

## Chapter - 6

# Triangle and its Properties



**6.1** In Class-VI, you have studied that a plane closed figure bounded by line segments only is called a polygon. The line segments are called sides of the polygon. If the number of sides of the polygon is three it is called a triangle. The three sides of the triangle form three angles by meeting two at a time at a point. These points are called vertices of the triangle. Thus a triangle has three sides, three angles and three vertices.

The word Triangle is derived from old Latin word 'Triangulum'. Triangulum is one among the 48 galaxies discovered by Astrologer Ptolemy of second century AD. Three bright stars of the triangulum form the shape of a Triangle.

In the adjoining diagram, AB, BC, CA are three sides of the triangle ABC.

$\angle ABC$ ,  $\angle BCA$ ,  $\angle ACB$  are three angles

A, B and C are its three vertices.

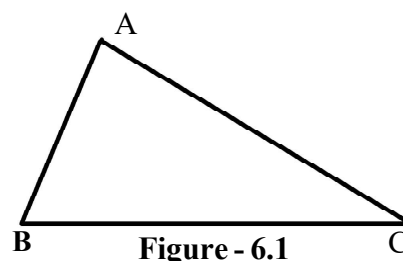


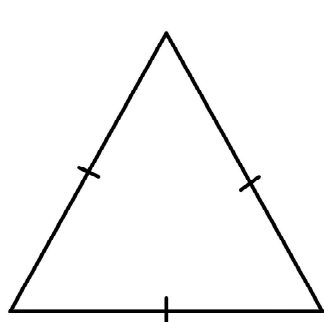
Figure - 6.1

In order to have a good idea about triangle, it is classified in two categories depending on the following features –

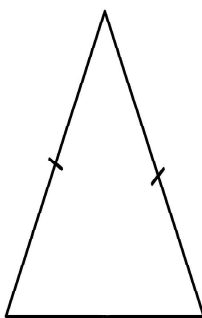
### 6.2 Triangles in terms of side –

- (i) **Scalene Triangle** : A Triangle in which three sides are of different lengths is called a Scalene Triangle.
- (ii) **Isosceles Triangle** : A Triangle in which two sides are of equal length is called an Isosceles Triangle.
- (iii) **Equilateral Triangle** : A Triangle in which all three sides are of equal lengths is called an Equilateral Triangle.

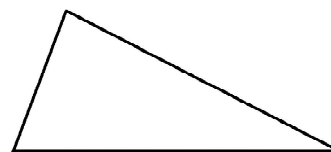
## Triangle and its Properties



(i) Equilateral Triangle



(ii) Isosceles Triangle

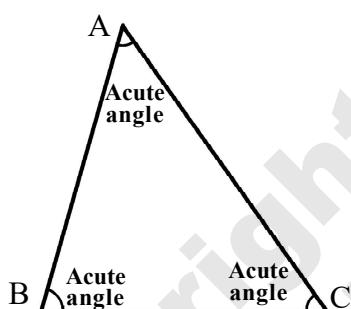


(iii) Scalene Triangle

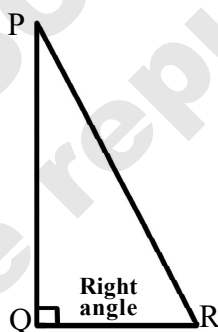
Figure - 6.2

### 6.3 Triangles in terms angles :

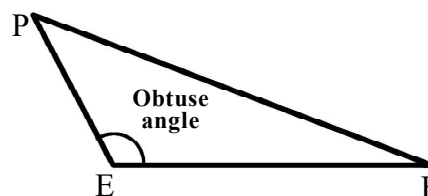
- (i) **Acute-angled Triangle** : A Triangle in which all the three angles are acute i.e. less than  $90^\circ$  is called an Acute-angled Triangle.
- (ii) **Right-angled Triangle** : A Triangle in which one angle is right angle, i.e.  $90^\circ$  is called a Right-angled Triangle.
- (iii) **Obtuse angled Triangle** : A Triangle in which one angle is Obtuse, that is greater than a right angle but smaller than a straight angle is called an Obtuse-angled Triangle.



(i) Acute angled Triangle



(ii) Right angled Triangle



(iii) Obtuse angled Triangle

Figure - 6.3

Activity : Classify the triangles in fig. 6.4 based on their sides and angles.

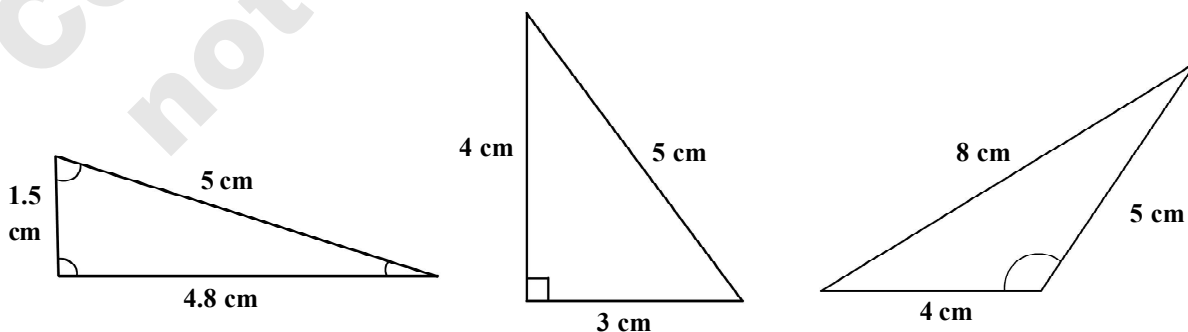


Figure - 6.4

## 6.4 Medians of a Triangle :

The line joining a vertex and the middle point of its opposite side is called the median of a triangle. Can you tell, how many median are there in a triangle ?

**Think :** How many medians can be drawn in a triangle ? As there are only three vertices and one side in each of the opposite vertex, only three medians can be drawn in a triangle.

### Activity :

Draw a triangle on a paper and cut it out by a pair of scissors. Take the two ends of a side together and fold the triangle. Will not the fold line go through the mid-point of the side ? Now fold the triangle through the mid point of the side and the opposite vertex. This fold line is median of the triangle. Same activity can be performed for other sides as well. How many medians have you got ?

Note that Any median of a triangle lies in the interior of the triangle.

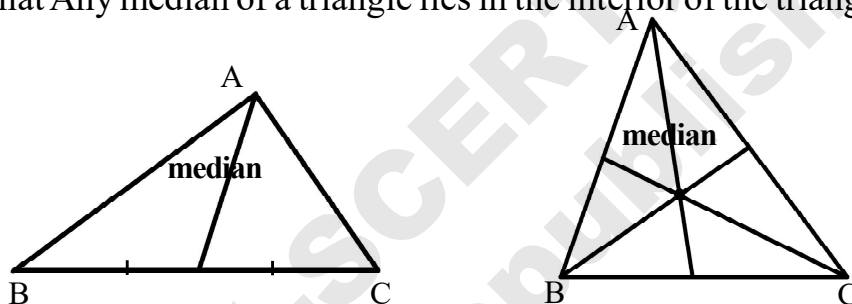


Figure - 6.5

## 6.5 Altitude of a Triangle :

A perpendicular drawn from a vertex to its opposite side is called an altitude of a triangle. Is there any relationship between our usual concept height and altitude of a triangle ? Let us examine it with the help of the following activity.

Draw a triangle on a art paper and separate it with a pair of scissors. Hold the triangle on a table so that one side rests on the table surface. How would you know the height of the triangle? The shortest length of the line segments drawn from the vertex to the side touching the table is the height of the triangle. Note that the shortest line segment drawn from the vertex of a triangle to its opposite side is perpendicular to the side.

Therefore, altitude of the triangle is the height of the triangle.

### Think :

How many altitudes can be drawn in a triangle? As there are three vertices and three sides each opposite to one vertex, there will be three altitudes in a triangle.

Does any altitude of a triangle always lie in its interior ? Look at the fig 6.6.

## Triangle and its Properties

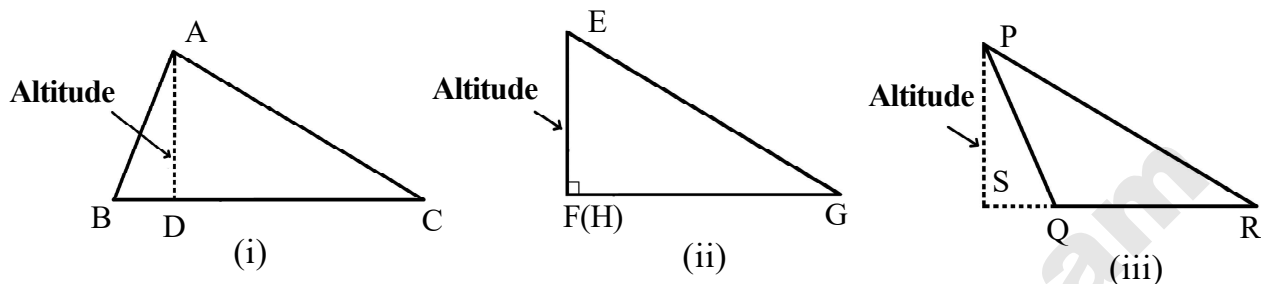


Figure - 6.6

What do you observe?

In Fig (i) the altitude AD is in the interior of the triangle.

In Fig (ii) the altitude EH is the same as the side EF.

In Fig (iii) the altitude PS lies exterior to the triangle.

So, altitude of a triangle may not lie in the interior of the triangle always.

You have seen that, the medians and altitudes are drawn from the vertices to their respective opposite sides.

Can a median and altitude of a triangle be coincident sometime?

Try to draw the median and altitude for the isosceles and equilateral triangles.

**Activity : Look for triangles :**

Join the dots in the following figure to form different types of triangles and discuss their difference.

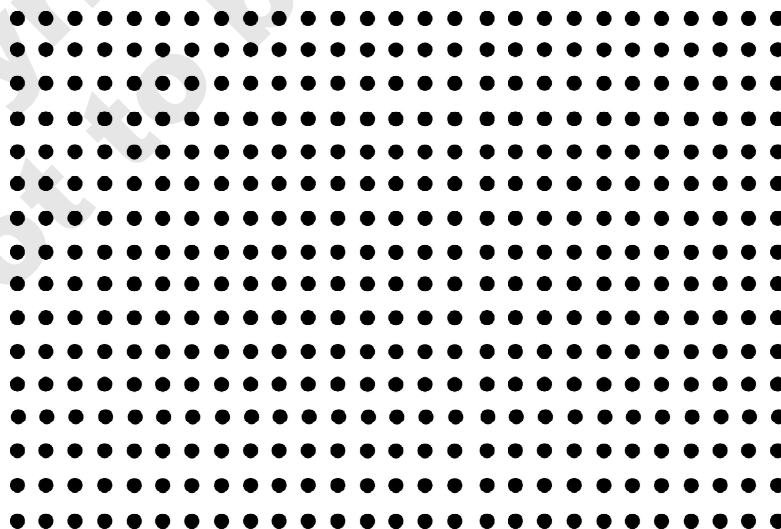
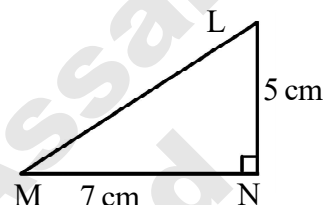


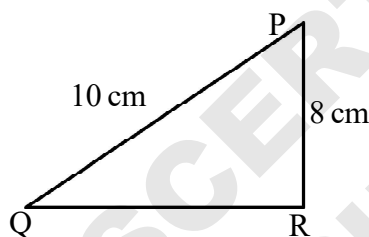
Figure - 6.8

## Exercise - 6.1

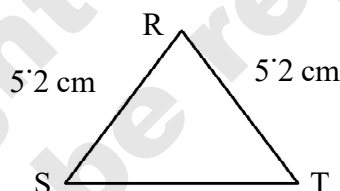
1. How many medians are there in a triangle?
2. How many altitudes are there in a triangle?
3. Draw a triangle and display its medians.
4. Draw a triangle and display its altitudes.
5. Name the angle opposite to side LM in  $\triangle LMN$ .



6. Name the side opposite to vertex Q of  $\triangle PQR$ .



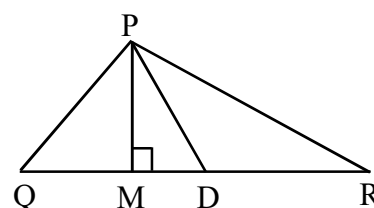
7. Name the vertex opposite to side RT of  $\triangle RST$ .



8. (i) Tick ( $\checkmark$ ) the correct answer

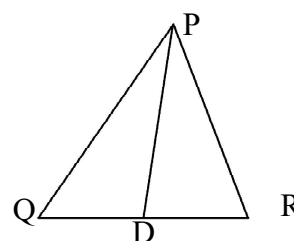
In  $\triangle PQR$ , PM is

- |                      |                 |
|----------------------|-----------------|
| (a) A median         | (b) An altitude |
| (c) A bisector of QR | (d) A side      |



- (ii) If D is the mid-point of side QR of  $\triangle PQR$ , then PD is

- |                                  |                         |
|----------------------------------|-------------------------|
| (a) perpendicular bisector on QR | (b) altitude            |
| (c) median                       | (d) opposite side of QR |



## Triangle and its Properties

### 6.6 Interior Angles in a Triangle :

The angles  $\angle A$ ,  $\angle B$  and  $\angle C$  are also called the interior angles in the  $\triangle ABC$

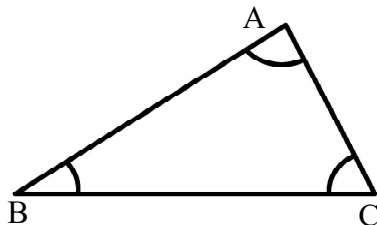


Figure - 6.9

### 6.7 Exterior Angle of a Triangle :

Draw a triangle  $ABC$  and extend its side  $BC$  towards  $\overline{BC}$  to  $D$ . The side  $AC$  and  $CD$  of extended  $BC$  form an angle  $\angle ACD$  at the vertex  $C$ .

$\angle ACD$  is clearly an angle formed outside of  $\triangle ABC$ . Therefore  $\angle ACD$  is called exterior angle of the  $\triangle ABC$  at vertex  $C$ .

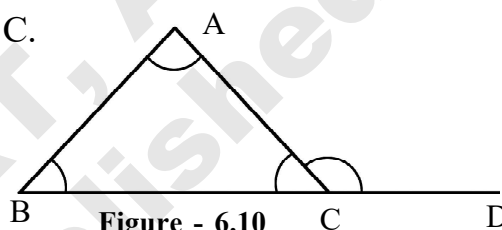


Figure - 6.10

You might note that by extending  $AC$  to  $E$  also we obtain an exterior angle  $\angle BCE$ . However,  $\angle ACD$  and  $\angle BCE$  are vertically opposite angle and so they are equal. That means, there are two exterior angles  $\angle ACD$  and  $\angle BCE$  at the vertex  $C$ .

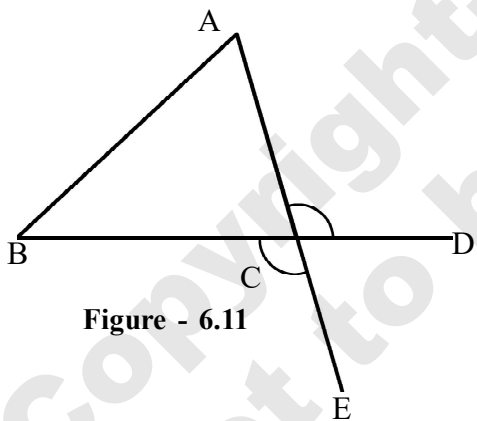


Figure - 6.11

Similarly, we can have two exterior angles at each of the vertices  $A$  and  $B$ . Consider the exterior  $\angle ACD$  of  $\triangle ABC$  at the vertex  $C$ . From the diagram it can be understood that the angle  $\angle ACB$  and the exterior angle  $\angle ACD$  are adjacent angle at  $C$ . In this case the angles  $\angle BAC$  and  $\angle ABC$  are called distinct interior angles with respect to the exterior angle  $\angle ACD$ .

#### Do yourself :

Draw  $\triangle ABC$  on a paper and extend the side  $BC$  to point  $D$ . Copy  $\triangle ABC$  on a tracing paper and cut it with a pair of scissors. Cut  $\angle A$ ,  $\angle B$  and place it on the exterior  $\angle ACD$ . You will find the  $\angle A$ ,  $\angle B$  completely cover the internal part of  $\angle ACD$ .

**An exterior angle of a triangle is equal to the sum of its interior opposite angles**

This property can be experimented on various triangles to prove that any exterior angle is equal to the sum of distant interior angles. But is this verification the only method to establish the truth of this fact ?

Let us see how the property of exterior and distant interior angles can be justified.

### 6.7.1 Property of exterior angle of a triangle :

The measure of any exterior angle of a triangle is equal to the sum of its interior opposite angles. The side BC of  $\triangle ABC$  is extended to the point D to form the exterior angle  $\angle ACD$ . EC is drawn parallel to AB through C.

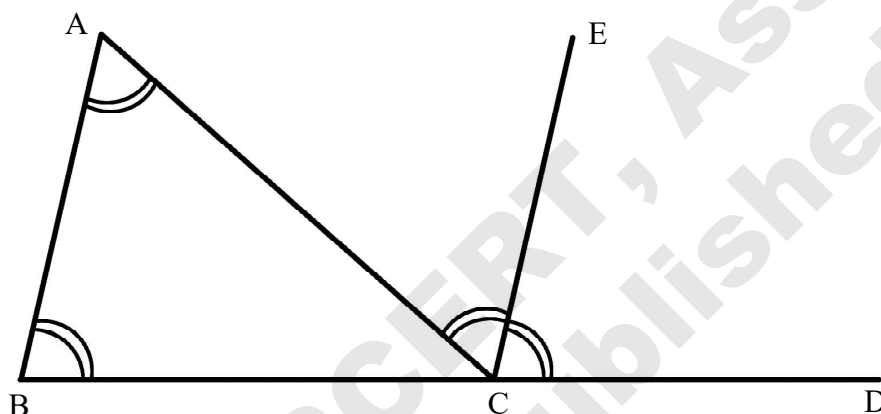


Figure - 6.12

Justification :

Steps	Reasons
1. $\angle BAC = \angle ACE$	$AB \parallel EC$ , AC is a transversal. $\therefore$ Alternate angles $\angle ACE$ and $\angle BAC$ are equal
2. $\angle ABC = \angle ECD$	$AB \parallel EC$ , BD is a transversal. Therefore corresponding angles $\angle ABC$ and $\angle ECD$ are equal.
3. $\angle BAC + \angle ABC = \angle ACE + \angle ECD = \angle ACD$	

**Think :**

- How many exterior angles can be drawn in a triangle?
- What will be the sum of any exterior angle of a triangle and its adjacent interior angle.

## Triangle and its Properties

**Example :** The measure of an exterior angle of a triangle is  $(3x - 10^\circ)$  and the measures of its interior opposite angles are  $(x + 15^\circ)$  and  $25^\circ$ . Find  $x$ .

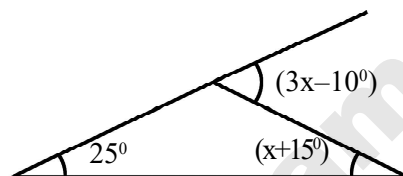
**Solution :** We know that an exterior angle of a triangle is equal to the sum of its interior opposite angles.

Therefore,

$$3x - 10^\circ = 25^\circ + x + 15^\circ$$

$$\text{Or, } 2x = 50^\circ$$

$$\text{So, } x = 25^\circ$$



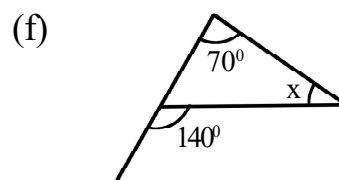
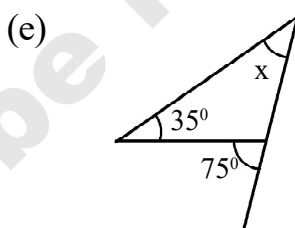
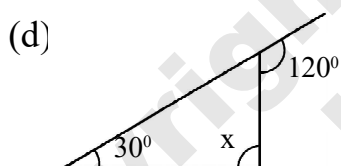
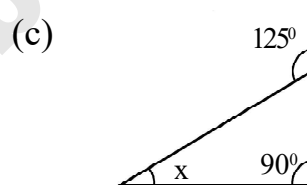
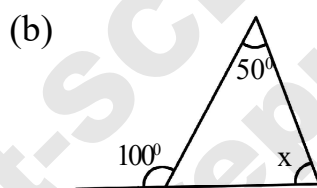
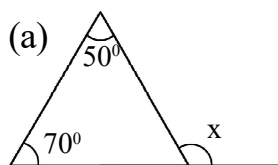
### Exercise - 6.2

1. Fill in the blanks :

(a) The angles in the interior of a triangle are called \_\_\_\_\_.

(b) The angles exterior to a triangle are called \_\_\_\_\_.

2. Find the value of  $x$  from the diagrams given below –



3. In a triangle, an exterior angle is  $70^\circ$  and one of its interior opposite angles is  $25^\circ$ . Find the value of the other interior opposite angle.

4. Two remote interior angles of a triangle are  $60^\circ$  and  $80^\circ$ . Find the value of the exterior angle.

5. In a triangle, an exterior angle is  $114^\circ$  and one of its interior opposite angles is  $25^\circ$ . Find the value of other distant interior angle.

6. Two remote interior angles of an exterior angle of a triangle are  $49^\circ$  and  $41^\circ$ . Find the value of the exterior angle. (Remote interior angles are also called opposite angles).



### 6.8 Angle sum Property of a Triangle :

Let us have a prior idea about the sum of the interior angles of a triangle through some activities

- (i) **Step 1 :** Draw a triangle on a coloured paper. Name the vertices as A, B and C. Its interior angles are  $\angle A$ ,  $\angle B$  and  $\angle C$ .

**Step 2 :** Cut the  $\triangle ABC$  with a pair of scissors.

**Step 3 :** Put number 1, 2, 3 in the angles as soon in the figure 6.13 and cut the three angles from the triangle.

**Step 4 :** Draw a straight line on a paper and adjust the sides of the angles such that one side of one angle coincide with the line, while the other side coincide with a side of the second angle and the other side of the second angle coincide with one side of the third angle. You will see that the other side of the third angle will coincide with the other direction of the line.

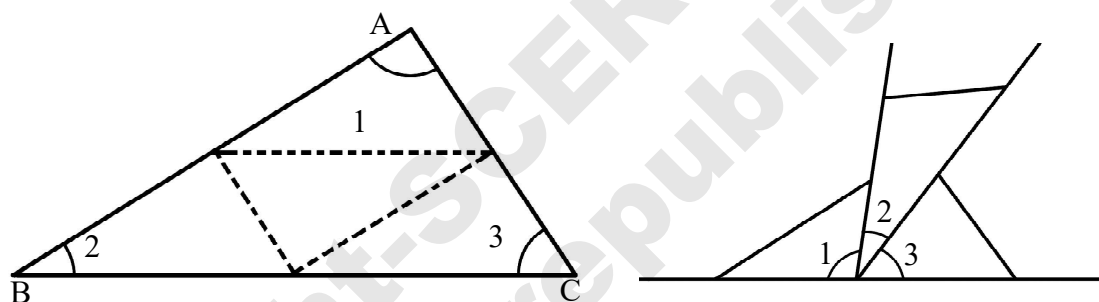


Figure - 6.13

- (ii) Draw a triangle  $\triangle ABC$  on a paper and make three copies of the same with tracing paper. Make three cut outs of the triangle from the tracing paper. Now match different vertices of the triangle, so that their arms touch each other.

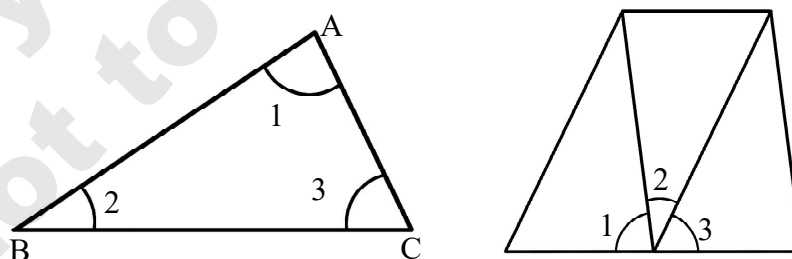


Figure - 6.14

You will observe that the exterior arms of a triangle are placed in a straight line. Hence it forms a straight angle, ie,  $180^\circ$ .

Similarly, by considering various triangles it can be seen that. **‘The total measure of the three angles of a triangle is  $180^\circ$ ’.**

## Triangle and its Properties

- (iii) By drawing a triangle ABC and measuring its three angles by protector also, we can obtain  $180^\circ$ . Thus we have, **the sum of three interior angles of a triangle is  $180^\circ$ .**

Draw a triangle ABC. Extend the arm BC to point D.

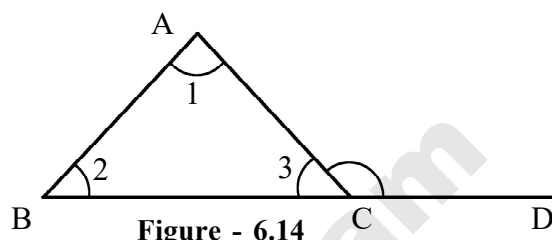


Figure - 6.14

### Justification

#### Steps

1.  $\angle 1 + \angle 2 = \angle ACD$
2.  $\angle 1 + \angle 2 + \angle 3 = \angle ACD + \angle 3$
3.  $\angle ACD + \angle 3 = 180^\circ$
4.  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

#### Reasons

$\angle ACD$  is the exterior angle at the vertex C.  $\angle 1, \angle 2$  are opposite interior angles of  $\triangle ABC$ . By property of exterior angles of a triangle the two sides are equal.

By adding  $\angle 3$  to both side of step 1.

$\angle ACD$  and  $\angle 3$  make a linear pair.

From steps 1 and 2.

Therefore, the total sum of the three angles of a triangle is  $180^\circ$  or two right angles. Hence, the property on sum of the angles of a triangle is justified.

**Example 1 :** Two interior angles of a triangle are  $75^\circ$  and  $35^\circ$ . What is the measure of the third angle?

**Solution :** Let in  $\triangle ABC$ ,  $\angle B = 75^\circ$  and  $\angle C = 35^\circ$ . To find out the measure of  $\angle A$

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Angle sum property of triangle)}$$

$$\text{or, } \angle A + 75^\circ + 35^\circ = 180^\circ$$

$$\text{or, } \angle A + 110^\circ = 180^\circ$$

$$\begin{aligned} \text{or, } \angle A &= 180^\circ - 110^\circ \\ &= 70^\circ \end{aligned}$$

**Example 2 :** If two angles of a triangle are twice and thrice of its smallest angle find out the angles.

**Solution :** Let the smallest angle  $= x$

Therefore, the other two angles are  $2x$  and  $3x$ .

$$\text{By the given condition, } x + 2x + 3x = 180^\circ \text{ (Angle sum property of triangle)}$$

$$\text{or, } 6x = 180^\circ$$

$$\text{or, } x = \frac{180^\circ}{6}$$

$$\text{or, } x = 30^\circ$$

So, the smallest angle is  $30^\circ$

The other two angles are  $2x = (2 \times 30)^\circ = 60^\circ$  and  $3x = (3 \times 30)^\circ = 90^\circ$

So, the measures of the three angles are,  $30^\circ, 60^\circ$  and  $90^\circ$ .

**Example 3 :** The ratio of three angles of a triangle is 2:3:4. Find out the measure of the angles.

**Solution :**

Let the angles be  $2x, 3x$ , and  $4x$

$$\text{Then, } 2x + 3x + 4x = 180^\circ$$

$$\text{or, } 9x = 180^\circ$$

$$\text{or, } x = 20^\circ$$

Therefore, the three angles are  $2x = 2 \times 20^\circ = 40^\circ$

$$3x = 3 \times 20^\circ = 60^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

## 6.9 Two special triangles – Equilateral and Isosceles triangles :

**6.9.1 Equilateral Triangle :** A triangle in which all the three sides are equal in length is called Equilateral Triangle.

Let ABC be an equilateral triangle. How will its interior angles be ? Let us examine. Draw a  $\triangle ABC$  on a paper whose sides are equal. Take a cutout of the triangle. Fold the triangle in such a way so that the vertex B falls on vertex C. You will see that sides AB, AC and the angles  $\angle B, \angle C$  coincide completely you will see  $\angle B = \angle C$ . From this

Similarly, we also see that  $\angle A = \angle C$  by folding the segment AC at its midpoint.

Therefore in an equilateral triangle  $\triangle ABC$ ,  $\angle A = \angle B = \angle C$ , hence all the three angles are equal.

But,  $\angle A + \angle B + \angle C = 180^\circ$  (Angle sum property of triangle)

Therefore  $\angle A = \angle B = \angle C = 60^\circ$

Thus, every angle of an **equilateral triangle is of measure  $60^\circ$** .

## Triangle and its Properties

### 6.9.2 Isosceles Triangle:

Let  $\triangle ABC$  be an isosceles triangle, where  $AB = AC$ .

Do you observe any special characteristic in its three angles?

Draw an Isosceles Triangle  $ABC$  on a paper where  $AB = AC$ . Make a copy of the triangle and take a cutout of the same.

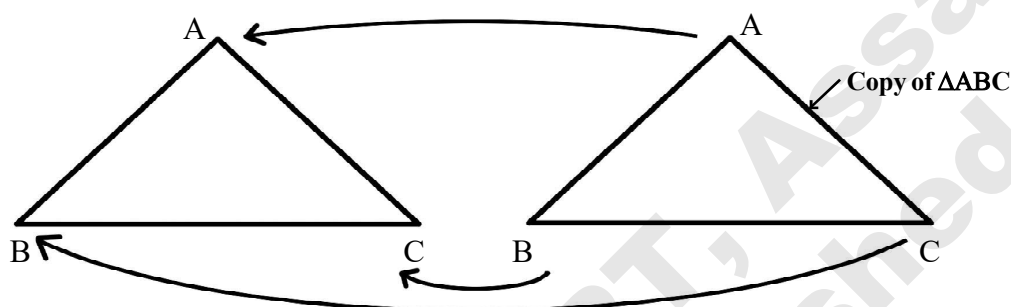


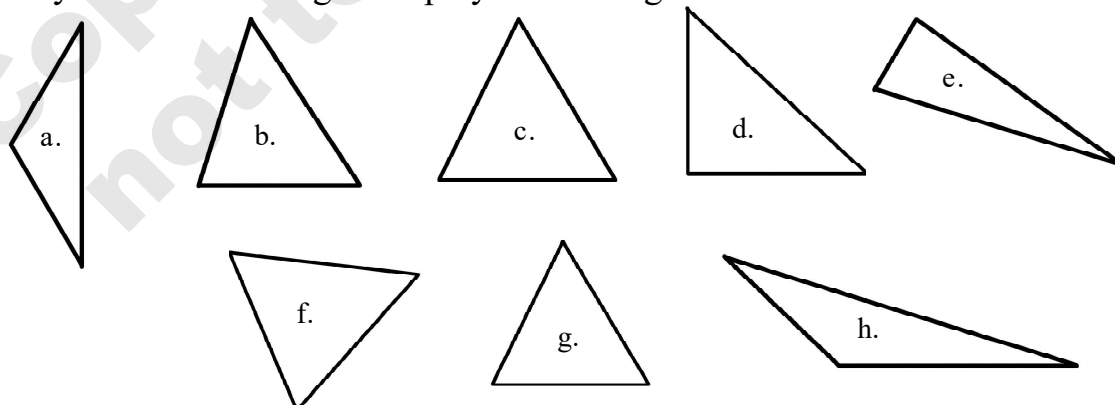
Figure - 6.15

Now, place the cutout on the original  $\triangle ABC$  so that vertices A, B, C of the cutout coincide exactly on the vertices A, C, B of the original triangle. In this case, sides AB, AC and BC of cutout coincide exactly on sides AC, AB and BC of the original triangle and then  $\angle A$ ,  $\angle B$  and  $\angle C$  of the cutout fit exactly on  $\angle A$ ,  $\angle C$  and  $\angle B$  of the original triangle.

From this, we can say that the angles opposite to the equal angles of an isosceles triangle are equal. Hence,  $\angle B = \angle C$ .

**Note :** All equilateral triangles are isosceles triangle but all isosceles triangles may not be equilateral triangles.

Activity : Name the triangles displayed in the fig 6.16 below and complete the table.



Figuer - 6.16

## Triangle and its Properties

Triangle	Triangles based on sides (Scalene, Isosceles, Equilateral)	Triangles based on Angles (Acute, Obtuse, Right angle)
a		
b		
c		
d		
e		
f		
g		
h		

Multiple Choice Question : Tick the correct option with (✓)

**Example 1 :** In  $\triangle ABC$ ,  $AB = AC = BC = 9$  cm, then the triangle is –

- A. Equilateral Triangle.
- B. Isosceles Triangle.
- C. Scalene Triangle.

**Solution :** Here,  $AB = AC = BC = 9$  cm, hence the triangle is a equilateral triangle.

**Example 2 :** Each angle of a triangle is  $60^\circ$ . The triangle is a

- A. Equilateral Triangle.
- B. Isosceles Triangle.
- C. Scalene Triangle.

**Solution :** In an equilateral triangle, each angle is  $60^\circ$ . Hence, it is an equilateral triangle.

**Example 3 :** In a triangle, two sides are of length 12 cm each and third side is of length 8cm. The triangle is an / a

- A. equilateral eriangle.
- B. isosceles triangle.
- C. scalene triangle.

**Solution :** Here two sides of the triangle are equal. Hence it is an isosceles triangle.

**Example 4 :** Two angles of a triangle are  $51^\circ$  and  $78^\circ$ . The triangle is an / a

- A. equilateral triangle.
- B. isosceles triangle.
- C. scalene triangle.

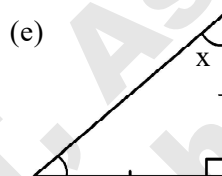
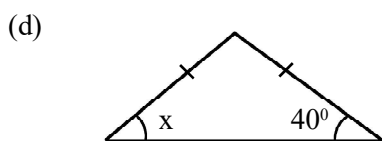
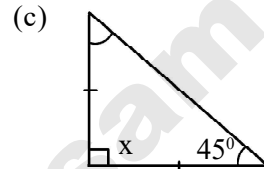
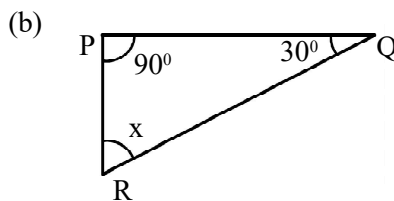
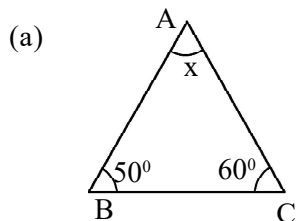
**Solution :** The third angle  $= 180^\circ - (51^\circ + 78^\circ) = 51^\circ$

It is seen that two angles of the triangle are equal. So this is an isosceles triagnle.

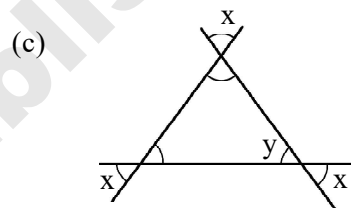
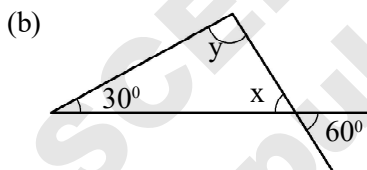
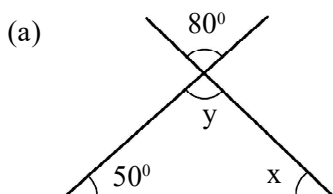
## Triangle and its Properties

### Exercise - 6.3

1. Find the value of  $x$  from the figs given below –



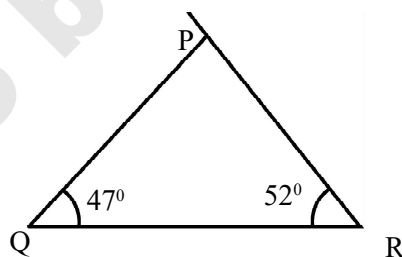
2. Find the values of  $x$  and  $y$  from the figs given below –



3. In a triangle, the measure of one angle is  $60^\circ$ . What will be the measure of other two angles

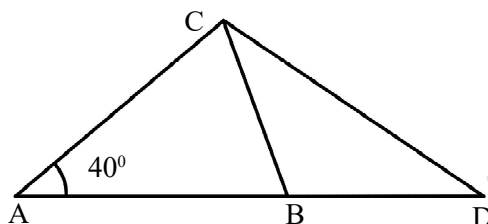
(a)  $50^\circ, 40^\circ$       (b)  $40^\circ, 60^\circ$       (c)  $60^\circ, 70^\circ$       (d)  $50^\circ, 70^\circ$

4. Find the value of  $\angle P$  from the figures given below –



- Two angles of a triangle are  $30^\circ$  and  $80^\circ$ . Find the measure of the third angle.
- One angle of a triangle is  $80^\circ$  and the other two angles are equal. What are the measures the other tow angles?
- The ratio of three angles of a triangle is  $1:2:1$ . Find the measures of the angles.
- The angles of a triangle are  $(x + 21^\circ)$ ,  $(x - 20^\circ)$  and  $(2x - 45^\circ)$ . Find  $x$ .
- The ratio of three angles of a triangle is  $1:2:3$ . Find the measures of the angles.

10. In  $\triangle ABC$ ,  $\angle A + \angle B = 116^\circ$ ,  $\angle B + \angle C = 126^\circ$ . Find out the measures of the interior angles of the triangle
11. In  $\triangle ABC$ ,  $2\angle A = 3\angle B = 6\angle C$ . Find out  $\angle A$ ,  $\angle B$ ,  $\angle C$ .
12. In the figure given below  $\angle CAB = 40^\circ$ ,  $AC = AB$  and  $BC = BD$   
Find out (a)  $\angle ACB$  and (b)  $\angle CDB$



### 6.10 Right-Angled Triangles and Pythagoras Property.

Think for a while! Can a triangle have more than one right angle?

As per angle sum property of a triangle, the total sum of three angles of a triangle is  $180^\circ$  or two right angles. So, in case one angle is right angle, sum of rest of the two angles is  $90^\circ$ . So, each of the other two angles must be less than a right angle.

Therefore, it is impossible for any triangle to have more than one right angle.

In Fig 6.17, ABC is right angled triangle. In a right angled triangle, the sides have some special names. The side opposite to the right angle is called the hypotenuse. The other two sides are known as the **legs of the right-angled triangle**.

A special property of sides of the right-angled triangle is called the Pythagoras Property.

According to the Pythagoras Property, **in a right-angled triangle, the square of the hypotenuse is equal to the sum of the square of the legs**

Hence, in a right-angled triangle if the hypotenuse is 'a' and legs are 'b' and 'c' then

$$a^2 = b^2 + c^2$$

This property can be verified by a simple activity.

Draw two identical squares on a paper with side length  $(b+c)$  and make identical copies of 8 right-angled triangles.

Now, you place 4 of these right triangles in one square and the remaining 4 right angled triangles in the other square as shown in the figure on the other page.

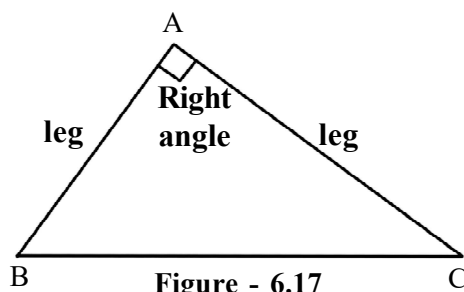


Figure - 6.17

## Triangle and its Properties

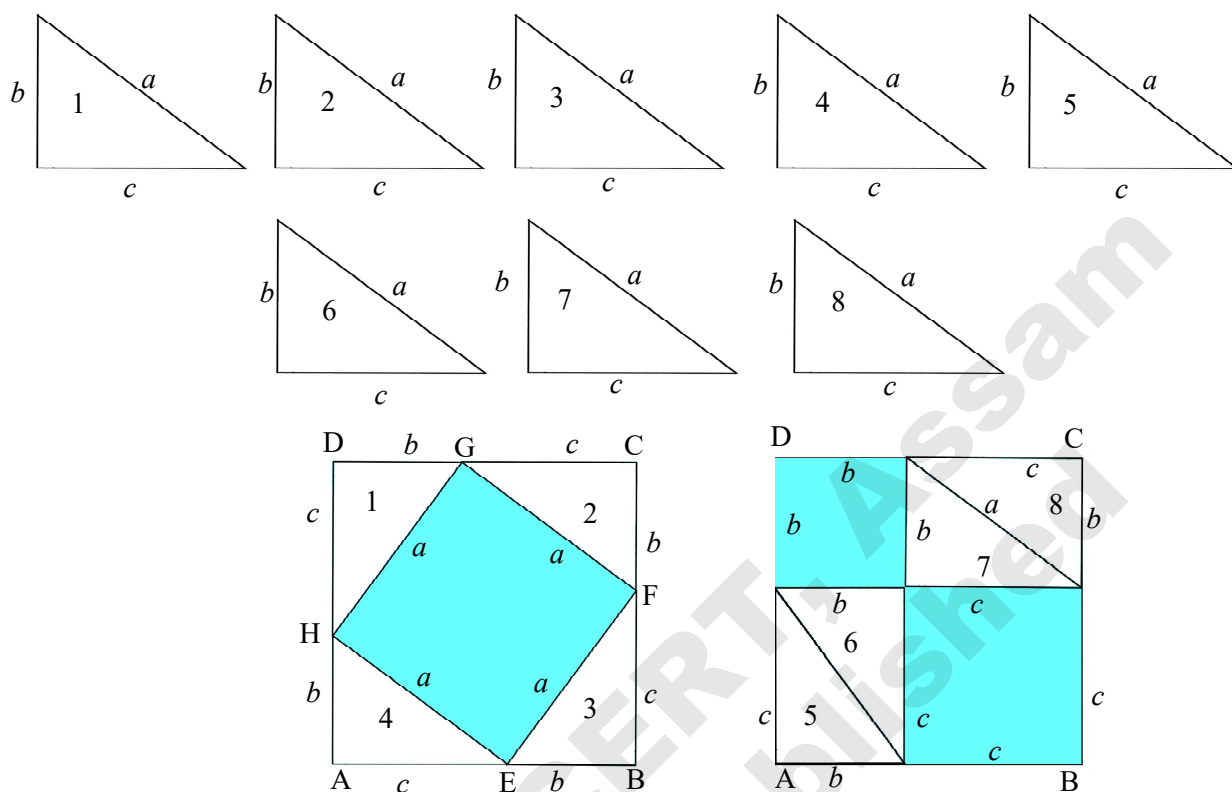


Figure - 6.18

What have you observed ? After placing the four right-angled triangles in the first square as shown in the diagram, the space equal to the area of a square of side  $a$  remains empty.

Similarly, after placing the four right-angled triangles in the second square as shown, the space equal to areas of the squares of sides  $b$  and  $c$  remains empty. Since the two square are same in size each having a sides equal to  $b+c$  and the number of right-angled triangles placed on both squares are also equal therefore, the empty parts in the two squares are also equal. In the first square, empty place is equal to  $a$  square having side  $a$ , ie,  $a^2$ . The uncovered place on the second square is the sum of two squares having sides  $b$  and  $c$ . Therefore,  $a^2 = b^2 + c^2$

**In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the legs.**

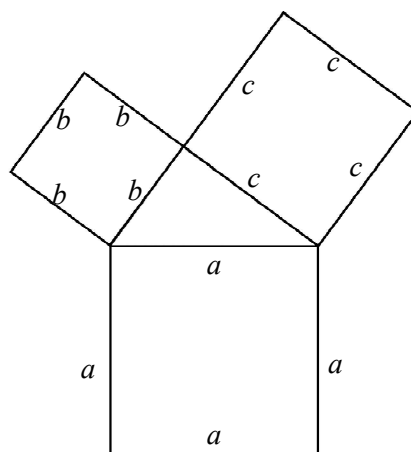


Figure - 6.19

With the help of fig. 6.19 Pythagoras Property is displayed ie, in a right-angled triangle, the area of the square drawn on the **hypotenuse is equal to the sum of areas of squares drawn on the legs.**



### Do practically :

Draw triangles having sides of different measure –

- (i) 2 cm, 3 cm, 4 cm
- (ii) 3 cm, 4 cm, 5 cm
- (iii) 2 cm, 3 cm, 5 cm
- (iv) 2 cm, 4 cm, 5 cm etc.

In case of (i)  $2^2 + 3^2 = 13$  and  $4^2 = 16$

Hence,  $2^2 + 3^2 < 4^2$

Now measure the angles of the triangle. Is there any right angle in the triangle? You will find that there is an obtuse angle in the triangle.

In case of (ii)  $3^2 + 4^2 = 9 + 16 = 25$  and  $5^2 = 25$ .

Hence  $3^2 + 4^2 = 5^2$

Now measure the angles. Angle opposite to the side 5 cm is a right angle. It means that Pythagoras Property is satisfied here.

In the same way, verify the other triangles.

From these activities you will understand that holds Pythagoras Property in opposite direction as well. That is means, in a triangle, if sum of squares of two sides are equal to the square of the third side, then the triangle is a right-angled triangle.

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the legs. This property of right-angled triangle was first by Mathematician Pythagoras (570 - 495BC). Therefore this property of a right-angled triangle is known as **Pythagoras Property**. He was a Mathematician and Philosopher of ancient Greek civilisation. But we can find reference of its use even before Pythagoras. In addition to Geometry, Pythagoras and his followers studied numbers ratios etc in details. This property was referred to Baudhayan Sulva Sutra much earlier to Pythagoras. Budhayan applied it in rectangle.

### 6.11 Inequalities in a Triangle :

Uptill now, you have studied about equality of one or more sides and angles of a triangle. At times we faces inequalities in certain cases. In these situation we often need comparison of things.

Activity : Let us compare sum of lengths of any two sides of a triangle to the length of the third side. Draw the triangle  $\triangle ABC$ ,  $\triangle DEF$  and  $\triangle GHI$  as shown the fig 6.20. Measure the lengths of sides with the help of a scale and fill up the table –

## Triangle and its Properties

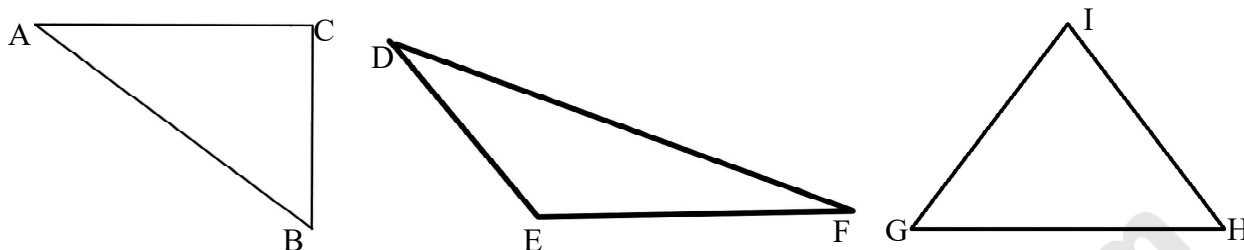


Figure - 6.20

Triangle	Length of side	Sum of two sides	Difference of two sides	Remarks
$\triangle ABC$	AB = BC = CA =	AB + BC = BC + AC = AC + AB =	AB - BC = BC - AC = AC - AB =	
$\triangle DEF$	DE = EF = DF =	DE + EF = EF + DF = DF + DE =	DE - EF = EF - DF = DF - DE =	
$\triangle GHI$	GH = HI = GI =	GH + HI = HI + GI = GI + GH =	GH - HI = HI - GI = GI - GH =	

### The sum of the lengths of two sides of a triangle

In each of the following groups, lengths of three segments are given. Can a triangle be formed with the segments given in each group? Same activity can be performed by taking sticks also of specific length in measure.

- (i) (3 cm, 5 cm, 7cm)      (ii) (4 cm, 6 cm, 2 cm)      (iii) (7cm, 6 cm, 5 cm)  
(iv) (6 cm, 8 cm, 3 cm)      (v) (3 cm, 2 cm, 6 cm)

You must have noticed that by joining the tip, of sticks as per measurements of (i), (iii) and (iv) triangles can be formed. But no triangles can be formed with sticks as per measurements in (ii) and (v). (Try to find out the reason by yourselves, otherwise ask your teacher) triangles, you may observe that

By looking at the sides forming

Length of sides of the triangle

- |                        |                       |
|------------------------|-----------------------|
| (i) (3 cm, 5 cm, 7cm)  | $3+5>7, 5+7>3, 3+7>5$ |
| (iii) (7cm, 6 cm, 5cm) | $7+6>5, 7+5>6, 6+5>7$ |
| (iv) (6 cm, 8 cm, 3cm) | $6+8>3, 8+3>6, 6+3>8$ |

Here, in each of (i), (iii) and (iv) the sum of any two sides are greater than the third side

Therefore, any sides  $\overline{AB}, \overline{BC}, \overline{CA}$  can make a triangle if–

$$AB + BC > AC$$

$$AC + AB > BC$$

$$BC + AC > AB$$

Now can you find out the reason why a triangle can't be formed by groups (ii) and (v)?

**Example 9 :** Is it possible to draw a triangle where the 3 sides measure 7cm, 9 cm and 13 cm?

**Solution :**  $7 + 9 = 16 > 13$

$$9 + 13 = 22 > 7$$

$$13 + 7 = 20 > 9$$

It is seen that sum of any two sides is greater than the third side. Hence these sides can be the sides will form triangle.

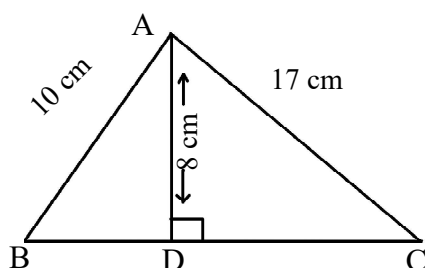
**Example 10 :** Will the sides measuring 4 cm, 8 cm, and 15 cm, be sides of a triangle?

**Solution :**  $4 + 8 = 12 < 15$

Therefore, the given measures can not be sides of a triangle.

## Exercise - 6.4

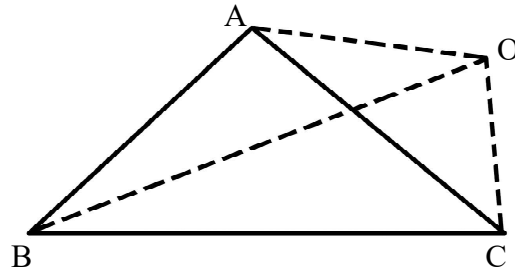
- In the following figure  $AB = 10$  cm,  $AC = 17$ cm and  $AD = 8$  cm. Determine  $BC$ .



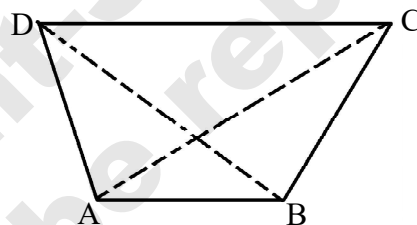
- If the perimeter of a triangle is 15cm and two sides are 5cm and 7cm then what is the length of the third side.

## Triangle and its Properties

- In a rectangle the lengths of two adjacent sides are 16 cm and 12 cm. Find out the lengths of two diagonals.
- O is an external point of  $\triangle ABC$ . Show that  $2(OA + OB + OC) > AB + BC + CA$



- Will the sides of the following measures form a right angled triangle ?  
 (a) 5, 12, 13                      (b) 3, 4, 5                      (c) 6, 8, 10                      (d) 6, 7, 8
- Is it possible to form four triangles with sides of the following measures ?  
 (a) 3 cm, 4 cm, 5 cm                      (b) 5 cm, 7 cm, 12 cm  
 (c) 3.4 cm, 2 cm, 5.8 cm                      (d) 6 cm, 7 cm, 14 cm
- Prove that in a quadrilateral ABCD.  
 $(AB + BC + CD + DA) > (AC + BD)$

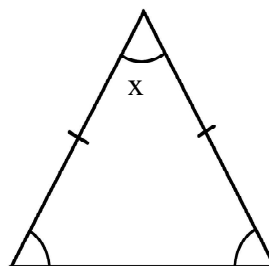


### Exercise - 6.5

Find out the correct answers for Q. No. 1 to Q. No. 12

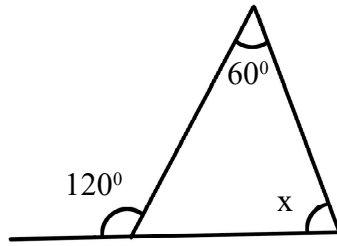
- Value of  $x$  in the adjoining triangle is

- $40^\circ$
- $60^\circ$
- $35^\circ$
- $180^\circ$



2. Value of 'x' in the adjoining figure is

- (a)  $180^\circ$
- (b)  $55^\circ$
- (c)  $90^\circ$
- (c)  $60^\circ$



3. In  $\triangle ABC$ , if  $\angle A = 35^\circ$ ,  $\angle B = 65^\circ$  then  $\angle C$  is

- (a)  $50^\circ$
- (b)  $80^\circ$
- (c)  $30^\circ$
- (d)  $60^\circ$

4. Hypotenuse of a right-angled triangle is 17cm. If one side is measures 8 cm, then the measure of the third side is

- (a) 15cm
- (b) 12 cm
- (c) 13 cm
- (d) 25 cm

5. In  $\triangle ABC$  if  $\angle A = 72^\circ$ ,  $\angle B = 63^\circ$  then  $\angle C$  is

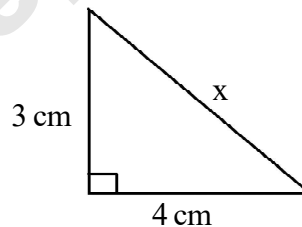
- (a)  $45^\circ$
- (b)  $80^\circ$
- (c)  $30^\circ$
- (d)  $60^\circ$

6. In a right-angled triangle, one of the acute angles is  $36^\circ$  then the other acute angle is

- (a)  $55^\circ$
- (b)  $54^\circ$
- (c)  $51^\circ$
- (d)  $52^\circ$

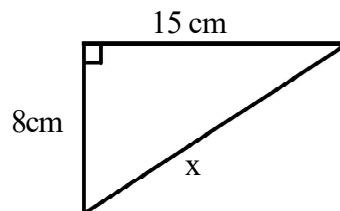
7. The value of 'x' in the adjoining figure is

- (a) 5cm
- (b) 7cm
- (c) 3 cm
- (d) 4 cm



8. The value of 'x' in the figure is

- (a) 15 cm
- (b) 17 cm
- (c) 13 cm
- (d) 14 cm



9. In a right-angled triangle ABC, if  $\angle C = 90^\circ$ .  $AC = 5\text{cm}$ ,  $BC = 12\text{ cm}$  then  $AB$  is

- (a) 7cm
- (b) 17cm
- (c) 13 cm
- (d) 14 cm

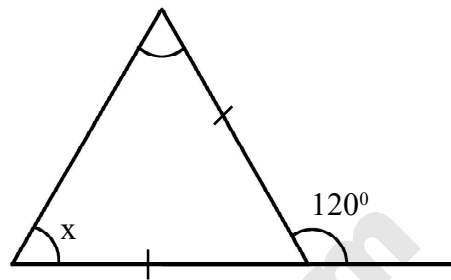
10. In triangle PQR, if  $\angle P = 90^\circ$ ,  $PQ = 3\text{ cm}$ ,  $PR = 4\text{ cm}$ , then  $QR$  is

- (a) 7cm
- (b) 17 cm
- (c) 5 cm
- (d) 13 cm

## Triangle and its Properties

11. The value of 'x' in the figure is

- (a)  $90^\circ$
- (b)  $60^\circ$
- (c)  $80^\circ$
- (d)  $40^\circ$



12. Pythagoras Property is satisfied if the triangle is ....

- (a) Obtuse-angled      (b) Right-angled      (c) Acute-angled

### What we have learned

1. Line segment joining a vertex to the mid point of opposite side of a triangle is called median.
2. The perpendicular drawn from a vertex to the opposite side of triangle is called altitude.
3. The sum of interior angles of a triangle is  $180^\circ$ .
4. Exterior angle formed by extending one side of a triangle is equal to the sum of two opposite interior angles.
5. The sum of two sides of a triangle is greater than the third side.
6. The square of hypotenuse of a right-angled triangle is equal to the sum of squares of the other two legs. It is known as Pythagoras Property.
7. If in a triangle, the square of a side is equal to the sum of the squares of the other two sides, then the angle converse of the first side is a right angle. This is called converse of Pythagoras Property.

