Chapter 2
Polynomials

Exercise No. 2.1

1. Which one of the following is a polynomial?

(A) $\frac{x^2}{2}-\frac{2}{x^2}$

(B) $\sqrt{2x}-1$

(C) $x^2+\frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

(D) $\frac{x-1}{x+1}$

Solution:

(A) $\frac{x^2}{2}-\frac{2}{x^2}$ can be written as $\frac{x^2}{2}-2x^{-2}$. The power of x^{-2} is -2, which is not a whole number. So, the given algebraic expression is not a polynomial.

(B) $\sqrt{2x}-1$ can be written as $\sqrt{2}x^{\frac{1}{2}}-1$. The power of $x^{\frac{1}{2}}$ is $\frac{1}{2}$, which is not a whole number. So, the given algebraic expression is not a polynomial.

(C) $x^2+\frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ can be written as x^2+3x. The power of the variable are whole number. So, the given algebraic expression is a polynomial.

(D) $\frac{x-1}{x+1}$ is not a standard from of a polynomial.
Hence, the correct option is (D).

2. $\sqrt{2}$ is a polynomial of degree.

(A)2
(B) 0
(C) 1
(D) $\frac{1}{2}$

Solution:

The term $\sqrt{2}$ can be written as $\sqrt{2}x^0$. Since, the power of x is 0. Therefore, the degree of the constant term polynomial is 0.
Hence, the correct option is (D).

3. **Degree of the polynomial** $4x^4 + 0\times x^3 + 0\times x^5 + 5x + 7$ **is.**
(A) 4
(B) 5
(C) 3
(D) 7

Solution:
We know that, the degree of given polynomial $4x^4 + 0\times x^3 + 0\times x^5 + 5x + 7$ will be the highest power of variable that is 4.
Hence, the correct option is (A).

4. **Degree of the zero polynomial is:**
(A) 0
(B) 1
(C) Any natural number
(D) Not defined

Solution:
The degree of the polynomial is equal to the highest power of the variable. We know that, the degree of zero polynomial is not defined.
Hence, the correct option is (D).

5. **If** $p(x) = x^2 - 2\sqrt{2}x + 1$**, then** $p(2\sqrt{2})$ **is equal to:**
(A) 0
(B) 1
(C) $4\sqrt{2}$
(D) $8\sqrt{2} + 1$

Solution
Consider the polynomial:
$p(x) = x^2 - 2\sqrt{2}x + 1$

The value of $p(2\sqrt{2})$ is calculated as follows:

$$p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}\times 2\sqrt{2} + 1$$
$$= 8 - 8 + 1$$
$$= 1$$

Hence, the correct option is (B).

6. **The value of the polynomial** $5x - 4x^2 + 3$**, when x = -1 is:**

(A) -6
(B) 6
(C) 2
(D) -2

Solution:
Let $f(x) = 5x - 4x^2 + 3$
The value of f(-1) will be:
$$f(-1) = 5\times(-1) - 4\times(-1)^2 + 3$$
$$= -5 - 4 + 3$$
$$= -9 + 3$$
$$= 6$$
Hence, the correct option is (B).

7. If p(x) = x + 3, then p(x) + p(–x) is equal to:
(A) 3
(B) 2x
(C) 0
(D) 6

Solution:
Given:
p(x) = x + 3
So,
$$p(-x) = (-x) + 3$$
$$p(-x) = -x + 3$$

Now, the sum of p(x) + p(-x) is:
$$p(x) + p(-x) = x + 3 + (-x + 3)$$
$$= x + 3 - x + 3$$
$$= 6$$
Hence, the correct option is (D).

8. Zero of the zero polynomial is:
(A) 0
(B) 1
(C) Any real number
(D) Not defined

Solution:
The zero degree of the zero polynomial is not defined.

9. Zero of the polynomial p(x) = 2x + 5 is

(A) $-\frac{2}{5}$

(B) $-\frac{5}{2}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$

Solution:

Zero of the polynomial p(x) = 2x + 5 is p(x) =0. So,

$2x+5=0$

$2x=-5$

$x=-\frac{5}{2}$

Hence, the correct option is (B).

10. One of the zeroes of the polynomial $2x^2+7x-4$ **is:**

(A) 2

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) -2

Solution:

Let $f(x)=2x^2+7x-4$

Zero of the polynomial is calculate as f(x) = 0.

$$f(x)=0$$

$$2x^2+7x-4=0$$

$$2x^2+8x-x-4=0$$

$$2x(x+4)-1(x+4)=0$$

$$(2x-1)(x+4)=0$$

$$2x-1=0 \text{ or } x+4=0$$

$$2x=1 \text{ or } x=-4$$

$$x=\frac{1}{2} \text{ or } x=-4$$

Hence, the correct option is (B).

11. If $x^{51}+51$ is divided by x + 1, the remainder is:

(A) 0

(B) 1

(C) 49

(D) 50

Solution:

Let $f\left(x\right)=x^{51}+51$

We know that when f(x) is divided by x + a, then the remainder is f(-a).

So, remainder is:

$$f\left(-1\right)=\left(-1\right)^{51}+51$$
$$=-1+51$$
$$=50$$

Hence, the correct option is (D).

12. If x + 1 is a factor of the polynomial $2x^2+kx$, then the value of k is:

(A) -3

(B) 4

(C) 2

(D) -2

Solution:

Let $f\left(x\right)=2x^2+kx$

Zero of x + 1 is x = -1.

By factor theorem f(-1) = 0. So,

$$f\left(x\right)=0$$
$$2\times\left(-1\right)^2+k\times\left(-1\right)=0$$
$$2-k=0$$
$$2=k$$

Hence, the correct option is (C).

13. x + 1 is a factor of the polynomial:

(A) x^3+x^2-x+1

(B) x^3+x^2+x+1

(C) $x^4+x^3+x^2+1$

(D) $x^4+3x^3+3x^2+x+1$

Solution:

We know that if x + a is a factor of f(x) then, f(-a) = 0.

(A) Let $f(x) = x^3 + x^2 - x + 1$

Now,

$$f(-1) = (-1)^3 + (-1)^2 - (-1) + 1$$
$$= -1 + 1 + 1 + 1$$
$$= 2 \neq 0$$

So, f(x) is not a factor of x + 1.

(B) Let $f(x) = x^3 + x^2 + x + 1$

Now,

$$f(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$
$$= -1 + 1 - 1 + 1$$
$$= 0$$

So, f(x) is a factor of x + 1.

(C) Let $f(x) = x^4 + x^3 + x^2 + 1$

Now,

$$f(-1) = (-1)^4 + (-1)^3 + (-1)^2 + 1$$
$$= 1 - 1 + 1 + 1$$
$$= 2 \neq 0$$

So, f(x) is not a factor of x + 1.

(D) Let $f(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Now,

$$f(-1) = (-1)^4 + 3 \times (-1)^3 + 3 \times (-1)^2 + (-1) + 1$$
$$= 1 - 3 + 3 - 1 + 1$$
$$= 1 \neq 0$$

So, f(x) is not a factor of x + 1.

Hence, the correct option is (B).

14. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is:

(A) 5 + x
(B) 5 – x
(C) 5x – 1
(D) 10x

Solution:

$$\left(25x^2-1\right)+\left(1+5x\right)^2=\left(5x\right)^2-1^2+\left(5x+1\right)^2$$
$$=\left(5x-1\right)\left(5x-1\right)+\left(5x+1\right)^2$$
$$=\left(5x+1\right)\left(5x-1+5x+1\right)$$
$$=\left(5x+1\right)10x$$
$$=10x\left(5x+1\right)$$

So, one of the factor of $\left(25x^2-1\right)+\left(1+5x\right)^2$ is 10x.

Hence, the correct option is (D).

15. The value of 249^2-248^2 is:

(A) 1^2
(B) **477**
(C) **487**
(D) **497**

Solution:

Consider the expression:

249^2-248^2

Use the identity: $a^2-b^2=\left(a+b\right)\left(a-b\right)$

$$249^2-248^2=\left(249+248\right)\left(249-248\right)$$
$$=497\times1$$
$$=497$$

Hence, the correct option is (D).

16. The factorisation of $4x^2+8x+3$ is:

(A) **(x + 1) (x + 3)**
(B) **(2x + 1) (2x + 3)**
(C) **(2x + 2) (2x + 5)**
(D) **(2x –1) (2x –3)**

Solution:

Do the factor of given expression as follows:

$$4x^2+8x+3=4x^2+6x+2x+3$$
$$=2x\left(2x+3\right)+1\left(2x+3\right)$$
$$=\left(2x+1\right)\left(2x+3\right)$$

Hence, the correct option is (B).

17. Which of the following is a factor of $\left(x+y\right)^3-\left(x^3+y^3\right)$?

(A) x^2+y^2+2xy

(B) x^2+y^2-xy

(C) xy^2

(D) $3xy$

Solution:

$$(x+y)^3-(x^3+y^3)=x^3+y^3+3xy(x+y)-x^3-y^3$$

$$=3xy(x+y)$$

Since, 3xy is factor of $(x+y)^3-(x^3+y^3)$.

Hence, the correct option is (D).

18. The coefficient of x in the expansion of $(x+3)^3$ is:

(A) **1**

(B) **9**

(C) **18**

(D) **27**

Solution:

Consider the expression:

$$(x+3)^3$$

Use the identity: $(a+b)^3=a^3+b^3+3ab(a+b)$

$$(x+3)^3=x^3+3^3+3\times x\times 3(x+3)$$

$$=x^3+27+9x^2+27x$$

Since, the coefficient of x is 27.

Hence, the correct option is (D).

19. If $\frac{x}{y}+\frac{y}{x}=-1(x,y\neq 0)$, the value of x^3-y^3 is:

(A) 1

(B) -1

(C) 0

(D) $\frac{1}{2}$

Solution:

Consider the equation:

$$\frac{x}{y}+\frac{y}{x}=-1$$

Simplify the above expression as follows:

$$\frac{x^2+y^2}{xy}=-1$$

$$x^2+y^2=-xy$$

Now,

$$x^3-y^3=(x-y)(x^2+y^2+xy)$$

$$=(x-y)(-xy+xy)\left[\text{Substitute: } x^2+y^2=-xy\right]$$

$$=(x-y)\times 0$$

$$=0$$

Hence, the correct option is (C).

20. If $49x^2-b=\left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$, then the value of b is:

(A) 0

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Solution:

Consider the equation:

$$49x^2-b=\left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$$

Solve the above equation as follows:

$$49x^2-b=\left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)\left[\text{Use the identity:}(a+b)(a-b)=a^2-b^2\right]$$

$$49x^2-b=(7x)^2-\left(\frac{1}{2}\right)^2$$

$$49x^2-b=49x^2-\frac{1}{4}$$

Compare the each term of both side, get:

$$b=\frac{1}{4}$$

Hence, the correct option is (C).

21. If a + b + c = 0, then $a^3+b^3+c^3$ is equal to:

(A) 0

(B) abc

(C) 3abc

(D) 2abc

Solution:

We know that:

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

As a + b + c = 0, So, $a^3+b^3+c^3-3abc=(0)(a^2+b^2+c^2-ab-bc-ca)=0$

Hence, $a^3+b^3+c^3=3abc$

Therefore, the correct option is (C).

Exercise No. 2.2

1. Which of the following expressions are polynomials? Justify your answer:

(i) **8**

(ii) $\sqrt{3}x^2 - 2x$

(iii) $1 - \sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$

(v) $\frac{(x-2)(x-4)}{x}$

(vi) $\frac{1}{1+x}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

(viii) $\frac{1}{2x}$

Solution:-

(i)
Consider the expression:
8
The expression 8 also can be written as $8x^0$.
Since, the power of x is 0, which is a whole number.
Therefore, the expression 8 is a polynomial.

(ii)
Consider the expression:
$\sqrt{3}x^2 - 2x$
In the above expression, the power of x are 2 and 1 respectively, and they are whole number.
Therefore, the expression $\sqrt{3}x^2 - 2x$ is a polynomial.

(iii)
Consider the expression:
$1 - \sqrt{5x}$
$1 - \sqrt{5x}$ can be written as $1 - \sqrt{5}x^{\frac{1}{2}}$.
Since, the power of x is 1/2, which is not a whole number.
Therefore, the expression $1 - \sqrt{5x}$ is not a polynomial.

(iv)

Consider the expression:

$$\frac{1}{5x^{-2}}+5x+7$$

$\frac{1}{5x^{-2}}+5x+7$ can be written as $5x^2+5x+7$.

Since, the power of x are 2 and 1 respectively, and they are whole number.

Therefore, the expression $\frac{1}{5x^{-2}}+5x+7$ is a polynomial.

(v)

Consider the expression:

$$\frac{(x-2)(x-4)}{x}$$

Simplify the above expression as follows:

$$\begin{aligned}\frac{(x-2)(x-4)}{x}&=\frac{x^2-4x-2x+8}{x}\\&=\frac{x^2-6x+8}{x}\\&=x-6+8x^{-1}\end{aligned}$$

Since, the power of $x=-1$ is not a whole number.

Therefore, the expression $\frac{(x-2)(x-4)}{x}$ is not a polynomial.

(vi)

Consider the expression:

$$\frac{1}{1+x}$$

Simplify the above expression as follows:

$$\frac{1}{1+x}=(1+x)^{-1}$$

Since, the power x is not a whole number.

Therefore, the expression $\frac{1}{1+x}$ is not a polynomial.

(vii)

Consider the expression:

$$\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$$

In the above expression, the power of a are 3, 2, and 1, which are whole number.

Therefore, the expression $\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$ is a polynomial.

(viii)

Consider the expression:

$\frac{1}{2x}$

The expression $\frac{1}{2x}$ can be written as $\frac{1}{2x}=\frac{1}{2}x^{-1}$.

Since, the power of x is -1, which is not a whole number.

Therefore, the expression $\frac{1}{2x}$ is not a polynomial.

2. Write whether the following statements are true or false. Justify your answer.

(i) A binomial can two have at most terms.

(ii) Every polynomial is a binomial.

(iii) A binomial may have degree 5.

(iv) Zero of a polynomial is always 0.

(v) A polynomial cannot have more than one zero.

(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

Solution:-

(i) We know that the binomial have only two term so, the given statement is **false**. For example: $(a+b)^n$.

(ii) A polynomial can be a power of 0, 1, 2, and 3 etc. that are also called respectively monomial, binomial, and trinomial. For example x^3+x^2+x+1 is a polynomial but not binomial. Therefore, the given statement is **false**.

(iii) We know that binomial have two term whose degree is a whole number greater than equal to 1. For example, x^5-1 is a binomial of degree 5. Hence, the given statement is **true**.

(iv) The zero of polynomial can be any real number. Since, the given statement is **false**.

(v) A polynomial can have any number of zero and it's dependent on the degree of polynomial. Hence, the given statement is **false**.

(vi) The given statement is **false** because it will be not always true. For example: $x^5+2x^4+3x^3+x^2+x+1$ and $-x^5-2x^4+x^3+x^2+x+1$, the degree of both polynomial is 5 but their sum is $4x^3+2x^2+2x+2$. Therefore, the degree of this polynomial is 3 not 5.

Exercise No. 2.3

1. Classify the following polynomials as polynomials in one variable, two variables etc.

(i) x^2+x+1

(ii) y^3-5y

(iii) $xy+yz+zx$

(iv) $x^2-2xy+y^2+1$

Solution:

(i)

The given polynomial x^2+x+1 has only one variable that is x.
Therefore, the given polynomial is a polynomial in **one** variable.

(ii)

The given polynomial y^3-5y has only one variable that is y.
Therefore, the given polynomial is a polynomial in **one** variable.

(iii)

The given polynomial $xy+yz+zx$ has two variable that are x and y.
Therefore, the given polynomial is a polynomial in **two** variable.

(iv)

The given polynomial $x^2-2xy+y^2+1$ has two variable that are x and y.
Therefore, the given polynomial is a polynomial in two variable.

2. Determine the degree of each of the following polynomials:

(i) 2x-1

(ii) -10

(iii) $x^3-9x+3x^5$

(iv) $y^3\left(1-y^4\right)$

Solution:

(i)

Consider the expression: 2x-1
The degree of a polynomial in one variable is equal to highest power of the variable in algebraic expression that is 1.

(ii)

Consider the expression: -10
-10 can be written as $-10x^0$.

The degree of a polynomial in one variable is equal to highest power of the variable in algebraic expression that is 0.

(iii)
Consider the expression: $x^3 - 9x + 3x^5$
The degree of a polynomial in one variable is equal to highest power of the variable in algebraic expression that is 5.

(iv)
Consider the expression: $y^3\left(1-y^4\right)$
Simplify the above expression as follows:
$y^3\left(1-y^4\right) = y^3 - y^7$
The degree of a polynomial in one variable is equal to highest power of the variable in algebraic expression that is 7.

3. For the polynomial $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$, write

(i) The degree of the polynomial
(ii) The coefficient of x^3
(iii) The coefficient of x^6
(iv) The constant term

Solution:

Consider the expression:

$$\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$$

Simplify the above expression as:

$$\begin{aligned}\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6 &= \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6 \\ &= -x^6 + \frac{1}{5}x^3 - \frac{7}{2}x^2 + \frac{2}{5}x + \frac{1}{5}\end{aligned}$$

(i)
The degree of a polynomial in one variable is equal to highest power of the variable in algebraic expression that is 6.

(ii)
The coefficient of x^3 in the given expression is $\frac{1}{5}$.

(iii)
The coefficient of x^3 in the given expression is -1.

(iv)

The constant term in the given expression is $\frac{1}{5}$ because it has no variable x associated with it.

4. Write the coefficient of x^2 in each of the following:

(i) $\frac{\pi}{6}x+x^2-1$

(ii) **3x-5**

(iii) **(x-1)(3x-4)**

(iv) $(2x-5)(2x^2-3x+1)$

Solution:

(i)

In the given polynomial $\frac{\pi}{6}x+x^2-1$, the coefficient of x^2 is 1.

(ii)

The polynomial 3x-5 can be written as $0x^2+3x-5$.

Therefore, the coefficient of x^2 is 0.

(iii)

Consider the expression:

$(x-1)(3x-4)$

Simplify the above expression as:

$$(x-1)(3x-4)=3x^2-4x-3x+4$$
$$=3x^2-7x+4$$

Therefore, the coefficient of x^2 is 3.

(iv)

Consider the expression:

$(2x-5)(2x^2-3x+1)$

Simplify the above expression as:

$$(2x-5)(2x^2-3x+1)=4x^3-6x^2+2x-10x^2+15x-5$$
$$=4x^3-16x^2+17x-5$$

Therefore, the coefficient of x^2 is -16.

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i) $2-x^2+x^3$

(ii) $3x^3$
(iii) $5t-\sqrt{7}$
(iv) $4-5y^2$
(v) **3**
(vi) **2+x**
(vii) y^3-y
(viii) $1+x+x^3$
(ix) t^3
(x) $\sqrt{2}x-1$

Solution:

(i)
Consider the polynomial - $2-x^2+x^3$
The highest power in the above polynomial is 3. So, the given polynomial is **cubic polynomials.**

(ii)
Consider the polynomial - $3x^3$
The highest power in the above polynomial is 3. So, the given polynomial is **cubic polynomials.**

(iii)
Consider the polynomial - $5t-\sqrt{7}$
The highest power in the above polynomial is 1. So, the given polynomial is **linear polynomials.**

(iv)
Consider the polynomial - $4-5y^2$
The highest power in the above polynomial is 2. So, the given polynomial is **quadratic polynomials.**

(v)
Consider the polynomial - **3**
3 can be written as $3x^0$.
The highest power in the above polynomial is 0. So, the given polynomial is **constant polynomials.**

(vi)
Consider the polynomial - **2+x**
The degree of the above polynomial is 1. So, the given polynomial is **linear polynomials.**

(vii)
Consider the polynomial - y^3-y

The degree of the above polynomial is 3. So, the given polynomial is **cubic polynomials.**

(viii)
Consider the polynomial - $1+x+x^3$
The degree of the above polynomial is 3. So, the given polynomial is **cubic polynomials.**

(ix)
Consider the polynomial - t^3
The degree of the above polynomial is 3. So, the given polynomial is **cubic polynomials.**

(x)
Consider the polynomial - $\sqrt{2}x-1$
The degree of the above polynomial is 1. So, the given polynomial is **linear polynomials.**

6. Give an example of a polynomial, which is:
(i) Monomial of degree 1
(ii) Binomial of degree 20
(iii) Trinomial of degree 2
Solution:

(i)
We know that monomial polynomial has only one term. For example of monomial polynomial with degree 1 is 2x.

(ii)
We know that binomial polynomial has only two term. For example of binomial polynomial with degree 20 is $x^{20}+10$.

(iii)
We know that Trinomial polynomial has only two term. For example of Trinomial polynomial with degree 2 is x^2+2x+5.

7. Find the value of the polynomial $3x^3-4x^2+7x-5$, when x = 3 and also when x = –3.

Solution:
Consider the polynomial:
$p(x)=3x^3-4x^2+7x-5$

The value of given polynomial at x = 3 is:

$$p(3) = 3\times 3^3 - 4\times 3^2 + 7\times 3 - 5$$
$$= 3\times 27 - 4\times 9 + 7\times 3 - 5$$
$$= 61$$

Now, the value of given polynomial at x = -3 is:

$$p(-3) = 3\times(-3)^3 - 4\times(-3)^2 + 7\times(-3) - 5$$
$$= 3\times(-27) - 4\times 9 + 7\times(-3) - 5$$
$$= -143$$

8. If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p\left(\frac{1}{2}\right)$.

Solution:

Consider the polynomial:

$$p(x) = x^2 - 4x + 3$$

The value of given polynomial at x = 2 is:

$$p(2) = 2^2 - 4\times 2 + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

When x = -1,

$$p(-1) = (-1)^2 - 4\times(-1) + 3$$
$$= 1 + 4 + 3$$
$$= 8$$

When $x = \frac{1}{2}$,

$$p(-1) = \left(\frac{1}{2}\right)^2 - 4\times\left(\frac{1}{2}\right) + 3$$
$$= \frac{1}{4} - 2 + 3$$
$$= \frac{5}{4}$$

Now, the value of $p(2) - p(-1) + p\left(\frac{1}{2}\right)$ will be:

$$p(2)-p(-1)+p\left(\frac{1}{2}\right)=-1-8+\frac{5}{4}$$
$$=-9+\frac{5}{4}$$
$$=\frac{-36+5}{4}$$
$$=-\frac{31}{4}$$

9. Find p(0), p(1), p(-2) for the following polynomial:
(i) $p(x)=10x-4x^2-3$**,**
(ii) $p(y)=(y+2)(y-2)$

Solution:
(i)
Consider the polynomial:
$p(x)=10x-4x^2-3$

The value of given polynomial at x = 0 is:
$$p(0)=10\times 0-4\times 0^2-3$$
$$=-3$$

When x = 1
$$p(1)=10\times 1-4\times 1^2-3$$
$$=10-4-3$$
$$=3$$

When x= -2
$$p(-2)=10\times(-2)-4\times(-2)^2-3$$
$$=-20-16-3$$
$$=-39$$

(ii)
Consider the polynomial:
$p(y)=(y+2)(y-2)$

The value of given polynomial at y = 0 is:
$$p(0)=(0+2)(0-2)$$
$$=2\times -2$$
$$=-4$$

When y = 1

$p(1) = (1+2)(1-2)$

$= 3 \times -1$

$= 3$

When y= -2

$p(1) = (-2+2)(-2-2)$

$= 0 \times (-2-2)$

$= 0$

10. Verify whether the following are true or False:

(i) –3 is a zero of x – 3

(ii) $-\frac{1}{3}$ is a zero of 3x + 1

(iii) $\frac{-4}{5}$ is a zero of 4 –5y

(iv) 0 and 2 are the zeroes of $t^2 - 2t$

(v) –3 is a zero of $y^2 + y - 6$

Solution:

(i)

Zero of x-3 is calculated as follows:

x - 3 = 0

x = 3

Hence, the given statement is false.

(ii)

Zero of 3x + 1 is calculated as follows:

$3x + 1 = 0$

$3x = -1$

$x = -\frac{1}{3}$

Hence, the given statement is true.

(iii)

Zero of 4 –5y is calculated as follows:

$4 \ -5y = 0$

$4 = 5y$

$y = \frac{4}{5}$

Hence, the given statement is false.

(iv)

Zero of $t^2 - 2t$ is calculated as follows:

$t^2 - 2t = 0$

$t(t-2) = 0$

$t = 0$ or $t - 2 = 0$

$t = 0$ or $t = 2$

Hence, the given statement is true.

(v)

Zero of $y^2 + y - 6$ is calculated as follows:

$$y^2 + y - 6 = 0$$

$$y^2 + 3y - 2y - 6 = 0$$

$$y(y+3) - 2(y+3) = 0$$

$$(y+3)(y-2) = 0$$

$$y + 3 = 0 \text{ or } y - 2 = 0$$

$$y = -3 \text{ or } y = 2$$

Hence, the given statement is true.

11. Find the zeroes of the polynomial in each of the following:

(i) p(x) = x – 4

(ii) g(x) = 3 – 6x

(iii) q(x) = 2x –7

(iv) h(y) = 2y

Solution:

(i)

Consider the polynomial: $p(x) = x - 4$

The zeroes of the given polynomial p(x) = x – 4 is calculated as follows:

$p(x) = 0$

$x - 4 = 0$

$x = 4$

(ii)

Consider the polynomial: $g(x) = 3 - 6x$

The zeroes of the given polynomial $g(x) = 3 - 6x$ is calculated as follows:

$g(x)=0$

$3-6x=0$

$3=6x$

$x=\frac{3}{6}$

$x=\frac{1}{2}$

(iii)
Consider the polynomial: $q(x)=2x-7$

The zeroes of the given polynomial $q(x)=2x-7$ is calculated as follows:

$$q(x)=0$$

$$2x-7=0$$

$$2x=7$$

$$x=\frac{7}{2}$$

(iv)
Consider the polynomial:
$h(y)=2y$

The zeroes of the given polynomial $h(y)=2y$ is calculated as follows:

$$h(y)=0$$

$$2y=0$$

$$y=0$$

12. Find the zeroes of the polynomial :

$p(x)=(x-2)^2-(x+2)^2$

Solution:

Consider the polynomial:
$p(x)=(x-2)^2-(x+2)^2$

The zeroes of the polynomial can be calculated as:
$p(x)=0$

$(x-2)^2-(x+2)^2=0$

Used the identity: $a^2-b^2=(a+b)(a-b)$

Exercise No. 2.4

1. **If the polynomials az^3+4z^2+3z-4 and z^3-4z+a leave the same remainder when divided by z – 3, find the value of a.**

Solution:

Let $f(z)=az^3+4z^2+3z-4$ and $g(z)=z^3-4z+a$

Both given polynomial have leave same remainder when divided by z-3. So, f(z) = g(z).

$$f(3)=a(3)^3+4\times(3)^2+3\times3-4$$
$$=27a+36+9-4$$
$$=27a+41$$

And:

$$q(3)=(3)^3-4\times3+a$$
$$=27-12+a$$
$$=15+a$$

Now, f(3) = g(3)

$$27a+41=15+a$$
$$26a=-26$$
$$a=-1$$

Therefore, the required value of $a=-1$.

2. **The polynomial $p(x)=x^4-2x^3+3x^2-ax+3a-7$ when divided by x + 1 leaves the remainder 19. Find the values of a. Also find the remainder when p(x) is divided by x + 2.**

Solution:

We know that when p(x) is divided by x + a, then the remainder = p(-a).

When given polynomial $p(x)=x^4-2x^3+3x^2-ax+3a-7$ is divided by x+1, then the remainder = p (-1)

Now,

$$p(-1)=(-1)^4-2\times(-1)^3+3\times(-1)^2-a\times(-1)+3a-7$$
$$=1-2\times(-1)+3\times(1)+a+3a-7$$
$$=1+2+3+4a-7$$
$$=-1+4a$$

Also, remainder = 19

$-1+4a=19$

$4a=20$

$a=\frac{20}{4}$

$a=5$

Now, when p(x) is divided by x + 2, then:

Remainder $=p(-2)$

$=(-2)^4-2\times(-2)^3+3\times(-2)^2-a\times(-2)+3a-7$

$=16+16+12+2a+3a-7$

$=37+5a$

$=37+5\times5$

$=37+25$

$=62$

3. If both x – 2 and $x-\frac{1}{2}$ are factors of px^2+5x+r, show that p = r.

Solution:

Let $p(x)=px^2+5x+r$

Given (x – 2) is a factor of p(x).

So, p(2) = 0

$p(2)^2+5\times2+r=0$

$4p+10+r=0...(1)$

Again, $\left(x-\frac{1}{2}\right)$ is a factor of p(x).

So, $p\left(\frac{1}{2}\right)=0$

Now,

$p\left(\frac{1}{2}\right)=p\left(\frac{1}{2}\right)^2+5\times\left(\frac{1}{2}\right)+r$

$=\frac{1}{4}p+\frac{5}{2}+r$

$p\left(\frac{1}{2}\right)=0$

$\frac{1}{4}p+\frac{5}{2}+r=0$

From (1), we have 4p+r=-10

From (2), we have p+10+4r=0

$p+4r=-10$

$4p+r=p+4r$

$3p=3r$

$p=r$

Hence, proved.

4. Without actual division, prove that $2x^4-5x^3+2x^2-x+2$ is divisible by x^2-3x+2. [Hint: Factorise x^2-3x+2].

Solution:

Let $p(x)=2x^4-5x^3+2x^2-x+2$

Consider the polynomial:

x^2-3x+2

Then, factorise the above expression as:

$x^2-3x+2=x^2-2x-x+2$

$=x(x-2)-1(x-2)$

$=(x-2)(x-1)$

We know that when p(x) is divided by x + a, then the remainder = p(-a).

Now, p(1) = $2\times1^4-5\times1^3+2\times1^2-1+2$

$=2-5+2-1+2$

$=0$

P(1) = 0

Therefore, (x -1) divides p(x). So,

$(x-1)(x-2)=x^2-3x+2$ Divides $2x^4-5x^3+2x^2-x+2$

5. Simplify $(2x-5y)^3-(2x+5y)^3$.

Solution:

Consider the expression:

$(2x-5y)^3-(2x+5y)^3$

Use the identity: $a^3-b^3=(a-b)(a^2+ab+b^2)$

$(2x-5y)^3-(2x+5y)^3=\{(2x-5y)-(2x+5y)\}\{(2x-5y)^2+(2x-5y)(2x+5y)+(2x+5y)^2\}$

$=(2x-5y-2x-5y)(4x^2+25y^2-20xy+4x^2-25y^2+4x^2+25y^2+20xy)$

$=(-10y)(2x^2+25y^2)$

$=-120x^2y-250y^3$

6. **Multiply** $x^2+4y^2+z^2+2xy+xz-2yz$ by $\left(-z+x-2y\right)$.

Solution:

According to the question:

$\left(x^2+4y^2+z^2+2xy+xz-2yz\right)\times\left(-z+x-2y\right)$

Now, multiply as follows:

$=\{x+(-2y)+(-z)\}\{(x)^2+(-2y)^2+(-z^2)-(x)(-2y)-(-2y)(-z)-(-z)(x)\}$

$=x^3+(-2y)^3+(-z)^3-3\times x\times(-2y)\times(-z)$

Use the identity: $(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)=a^3+b^3+c^3-3abc$

$=x^3-8y^3-z^3-6xyz$

7. **If a, b, c are all non-zero and a + b + c = 0, prove that** $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=3$.

Solution:

Given in the question, a, b and c is non-zero and a + b + c=0.

Therefore, $a^3+b^3+c^3=3abc$

Now,

$$\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{a^2+b^2+c^2}{abc}$$

$$=\frac{3abc}{abc}$$

$$=3$$

8. **If a + b + c = 5 and ab + bc + ca = 10, then prove that** $a^3+b^3+c^3-3abc=-25$.

Solution:

Given:

a + b + c = 5 and ab + bc + ca = 10

We know that:

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$$

$$=(a+b+c)\left[a^2+b^2+c^2-(ab+bc+ca)\right]$$

$$=5\left\{a^2+b^2+c^2-(ab+bc+ca)\right\}$$

$$=5\left(a^2+b^2+c^2-10\right)$$

Given: a + b + c = 5

Now, squaring both sides, get:

$$(a+b+c)^2 = 5^2$$
$$a^2+b^2+c^2+2(ab+bc+ca) = 25$$
$$a^2+b^2+c^2+2\times 10 = 25$$
$$a^2+b^2+c^2 = 25-20$$
$$= 5$$

Now,

$$a^3+b^3+c^3-3abc = 5(a^2+b^2+c^2-10)$$
$$= 5\times(5-10)$$
$$= 5\times(-5)$$
$$= -25$$

Hence, proved.

9. Prove that $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$**.**

Solution:

$$(a+b+c)^3 = [a+(b+c)]^3$$
$$= a^3+3a^2(b+c)+3a(b+c)^2+(b+c)^3$$
$$= a^3+3a^2b+3a^2c+3a(b^2+2bc+c^2)+(b^3+3b^2c+3bc^2+c^3)$$
$$= a^3+3a^2b+3a^2c+3ab^2+6abc+3ac^2+b^3+3b^2c+3bc^2+c^3$$
$$= a^3+b^3+c^3+3a^2b+3a^2c+3b^2c+3b^2a+3c^2a+3c^2b+6abc$$
$$= a^3+b^3+c^3+3a^2(b+c)+3b^2(c+a)+3c^2(a+b)+6abc$$

Since, the above result can be written as:

$$(a+b+c)^3 = a^3+b^3+c^3+3(a+b)(b+c)(c+a)$$

Therefore, $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$