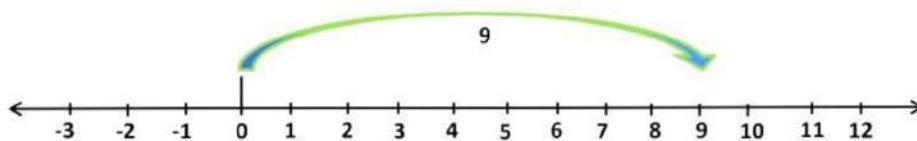When we subtract a negative integer from other integer, it is same as adding the two integers.

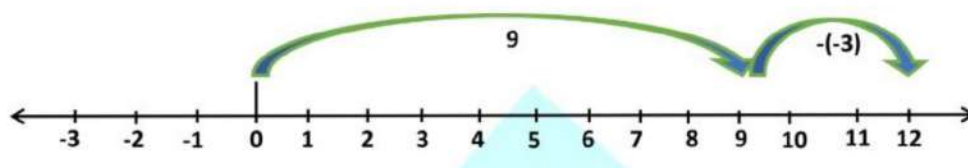a and b are two integers, where a is positive and b is negative.
$$a - (-b) = a + b$$

Subtract -3 from 9.

$9 - (-3) = 9 + 3 = 12$

On a number line we jump 9 places to the right of 0.

We reach 9 on the number line. When we subtract a negative integer from any integer we move towards right. So we jump 3 places to the right of 9.

We reach 12 on the number line.

So, $9 - (-3) = 9 + 3 = 12$

Example: Subtract -26 from 48

Here, we are subtracting the negative integer, -26 from 48. So we simply add the two integers.

48 – (-26)

48 – (-26) = 48 + 26 = 74

Example: Subtract -89 from -67

Now, we have to subtract a negative integer, -89 from another negative integer, -67.

-67 – (-89) = -67 + 89

We know that when we add a positive and a negative integer, then we find their difference and put the sign of the greater integer.

-67 + 89 = 22 (sign will be positive as the greater integer, 89 has a positive sign)