PROBABILITY

IMPORTANT FACTS AND FORMULAE :

◈ **Experiment :** An operation in which can produce some well-defined outcomes is called an experiment.

◈ **Random Experiment :** An Experiment in which all possible outcomes are known and the exact output cannot be predicated in advance, is called a random experiment.

◈ **Examples of Performing a Random Experiment :**

(i) Rolling an unbiased dice.
(ii) Tossing a fair coin.
(iii) Drawing a card from a pack of well-shuffled cards.
(iv) Picking up a ball of certain colour from a bag containing balls of different colours.

◈ **Details :**

(i) When we throw a coin. Then either a Head (H) or a Tail (T) appears.
(ii) A dice is a solid cube, having 6 faces, marked 1,2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.
(iii) A pack of cards has 52 cards.
It has 13 cards of each suit, namely Spades, Clubs, Hearts and Diamonds.
Cards of spades and clubs are black cards.
Cards of hearts and diamonds are red cards.
There are 4 honours of each suit.
These are Aces, Kings, Queens and Jacks.
These are called face cards.

◈ **Sample Space :** When we perform an experiment, then the set S of all possible outcomes is called the Sample Space.

◈ **Examples of Sample Spaces :**

(i) In tossing a coin, S = {H, T}.
(ii) If two coins are tossed, then S = {HH, HT, TH, TT}.
(iii) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

◈ **Event :** Any subset of a sample space is called an event.

◈ **Probability of Occurrence of an Event :**

Let S be the sample space and let E be an event.

Then, $E \subseteq S$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$.

❖ EXAMPLES ❖

Ex.1 In a throw of a coin, find the probability of getting a head.

Sol. Here S = {H, T} and E = {H}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}$.

Ex.2 Two unbiased coins are tossed. What is the probability of getting at most one head ?

Sol. Here S = {HH, HT, TH, TT}

Let E = event of getting at most one head.

$\therefore$ E = {TT, HT, TH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$.

Ex.3 An unbiased die is tossed. Find the probability of getting a multiple of 3.

Sol. Here S = {1, 2, 3, 4, 5, 6}.

Let E be the event of getting a multiple of 3.

Then E = {3, 6}

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

Ex.4 In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.

Sol. Here, $n(S) = (6 \times 6) = 36$.

Let E = Event of getting a total more than 7

= {(2, 6), (3, 5) (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5) (6, 6)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.

Ex.5 Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6 ?

Sol. Clearly, $n(S) = 6 \times 6 = 36$.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}

$\therefore$ n (E) = 14.

Hence, $P(E) = \left(\frac{14}{36}\right) = \frac{7}{18}$.

EXERCISE

Q.1 In a simultaneous throw of two coins, the probability of getting at least one head is:

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.2 Three unbiased coins are tossed. What is the probability of getting at least 2 heads?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{8}$

Q.3 Three unbiased coins are tossed. What is the probability of getting at most two heads?

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{7}{8}$

Q.4 In a single throw of a die, what is the probability of getting a number greater than 4 ?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$

Q.5 In a simultaneous throw of two dice, what is the probability of getting a total of 7 ?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.6 What is the probability of getting a sum 9 from two throws of a dice?

(A) $\frac{1}{6}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

Q.7 In a simultaneous throw of two dice, what is the probability of getting a doublet?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{7}$

Q.8 In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11?

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{7}{12}$ (D) $\frac{5}{36}$

Q.9 Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ?

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{3}{8}$ (D) $\frac{5}{16}$

Q.10 Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(A) $\frac{3}{10}$ (B) $\frac{3}{20}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$

Q.11 Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ?

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{8}{15}$ (D) $\frac{9}{20}$

Q.12 In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(A) $\frac{1}{10}$ (B) $\frac{2}{5}$ (C) $\frac{2}{7}$ (D) $\frac{5}{7}$

Q.13 One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(A) $\frac{1}{13}$ (B) $\frac{4}{13}$ (C) $\frac{1}{4}$ (D) $\frac{9}{52}$

Q.14 A card is from a pack of 52 cards. The probability of getting a queen of club or a king of heart is:

(A) $\frac{1}{13}$ (B) $\frac{2}{13}$ (C) $\frac{1}{26}$ (D) $\frac{1}{52}$

Q.15 One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or king ?

(A) $\frac{1}{2}$ (B) $\frac{6}{13}$ (C) $\frac{7}{13}$ (D) $\frac{27}{52}$

Q.16 From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(A) $\frac{4}{13}$ (B) $\frac{1}{4}$ (C) $\frac{1}{13}$ (D) $\frac{1}{26}$

Q.17 The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is :

(A) $\frac{2}{13}$ (B) $\frac{4}{13}$ (C) $\frac{1}{13}$ (D) $\frac{1}{52}$

Q.18 A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white ?

(A) $\frac{3}{4}$ (B) $\frac{4}{7}$ (C) $\frac{1}{8}$ (D) $\frac{3}{7}$

Q.19 In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither blue nor green?

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{7}{19}$ (D) $\frac{8}{21}$

Q.20 Two dice are tossed. The probability that the total score is a prime number is :

(A) $\frac{1}{6}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{9}$

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	B	D	B	A	C	A	D	B	A	D	C	B	C	C	A	B	B	D	B

HINTS & SOLUTION

Sol. 1 Here S = {HH, HT, TH, TT}.

Let E = event of getting at least one head

= {HT, TH, HH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

Sol. 2 Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two heads

= {THH, HTH, HHT, HHH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}.$

Sol. 3 Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at most two heads.

Then, E = {TTT, TTH, THT, HTT, THH, HTH, HHT}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}.$

Sol. 4 When a die is thrown, we have

S = {1, 2, 3, 4, 5, 6}.

Let E = event of getting a number greater than 4 = {5, 6}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$

Sol. 5 We know that in a simultaneous throw of two dice, $n(S) = 6 \times 6 = 36$.

Let E = event of getting a total of 7 = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$

Sol. 6 In two throws of a die, $n(S) = (6 \times 6) = 36$.

Let E = event of getting a sum 9 = {(3, 6), (4, 5), (5, 4), (6, 3)}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

Sol. 7 In a simultaneous throw of two dice,

$n(S) = (6 \times 6) = 36$.

Let E = event of getting a doublet = {(1, 1), (2, 2), (3, 3),(4, 4), (5, 5), (6, 6)}

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.

Sol. 8 In a simultaneous throw of two dice, we have

$n(S) = (6 \times 6) = 36$.

Let E = event of getting a total of 10 or 11 = {(4, 6), (5, 5), (6, 4),(5, 6), (6, 5)}

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$.

Sol. 9 In a simultaneous throw of two dice, we have

$n(S) = (6 \times 6) = 36$.

Let E = event of getting two numbers whose product is even.

Then, E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore$ $n(E) = 27$.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$.

Sol.10 Here, S= {1, 2, 3, 4, ..., 19, 20}.

Let E = event of getting a multiple of

3 = {3, 6, 9, 12, 15, 18}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$.

Sol.11 Here, S= {1, 2, 3, 4, ..., 19, 20}.

Let E = event of getting multiple of

3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$.

Sol.12 P (getting a prize) $= \frac{10}{(10+25)} = \frac{10}{35} = \frac{2}{7}$.

Sol.13 Clearly, there are 52 cards, out of which there are 16 face cards.

$\therefore$ P (getting a face card) $= \frac{16}{52} = \frac{4}{13}$.

Sol.14 Here, n(S) = 52.

Let E = event of getting a queen of club or a king or heart.

Then, n(E) = 2.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$.

Sol.15 Here, n(S) = 52.

There are 26 red cards (including 2 kings) and there are 2 more kings.

Let E = event of getting a red card or a king

Then, n(E) = 28.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$.

Sol.16 Here, n (S) = 52.

There are 13 spades (including one ten) and there are 3 more tens.

Let E = event of getting a ten or a spade

Then, n(E) = (13 + 3) = 16.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$.

Sol.17 Here, n (S) = 52.

There are 13 cards of diamond (includng one king) and there are 3 more kings.

Let E = event of getting a diamond or a king.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$.

Sol.18 Total number of balls = (6 + 8) = 14

Number of white balls = 8.

P (drawing a white ball) $= \frac{8}{14} = \frac{4}{7}$.

Sol.19 Total number of balls = (8 + 7 + 6) = 21.

Let E = event that the ball drawn is neither red nor green.

= event that the ball drawn is red.

$\therefore$ n (E) = 8

$\therefore$ P (E) $= \frac{8}{21}$.

Sol.20 Clearly, n (S) = (6 × 6) = 36.

Let E = Event that the sum is a prime number.

Then, E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

$\therefore$ n (E) = 15

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.