CHAPTER

1

Indices and Surds

Before we proceed to Exponents (Indices) and Surds, it is proper to learn about Real Numbers.

Numbers are collection of certain symbols or figures called digits. The common Number System in use is Decimal System. In this system we use ten symbols each representing a digit. These are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. A combination of these figures representing a number is called a numeral.

We also have Binary Number System. It uses only 0 and 1. There are other Number Systems too.

Every digit has a face value which equals the value of the digit itself, irrespective of its place in the numeral.

Each digit in a numeral/number has a place value besides its face value. For a given number/numeral we begin from the extreme right as Unit's place, Ten's place, Hundred's place, Thousand's place and so on. This is illustrated with an example below :

The number 2465971385 may be represented as :

Billion	*Ten crores*	*Crores*	*Ten Lacs (million)*	*Lacs*	*Ten Thousands*	*Thousands*	*Hundreds*	*Tens*	*Units*
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
2	4	6	5	9	7	1	3	8	5

We may also write it as

$2465971385 = 2 \times 10^9 + 4 \times 10^8 + 6 \times 10^7 + 5 \times 10^6 + 9 \times 10^5 + 7 \times 10^4 + 1 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 5 \times 10^0$

Here the place value of various digits are

Digit	*Place Value*
5	$5 \times 10^0 = 5$
8	$8 \times 10^1 = 80$
3	$3 \times 10^2 = 300$
1	$1 \times 10^3 = 1,000$
7	$7 \times 10^4 = 70,000$
9	$9 \times 10^5 = 900,000$
5	$5 \times 10^6 = 5,000,000$
6	$6 \times 10^7 = 70,000,000$
4	$4 \times 10^8 = 400,000,000$
2	$2 \times 10^9 = 2,000,000,000$

Thus we see that the place value of a digit depends on its location in the number. We can see it for digits 5 in the above example. Its face value remains five at both the places while its place value at two places are 5 and 5,000,000.

NUMBER SYSTEM

The development of the number system started with natural numbers. These are generally known as counting numbers. 'Kronecker' a German mathematician termed them as 'God's gift to mankind'. However, we should know that the number system is far from being simple and that numbers of various kinds are used in Mathematics. They have been mentioned below :

Natural Numbers : The counting numbers 1, 2, 3, 4... are called natural numbers.

The smallest natural number is 1 but there is no largest, because regardless of how large a number is chosen, there exist larger ones. The set of natural numbers is denoted by N.

N = {1, 2, 3, 4 ......}

Whole Numbers : In the set of natural numbers, if we include the number 0, the resulting set is called the set of whole numbers. The set of whole numbers is represented by W.

Thus, W = {0, 1, 2, 3, ...}

Remarks : (i) The number 0 has the property that when it is added to or subtracted from a given number, it gives back the same number.

For example : $7 + 0 = 7$
$7 - 0 = 7$

(ii) Zero is the only number which can be divided by any other number but can divide no other number.

For example : $\frac{0}{5} = 0$, $\frac{5}{0}$ = Meaningless (∞)

Integers : Natural numbers along with 0 and their negatives are known as integers. The set of integers is denoted by I. Thus,

I = {.... – 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5 ....}

Zero : 0 is also an integer but it is neither a positive integer nor a negative integer. But 0 is definitely a non-negative integer as well as a non-positive integer.

Thus, **(i)** Set of positive integers = {1, 2, 3, ...}

(ii) Set of negative integers = {–1, – 2, – 3...}

(iii) Set of non-negative integers = {0, 1, 2, 3, ...}

(iv) Set of non-positive integers = {0, – 1, – 2, – 3, –4, –5, –6 ....}

(v) If $a \neq 0$, then $\frac{0}{a} = 0$ (0 is divisible by all non-zero numbers).

(vi) No number is divisible by zero. Thus denominator can never be zero.

Rational Numbers : A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is known as a rational number.

Ex. : (i) $\frac{2}{5}$ (ii) $\frac{-5}{2}$

(iii) $5\left(\text{as } 5 = \frac{5}{1}\right)$

Without loss of generality we shall assume that there is no common factor in p and q; for if they have a common factor, we can divide each of them by it without affecting the value of $\frac{p}{q}$. We may also assume that $q > 0$.

Standard Form of a Rational Number : A rational number $\frac{p}{q}$ is said to be in standard form if p and q are integers and have no 0 common factor other than 1 and $q > 0$.

Ex. : $\frac{4}{6}$ is a rational number but it is not in standard form. Its standard form is $\frac{2}{3}$. A rational number may be positive, negative or zero. The rational number $\frac{p}{q}$ is positive if p and q have like signs and negative if p and q have not like signs.

Representation of Rational Numbers as Decimals : We know that the decimal form of a rational number is either recurring or terminating.

An Important Result : If the denominator of a rational number has no prime factors other than 2 or 5, then and only then it is expressible as a terminating decimal.

Ex. : (i) $\frac{2}{3} = 0.\bar{6}$ (Recurring decimal).

(ii) $-\frac{5}{2} = -2.5$ (Terminating decimal).

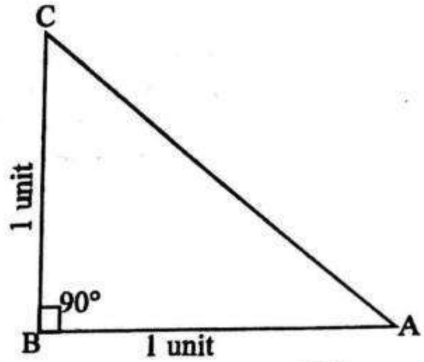
(iii) $\frac{3}{4} = 0.75$

Here, denominator of $\frac{3}{4}$ is 4. The only prime factor of 4 is 2. Thus 4 has no prime factor other than 2. So $\frac{3}{4}$ is expressible as a terminating decimal.

Irrational Numbers : Let us consider a right angled isosceles triangle ABC with sides AB and BC of unit length.

Then,

$$AC^2 = AB^2 + BC^2 = 1^2 + 1^2 = 2$$



Now, the question arises can AC be represented by a rational number? If yes,

Let $AC = \frac{p}{q}$, where $p, q \in I$ and $q \neq 0$.

We assume that there is no common factor other than 1 in p and q; for if they have a common factor, we cancel it out.

Now, $AC = \frac{p}{q}$ $\quad \therefore AC^2 = \frac{p^2}{q^2}$

or, $2 = \frac{p^2}{q^2}$ $\quad$ or, $p^2 = 2q^2$ ....(i)

From, (i) it is clear that p^2 is divisible by 2 and hence p is also divisible by 2.

Let $p = 2m,\ m \in I$

Then, $p^2 = 4m^2 = 2q^2$

$\therefore\ q^2 = 2m^2$ ...(ii)

From (ii), it follows that q is also divisible by 2.

Thus both p and q are divisible by 2, i.e., they have a common factor 2, which contradicts our assumptions that p and q have no common factor.

Hence AC cannot be represented by a rational number. We will say that AC represents an irrational number $\sqrt{2}$. There are infinite number of irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $2+\sqrt{3}$, $2-\sqrt{3}$, $3-2\sqrt{2}$ ... etc.

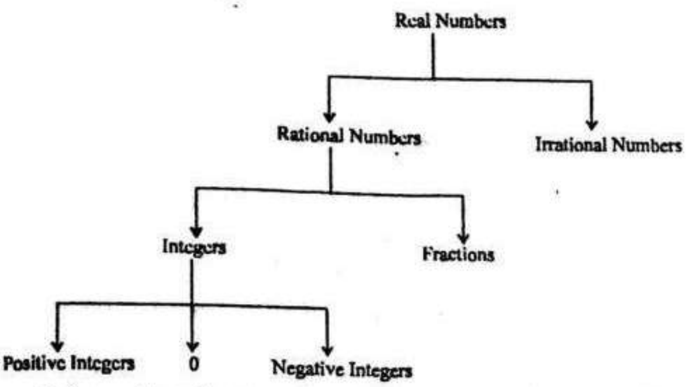
Decimal form of an irrational number is neither recurring nor terminating. Whenever an irrational number is written in decimal form it is its approximate value.

Hence, (i) $\sqrt{2} = 1.4142135.....$

(ii) $\sqrt{5} = 2.236.....$

(iii) If we take $\pi = \frac{22}{7}$ or 3.14, then it is its approximate value. π is an irrational number.

Real Numbers : Real numbers are those which are either rational or irrational. The set of real numbers is denoted by R.



Prime Numbers : A prime number is a number which has no factors besides itself and unity i.e. it is divisible only by itself and 1 but not by any other number.

For example : 2, 3, 5, 7, 11, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107.

Note : (*i*) 2 is the only even number which is prime.

(*ii*) All prime numbers other than 2 are odd numbers but all odd numbers are not prime numbers, e.g., 9 is an odd number but it is not a prime number as it is divisible by 3.

Composite Numbers : A composite number is one which has other factors besides itself and unity. Thus it is a non-prime number. For example, 4, 6, 9, 14, 15 etc.

Note : (*i*) 1 is neither prime nor composite.

(*ii*) A composite number may be even or odd.

The number of ways in which a number N can be expressed as product of two factors which are relatively prime to each other is $\mathbf{2^{m-1}}$ where m is the number of different prime factors of N.

For example, $540 = 2^2 \times 3^3 \times 5$ ($= 4 \times 27 \times 5$)

$\therefore$ $m = 3$(i.e., 2, 3, 5)

$\therefore$ No. of ways $= 2^{3-1}$

$= 4$ i.e., 20×27, 4×135, 108×5, 540×1.

The largest prime number known so far is $2^{2281}-1$ which is of about 700 digits.

Consecutive Integers : These are series of numbers differing by 1 in ascending or descending order. For example, 12, 13, 14, 15.....

Similarly, examples of consecutive even numbers will be 4, 6, 8, 10 .... .

22, 24, 26, 28 ..... and so on.

Examples of consecutive prime numbers may be written as 7, 11, 13, 17 ....

TEST FOR PRIME NUMBERS

For numbers less than 100, it is not very difficult to determine whether it is a prime number or not. We can also check it up from the list given under prime numbers.

For testing any number greater than 100 whether it is a prime number or not:

I. We take the nearest integer larger than the approximate square root of that number. Suppose it is x.

II. We test the divisibility of the given number by every prime number less than x.

III. If the number is not divisible by any of them, then it is a prime number, otherwise it is a composite number.

Let us test the three numbers : (*a*) 331, (*b*) 481 and (*c*) 881

(*a*) For 331, $x = 19$. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13 and 17.

Now try to divide 331 by these prime numbers. We see that 331 is not completely divisible by any of these prime numbers. Therefore, 331 is a prime number.

(*b*) For 481, $x = 22$.

The prime numbers less than 22 are 2, 3, 5, 7, 11, 13, 17, 19. We find that 481 is divisible by 13.

Therefore, 481 is not a prime number.

(*c*) For 881, $x = 30$.

The prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

We see that 881 is not divisible any of the above prime numbers. Therefore, 881 is a prime number.

Any whole number can be written as a product of factors which are prime numbers. To write a number as a product of prime factors :

(*a*) Divide the number by 2 if possible; continue to divide by 2 until the factor you get is not divisible by 2.

(*b*) Divide the result from (*a*) by 3 if possible; continue to divide by 3 until the factor you get is not divisible by 3.

(*c*) Divide the result from (*b*) by 5 if possible; continue to divide by 5 until the factor you get is not divisible by 5.

(*d*) Continue the procedure with 7, 11 and so on, until all the factors are primes. For example, $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$.

Perfect Numbers : If the sum of the divisors of a number N excluding N itself is equal to N, then N is called a perfect number. For example, 6, 28, 496, 8128

For 6, divisors are 1, 2, and 3.

6 : $1 + 2 + 3 = 6$

28 : $1 + 2 + 4 + 7 + 14 = 28$

496 : $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$

8128 : $1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064 = 8128$

Note : The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2.

Ex. For 28 the factors are 1, 2, 4, 7 and 14.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}$$

$$=\frac{28+14+7+4+2+1}{28} = \frac{56}{28} = 2$$

Decimal Numbers : A collection of digits (0, 1, 2, 3, ...9) after a period (called the decimal point) is called a decimal fraction.

For example, 0.629, 0.53, 0.023 etc.

0.53 is read as decimal five three and not as decimal fifty three. 12.142 is read as twelve decimal one four two.

Every decimal fraction represents a fraction. These fractions have denominators with powers of 10.

For example, $0.5 = \frac{5}{10}$.

$$0.42 = \frac{4}{10} + \frac{2}{100} = \frac{42}{100}$$

$$0.429 = \frac{4}{10} + \frac{2}{100} + \frac{9}{1000} = \frac{429}{1000}$$

A number containing a decimal point is called a decimal number.

$$35.467 = (3\times10^1) + (5\times10^0) + \frac{4}{10^1} + \frac{6}{10^2} + \frac{7}{10^3}$$

$$= (3\times10) + (5\times1) + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000}$$

$$= 30 + 5 + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000}$$

$$= 35 + \frac{400+60+7}{1000} = 35 + \frac{467}{100} = \frac{35467}{1000}$$

Mixed Numbers : A mixed number consists of a whole number and a fraction.

For example, $3\frac{4}{5}$ is a mixed number. This is equivalent to and hence can be written as : $3\frac{4}{5} = 3 + \frac{4}{5}$. Here 3 is the whole number and $\frac{4}{5}$ is the fraction.

A mixed number can be changed into a fraction as follows :

(*i*) Multiply the whole number by the denominator of the fraction.

(*ii*) Add the numerator of the fraction to the resultant product.

(*iii*) The resultant fraction of the given mixed number will have the result of (*ii*) as numerator and the denominator of the fractional part as the denominator.

In the above example, $3\frac{4}{5} = \frac{(3\times5)+4}{5} = \frac{19}{5}$.

We can also do the reverse of it, i.e, when the numerator of a given fraction is greater than the denominator, we can change it into a mixed number using the following procedure :

(*i*) Divide the numerator by the denominator.

(*ii*) The quotient, gives the whole number and the remainder gives the numerator of the fractional part. The denominator of the fractional part will be the same as the denominator of the given fraction.

For example, $\frac{21}{4} = 5\frac{1}{4}$.

Signed Numbers : A number preceded by either a plus or a minus sign is called *Signed Number*. Such numbers are also called *Directed Numbers*.

For example, $+3, -6, -7, +\frac{5}{2}, -\frac{6}{7}, +3\frac{1}{2}, -4\frac{1}{3}$ etc.

If no sign is given with a number, a plus sign is assumed and *vice versa*.

Signed numbers are often used to distinguish different concepts. For example, a profit of Rs. 25 may be indicated by + Rs. 25 and a loss of Rs. 25 by – Rs. 25. Thus plus and minus signs have opposite meanings. A height of 2500 m above sea level may be denoted by +2500 m while a depth of 2500 m below sea level is denoted by – 2500 m. Similarly, we can write for temperature above and below 0°C. So, we see that in all cases we have to have a datum or reference with respect to which the given quantity is measured and expressed. In the above examples, zero, profit/loss, sea level, 0°C are the reference points. The quantity on the positive side of the datum is given plus sign and its absolute value is obtained by adding the quantity to the datum/reference value. On the other hand, the other (negative) side of the datum is given minus sign and its absolute value is given by subtracting the given quantity from the datum which, in other words, is equivalent to adding the quantity with minus sign to the datum.

For example, $0 + 25 = +25 = 25$

$$0 - 25 = 0 + (-25) = -25$$

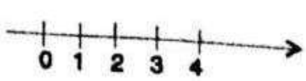
Thus, we see that adding a negative quantity to any number is equivalent to subtracting its absolute (positive) value from the given number.

You can imagine numbers to be arranged on a line called a number line.

Draw a line which extends indefinitely (i.e., upto infinity) in both directions. Mark a reference point on this line and call it zero. The portion on the right of zero is called positive side and that on the left of zero as negative side. (Instead of a horizontal line we can alternatively draw a vertical line in which case the portion above zero is equal to positive and that below zero as negative).

Mark another point on the line to the right of zero and call it 1.

The point to the right of 1 which is exactly as far away from 1 as 1 is from 0 is marked 2. The point to the right of 2 just as far from 2 as 1 is from 0 (or 2 is from 1) is called 3. Similarly, we have 4, 5, 6 and so on.

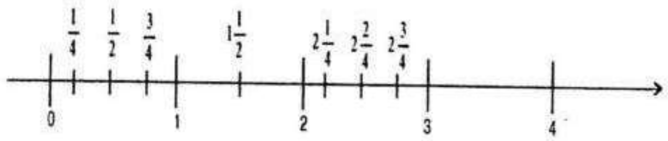


The point mid-way or half-way between 0 and 1 is called $\frac{1}{2}$, the point midway between 1 and 2 is called $1\frac{1}{2}$ or $\frac{3}{2}$ and so on. The point midway between 0 and $\frac{1}{2}$ is called $\frac{1}{4}$ and the point midway between $\frac{1}{2}$ and 1 is called $\frac{3}{4}$. Thus any whole number or fraction can be identified.

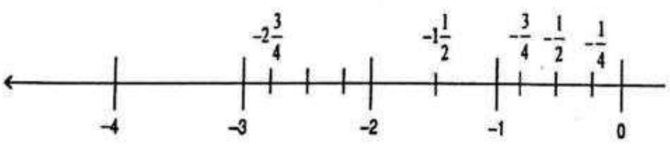
For example, $2\frac{3}{4}$.

It lies between 2 and 3.

Divide the distance between 2 and 3 into 4 equal parts. The point at the third mark between 2 and 3 represents $2\frac{3}{4}$.



If we go to the left of zero, the same distance as we did from 0 to 1, the point is called –1. In a similar manner we can represent – 2, –3, – 4, $-\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{3}{4}$, $-1\frac{1}{2}$, $-2\frac{3}{4}$ and so on.



Zero is neither positive nor negative and therefore we can write,

$$0 = +0 = -0$$

Any non-zero number is either positive or negative but can't be both.

It is not always necessary to take zero as datum/reference point. Datum or Reference Point is chosen as per convenience and depends on the problem to be solved. Same hold true for positive and negative numbers. But once chosen, their uniformity must be maintained throughout the solution. It is usual practice to assign zero to the reference point/datum, whatever its value might be.

For instance, we take ground level of a particular place instead of sea level as datum for expressing the height of a building. Similarly, the datum for local time is different in different countries.

Modulus or Absolute Value : This Absolute value must be differentiated from the absolute value discussed above. The Modulus or Absolute value of any signed number (be it positive or negative) is always taken as positive and is equal to the distance of the number from zero datum. It is denoted by writing the number between two vertical lines. In other words, it denotes only the magnitude of the number. If A is any signed number, its Modulus or Absolute value is written as $|A|$.

If A is positive, $|A| = A$

If A is negative, $|A| = -A$

So, what we do effectively is that we simply drop the sign attached to the number, when we are taking its Modulus or Absolute value.

For example,

$|+3| = +3 = 3$

$|-8| = -(-8) = +8 = 8$

$|+8| = +8 = 8$

$|-12| = -(-12) = +12 = 12$

Absolute value of zero will always be zero.

Mathematical Operations on Signs :

$$\left.\begin{aligned} +(+20) &= +20 \\ +(-20) &= -20 \\ -(+20) &= -20 \\ -(-20) &= +20 \end{aligned}\right\}$$

We notice that the operation of plus does not change the sign of the number whereas the operation of minus changes the sign of the number.

$$\left.\begin{aligned} (+)\times(+) &= + \\ (+)\times(-) &= - \\ (-)\times(+) &= - \\ (-)\times(-) &= + \\ (+)\div(+) &= + \\ (+)\div(-) &= - \\ (-)\div(+) &= - \\ (-)\div(-) &= + \end{aligned}\right\}$$

In multiplication and division, when both the numbers carry similar sign, we get positive sign in the result, otherwise, we get negative sign in the result.

Some Other Important Terms Which Are Related To Numbers

1. Identity Element of Addition : Addition of '0' (zero) in any number does not affect that number. So, '0' is called identity element of addition. For example, $a + 0 = a$ (where 'a' is a Rational Number).

Here, '0' is called Identity Element of Addition, because 'a' is unaffected after the addition of '0'.

2. Identity Element of Multiplication : When a number is multiplied by 1, then there is no any change in that number, so, '1' is called Identity Element of Multiplication.

For example, $a \times 1 = a$

Here, '1' is called Identity Element of Multiplication.

3. Inverse Element of Addition/Negative Element of Addition/Additive Inverse : The number is called "Additive Inverse" of a certain number, when it is added to the certain number and result becomes '0' (zero).

For example, (i) $a + (-a) = 0$

Here $(-a)$ is Additive Inverse of 'a'

(ii) $5 + (-5) = 0$

Here, '-5' is Additive Inverse of '5'.

4. Inverse Element of Multiplication/Reciprocal Element/Multiplicative Inverse : Multiplicative inverse is the number of a certain number when product of these two numbers (M. I. × certain number) is '1'.

For example, $a \times \frac{1}{a} = 1$

Here, $\frac{1}{a}$ is Multiplicative Inverse of 'a'

EXPONENTS

We are familiar with real numbers and throughout this text book we are concerned with real numbers only. So, it is desirable to define powers of real numbers. The powers of real numbers are defined in the same manner as powers of rational numbers.

INTEGRAL EXPONENTS OF A REAL NUMBER

Positive Integral Power : For any real number 'a' and a positive integer 'n', we define a^n as

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{(n \text{ factors})}$$

Just as in case of rational numbers, here also number 'a' is called the base and n is called the exponent or index of the nth power of a. a^n is called the nth power of a or a raised to power n.

We have specific terms for $n = 2$ known as **square** and for $n = 3$ known as **cube.**

For example, (i) $3^2 = 3 \times 3 = 9$

(ii) $(-3)^2 = (-3) \times (-3) = 9$

(iii) $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

(iv) $\left(\frac{3}{2}\right)^4 = \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{81}{16}$

(v) $\left(\frac{-3}{2}\right)^4 = \left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right) \times \left(\frac{-3}{2}\right) = \frac{81}{16}$

For any non-zero real number a, we define $a^0 = 1$

Thus, $5^0 = 1$, $7^0 = 1$, $\left(\frac{5}{7}\right)^0 = 1$ and so on.

Negative Integral Power : For any non-zero real number a and a positive integer n, we define

$$a^{-n} = \frac{1}{a^n}$$

For example,

(i) $(5)^{-2} = \frac{1}{5^2} = \frac{1}{5 \times 5} = \frac{1}{25}$

(ii) $(4)^{-3} = \frac{1}{4^3} = \frac{1}{4 \times 4 \times 4} = \frac{1}{64}$

(iii) $\left(\frac{3}{2}\right)^{-3} = \frac{1}{\left(\frac{3}{2}\right)^3} = \frac{1}{\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}} = \frac{1}{\frac{27}{8}} = \frac{8}{27}$

LAWS OF INTEGRAL EXPONENTS

1st Law : If a is any real number and m, n are positive integers, then

$a^m \times a^n = a^{m+n}$

Proof : By definition, we have

$a^m \times a^n = (a \times a \times \ldots$ to m factors$) \times (a \times a \times \ldots$ to n factors$)$

$= a \times a \times a \times \ldots$ to $(m + n)$ factors

$= a^{m+n}$

Conversely, $a^{m+n} = a^m \times a^n$

Corollary : If m, n, p are positive integers, then $a^m \times a^n \times a^p = a^{m+n} \times a^p = a^{m+n+p}$

Similarly, $a^m \times a^n \times a^p \times a^q = a^{m+n+p+q}$

For example, (i) $2^4 \times 2^3 = 2^{4+3} = 2^7$

(ii) $3^2 \times 3^5 \times 3^7 = 3^{2+5+7} = 3^{14}$

2nd Law : If a is a non-zero real number and m, n are positive integers, then

$$\frac{a^m}{a^n} = a^{m-n}$$

Proof : Here, we will consider three different cases to prove it.

Case I : When $m > n$

In this case, $\frac{a^m}{a^n} = \frac{a \times a \times a \times \ldots\ldots \text{to } m \text{ factors}}{a \times a \times a \times \ldots\ldots \text{to } n \text{ factors}}$

$= a \times a \times a \times \ldots\ldots$ to $(m - n)$ factors

$= a^{m-n}$

Case II : When $m = n$

In this case, $\frac{a^m}{a^n} = \frac{a^m}{a^m}$

$= 1 = a^0$

$= a^{m-m} = a^{m-n}$ $\quad [\because a^0 = 1]$ $[\because n = m]$

Case III : When $m < n$

In this case, $\frac{a^m}{a^n} = \frac{a \times a \times a \times \ldots\ldots \text{to } m \text{ factors}}{a \times a \times a \times \ldots\ldots \text{to } n \text{ factors}}$

$= \frac{1}{a \times a \times a \times \ldots\ldots to\ (n-m) \text{ factors}}$

$= \frac{1}{a^{n-m}} = a^{-(n-m)} = a^{m-n}$

Therefore, $\frac{a^m}{a^n} = a^{m-n}$ whether $m > n$, or $m = n$ or $m < n$

For example, (i) $3^3 \div 3^2 = 3^{3-2} = 3$

(ii) $\left(\frac{2}{5}\right)^5 \div \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{5-2} = \left(\frac{2}{5}\right)^3$

3rd Law : If a is any real number and m, n are positive integers, then

$(a^m)^n = a^{mn} = (a^n)^m$

Proof : $(a^m)^n = a^m \times a^m \times a^m \times \ldots$ to 'n' factors

$= (a \times a \times \ldots$ to m factors$) \times (a \times a \times \ldots$ to m factors$) \times$

$(a \times a \times \ldots$ to m factors$) \ldots$ to n factors

$= a \times a \times a \ldots$ to (mn) factors.

$= a^{mn}$

Similarly, $(a^n)^m = a^{mn}$

Corollary : $\{(a^m)^n\}^p = (a^{mn})^p = a^{mnp}$ and so on.

Conversely, $a^{mnp} = \{(a^m)^n\}^p$

For example, (i) $(2^2)^5 = 2^{2\times5} = 2^{10}$

(ii) $\{(3^4)^3\}^5 = 3^{4\times3\times5} = 3^{60}$

4th Law : If a, b are real numbers and m, n are positive integers, then

(i) $(ab)^n = a^n b^n$ (ii) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

Proof : (i) $(ab)^n = (ab) \times (ab) \times \ldots$ to n factors

$= (a \times a \times \ldots$ to n factors$) \times (b \times b \times b \times \ldots$ to n factors$)$

$= a^n \times b^n = a^n b^n$

(ii) $\left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \ldots$ to n factors

$= \frac{a \times a \times a \times \ldots\ldots \text{to } n \text{ factors}}{b \times b \times b \times \ldots\ldots \text{to } n \text{ factors}} = \frac{a^n}{b^n}$

For example, (i) $6^5 = (2 \times 3)^5 = 2^5 \times 3^5$

(ii) $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

We now come to the meaning of the rational power of a positive real number. We recall here the following results :

Principal nth Root of a Positive Real Number : If a is a positive real number and n is a positive integer, then the principal nth root of a is the unique positive real number x such that $x^n = a$.

The principal nth root of a positive real number a is denoted by the symbol $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$.

For example,

(i) $(4)^{\frac{1}{2}} = 2$ because $2^2 = 4$

(ii) $(64)^{\frac{1}{3}} = 4$ because $4^3 = 64$

Principal nth Root of a Negative Real Number : If a is a negative real number and n is an odd positive integer, then the principal nth root of a is defined as $-|a|^{\frac{1}{n}}$, i.e., the principal nth root of a is minus (negative) of the principal nth root of $|a|$.

For example,

(i) $(-8)^{\frac{1}{3}} = -|-8|^{\frac{1}{3}} = -8^{\frac{1}{3}} = -2$

(ii) $(-64)^{\frac{1}{3}} = -|-64|^{\frac{1}{3}} = -64^{\frac{1}{3}} = -4$

Remark : If a is a negative real number and n is an even integer, then the principal nth root of a is not defined, because an even power of a real number is always positive. Therefore, $(-16)^{\frac{1}{2}}$ is a meaningless quantity, if we confine ourselves to the set of real numbers.

Rational Exponents : For any positive real number a and a rational number $\frac{p}{q}$, where $q > 0$ we define

$$a^{\frac{p}{q}} = \left(a^p\right)^{\frac{1}{q}}$$

i.e., $a^{\frac{p}{q}}$ is the principal qth root of a^p.

For example,

(i) $(9)^{\frac{3}{2}} = \left(9^3\right)^{\frac{1}{2}} = (729)^{\frac{1}{2}} = 27$

(ii) $(-8)^{\frac{2}{3}} = \left[(-8)^2\right]^{\frac{1}{3}} = (64)^{\frac{1}{3}} = 4$

LAWS OF RATIONAL EXPONENTS

For a and b to be positive real numbers and $\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers with $q > 0$ and $s > 0$, the following laws hold good :

1. $a^{\frac{p}{q}}.a^{\frac{r}{s}} = a^{\frac{p}{q}+\frac{r}{s}}$

2. $\frac{a^{\frac{p}{q}}}{a^{\frac{r}{s}}} = a^{\left(\frac{p}{q}-\frac{r}{s}\right)}$

3. $\left(a^{\frac{p}{q}}\right)^{\frac{r}{s}} = a^{\frac{pr}{qs}}$

4. $(ab)^{\frac{p}{q}} = a^{\frac{p}{q}}b^{\frac{p}{q}}$

5. $\left(\frac{a}{b}\right)^{\frac{p}{q}} = \frac{a^{\frac{p}{q}}}{b^{\frac{p}{q}}}$

SURDS

We have already studied that a real number is either rational or irrational. In fact, if a real number is expressible in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$, then it is called a rational number. For example $\frac{3}{5}, \frac{2}{7}, -\frac{4}{5}$ etc. are rational numbers. A real number is an irrational number if it is not expressible as the quotient of two integers. For example, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$, π etc. are irrational numbers. The decimal representation of an irrational number is non-terminating and non-repeating. In this chapter we shall deal with a particular type of irrational numbers whose nth power, for some positive integer 'n', is a rational number. Such irrational numbers are called **SURDS**.

Surds or Radicals

Before introducing the concept of a surd and its order, let us first introduce the concept of positive nth root of a real number.

Positive n th root of a real number : Let 'a' be a real number and 'n' be a positive integer. Then a number which when raised to the power 'n' gives 'a' is called the nth root of 'a' and it is expressed as $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$.

Thus nth root of a real number a is a real number b such that $b^n = a$. The real number b is denoted by $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$.

The cube root of 3 is the real number whose cube is 3. The cube root of 3 is denoted by the symbol $3^{\frac{1}{3}}$ or $\sqrt[3]{3}$.

Surds or Radicals :

If 'a' is a rational number and n is a positive integer such that the nth root of 'a', i.e., $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$ is an irrational number, then $a^{\frac{1}{n}}$ is called a surd or radical. In other words, an irrational root of a rational number is called a surd.

For the surd $\sqrt[n]{a}$, n is called the surd-index or the order of the surd and a is called the radicand. The symbol '$\sqrt{\ }$' is called the radical sign.

Ex. Let us consider the real number $\sqrt{5}$. It may also be written as $5^{\frac{1}{2}}$, since 5 is a rational number and 2 is a positive integer such that $5^{\frac{1}{2}}$ or $\sqrt{5}$ is an irrational number. So, $\sqrt{5}$ is a surd.

Ex. Let us consider the real number $\sqrt{2+\sqrt{5}}$. Since $2+\sqrt{5}$ is not a rational number, therefore, $\sqrt{2+\sqrt{5}}$ is not a surd.

Remark : If $\sqrt[n]{a}$ is a surd it implies :

(i) a is a rational number.

(ii) $\sqrt[n]{a}$ is an irrational number.

Quadratic Surd : A surd of order 2 is known as a quadratic surd.

For example, $\sqrt{2} = 2^{\frac{1}{2}}$ is a quadratic surd but $\sqrt{4} = 4^{\frac{1}{2}}$ is not a quadratic surd, because $\sqrt{4} = 4^{\frac{1}{2}} = 2$ is a rational number. Therefore, $\sqrt{4}$ is not a surd.

Cubic Surd : A surd of order 3 is called a cubic surd.

For example, The real number $\sqrt[3]{9}$ is a cubic surd but the real number $\sqrt[3]{27}$ is not a cubic surd because it is not a surd.

Biquadratic Surd : A surd of order 4 is called a biquadratic surd. It is a also known as quartic surd.

For example, $\sqrt[4]{3}$ is a biquadratic surd but $\sqrt[4]{81}$ is not a biquadratic surd because it is not a surd.

Laws of Surds

As we have seen earlier that surds can be expressed with fractional exponents or indices, the laws of surds follow directly from those of indices. The laws of surds are very useful to simplify a given surd or to reduce two given surds to the same form. The laws are given below :

First Law : For any positive integer 'n' and a positive rational number 'a'

$$\left(\sqrt[n]{a}\right)^n = a$$

Proof : We have, $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n}\times n} = a$

Hence, $\left(\sqrt[n]{a}\right)^n = a$

For example, $\left(\sqrt[3]{3}\right)^3 = \left(3^{\frac{1}{3}}\right)^3 = 3$

Ex. If $\sqrt[3]{4x-7}-5=0$, find the value of x.

Sol. : Here $\sqrt[3]{4x-7}-5=0$

$\Rightarrow \sqrt[3]{4x-7} = 5$

$\Rightarrow \left(\sqrt[3]{4x-7}\right)^3 = 5^3$

$\Rightarrow 4x-7 = 125$ $\qquad \left[\because \left(\sqrt[n]{a}\right)^n = a\right]$

$\Rightarrow 4x = 125+7 = 132$

$\Rightarrow x = 132 \div 4 = 33$

Second Law : If n is a positive integer and a, b are rational numbers, then,

$$\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{ab}$$

Proof : We have

$\left(\sqrt[n]{a}.\sqrt[n]{b}\right)^n = \left(\sqrt[n]{a}\right)^n\left(\sqrt[n]{b}^n\right)$ *[By Laws of Indices]*

$= ab$

Now, by using the definition of nth root of a real number,

$\sqrt[n]{a}.\sqrt[n]{b} = (ab)^{\frac{1}{n}} = \sqrt[n]{ab}$

For example, $\sqrt[3]{5}.\sqrt[3]{7} = \sqrt[3]{5\times 7} = \sqrt[3]{35}$

Third Law : If n is a positive integer and a, b are rational numbers, then

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Proof : We have $\left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \frac{\left(\sqrt[n]{a}\right)^n}{\left(\sqrt[n]{b}\right)^n} = \frac{a}{b}$

[By Laws of Indices]

$\therefore \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

For example, $\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}}$

$= \frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{5\times5\times5}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}} = \frac{2}{5}$

Fourth Law : If m, n are positive integers and 'a' is a positive rational number, then

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$$

Proof : We have

$\left(\sqrt[m]{\sqrt[n]{a}}\right)^{mn} = \left[\left(\sqrt[m]{\sqrt[n]{a}}\right)^m\right]^n = \left(\sqrt[n]{a}\right)^n = a$

$\therefore \quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Similarly, $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$

Hence, $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

For example, (a) $\sqrt[3]{\sqrt[4]{5}} = \sqrt[12]{5}$ (b) $\sqrt[3]{\sqrt[2]{3}} = \sqrt[6]{6}$

Fifth Law : If m, n are positive integers and 'a' is a positive rational number, then

$$\sqrt[n]{\sqrt[m]{\left(a^p\right)^m}} = \sqrt[n]{a^p} = \sqrt[mn]{a^{pm}}$$

Proof : We have,

$\left(\sqrt[n]{\sqrt[m]{\left(a^p\right)^m}}\right)^n = \sqrt[m]{\left(a^p\right)^m}$ [Using First Law]

$= \sqrt{a^p}$ ....(i)

and, $\left(\sqrt[n]{\sqrt[m]{\left(a^p\right)^m}}\right)^{mn} = \left(\sqrt[mn]{\left(a^p\right)^m}\right)^{mn}$

[Using Fourth Law]

$= \sqrt{\left(a^p\right)^m}$ [Using First Law]

$= \sqrt{a^{pm}}$ ....(ii)

From equations (i) and (ii), we have

$\sqrt[n]{\sqrt[m]{\left(a^p\right)^m}} = \sqrt[n]{a^p} = \sqrt[mn]{a^{pm}}$

For example, $\sqrt[5]{\sqrt[3]{\left(2^3\right)^3}}$.

$= \sqrt[5]{2^3} = \sqrt[5]{8}$

Surd in Simplest Form

By simplification of a surd we mean that it has :

(i) no fraction under the radical sign.

(ii) no factor which is nth power of a rational number under the radical sign of index n.

(iii) the smallest possible index of this radical, i.e., the order of the surd is the smallest possible.

For example, The surd $\sqrt[4]{2\times 3^4}$ is not in its simplest form since the number under the radical sign has factor 3^4 such that its index is equal to the order of the surd. This surd can be expressed in simplest form as

$$\sqrt[4]{2\times 3^4} = \sqrt[4]{2}\cdot\sqrt[4]{3^4} = \left(\sqrt[4]{2}\right)(3) = 3.\left(\sqrt[4]{2}\right)$$

Pure and Mixed Surds

Pure Surd : A surd which has unity only as rational factor, the other factor being irrational, is called a pure surd.

For example, $\sqrt{2}, \sqrt[3]{3}, \sqrt[5]{3}$ are pure surds.

Mixed Surd : A surd which has a rational factor other than unity, the other factor being irrational, is called a mixed surd.

For example, $2\sqrt{5}, 3\sqrt{7}, 5\sqrt[3]{7}$ are mixed surds.

Ex. Express the following as a pure surd

(i) $2\sqrt[3]{4}$ (ii) $3\sqrt[4]{5}$

Sol. : (i) $2\sqrt[3]{4} = \sqrt[3]{2^3}.\sqrt[3]{4} = \sqrt[3]{8\times 4} = \sqrt[3]{32}$

(ii) $3\sqrt[4]{5} = \sqrt[4]{3^4}.\sqrt[4]{5} = \sqrt[4]{3^4\times 5} = \sqrt[4]{81\times 5} = \sqrt[4]{405}$

Ex. Express the following as a mixed surd in its simplest form.

(i) $\sqrt[5]{320}$ (ii) $\sqrt[4]{1280}$

Sol. : (i) $\sqrt[5]{320} = \sqrt[5]{32\times 10} = \sqrt[5]{32}\times\sqrt[5]{10}$

$= \sqrt[5]{(2)^5}\,\sqrt[5]{10} = 2\sqrt[5]{10}$

(ii) $\sqrt[4]{1280} = \sqrt[4]{256\times 5} = \sqrt[4]{256}\times\sqrt[4]{5}$

$= \sqrt[4]{4^4}\times\sqrt[4]{5} = 4\sqrt[4]{5}$

Comparison of Surds

To compare the magnitude of surds of distinct orders, we change them into the surds of the same order.

This order is the L.C.M. of the orders of the given surds.

Then two surds of the same order can be easily compared by just comparing their radicands. The surds with larger radicand is the larger of the given surds.

Following examples will illustrate the above procedure :

Ex. Which surd is larger $\sqrt[3]{3}$ or $\sqrt[4]{4}$?

Sol. : The order of the given surds are 3 and 4 respectively.

Now, LCM of 3 and 4 = 12

Therefore, we convert each surd into a surd of order 12.

Now, $\sqrt[3]{3} = (3)^{\frac{1}{3}} = (3)^{\frac{4}{3\times 4}} = (3)^{\frac{4}{12}}$

$= \left(3^4\right)^{\frac{1}{12}} = (81)^{\frac{1}{12}} = \sqrt[12]{81}$

and, $\sqrt[4]{4} = (4)^{\frac{1}{4}} = (4)^{\frac{3}{4\times 3}} = (4)^{\frac{3}{12}}$

$= \left(4^3\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}$

Clearly, $81 > 64$

$\therefore \sqrt[12]{81} > \sqrt[12]{64} \Rightarrow \sqrt[3]{3} > \sqrt[4]{4}$

Note : $\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$, and $\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$

Ex. Arrange the following surds in ascending order of their magnitude:

(i) $\sqrt{5}, \sqrt[3]{9}, \sqrt[6]{105}$ (ii) $\sqrt[4]{3}, \sqrt[6]{10}, \sqrt[12]{25}$

Sol. : (i) The given surds are $\sqrt{5}, \sqrt[3]{9}, \sqrt[6]{105}$.

The order of these surds are 2, 3 and 6 respectively.

The L.C.M. of 2, 3, 6 is 6.

So, we convert each surds into a surd of order 6.

Now, $\sqrt{5} = (5)^{\frac{1}{2}} = 5^{\frac{1\times 3}{2\times 3}} = 5^{\frac{3}{6}}$

$= \left(5^3\right)^{\frac{1}{6}} = (125)^{\frac{1}{6}} = \sqrt[6]{125}$

or, we can write simply $\sqrt{5}$ because

$\sqrt{5} = \sqrt[6]{5^3} = \sqrt[6]{125}$

$\Rightarrow \sqrt[3]{9} = \sqrt[6]{9^2} = \sqrt[6]{81}$

and, $\sqrt[6]{105}$ is already a surd of order 6.

Since $81 < 105 < 125$

$\therefore \sqrt[6]{81} < \sqrt[6]{105} < \sqrt[6]{125} \Rightarrow \sqrt[3]{9} < \sqrt[6]{105} < \sqrt{5}$

(ii) The given surds are of order 4, 6 and 12 respectively.

The L.C.M. of 4, 6 and 12 is 12.

So, we shall convert each one of the given surds into a surd of order 12.

Now, $\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$

$\sqrt[6]{10} = \sqrt[12]{10^2} = \sqrt[12]{100}$

and, $\sqrt[12]{25}$ is already a surd of order 12.

Since, $25 < 27 < 100$

Therefore,

$\sqrt[12]{25} < \sqrt[12]{27} < \sqrt[12]{100}$

$\Rightarrow \sqrt[12]{25} < \sqrt[4]{3} < \sqrt[6]{10}$

Addition and Subtraction of Surds

Similar or Like Surds : Surds having same irrational factors are called similar or like surds.

For example, $4\sqrt{3}, 5\sqrt{3}, \sqrt{3}, \frac{1}{5}\sqrt{3}$ etc. are similar surds.

Unlike Surds : Surds having no common irrational factors are called unlike surds.

For example, $\sqrt{3}, 3\sqrt{2}, 6\sqrt{5}$ etc. are unlike surds.

SINCE surds are real numbers and multiplication of real numbers is distributive over the addition of real numbers. Therefore, surds also satisfy the same property. By using the distributive law, like surds can be added and subtracted.

To add or subtract unlike surds, we first reduce each of them to its simplest form and introduce the same irrational factor in each of them.

Symbolically, if $\sqrt[n]{a}$ is a surd, then

$K\sqrt[n]{a} \pm l\sqrt[n]{a} \pm m\sqrt[n]{a} = (K \pm l \pm m)\sqrt[n]{a}$

Ex. Simplify

(i) $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

(ii) $\sqrt{50} - \sqrt{98} + \sqrt{162}$

(iii) $\sqrt{294} - \sqrt{150} + 2\sqrt{6} - 3\sqrt{\frac{1}{6}}$

Sol. : (i) First we reduce each term to its simplest form.

$3\sqrt{12} = 3\sqrt{2\times2\times3} = 3\times2\sqrt{3} = 6\sqrt{3}$

$2\sqrt{75} = 2\sqrt{5\times5\times3} = 2\times5\sqrt{3} = 10\sqrt{3}$

$\therefore 4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

$= 4\sqrt{3} - 6\sqrt{3} + 10\sqrt{3} = (4-6+10)\sqrt{3} = 8\sqrt{3}$

(ii) Reducing each term in the given expression to its simplest form, we have:

$\sqrt{50} = \sqrt{2\times25} = \sqrt{2\times5\times5} = \sqrt{2\times5^2} = 5\sqrt{2}$

$\sqrt{98} = \sqrt{2\times49} = \sqrt{2\times7\times7} = \sqrt{2\times7^2} = 7\sqrt{2}$

$\sqrt{162} = \sqrt{2\times81} = \sqrt{2\times9\times9} = \sqrt{2\times9^2} = 9\sqrt{2}$

$\therefore \sqrt{50} - \sqrt{98} + \sqrt{162} = 5\sqrt{2} - 7\sqrt{2} + 9\sqrt{2}$

$= (5-7+9)\sqrt{2} = 7\sqrt{2}$

(iii) Reducing each term to its simplest form, we get

$\sqrt{294} = \sqrt{7\times7\times6} = 7\sqrt{6}$

$\sqrt{150} = \sqrt{5\times5\times6} = 5\sqrt{6}$

$3\sqrt{\frac{1}{6}} = 3\times\sqrt{\frac{6}{36}} = \frac{3}{6}\sqrt{6} = \frac{1}{2}\sqrt{6}$

$\therefore \sqrt{294} - \sqrt{150} + 2\sqrt{6} - 3\sqrt{\frac{1}{6}}$

$= 7\sqrt{6} - 5\sqrt{6} + 2\sqrt{6} - \frac{1}{2}\sqrt{6}$

$= \left(7-5+2-\frac{1}{2}\right)\sqrt{6} = \frac{7}{2}\sqrt{6}$

Multiplication of Surds

Two and more surds can be multiplied if these are of the same order. Surds of the same order can be multiplied according to the following laws :

(i) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

(ii) $K.\ \sqrt[n]{a} \times l\sqrt[n]{b} \times m\ \sqrt[n]{c} = Klm\ \sqrt[n]{abc}$

For example, (a) $\sqrt{5}\times\sqrt{14} = \sqrt{5\times14} = \sqrt{70}$

(b) $\sqrt[3]{36}\times\sqrt[3]{30} = \sqrt[3]{36\times30} = \sqrt[3]{6^3\times5} = 6\sqrt[3]{5}$

Division of Surds

Surds of the same order may be divided according to the following laws :

(i) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(ii) $\frac{K\sqrt[n]{a}}{l\sqrt[n]{b}} = \frac{K}{l}\sqrt[n]{\frac{a}{b}}$

For example, (a) $\sqrt{24} \div \sqrt{6} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$

(b) $2\sqrt{28} \div 3\sqrt{7} = \frac{2\sqrt{28}}{3\sqrt{7}} = \frac{2\sqrt{7\times2\times2}}{3\sqrt{7}} = \frac{4\sqrt{7}}{3\sqrt{7}} = \frac{4}{3}$

Remark : If the surds to be multiplied or divided are not of the same order, we first reduce them to the same order before applying the above laws. Thus,

(i) $K.\sqrt[n]{a} \times l.\sqrt[m]{b} = K.\sqrt[mn]{a^m} \times l.\sqrt[mn]{b^n}$

$= Kl\sqrt[mn]{a^m b^n}$

(ii) $K.\sqrt[n]{a} \div l.\sqrt[m]{b} = K.\sqrt[mn]{a^m} \div l.\sqrt[mn]{b^n}$

$= \left(\frac{K}{l}\right)\sqrt[mn]{\frac{a^m}{b^n}}$

Ex. Divide $5\sqrt[3]{4}$ by $3\sqrt{2}$.

Sol. : $5\sqrt[3]{4} \div 3\sqrt{2} = \frac{5\sqrt[3]{4}}{3\sqrt{2}}$

We convert the surds to the same order

$= \frac{5\sqrt[6]{4^2}}{3\sqrt[6]{2^3}} = \frac{5\sqrt[6]{16}}{3\sqrt[6]{8}} = \frac{5}{3}\sqrt[6]{\frac{16}{8}} = \frac{5}{3}\sqrt[6]{2}$

Ex. Simplify $\sqrt{8x^5y} \times \sqrt[3]{4x^2y^2}$

Sol. : Since surds are not of the same order, we first convert them into the surds of the same order.

L.C.M. of 2 and 3 = 6

Now, $\sqrt{8x^5y} = \sqrt[6]{(8x^5y)^3}$

$= \sqrt[6]{(2^3)^3 x^{15} y^3} = \sqrt[6]{2^9 x^{15} y^3}$

and, $\sqrt[3]{4x^2y^2} = \sqrt[6]{(4x^2y^2)^2}$

$= \sqrt[6]{(2^2)^2 (x^2)^2 (y^2)^2} = \sqrt[6]{2^4 x^4 y^4}$

$\therefore \sqrt{8x^5y} \times \sqrt[3]{4x^2y^2} = \sqrt[6]{2^9 x^{15} y^3} \times \sqrt[6]{2^4 x^4 y^4}$

$= \sqrt[6]{2^{13} x^{19} y^7} = \sqrt[6]{(2^{12} x^{18} y^6) 2xy}$

$= 2^2 x^3 y \sqrt[6]{2xy} = 4x^3 y \sqrt[6]{2xy}$

Kinds of Surds

Monomial Surd : A surd consisting of one and only one term is called a monomial surd.

For example, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{75}$ etc. are monomial surds.

Now, $\sqrt{2} + \sqrt{5 - 2\sqrt{6}}$ is a monomial surd because

$\sqrt{2} + \sqrt{5 - 2\sqrt{6}} = \sqrt{2}\sqrt{5 - 2.\sqrt{3}.\sqrt{2}}$

$= \sqrt{2} + \sqrt{3 + 2 - 2.\sqrt{3}.\sqrt{2}}$

$= \sqrt{2} + \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 - 2.\sqrt{3}.\sqrt{2}}$

$= \sqrt{2} + \sqrt{(\sqrt{3} - \sqrt{2})^2} = \sqrt{2} + \sqrt{3} - \sqrt{2} = \sqrt{3}$

Binomial Surd : An expression consisting of the sum or difference of two monomial surds or the sum of a monomial surd and a rational number is known as a binomial surd.

For example, $\sqrt{3} + \sqrt{5}$, $\sqrt{7} + 5$, $\sqrt{11} - 2$, $\sqrt{3} - \sqrt{5}$ etc. are binomial surds.

Trinomial Surd : An expression consisting of three terms at least two of which are monomial surds, is called a trinomial surds.

For example, $\sqrt{3} + \sqrt{7} - \sqrt{8}$, $5 + \sqrt{3} + \sqrt{6}$ are trinomial surds.

Rationalisation of Surds

When the product of two surds is a rational number, then each of them is called the Rationalising Factor (R.F.) of the other.

For example, $3\sqrt{5} \times \sqrt{5} = 3 \times \sqrt{5 \times 5} = 3 \times 5 = 15$

$\therefore \sqrt{5}$ is a rationalising factor of $3\sqrt{5}$.

In monomial surds, the rationalising factor of the surds $\sqrt[n]{a}$ is $a^{1-\frac{1}{n}}$.

In a binomial surd of the form $\sqrt{a} \pm \sqrt{b}$, the rationalising factors are $\sqrt{a} \mp \sqrt{b}$.

If a binomial surd is of the form $a \pm \sqrt{b}$, then the rationalising factors are $a \pm \sqrt{b}$.

For example, $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

$= (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

$\therefore \sqrt{5} - \sqrt{3}$ is a R.F. of $\sqrt{5} + \sqrt{3}$.

Remark : The rationalising factor of a given surd is not unique.

Conjugate Surds : Two binomial surds which differ only in sign (+ or –) between the terms connecting them, are known as conjugate surds.

For example, $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are conjugate surds.

Rationalisation of Trinomial Surds

The rationalisation of trinomial surds is done by first grouping two of the terms and then proceeding in the same manner as the rationalisation of binomial surds.

The rationalisation of a trinomial surd $\sqrt{x}+\sqrt{y}+\sqrt{z}$ is done as follows :

$$\left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)\left(\sqrt{x}+\sqrt{y}-\sqrt{z}\right)$$

$$=\left(\sqrt{x}+\sqrt{y}\right)^2-\left(\sqrt{z}\right)^2=\left(x+y+2\sqrt{xy}\right)-z$$

$$=(x+y-z)+2\sqrt{xy}$$

Again multiply by $(x+y-z)-2\sqrt{xy}$

$$\left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)\left(\sqrt{x}+\sqrt{y}-\sqrt{z}\right)\left[(x+y-z)-2\sqrt{xy}\right]$$

$$=\left[(x+y-z)+2\sqrt{xy}\right]\left[(x+y-z)-2\sqrt{xy}\right]$$

$$=(x+y-z)^2-\left(2\sqrt{xy}\right)^2$$

$$=x^2+y^2+z^2+2xy-2yz-2zx-4xy$$

$$=x^2+y^2+z^2-2xy-2yz-2zx$$

SOLVED OBJECTIVE QUESTIONS

1. $\left(\frac{81}{16}\right)^{-\frac{3}{4}}\times\left[\left(\frac{25}{9}\right)^{-\frac{3}{2}}\div\left(\frac{5}{2}\right)^{-3}\right]=?$

(1) 1 (2) 3

(3) $\frac{3}{4}$ (4) $\frac{4}{3}$

2. $\left(\frac{1}{4}\right)^{-2}-3\times(8)^{\frac{2}{3}}\times(4)^0+\left(\frac{9}{16}\right)^{-\frac{1}{2}}=?$

(1) $4\frac{1}{3}$ (2) $5\frac{1}{3}$

(3) $4\frac{1}{2}$ (4) $5\frac{1}{2}$

3. $\frac{16\times2^{n+1}-4\times2^n}{16\times2^{n+2}-2\times2^{n+2}}=?$

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

4. If $\frac{9^n\times3^2\times\left(3^{-\frac{n}{2}}\right)^{-2}-(27)^n}{3^{3m}\times2^3}=\frac{1}{27}$,

then $n-m+1=?$

(1) 1 (2) $\frac{1}{2}$

(3) 0 (4) 2

5. $\frac{2^{2^3}\div(2^2)^3\times2^{-2}}{4^{2^3}\div(4^2)^3\times4^{-2}}=?$

(1) 2 (2) $\frac{1}{2}$

(3) 1 (4) –2

6. $\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}}+\left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0=?$

(1) $3\frac{13}{16}$ (2) $2\frac{13}{16}$

(3) $4\frac{11}{13}$ (4) $5\frac{13}{16}$

7. $\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}+\left(\frac{8}{27}\right)^{-\frac{1}{3}}=?$

(1) 1 (2) 0

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

8. If $5\sqrt{5}\times5^3\div5^{-\frac{3}{2}}=5^{x+2}$, then find the value of x.

(1) 2 (2) 3

(3) 4 (4) 5

9. If $\frac{(81)^{4x}\times(27)^x\times9^7}{(729)^{x+2}}=3^9$, then find the value of x.

(1) $\frac{6}{13}$ (2) $\frac{5}{13}$

(3) $\frac{8}{13}$ (4) $\frac{7}{13}$

10. If x is a positive real number and the index be a rational number, then

(i) $\left(\frac{x^a}{x^b}\right)^{a+b-c}\times\left(\frac{x^b}{x^c}\right)^{b+c-a}\times\left(\frac{x^c}{x^a}\right)^{c+a-b}=?$

(1) $\frac{1}{2}$ (2) 0

(3) 1 (4) 3

10 (ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2+a^2-ca} = ?$

(1) $x^{a^3+b^3+c^3}$ (2) $x^{a^2+b^2+c^2}$

(3) $x^{2(a^3+b^3)}$ (4) $x^{2(a^3+b^3+c^3)}$

11. If abc = 1, then

$\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}} = ?$

(1) 0 (2) $\frac{1}{2}$

(3) $\frac{1}{3}$ (4) 1

12. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{4z}=4$, then x = ?

(1) $\frac{7}{15}$ (2) $\frac{7}{16}$

(3) $\frac{8}{15}$ (4) $\frac{1}{2}$

13. If $9^x - 10\times3^x + 9 = 0$, the value of x is :

(1) 0 (2) 2

(3) 0 If 2 (4) 7

14. If $25^{x-1} = 5^{2x-1} - 100$, the value of x is :

(1) 2 (2) 3

(3) –2 (4) –3

15. If $\left[3^{m^2} \div (3^m)^2\right]^{\frac{1}{m}} = 81$, the value of m is :

(1) 5 (2) 7

(3) 6 (4) 8

16. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, the values of a and b are :

(1) $a = 2$, $b = 3$ (2) $a = 4$, $b = 6$

(3) $a = 3$, $b = 2$ (4) $a = -3$, $b = -2$

17. If $x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then the value of $x^3 - 6x$ is :

(1) 4 (2) 5

(3) –6 (4) 6

18. If $2^{x-7} \times 5^{x-4} = 1250$, then the value of x is :

(1) –8 (2) 8

(3) –4 (4) 4

19. If $3^x = 5^y = (75)^z$, then z = ?

(1) $\frac{xy}{2x+2y}$ (2) $\frac{x}{x+y}$

(3) $\frac{xy}{2x+y}$ (4) $\frac{xy}{x+2y}$

20. If $x+y+z=0$, then the value of

$2^{x^2y^{-1}z^{-1}} . 2^{x^{-1}y^2z^{-1}} . 2^{x^{-1}y^{-1}z^2}$ is :

(1) 2 (2) 8

(3) 4 (4) 6

21. $\frac{\left(2^{2n}-3.2^{2n-2}\right)\left(3^n-2.3^{n-2}\right)}{3^{n-4}\left(4^{n+3}-2^{2n}\right)} = ?$

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $-\frac{1}{2}$ (4) $\frac{1}{6}$

22. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7..........\infty}}}} = ?$

(1) 7 (2) –7

(3) 4 (4) – 4

23. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}} = ?$

(1) $7^{\frac{11}{32}}$ (2) $7^{\frac{13}{32}}$

(3) $7^{\frac{31}{32}}$ (4) $7^{\frac{26}{32}}$

24. $\sqrt{20+\sqrt{20+\sqrt{20+..........\infty}}} = ?$

(1) 2 (2) 3

(3) – 4 (4) 5

25. The simplified value of $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$ is

(1) $\frac{114+41\sqrt{6}}{30}$ (2) $\frac{114-41\sqrt{6}}{30}$

(3) $\frac{115-41\sqrt{6}}{30}$ (4) $\frac{116-41\sqrt{6}}{30}$

26. If $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=a-b\sqrt{6}$, then the value of $(a+b)^2$ is :

(1) $\frac{13}{36}$ (2) $\frac{49}{36}$

(3) $\frac{7}{6}$ (4) $\frac{16}{39}$

27. The simplified value $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}$ is :

(1) $8\sqrt{5}$ (2) $-8\sqrt{5}$

(3) $4\sqrt{5}$ (4) $-4\sqrt{5}$

28. The simplifed value of

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$ is :

(1) 1 (2) 2

(3) 4 (4) 6

29. The simplified value of

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$ is :

(1) 3 (2) 4

(3) 5 (4) 6

30. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of x^2+xy+y^2 is :

(1) 100 (2) 98

(3) 97 (4) 99

31. If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$, then the value of

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$ is :

(1) 4.398 (2) 5.398

(3) 4.938 (4) 5.938

32. If $x=5-\sqrt{24}$, then the value of

$$\left(x^3+\frac{1}{x^3}\right)-10\left(x^2+\frac{1}{x^2}\right)+4\left(x+\frac{1}{x}\right)-30$$ is

(1) 0 (2) 3

(3) 4 (4) -2

33. If $x=\frac{1}{2-\sqrt{3}}$, then the value of x^3-2x^2-7x+5 is :

(1) 2 (2) 3

(3) -3 (4) -2

34. If $x=\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, then the value of bx^2-ax+b is :

(1) 0 (2) -1

(3) 1 (4) -2

35. If $x=\sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}}$, then the value of $x^2(x-10)^2$ is

(1) 0 (2) 1

(3) -1 (4) 2

36. Find the value of $\frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$.

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $-\frac{2}{3}$ (4) $\frac{4}{3}$

37. $\sqrt{14\sqrt{5}-30}$ = ?

(1) $\sqrt{5}(3-\sqrt{5})$ (2) $\sqrt{5}(3+\sqrt{5})$

(3) $\sqrt[3]{5}(3-\sqrt{5})$ (4) $\sqrt[4]{5}(3-\sqrt{5})$

38. $\sqrt{\left(\frac{6+2\sqrt{3}}{33-19\sqrt{3}}\right)}$ = ?

(1) $5+3\sqrt{3}$ (2) $5-3\sqrt{3}$

(3) $5+2\sqrt{3}$ (4) $5-2\sqrt{3}$

39. Find the value of $\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}$

(1) $\sqrt{2}$ (2) $\sqrt{2}+1$

(3) 1 (4) $\sqrt{2}+2$

40. Find the value of $\sqrt{4-\sqrt{7}}+\sqrt{8+3\sqrt{7}}$.

(1) $\sqrt{2}(\sqrt{7}+1)$ (2) $\sqrt{3}(\sqrt{7}-1)$

(3) $\sqrt{2}(\sqrt{7}-1)$ (4) $\sqrt{3}(\sqrt{7}+1)$

41. Find the value of $\sqrt{38+5\sqrt{3}}+\sqrt{3-\sqrt{5}}$.

(1) $\frac{5\sqrt{6}-\sqrt{10}}{2}$ (2) $\frac{5\sqrt{6}+\sqrt{10}}{2}$

(3) $\frac{5\sqrt{6}-\sqrt{10}}{4}$ (4) $\frac{5\sqrt{6}+\sqrt{10}}{4}$

42. Find the value of $\sqrt{8+3\sqrt{7}}-\sqrt{7+3\sqrt{5}}$.

(1) $\frac{\sqrt{14}+\sqrt{10}}{2}$ (2) $\frac{\sqrt{14}-\sqrt{10}}{2}$

(3) $\frac{\sqrt{14}+\sqrt{10}}{4}$ (4) $\frac{\sqrt{14}-\sqrt{10}}{4}$

43. $\sqrt{11+4\sqrt{7}}-\sqrt{11-4\sqrt{7}}+\frac{\sqrt{8}}{\sqrt{33+10\sqrt{8}}-\sqrt{33-10\sqrt{8}}}=?$

(1) $2\frac{1}{2}$ (2) $3\frac{1}{2}$

(3) $1\frac{1}{2}$ (4) $4\frac{1}{2}$

44. $\left(18-8\sqrt{2}\right)^{\frac{1}{2}}+\left(6-4\sqrt{2}\right)^{-\frac{1}{2}}+\frac{\sqrt{3}}{\sqrt{19+8\sqrt{3}}-\sqrt{19-8\sqrt{3}}}=?$

(1) $\frac{11-\sqrt{2}}{2}$ (2) $\frac{11+\sqrt{2}}{2}$

(3) $\frac{11-2\sqrt{2}}{2}$ (4) $\frac{11+2\sqrt{2}}{2}$

45. $\sqrt{7+4\sqrt{3}}-\sqrt{28+10\sqrt{3}}+\frac{\sqrt{11}}{\sqrt{20+6\sqrt{11}}+\sqrt{20-6\sqrt{11}}}=?$

(1) $2\frac{1}{2}$ (2) $1\frac{1}{2}$

(3) $-1\frac{1}{2}$ (4) $-2\frac{1}{2}$

46. $(9)^{-3}\times\frac{(16)^{\frac{1}{4}}}{(6)^{-2}}\times\left(\frac{1}{27}\right)^{-\frac{4}{3}}=?$

(1) 2 (2) 4

(3) 6 (4) 8

47. $\frac{\sqrt[6]{2}\left[(625)^{\frac{3}{5}}\times(1024)^{\frac{-6}{5}}+(25)^{\frac{3}{5}}\right]^{\frac{1}{2}}}{\left(\sqrt[3]{128}\right)^{\frac{-5}{2}}\times(125)^{\frac{1}{5}}}+\frac{\left(10^{3}\right)^{2}+\left(10^{3^{2}}\right)}{\left(10^{2}\right)^{3}+10^{2^{3}}}$

(1) 1.2 (2) 1.4

(3) 1.6 (4) 1.1

48. If $10^{0.48}=x,\ 10^{0.7}=y$ and $x^{z}=y^{2}$, then $z=?$

(1) $1\frac{11}{12}$ (2) $2\frac{11}{12}$

(3) $3\frac{11}{12}$ (4) $-2\frac{11}{12}$

49. The simplifed value of $(27)^{\frac{-2}{3}}+\left[\left(2^{\frac{-2}{3}}\right)^{\frac{-5}{3}}\right]^{\frac{-9}{10}}$ is :

(1) $\frac{11}{18}$ (2) $\frac{-11}{18}$

(3) $\frac{17}{18}$ (4) $\frac{-17}{18}$

50. The simplified value of $\frac{(0.3)^{\frac{1}{3}}\left(\frac{1}{27}\right)^{\frac{1}{4}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}}}{(0.9)^{\frac{2}{3}}(3)^{-\frac{1}{2}}(243)^{-\frac{1}{4}}}$ is :

(1) 2.2 (2) 2.7

(3) 2.4 (4) 2.6

51. The simplified value of

$$\sqrt{\frac{(12.12)^{2}-(8.12)^{2}}{(0.25)^{2}+(0.25)\times(19.99)}}+\frac{\left[\left(8^{\frac{-3}{4}}\right)^{\frac{5}{2}}\right]^{\frac{8}{15}}\times16^{\frac{3}{4}}}{\sqrt[3]{\left[\left\{\left(128\right)^{-5}\right)^{\frac{3}{7}}\right\}^{\frac{-1}{5}}\right]}}$$ is :

(1) $3\frac{1}{2}$ (2) $2\frac{1}{2}$

(3) $4\frac{1}{2}$ (4) $5\frac{1}{2}$

52. $\frac{2^{n+4}-2\times2^{n}}{2\times2^{n+3}}+2^{-3}=?$

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) $\frac{1}{3}$

53. $\left[\frac{(1.331)^{-1}+(1.331)^{-2}+(1.331)^{-3}+....+(1.331)^{-6}}{(1.331)^{-2}+(1.331)^{-3}+(1.331)^{-4}+...+(1.331)^{-7}}\right]^{\frac{1}{3}}+1.1=?$

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 2 (4) 1

54. $\left[\frac{1.2.4+2.4.8+3.6.12+......}{1.3.9.+2.6.18+3.9.27+......}\right]^{\frac{1}{3}}=?$

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $\frac{3}{4}$ (4) $\frac{3}{5}$

55. $\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{\sqrt{7-2\sqrt{10}}}-\frac{4}{\sqrt{8+4\sqrt{3}}}=?$

(1) 0 (2) 1

(3) –1 (4) $-2\sqrt{3}$

56. $\frac{4\sqrt{3}}{2-\sqrt{2}}-\frac{30}{4\sqrt{3}-\sqrt{18}}-\frac{\sqrt{18}}{3-2\sqrt{3}}=?$

(1) $2\sqrt{6}$ (2) $\sqrt{6}$

(3) $3\sqrt{6}$ (4) $4\sqrt{6}$

57. The simplified value of

$(28-10\sqrt{3})^{\frac{1}{2}}-(7+4\sqrt{3})^{-\frac{1}{2}}+\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$ is :

(1) $1\frac{1}{2}$ (2) $2\frac{1}{2}$

(3) $3\frac{1}{2}$ (4) $4\frac{1}{2}$

58. The simplified value of $\frac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}$ is :

(1) 1 (2) 2

(3) –1 (4) –2

59. The simplifed value of

$\frac{26-15\sqrt{3}}{\left[5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right]^2}+\frac{\sqrt{10}+\sqrt{18}}{\sqrt{8}+\sqrt{(3-\sqrt{5})}}$ is :

(1) $1\frac{1}{3}$ (2) $2\frac{1}{3}$

(3) $3\frac{1}{3}$. (4) $4\frac{1}{3}$

60. The simplified value of $(28+10\sqrt{3})^{\frac{1}{2}}-(7-4\sqrt{3})^{-\frac{1}{2}}$ is :

(1) 1 (2) 2

(3) 3 (4) 4

61. The simplified value of $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ is :

(1) 0 (2) 2

(3) 3 (4) –2

62. The simplified value of $\sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}$ is :

(1) $\sqrt{3}$ (2) $\sqrt{3}+1$

(3) $\sqrt{3}-1$ (4) $\sqrt{3}+\sqrt{2}$

63. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=?$

(1) $-\sqrt{3}$ (2) $\sqrt{3}$

(3) $-\sqrt{2}$ (4) $\sqrt{2}$

64. $\left[\frac{1\times3\times9+2\times6\times18+3\times9\times27+......}{1\times5\times25+2\times10\times50+3\times15\times75+......}\right]^{\frac{1}{3}}=?$

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$

(3) $\frac{3}{5}$ (4) $\frac{4}{5}$

65. $(-1)^{(-1)^{(-1)......}}=?$

(1) 1 (2) –1

(3) 2 (4) –2

66. $\frac{(625)^{6.25}\times(25)^{2.6}}{(625)^{6.75}\times(5)^{1.2}}=?$

(1) 5 (2) 10

(3) 15 (4) 25

67. $(16)^{0.16}\times(2)^{0.36}=?$

(1) 2 (2) 4

(3) –2 (4) –4

68. The simplified value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ is :

(1) $7+4\sqrt{3}$ (2) $7-4\sqrt{3}$

(3) $7+2\sqrt{3}$ (4) $7-2\sqrt{3}$

69. The simplifed value of $\sqrt{\frac{19+8\sqrt{3}}{7-4\sqrt{3}}}$ is :

(1) $11-6\sqrt{3}$ (2) $11+6\sqrt{3}$
(3) $10+5\sqrt{3}$ (4) $10-5\sqrt{3}$

70. $\sqrt{\frac{(2.4)^6+9(5.76)+6(2.4)^4}{(2.4)^4+6(5.76)+9}}$

$$+\frac{\left[(3^{-2})^{-5}\right]^{\frac{1}{5}}+\left[(4^{-3})^{-6}\right]^{\frac{1}{6}}-(3^{-4})^{\frac{-1}{2}}}{+\left[(2^{-3})^{-4}\right]^{\frac{1}{4}}}=?$$

(1) 10.2 (2) 9.4
(3) 10.4 (4) 10.8

71. $\sqrt{11-2\sqrt{30}}+\sqrt{7-2\sqrt{10}}-\frac{4}{\sqrt{6}+\sqrt{2}}=?$

(1) 0 (2) 3
(3) −1 (4) 2

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

72. If $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$, then x is equal to

(1) −2 (2) 2
(3) −1 (4) 1

[SSC Graduate Level Tier-I Exam, 2010]

73. $\sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}$ is equal to

(1) $(\sqrt{2}+\sqrt{3}-\sqrt{5})$ (2) $(\sqrt{3}+\sqrt{5}-\sqrt{2})$
(3) $(\sqrt{2}+\sqrt{5}-\sqrt{3})$ (4) $(\sqrt{2}+\sqrt{3}+\sqrt{5})$

[SSC Graduate Level Prelim Exam, 2007]

74. If $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$, then the value of a is

(1) $\frac{11}{3}$ (2) $-\frac{4}{3}$
(3) $\frac{4}{3}$ (4) $\frac{-4\sqrt{7}}{3}$

[SSC CPO Sub-Inspector Exam, 2007]

75. $\left[\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\right]$ is simplified to

(1) 0 (2) 1
(3) $\sqrt{3}$ (4) $\sqrt{6}$

[SSC CPO Sub-Inspector Exam, 2007]

76. For what value(s) of a is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square?

(1) $\pm\frac{1}{18}$ (2) $\pm\frac{1}{8}$
(3) $-\frac{1}{5}$ (4) $\frac{1}{4}$

[SSC CPO Sub-Inspector Exam, 2006]

77. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is equal to

(1) 1 (2) $2/\sqrt{3}$
(3) $2-\sqrt{3}$ (4) 2

[SSC CPO Sub-Inspector Exam, 2006]

78. If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $(x+y)$ equals :

(1) 8 (2) 16
(3) $2\sqrt{15}$ (4) $2(\sqrt{5}+\sqrt{3})$

[SSC Graduate Level Prelim Exam, 2005]

79. If $\sqrt{2}=1.414$, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to

(1) 0.172 (2) 0.414
(3) 0.586 (4) 1.414

[SSC CPO Sub-Inspector Exam, 2003]

80. Which of the following numbers is the least ?

$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23

(1) $(0.5)^2$ (2) $\sqrt{0.49}$
(3) $\sqrt[3]{0.008}$ (4) 0.23

[SSC Graduate Level Prelim Exam, 2002]

81. The value of $\sqrt{\frac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}}$ is

(1) $\sqrt{6}-\sqrt{2}$ (2) $\sqrt{6}+\sqrt{2}$
(3) $\sqrt{6}-2$ (4) $2-\sqrt{6}$

[SSC Graduate Level Prelim Exam, 2002]

82. Arrange the following in descending order : $\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$

(1) $\sqrt[3]{4}>\sqrt[4]{5}>\sqrt{2}>\sqrt[6]{3}$ (2) $\sqrt[4]{5}>\sqrt[3]{4}>\sqrt[6]{3}>\sqrt{2}$
(3) $\sqrt{2}>\sqrt[6]{3}>\sqrt[3]{4}>\sqrt[4]{5}$ (4) $\frac{3.8}{4+0.632}=\frac{3.8}{4.632}=0.82$

[SSC Graduate Level Prelim Exam, 2003]

83. $\left[\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right]$

simplifies to

(1) $2\sqrt{6}$ (2) s $4\sqrt{6}$

(3) $2\sqrt{3}$ (4) $3\sqrt{2}$

[SSC Graduate Level Prelim. Exam., 2008]

84. $\left[\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}\right]$

is simplified to

(1) $\sqrt{6}$ (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) 0

[SSC Graduate Level Prelim. Exam., 2008]

85. By how much does $\left(\sqrt{12}+\sqrt{18}\right)$ exceed $\left(2\sqrt{3}+2\sqrt{2}\right)$?

(1) 2 (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) 3

[SSC Graduate Level Prelim. Exam., 2008]

86. The greatest number among $\sqrt{5}, \sqrt[3]{4}, \sqrt[5]{2}, \sqrt[7]{3}$ is

(1) $\sqrt[3]{4}$ (2) $\sqrt[7]{3}$

(3) $\sqrt{5}$ (4) $\sqrt[5]{2}$

[SSC Graduate Level Tier-II Exam, 2010]

87. $\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$

[SSC Graduate Level Tier-II Exam, 2010]

88. The largest among the numbers

$\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$ is

(1) $\sqrt{7}-\sqrt{5}$ (2) $\sqrt{5}-\sqrt{3}$

(3) $\sqrt{9}-\sqrt{7}$ (4) $\sqrt{11}-\sqrt{9}$

[SSC Graduate Level Tier-II Exam, 2010]

89. If $\sqrt{7\sqrt{7\sqrt{7\sqrt{7....}}}} = (343)^{y-1}$, then y is equal to

(1) $\frac{2}{3}$ (2) 1

(3) $\frac{4}{3}$ (4) $\frac{3}{4}$

[SSC Graduate Level Tier-II Exam, 2010]

90. $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}$ is equal to

(1) $\sqrt[3]{2}+1$ (2) $\sqrt[3]{2}-1$

(3) $-\sqrt[3]{2}-1$ (4) $1-\sqrt[3]{2}$

[SSC SAS Exam, 2010]

91. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+ .... + \frac{1}{\sqrt{99}+\sqrt{100}}$

is equal to

(1) $10-\sqrt{99}$ (2) $\sqrt{2}-10$

(3) 7 (4) 9

[SSC SAS Exam, 2010]

92. If $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}$, then x^4+x^{-4} is

(1) a surd

(2) a rational number but not an integer

(3) an integer

(4) an irrational number but not a surd

[SSC Graduate Level Tier-I Exam, 2012]

93. If $x+\frac{1}{x} = 3$, the value of $x^5+\frac{1}{x^5}$ is

(1) 123 (2) 126

(3) 113 (4) 129

[SSC Graduate Level Tier-I Exam, 2012]

94. If $\sqrt{6}\times\sqrt{15} = x\sqrt{10}$, then the value of x is

(1) 3 (2) ± 3

(3) $\sqrt{3}$ (4) $\sqrt{6}$

95. If $x=\frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$, then the value of $\frac{x+\sqrt{8}}{x-\sqrt{8}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$ is :

(1) 1 (2) 2

(3) −1 (4) −2

[SSC Graduate Level Tier-I Exam, 2012]

96. If $a=\frac{2+\sqrt{3}}{2-\sqrt{3}}$ and $b=\frac{2-\sqrt{3}}{2+\sqrt{3}}$, then the value of (a^2+b^2+ab) is :

(1) 185 (2) 195

(3) 200 (4) 175

[SSC Graduate Level Tier-I Exam, 2012]

97. If $x=\frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$, then the value of $\frac{x+\sqrt{2}}{x-\sqrt{2}}+\frac{x+\sqrt{3}}{x-\sqrt{3}}$ is :

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $\sqrt{6}$ (4) 2

[SSC Graduate Level Tier-I Exam, 2012]

98. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, $b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of $\frac{a^2}{b}+\frac{b^2}{a}$ is

(1) 900 (2) 970
(3) 1030 (4) 930

[SSC Graduate Level Tier-I Exam, 2012]

99. If $2\sqrt{x} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then the value of x is

(1) 6 (2) 30
(3) $\sqrt{15}$ (4) 15

[SSC Graduate Level Tier-I Exam, 2012]

100. If $x = 2 + \sqrt{3}$, then the value, of $\sqrt{x}+\frac{1}{\sqrt{x}}$ is:

(1) $\sqrt{3}$ (2) $\sqrt{6}$
(3) $2\sqrt{6}$ (4) 6

[SSC Graduate Level Tier-I Exam, 2012]

101. If $\sqrt{3} = 1.732$, the value of

$\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is

(1) 4.899 (2) 2.551
(3) 1.414 (4) 1.732

[SSC Graduate Level Tier-I Exam, 2012]

102. If $x = \frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+x}+\sqrt{1-x}$ will be

(1) $\frac{1}{\sqrt{3}}$ (2) $2\sqrt{3}$
(3) $\sqrt{3}$ (4) 2

[SSC Graduate Level Tier-I Exam, 2012]

103. The value of $\frac{(81)^{3.6}\times(9)^{2.7}}{(81)^{4.2}\times(3)}$ is

(1) 3 (2) 6
(3) 9 (4) 8.2

[SSC Graduate Level Tier-II Exam, 2011]

104. $\sqrt{6+\sqrt{6+\sqrt{6+.....}}}$ is equal to

(1) 2 (2) 5
(3) 4 (4) 3

[SSC Graduate Level Tier-I Exam, 2012 & SSC Graduate Level Tier-II Exam, 2011]

105. The largest among the numbers $(0.1)^2$, $\sqrt{0.0121}$, 0.12 and $\sqrt{0.0004}$ is

(1) $(0.1)^2$ (2) $\sqrt{0.0121}$
(3) 0.12 (4) $\sqrt{0.0004}$

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

ANSWERS

1. (1)	**2.** (2)	**3.** (3)	**4.** (3)	**5.** (1)
6. (1)	**7.** (2)	**8.** (3)	**9.** (4)	**10.** (1)
11. (4)	**12.** (2)	**13.** (3)	**14.** (1)	**15.** (3)
16. (3)	**17.** (4)	**18.** (2)	**19.** (3)	**20.** (2)
21. (2)	**22.** (1)	**23.** (3)	**24.** (4)	**25.** (2)
26. (2)	**27.** (2)	**28.** (1)	**29.** (3)	**30.** (4)
31. (2)	**32.** (1)	**33.** (2)	**34.** (1)	**35.** (2)
36. (4)	**37.** (4)	**38.** (1)	**39.** (3)	**40.** (1)
41. (2)	**42.** (2)	**43.** (4)	**44.** (1)	**45.** (4)
46. (4)	**47.** (4)	**48.** (2)	**49.** (1)	**50.** (2)
51. (3)	**52.** (1)	**53.** (4)	**54.** (2)	**55.** (1)
56. (4)	**57.** (3)	**58.** (1)	**59.** (2)	**60.** (3)
61. (2)	**62.** (3)	**63.** (2)	**64.** (3)	**65.** (2)
66. (4)	**67.** (1)	**68.** (2)	**69.** (2)	**70.** (3)
71. (1)	**72.** (3)	**73.** (4)	**74.** (2)	**75.** (1)
76. (2)	**77.** (2)	**78.** (1)	**79.** (2)	**80.** (3)
81. (3)	**82.** (1)	**83.** (2)	**84.** (4)	**85.** (3)
86. (3)	**87.** (1)	**88.** (2)	**89.** (3)	**90.** (2)
91. (4)	**92.** (3)	**93.** (1)	**94.** (1)	**95.** (2)
96. (2)	**97.** (4)	**98.** (2)	**99.** (4)	**100.** (2)
101. (4)	**102.** (3)	**103.** (3)	**104.** (4)	**105.** (3)

EXPLANATIONS

1. (1) Expression $= \left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

$= \left(\frac{16}{81}\right)^{\frac{3}{4}} \times \left[\left(\frac{9}{25}\right)^{\frac{3}{2}} \div \left(\frac{2}{5}\right)^{3}\right]$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} \times \left[\left(\frac{3^2}{5^2}\right)^{\frac{3}{2}} \div \left(\frac{2}{5}\right)^{3}\right] = \left(\frac{2}{3}\right)^{4\times\frac{3}{4}} \times \left[\left(\frac{3}{5}\right)^{2}\right]^{\frac{3}{2}} \div \left[\left(\frac{2}{5}\right)^{3}\right]$

$= \left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{3}{5}\right)^{2\times\frac{3}{2}} \div \left(\frac{2}{5}\right)^{3}\right] = \left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right]$

$$= \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \div \frac{2^3}{5^3}\right] = \frac{2^3}{3^3} \times \frac{3^3}{5^3} \times \frac{5^3}{2^3} = 1$$

2. (2) Expression = $\left(\frac{1}{4}\right)^{-2} - 3 \times (8)^{\frac{2}{3}} \times (4)^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$

$$= \left[\left(\frac{1}{2}\right)^2\right]^{-2} - 3\left[(2^3)^{\frac{2}{3}} \times 1\right] + \left[\left(\frac{3}{4}\right)^2\right]^{-\frac{1}{2}}$$

$$= \left(\frac{1}{2}\right)^{-4} - 3 \times 2^2 + \left(\frac{3}{4}\right)^{-1}$$

$$= 2^4 - 3 \times 2^2 + \frac{4}{3} = 16 - 12 + \frac{4}{3}$$

$$= \frac{48 - 36 + 4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

3. (3) Expression = $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

$$= \frac{2^4 \times 2^{n+1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+2}} = \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n+3}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2 \times 2^{n+5} - 2 \times 2^{n+2}} = \frac{2^{n+5} - 2^{n+2}}{2(2^{n+5} - 2^{n+2})} = \frac{1}{2}$$

4. (3) $\frac{9^n \times 3^2 \times \left(3^{-\frac{n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$$\Rightarrow \frac{(3^2)^n \times 3^2 \times (3)^{-\frac{n}{2} \times -2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27} \quad \Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{27} \quad \Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow 3^{3n-3m} = \frac{1}{3^3} \quad \Rightarrow 3^{3n-3m} = 3^{-3}$$

$\Rightarrow 3n - 3m = -3$ (घातों को बराबर करने पर)

$\Rightarrow n - m = -1 \quad \Rightarrow n - m + 1 = 0$

5. (1) Expression = $\frac{2^{2^3} \div (2^2)^3 \times 2^{-2}}{4^{2^3} \div (4^2)^3 \times 4^{-2}}$

$$= \frac{2^{2\times2\times2} \div 2^{2\times3} \times 2^{-2}}{4^{2\times2\times2} \div 4^{2\times3} \times 4^{-2}}$$

$$= \frac{2^8 \div 2^6 \times 2^{-2}}{4^8 \div 4^6 \times 4^{-2}} = \frac{2^8 \times \frac{1}{2^6} \times \frac{1}{2^2}}{4^8 \times \frac{1}{4^6} \times \frac{1}{4^2}} = 1$$

6. (1) Expression = $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$

$$= \left\{\left(\frac{4}{5}\right)^3\right\}^{\frac{-2}{3}} + \frac{1}{\left\{\left(\frac{4}{5}\right)^4\right\}^{\frac{1}{4}}} + 1 \qquad [\because a^0 = 1]$$

$$= \left(\frac{4}{5}\right)^{3\times\frac{-2}{3}} + \frac{1}{\left(\frac{4}{5}\right)^{4\times\frac{1}{4}}} + 1 = \left(\frac{4}{5}\right)^{-2} + \frac{1}{\frac{4}{5}} + 1$$

$$= \left(\frac{5}{4}\right)^2 + \frac{5}{4} + 1 \qquad \left[\because \left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x\right]$$

$$= \frac{25}{16} + \frac{5}{4} + 1 = \frac{25 + 20 + 16}{16} = \frac{61}{16} = 3\frac{13}{16}$$

7. (2) Expression

$$= \frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} + \left(\frac{8}{27}\right)^{-\frac{1}{3}}$$

$$= \frac{1 - \left(\frac{1}{10}\right)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} + \left(\frac{8}{27}\right)^{-\frac{1}{3}}$$

$$= \frac{1 - 10}{\frac{8}{3} \times \frac{27}{8} + (-3)} + \left(\frac{27}{8}\right)^{\frac{1}{3}}$$

$$= \frac{-9}{9 - 3} + \left[\left(\frac{3}{2}\right)^3\right]^{\frac{1}{3}} = \frac{-9}{6} + \frac{3}{2} = \frac{-3}{2} + \frac{3}{2} = 0$$

8. (3) $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{x+2}$

$$\Rightarrow 5 \times 5^{\frac{1}{2}} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{x+2}$$

$$\Rightarrow \frac{5 \times 5^{\frac{1}{2}} \times 5^3}{5^{-\frac{3}{2}}} = 5^{x+2} \Rightarrow \frac{5^{1+\frac{1}{2}+3}}{5^{-\frac{3}{2}}} = 5^{x+2}$$

$$\Rightarrow \frac{5^{\frac{9}{2}}}{5^{-\frac{3}{2}}} = 5^{x+2} \qquad \Rightarrow 5^{\frac{9}{2}+\frac{3}{2}} = 5^{x+2}$$

$$\Rightarrow 5^6 = 5^{x+2} \Rightarrow x+2=6 \Rightarrow x = 6-2 = 4$$

9. (4) $\frac{(81)^{4x}\times(27)^x\times 9^7}{(729)^{x+2}} = 3^9$

$$\Rightarrow \frac{(3^4)^{4x}\times(3^3)^x\times(3^2)^7}{(3^6)^{x+2}} = 3^9$$

$$\Rightarrow \frac{3^{16x}\times 3^{3x}\times 3^{14}}{3^{6x+12}} = 3^9$$

$$\Rightarrow \frac{3^{16x+3x+14}}{3^{6x+12}} = 3^9 \qquad \Rightarrow 3^{19x+14-6x-12} = 3^9$$

$$\Rightarrow 3^{13x+2} = 3^9 \qquad \Rightarrow 13x+2 = 9$$

$$\Rightarrow 13x = 9-2 = 7 \qquad \Rightarrow x = \frac{7}{13}$$

10. (i) (3) $\left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b}$

$$= (x)^{(a-b)(a+b-c)} \times (x)^{(b-c)(b+c-a)} \times (x)^{(c-a)(c+a-b)}$$

$$= x^{(a-b)(a+b)-c(a-b)} \times x^{(b-c)(b+c)-a(b-c)} \times x^{(c-a)(c+a)-b(c-a)}$$

$$= x^{a^2-b^2-ac+bc} \times x^{b^2-c^2-ab+ac} \times x^{c^2-a^2-bc+ab}$$

$$= x^{a^2-b^2-ac+bc+b^2-c^2-ab+ac+c^2-a^2-bc+ab}$$

$$= x^0 = 1$$

(ii) (4) $\left(\frac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2+a^2-ca}$

$$= \left(x^{a+b}\right)^{a^2+b^2-ab} \times \left(x^{b+c}\right)^{b^2+c^2-bc} \times \left(x^{c+a}\right)^{c^2+a^2-ca}$$

$$= x^{(a+b)(a^2+b^2-ab)} \times x^{(b+c)(b^2+c^2-bc)} \times x^{(c+a)(c^2+a^2-ca)}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3} \qquad = x^{2(a^3+b^3+c^3)}$$

11. (4) $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$$= \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$$

$$= \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{1}{1+\frac{1}{ab}+\frac{1}{a}}$$

$$\left[\because abc = 1 \therefore \frac{1}{c} = ab \text{ and } c = \frac{1}{ab}\right]$$

$$= \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{ab}{ab+1+b} = \frac{b+1+ab}{b+ab+1} = 1$$

12. (2) It is given $2^x = 4^y = 8^z = k$ (Let)

$$\Rightarrow 2 = k^{\frac{1}{x}}, 4 = k^{\frac{1}{y}}, 8 = k^{\frac{1}{z}}$$

Now, $4\times 8 = 32 = 2^5$

$$\Rightarrow k^{\frac{1}{y}} \times k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^5 \qquad \Rightarrow k^{\frac{1}{y}+\frac{1}{z}} = k^{\frac{5}{x}}$$

$$\Rightarrow \frac{1}{y}+\frac{1}{z} = \frac{5}{x}$$

Now, $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{4z} = 4$

$$\Rightarrow \frac{1}{2x}+\frac{1}{4}\left(\frac{1}{y}+\frac{1}{z}\right) = 4 \qquad \Rightarrow \frac{1}{2x}+\frac{1}{4}\left(\frac{5}{x}\right) = 4$$

$$\Rightarrow \frac{1}{2x}+\frac{5}{4x} = 4 \qquad \Rightarrow \frac{2+5}{4x} = 4$$

$$\Rightarrow \frac{7}{4x} = 4 \Rightarrow 16x = 7 \qquad \Rightarrow x = \frac{7}{16}$$

13. (3) $9^x - 10\times 3^x + 9 = 0$

$$\Rightarrow (3^2)^x - 10\times 3^x + 9 = 0$$

$$\Rightarrow 3^{2x} - 10\times 3^x + 9 = 0$$

Let, $3^x = y$

then, $y^2 - 10y + 9 = 0$

$$\Rightarrow y^2 - 9y - y + 9 = 0$$

$$\Rightarrow y(y-9) - 1(y-9) = 0$$

$$\Rightarrow (y-9)(y-1) = 0 \qquad \Rightarrow y = 9 \text{ or } 1$$

Now, $y = 9 \Rightarrow 3^x = 9 = 3^2 \Rightarrow x = 2$

and, $y = 1 \Rightarrow 3^x = 1 = 3^0 \Rightarrow x = 0$

Hence, $x = 0$ and 2

14. (1) $25^{x-1} = 5^{2x-1} - 100$

$$\Rightarrow (5^2)^{x-1} = 5^{2x-1} - 100$$

$$\Rightarrow 5^{2x-2} - 5^{2x-1} = -100$$

$$\Rightarrow 5^{2x-2} - 5^{2x-2}5^1 = -100$$

$$\Rightarrow 5^{2x-2}(1-5) = -100$$

$\Rightarrow 5^{2x-2} \times -4 = -100$

$\Rightarrow 5^{2x-2} = 25$ $\Rightarrow 5^{2x-2} = 5^2$

$\Rightarrow 2x-2=2$ $\Rightarrow 2x=4 \Rightarrow x=2$

15. (3) $\left[3^{m^2} + (3^m)^2\right]^{\frac{1}{m}} = 81 \Rightarrow \left[3^{m^2} \div 3^{2m}\right]^{\frac{1}{m}} = 81$

$\Rightarrow \left[3^{m^2-2m}\right]^{\frac{1}{m}} = 3^4 \Rightarrow 3^{m(m-2)\times\frac{1}{m}} = 3^4$

$\Rightarrow 3^{m-2} = 3^4 \Rightarrow m-2=4 \Rightarrow m=4+2=6$

16. (3) The equations are

$2^a + 3^b = 17$ and $2^a \times 2^2 - 3^b \times 3 = 5$

Let $x = 2^a$ and $y = 3^b$

then, $x+y=17$............(i)

and $4x-3y=5$..........(ii)

By equation (i) × 3 + (ii),

$$\begin{array}{r} 3x+3y=51 \\ 4x-3y=5 \\ \hline 7x \quad =56 \end{array}$$

$$\therefore x = \frac{56}{7} = 8$$

From equation (i),

$y = 17 - x = 17 - 8 = 9$

$\therefore$ $x=8$ and $y=9$

$\therefore$ $x = 2^a = 8 \Rightarrow 2^a = 2^3 \Rightarrow a=3$

and $y = 3^b = 9 \Rightarrow 3^b = 3^2 \Rightarrow b=2$

$\therefore$ $a=3$ and $b=2$

17. (4) $x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

Cubing both sides,

$$x^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

$$\Rightarrow x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3.2^{\frac{1}{3}}.2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$$

$\left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$

$\Rightarrow x^3 = 2 + 2^2 + 3\times 2 \times x$

$\Rightarrow x^3 = 6 + 6x$ $\Rightarrow x^3 - 6x = 6$

18. (2) $2^{x-7} \times 5^{x-4} = 1250$

$\Rightarrow 2^{x-7} \times 5^{x-4} = 2 \times 625$

$\Rightarrow 2^{x-7} \times 5^{x-4} = 2^1 \times 5^4$

$\Rightarrow x-7=1$ and $x-4=4$ $\Rightarrow x=8$

19. (3) Let $3^x = 5^y = (75)^z = k$

Then, $3 = k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 75 = k^{\frac{1}{z}}$

Now, $75 = k^{\frac{1}{z}} \Rightarrow 25 \times 3 = k^{\frac{1}{z}}$

$$\Rightarrow 5^2 \times 3 = k^{\frac{1}{z}} \Rightarrow \left(k^{\frac{1}{y}}\right)^2 \times \left(k^{\frac{1}{x}}\right) = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}+\frac{1}{x}} = k^{\frac{1}{z}} \Rightarrow \frac{2}{y} + \frac{1}{x} = \frac{1}{z} \Rightarrow \frac{2x+y}{xy} = \frac{1}{z}$$

$$\Rightarrow z = \frac{xy}{2x+y}$$

20. (2) Expression

$= 2^{x^2y^{-1}z^{-1}} . 2^{x^{-1}y^2z^{-1}} . 2^{x^{-1}y^{-1}z^2}$

$$= 2^{\frac{x^2}{yz}} \times 2^{\frac{y^2}{xz}} \times 2^{\frac{z^2}{xy}} = 2^{\frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}} = 2^{\frac{x^3+y^3+z^3}{xyz}}$$

We know that if $x+y+z=0$,

then, $x^3+y^3+z^3 = 3xyz$

$$\therefore \quad 2^{\frac{x^3+y^3+z^3}{xyz}} = 2^{\frac{3xyz}{xyz}} = 2^3 = 8$$

21. (2) Expression $= \dfrac{\left(2^{2n} - 3.\dfrac{2^{2n}}{2^2}\right)\left(3^n - \dfrac{2.3^n}{3^2}\right)}{\dfrac{3^n}{3^4}\left(4^n.4^3 - 2^{2n}\right)}$

$$= \frac{3^n.2^{2n}\left(1-\frac{3}{2^2}\right)\left(1-\frac{2}{3^2}\right)}{3^n.2^{2n}\left(\frac{2^6-1}{3^4}\right)}$$

$$= \frac{\left(1-\frac{3}{4}\right)\left(1-\frac{2}{9}\right)}{\frac{64-1}{81}} = \frac{\frac{1}{4}\times\frac{7}{9}}{\frac{63}{81}} = \frac{7}{36}\times\frac{81}{63} = \frac{1}{4}.$$

22. (1) Let $x=\sqrt{7\sqrt{7\sqrt{7..........\infty}}}$

Squaring both sides,

$x^2=7\sqrt{7\sqrt{7\sqrt{7..........\infty}}}$

$\Rightarrow x^2=7x \qquad \Rightarrow x^2-7x=0$

$\Rightarrow x(x-7)=0 \qquad \Rightarrow x=0$ or, $x=7$

But $x\neq 0$

$\therefore$ Required answer = 7

23. (3) Let $x=\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}$

Squaring both sides, $x^2=7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}$

Squaring both sides again, $x^4=7^2.7\sqrt{7\sqrt{7\sqrt{7}}}$

Squaring both sides again, $x^8=7^4.7^2.7\sqrt{7\sqrt{7}}$

Squaring both sides again, $x^{16}=7^8.7^4.7^2.7\sqrt{7}$

Squaring both sides again, $x^{32}=7^{16}.7^8.7^4.7^2.7$

$\Rightarrow x^{32}=7^{16+8+4+2+1} \qquad \Rightarrow x^{32}=7^{31}$

$\therefore x=\sqrt[32]{7^{31}}=7^{\frac{31}{32}}$

24. (4) Let $x=\sqrt{20+\sqrt{20+\sqrt{20+.........\infty}}}$

Squaring both sides,

$x^2=20+\sqrt{20+\sqrt{20+\sqrt{20+......\infty}}}$

$\Rightarrow x^2=20+x \qquad \Rightarrow x^2-x-20=0$

$\Rightarrow x^2-5x+4x-20=0 \quad \Rightarrow x(x-5)+4(x-5)=0$

$\Rightarrow (x-5)(x+4)=0 \qquad \Rightarrow x=5$ Or -4

But the given expression is positive hence its value cannot be negative.

$\therefore$ Value of the expression = 5

25. (2) $\dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{4^2\times 3}+\sqrt{3^2\times 2}}$

$=\dfrac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}=\dfrac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\dfrac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

(Retionalising the denominator)

$=\dfrac{7\sqrt{3}\times 4\sqrt{3}-7\sqrt{3}\times 3\sqrt{2}-5\sqrt{2}\times 4\sqrt{3}+5\sqrt{2}\times 3\sqrt{2}}{\left(4\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}$

$=\dfrac{28\sqrt{3\times 3}-21\sqrt{3\times 2}-20\sqrt{2\times 3}+15\sqrt{2\times 2}}{16\times 3-9\times 2}$

$=\dfrac{28\times 3-21\sqrt{6}-20\sqrt{6}+15\times 2}{48-18}$

$=\dfrac{84-(21+20)\sqrt{6}+30}{30}=\dfrac{114-41\sqrt{6}}{30}$

26. (2) Rationalising the denominator,

$\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$=\dfrac{\sqrt{2}\times 3\sqrt{2}+\sqrt{2}\times 2\sqrt{3}+\sqrt{3}\times 3\sqrt{2}+\sqrt{3}\times 2\sqrt{3}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$

$=\dfrac{3\sqrt{2\times 2}+2\sqrt{2\times 3}+3\sqrt{2\times 3}+2\sqrt{3\times 3}}{9\times 2-4\times 3}$

$=\dfrac{3\times 2+2\sqrt{6}+3\sqrt{6}+2\times 3}{18-12}$

$=\dfrac{12+5\sqrt{6}}{6}=\dfrac{12}{6}+\dfrac{5\sqrt{6}}{6}=2+\dfrac{5\sqrt{6}}{6}$

$\therefore \dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=a-b\sqrt{6}$

$\Rightarrow 2+\dfrac{5\sqrt{6}}{6}=a-b\sqrt{6}$

$\Rightarrow a=2$ and $b=\dfrac{-5}{6}$

$\therefore (a+b)^2=\left(2-\dfrac{5}{6}\right)^2=\left(\dfrac{12-5}{6}\right)^2=\left(\dfrac{7}{6}\right)^2=\dfrac{49}{36}$

27. (2) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2}-\dfrac{\sqrt{5}+2}{\sqrt{5}-2}=\dfrac{\left(\sqrt{5}-2\right)^2-\left(\sqrt{5}+2\right)^2}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}$

$=\dfrac{\left[\left(\sqrt{5}\right)^2+2^2-2\times\sqrt{5}\times 2\right]-\left[\left(\sqrt{5}\right)^2+2^2+2\times\sqrt{5}\times 2\right]}{\left(\sqrt{5}\right)^2-(2)^2}$

$=\dfrac{(5+4-4\sqrt{5})-(5+4+4\sqrt{5})}{5-4}$

$=9-4\sqrt{5}-9-4\sqrt{5}$

$=-8\sqrt{5}$

28. (1) Rationalising the denominator,

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}-$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$=\frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}\right)^2-\left(\sqrt{3}\right)^2}-\frac{2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}-\frac{3\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)}{\left(\sqrt{15}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{10-3}-\frac{2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)}{6-5}-\frac{3\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)}{15-9\times2}$$

$$=\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)-2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)+\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)$$

$$=\sqrt{30}-3-2\sqrt{30}+2\times5+\sqrt{30}-3\times2$$

$$=-3+10-6+\sqrt{30}-2\sqrt{30}+\sqrt{30}$$

$$=1+(1-2+1)\sqrt{30}=1$$

29. (3) Rationalising the denominator,

$$\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$-\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$=\frac{3+\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2}-\frac{\sqrt{8}+\sqrt{7}}{\left(\sqrt{8}\right)^2-\left(\sqrt{7}\right)^2}+\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$-\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}+\frac{\sqrt{5}+2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$=\frac{3+\sqrt{8}}{9-8}-\frac{\sqrt{8}+\sqrt{7}}{8-7}+\frac{\sqrt{7}+\sqrt{6}}{7-6}-\frac{\sqrt{6}+\sqrt{5}}{6-5}+\frac{\sqrt{5}+2}{5-4}$$

$$=\left(3+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$=3+2=5$$

30. (4) Rationalising the denominator

$$x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3+2-2\times\sqrt{3}\times\sqrt{2}}{3-2}=5-2\sqrt{6}$$

$$\text{and } y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3+2+2\times\sqrt{3}\times\sqrt{2}}{3-2}=5+2\sqrt{6}$$

$$\therefore x+y=5-2\sqrt{6}+5+2\sqrt{6}=10$$

$$\text{and } xy=\left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right)$$

$$=(5)^2-\left(2\sqrt{6}\right)^2=25-24=1$$

$$\therefore x^2+xy+y^2=(x+y)^2-xy$$

$$=(10)^2-1=100-1=99$$

31. (2) Denominator of expression

$$=\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$$

$$=\sqrt{10}+\sqrt{2^2\times5}+\sqrt{2^2\times10}-\sqrt{5}-\sqrt{2^4\times5}$$

$$=\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$$

$$=(1+2)\sqrt{10}+(2-1-4)\sqrt{5}$$

$$=3\sqrt{10}-3\sqrt{5}=3\left(\sqrt{10}-\sqrt{5}\right)$$

$$\therefore \text{Expression}=\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

$$=\frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)}=\frac{5}{\sqrt{10}-\sqrt{5}}\times\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$$

$$=\frac{5\left(\sqrt{10}+\sqrt{5}\right)}{10-5}=\sqrt{10}+\sqrt{5}=3.162+2.236=5.398$$

32. (1) $x=5-\sqrt{24}$

$$\therefore \frac{1}{x}=\frac{1}{5-\sqrt{24}}$$

$= \frac{1}{5-\sqrt{24}} \times \frac{5+\sqrt{24}}{5+\sqrt{24}} = \frac{5+\sqrt{24}}{25-24} = 5+\sqrt{24}$

$\therefore\ x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= \left(5-\sqrt{24}+5+\sqrt{24}\right)^3 - 3\left(5-\sqrt{24}+5+\sqrt{24}\right)$

$= 10^3 - 3\times 10 = 1000 - 30 = 970$

$x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$

$= \left(5-\sqrt{24}+5+\sqrt{24}\right)^2 - 2 = 100 - 2 = 98$

and $x + \frac{1}{x} = 5 - \sqrt{2}4 + 5 + \sqrt{24} = 10$

$\therefore$ Expression = $970 - 10 \times 98 + 4 \times 10 - 30$
$= 970 - 980 + 40 - 30 = 0$

33. (2) $x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

Now, $x = 2 + \sqrt{3}$ $\Rightarrow x - 2 = \sqrt{3}$
Squaring both sides,

$(x-2)^2 = \left(\sqrt{3}\right)^2$ $\Rightarrow x^2 - 4x + 4 = 3$

$\Rightarrow x^2 - 4x + 1 = 0$

Now, $x^3 - 2x^2 - 7x + 5$

$= x^3 - 4x^2 + x + 2x^2 - 8x + 2 + 3$

$= x\left(x^2 - 4x + 1\right) + 2\left(x^2 - 4x + 1\right) + 3$

$= x \times 0 + 2 \times 0 + 3 = 3$

34. (1) $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

$= \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}} \times \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}}$

$= \frac{\left(\sqrt{a+2b}+\sqrt{a-2b}\right)^2}{\left(\sqrt{a+2b}\right)^2 - \left(\sqrt{a-2b}\right)^2}$

$= \frac{a+2b+a-2b+2\sqrt{a+2b}\times\sqrt{a-2b}}{(a+2b)-(a-2b)}$

$= \frac{2a+2\sqrt{a^2-4b^2}}{4b} = \frac{a+\sqrt{a^2-4b^2}}{2b}$

$\therefore\ 2bx = a + \sqrt{a^2-4b^2}$

$\Rightarrow 2bx - a = \sqrt{a^2-4b^2}$

$\Rightarrow (2bx-a)^2 = \left(\sqrt{a^2-4b^2}\right)^2$

(Squaring both sides)

$\Rightarrow 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$\Rightarrow 4b^2x^2 - 4abx + 4b^2 = 0$

$\Rightarrow 4b(bx^2 - ax + b) = 0$

$\Rightarrow bx^2 - ax + b = 0$

(Dividing both sides by 4b)

35. (2) $x = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}}$

Rationalising,

$x = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}}$

$= \sqrt{\frac{\left(5+2\sqrt{6}\right)^2}{25-24}} = 5+2\sqrt{6}$

$\therefore\ x^2(x-10)^2$

$= \left(5+2\sqrt{6}\right)^2\left(5+2\sqrt{6}-10\right)^2$

$= \left(5+2\sqrt{6}\right)^2\left(2\sqrt{6}-5\right)^2$

$= \left(25+24+20\sqrt{6}\right)\left(24+25-20\sqrt{6}\right)$

$= \left(49+20\sqrt{6}\right)\left(49-20\sqrt{6}\right)$

$= (49)^2 - \left(20\sqrt{6}\right)^2 = 2401 - 2400 = 1$

36. (4) Let $x = \frac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}$

Squaring both sides, $x^2 = \left[\frac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}\right]^2$

$$\Rightarrow x^2 = \frac{4\left(\sqrt{2}+\sqrt{6}\right)^2}{\left(3\sqrt{2+\sqrt{3}}\right)^2} \quad \Rightarrow x^2 = \frac{4\left(\sqrt{2}+\sqrt{6}\right)^2}{9\left(2+\sqrt{3}\right)}$$

$$\Rightarrow x^2 = \frac{4\left(2+6+2\times\sqrt{2}\times\sqrt{6}\right)}{9\left(2+\sqrt{3}\right)}$$

$$\Rightarrow x^2 = \frac{4\left(8+2\sqrt{12}\right)}{9\left(2+\sqrt{3}\right)} \Rightarrow x^2 = \frac{4\left(8+2\sqrt{2^2\times 3}\right)}{9\left(2+\sqrt{3}\right)}$$

$$\Rightarrow x^2 = \frac{4\left(8+4\sqrt{3}\right)}{9\left(2+\sqrt{3}\right)} \Rightarrow x^2 = \frac{16\left(2+\sqrt{3}\right)}{9\left(2+\sqrt{3}\right)} = \frac{16}{9}$$

$$\Rightarrow x = \frac{4}{3}$$

37. (4) Expression $= 14\sqrt{5}-30$

$= 14\sqrt{5}-6\times 5 \quad = \sqrt{5}\left(14-6\sqrt{5}\right)$

$= \sqrt{5}\left(14-2\times 3\times\sqrt{5}\right) \quad = \sqrt{5}\left(9+5-2\times 3\times\sqrt{5}\right)$

$= \sqrt{5}\left[(3)^2+\left(\sqrt{5}\right)^2-2\times 3\times\sqrt{5}\right]$

$= \sqrt{5}\left(3-\sqrt{5}\right)^2$

$\therefore \sqrt{14\sqrt{5}-30} \quad = \sqrt{\sqrt{5}\left(3-\sqrt{5}\right)^2}$

$= \left(3-\sqrt{5}\right)\sqrt{\sqrt{5}} \quad = \sqrt[4]{5}\left(3-\sqrt{5}\right)$

Note : Express $x+y\pm 2\sqrt{xy}$

in the form of $= \left(\sqrt{x}\right)^2+\left(\sqrt{y}\right)^2 \pm 2\times\sqrt{x}\times\sqrt{y}$

$= \left(\sqrt{x}\pm\sqrt{y}\right)^2$

38. (1) $\sqrt{\left(\frac{6+2\sqrt{3}}{33-19\sqrt{3}}\right)} \quad = \sqrt{\frac{\sqrt{3}\left(2\sqrt{3}+2\right)}{\sqrt{3}\left(11\sqrt{3}-19\right)}}$

$$= \sqrt{\frac{2\left(\sqrt{3}+1\right)}{\left(11\sqrt{3}-19\right)}\times\frac{11\sqrt{3}+19}{11\sqrt{3}+19}}$$

$$= \frac{2\left(33+19\sqrt{3}+11\sqrt{3}+19\right)}{\left(11\sqrt{3}\right)^2-(19)^2}$$

$$= \sqrt{\frac{2\left(52+30\sqrt{3}\right)}{363-361}} \quad = \sqrt{52+30\sqrt{3}}$$

$= \sqrt{52+2\times 15\times\sqrt{3}} \quad = \sqrt{52+2\times 5\times 3\sqrt{3}}$

$= \sqrt{25+27+2\times 5\times 3\sqrt{3}}$

$= \sqrt{(5)^2+\left(3\sqrt{3}\right)^2+2\times 5\times 3\sqrt{3}}$

$= \sqrt{\left(5+3\sqrt{3}\right)^2} = 5+3\sqrt{3}$

39. (3) Let $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

On squaring both sides,

$$\Rightarrow x^2 = \left[\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}\right]^2$$

$$= \frac{\left[\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}\right]^2}{\left[\sqrt{\sqrt{5}+1}\right]^2}$$

$$= \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{\sqrt{5}+2}.\sqrt{\sqrt{5}-2}}{\sqrt{5}+1}$$

$$= \frac{2\sqrt{5}+2\sqrt{\left(\sqrt{5}\right)^2-(2)^2}}{\sqrt{5}+1} = \frac{2\sqrt{5}+2}{\sqrt{5}+1} = \frac{2\left(\sqrt{5}+1\right)}{\sqrt{5}+1} = 2$$

$\therefore x = \sqrt{2}$

Again, $\sqrt{3-2\sqrt{2}} = \sqrt{2+1-2\times\sqrt{2}\times 1}$

$= \sqrt{\left(\sqrt{2}\right)^2+(1)^2-2\times\sqrt{2}\times 1} \quad = \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2}-1$

$$\therefore \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$$

$= \sqrt{2}-\left(\sqrt{2}-1\right) = 1$

40. (1) First term $=\sqrt{4-\sqrt{7}}=\sqrt{\frac{(4-\sqrt{7})\times 2}{2}}$

$=\sqrt{\frac{8-2\sqrt{7}}{2}}=\sqrt{\frac{(\sqrt{7})^2+1-2\sqrt{7}}{2}}$

$=\sqrt{\frac{(\sqrt{7}-1)^2}{2}}=\frac{\sqrt{7}-1}{\sqrt{2}}$

Second term $=\sqrt{8+3\sqrt{7}}=\sqrt{\frac{(8+3\sqrt{7})\times 2}{2}}$

$=\sqrt{\frac{16+6\sqrt{7}}{2}}=\sqrt{\frac{3^2+(\sqrt{7})^2+2\times 3\times\sqrt{7}}{2}}$

$=\sqrt{\frac{(3+\sqrt{7})^2}{2}}=\frac{3+\sqrt{7}}{\sqrt{2}}$

$\therefore$ Expression $=\frac{\sqrt{7}-1}{\sqrt{2}}+\frac{3+\sqrt{7}}{\sqrt{2}}$

$\frac{5\sqrt{6}+\sqrt{10}}{4}$

$=\frac{\sqrt{2}(\sqrt{2}\times\sqrt{7}+\sqrt{2})}{\sqrt{2}}=\sqrt{2}(\sqrt{7}+1)$

41. (2) First term $=\sqrt{38+5\sqrt{3}}=\sqrt{\frac{(38+5\sqrt{3})\times 2}{2}}$

$=\sqrt{\frac{76+10\sqrt{3}}{2}}=\sqrt{\frac{75+1+2\times 5\sqrt{3}\times 1}{2}}$

$=\sqrt{\frac{(5\sqrt{3}+1)^2}{2}}=\frac{5\sqrt{3}+1}{\sqrt{2}}$

Second term $=\sqrt{3-\sqrt{5}}=\sqrt{(3-\sqrt{5})\times\frac{2}{2}}$

$=\sqrt{\frac{6-2\sqrt{5}}{2}}=\sqrt{\frac{(\sqrt{5})^2+1-2\sqrt{5}}{2}}$

$=\sqrt{\frac{(\sqrt{5}-1)^2}{2}}=\frac{\sqrt{5}-1}{\sqrt{2}}$

$\therefore$ Expression $=\frac{5\sqrt{3}+1}{\sqrt{2}}+\frac{\sqrt{5}-1}{\sqrt{2}}$

$=\frac{5\sqrt{3}+1+\sqrt{5}-1}{\sqrt{2}}=\frac{5\sqrt{3}+\sqrt{5}}{\sqrt{2}}$

$=\frac{5\sqrt{3}+\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{5\sqrt{6}+\sqrt{10}}{2}$

42. (2) First term $=\sqrt{8+3\sqrt{7}}=\sqrt{\frac{(8+3\sqrt{7})\times 2}{2}}$

$=\sqrt{\frac{16+6\sqrt{7}}{2}}=\sqrt{\frac{3^2+(\sqrt{7})^2+2\times 3\times\sqrt{7}}{2}}$

$=\sqrt{\frac{(3+\sqrt{7})^2}{2}}=\frac{3+\sqrt{7}}{\sqrt{2}}$

Second term $=\sqrt{7+3\sqrt{5}}=\sqrt{\frac{(7+3\sqrt{5})\times 2}{2}}$

$=\sqrt{\frac{14+6\sqrt{5}}{2}}=\sqrt{\frac{3^2+(\sqrt{5})^2+2\times 3\times\sqrt{5}}{2}}$

$=\sqrt{\frac{(3+\sqrt{5})^2}{2}}=\frac{3+\sqrt{5}}{\sqrt{2}}$

$\therefore$ Expression $=\frac{3+\sqrt{7}}{\sqrt{2}}-\frac{3+\sqrt{5}}{\sqrt{2}}$

$=\frac{3+\sqrt{7}-3-\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{7}-\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{14}-\sqrt{10}}{2}$

43. (4) First term $=\sqrt{11+4\sqrt{7}}$

$=\sqrt{2^2+(\sqrt{7})^2+2\times 2\times\sqrt{7}}=\sqrt{(2+\sqrt{7})^2}=2+\sqrt{7}$

Second term

$=\sqrt{11-4\sqrt{7}}=\sqrt{(2)^2+(\sqrt{7})^2-2\times 2\times\sqrt{7}}$

$=\sqrt{(\sqrt{7}-2)^2}=\sqrt{7}-2$

Third term $=\frac{\sqrt{8}}{\sqrt{33+10\sqrt{8}}-\sqrt{33-10\sqrt{8}}}$

$$=\frac{\sqrt{8}}{\sqrt{(5)^2+\left(\sqrt{8}\right)^2+2\times5\times\sqrt{8}}-\sqrt{(5)^2+\left(\sqrt{8}\right)^2-2\times5\times\sqrt{8}}}$$

$$=\frac{\sqrt{8}}{\sqrt{\left(5+\sqrt{8}\right)^2}-\sqrt{\left(5-\sqrt{8}\right)^2}}$$

$$=\frac{\sqrt{8}}{5+\sqrt{8}-5+\sqrt{8}}=\frac{\sqrt{8}}{2\sqrt{8}}=\frac{1}{2}$$

$\therefore$ Expression $= 2+\sqrt{7}-\sqrt{7}+2+\frac{1}{2}=4\frac{1}{2}$

44. (1) First term $=\left(18-8\sqrt{2}\right)^{\frac{1}{2}}$

$$=\left(16+2-8\sqrt{2}\right)^{\frac{1}{2}}$$

$$=\left\{(4)^2+\left(\sqrt{2}\right)^2-2\times4\times\sqrt{2}\right\}^{\frac{1}{2}}$$

$$=\left(4-\sqrt{2}\right)^{2\times\frac{1}{2}}=4-\sqrt{2}$$

Second term $=\left(6-4\sqrt{2}\right)^{\frac{-1}{2}}=\left(4+2-4\sqrt{2}\right)^{-\frac{1}{2}}$

$$=\left\{2^2+\left(\sqrt{2}\right)^2-2\times2\times\sqrt{2}\right\}^{-\frac{1}{2}}$$

$$=\left(2-\sqrt{2}\right)^{2\times\frac{-1}{2}}=\left(2-\sqrt{2}\right)^{-1}=\frac{1}{2-\sqrt{2}}$$

$$=\frac{1}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{2+\sqrt{2}}{2}$$

Third term $=\frac{\sqrt{3}}{\sqrt{19+8\sqrt{3}}-\sqrt{19-8\sqrt{3}}}$

$$=\frac{\sqrt{3}}{\sqrt{16+3+8\sqrt{3}}-\sqrt{16+3-8\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{\sqrt{(4)^2+\left(\sqrt{3}\right)^2+2\times4\times\sqrt{3}}-\sqrt{(4)^2+\left(\sqrt{3}\right)^2-2\times4\times\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{\sqrt{\left(4+\sqrt{3}\right)^2}-\sqrt{\left(4-\sqrt{3}\right)^2}}$$

$$=\frac{\sqrt{3}}{4+\sqrt{3}-4+\sqrt{3}}=\frac{\sqrt{3}}{2\sqrt{3}}=\frac{1}{2}$$

$\therefore$ Expression $=4-\sqrt{2}+\frac{2+\sqrt{2}}{2}+\frac{1}{2}$

$$=\frac{8-2\sqrt{2}+2+\sqrt{2}+1}{2}=\frac{11-\sqrt{2}}{2}$$

45. (4)

First term $=\sqrt{7+4\sqrt{3}}=\sqrt{2^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}}$

$$=\sqrt{\left(2+\sqrt{3}\right)^2}=2+\sqrt{3}$$

Second term

$$=\sqrt{28+10\sqrt{3}}=\sqrt{(5)^2+\left(\sqrt{3}\right)^2+2\times5\times\sqrt{3}}$$

$$=\sqrt{\left(5+\sqrt{3}\right)^2}=5+\sqrt{3}$$

Third term $=\frac{\sqrt{11}}{\sqrt{20+6\sqrt{11}}+\sqrt{20-6\sqrt{11}}}$

$$=\frac{\sqrt{11}}{\sqrt{3^2+\left(\sqrt{11}\right)^2+2\times3\times\sqrt{11}}+\sqrt{3^2+\left(\sqrt{11}\right)^2-2\times3\times\sqrt{11}}}$$

$$=\frac{\sqrt{11}}{\sqrt{\left(\sqrt{11}+3\right)^2}+\left(\sqrt{11}-3\right)^2}$$

$$=\frac{\sqrt{11}}{\sqrt{11}+3+\sqrt{11}-3}=\frac{\sqrt{11}}{2\sqrt{11}}=\frac{1}{2}$$

Expression $=2+\sqrt{3}-5-\sqrt{3}+\frac{1}{2}$

$$=-3+\frac{1}{2}=\frac{-6+1}{2}=\frac{-5}{2}=-2\frac{1}{2}$$

46. (4) Expression $=(9)^{-3}\times\frac{(16)^{\frac{1}{4}}}{(6)^{-2}}\times\left(\frac{1}{27}\right)^{-\frac{4}{3}}$

$$=(3^2)^{-3}\times\frac{(2^4)^{\frac{1}{4}}}{(2\times3)^{-2}}\times\left(\frac{1}{3^3}\right)^{-\frac{4}{3}}$$

$$=3^{-6}\times\frac{2}{2^{-2}\times3^{-2}}\times(3^3)^{\frac{4}{3}} \qquad \left[\because\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\right]$$

$$\left[\because a^n=\frac{1}{a^{-n}}\right]$$

$$=3^{-6}\times2\times3^4\times2^2\times3^2$$

$$=3^{-6}\times3^4\times3^2\times2^2\times2$$

$$=3^{-6+6}\times2^{2+1} \qquad \left[\because a^m.a^n=a^{m+n}\right]$$

$$=3^0\times2^3=1\times8=8$$

47.(4) First term $=\dfrac{\sqrt[6]{2}\left[(625)^{\frac{3}{5}}\times(1024)^{-\frac{6}{5}}\div(25)^{\frac{3}{5}}\right]^{\frac{1}{2}}}{\left(\sqrt[3]{128}\right)^{\frac{-5}{2}}\times(125)^{\frac{1}{5}}}$

$$=\frac{2^{\frac{1}{6}}\left[\left(5^4\right)^{\frac{3}{5}}\times\left(2^{10}\right)^{-\frac{6}{5}}\div\left(5^2\right)^{\frac{3}{5}}\right]^{\frac{1}{2}}}{\left\{(128)^{\frac{1}{3}}\right\}^{\frac{-5}{2}}\times\left(5^3\right)^{\frac{1}{5}}}$$

$$=\frac{2^{\frac{1}{6}}\left[5^{4\times\frac{3}{5}}\times2^{10\times\frac{-6}{5}}\div5^{2\times\frac{3}{5}}\right]^{\frac{1}{2}}}{\left\{\left(2^7\right)^{\frac{1}{3}}\right\}^{-\frac{5}{2}}\times5^{3\times\frac{1}{5}}}$$

$$=\frac{2^{\frac{1}{6}}\left[5^{\frac{12}{5}}\times2^{-12}\div5^{\frac{6}{5}}\right]^{\frac{1}{2}}}{2^{\left(7\times\frac{1}{3}\times\frac{-5}{2}\right)}\times5^{\frac{3}{5}}}=\frac{2^{\frac{1}{6}}\left[5^{\frac{6}{5}}\times2^{-12}\right]^{\frac{1}{2}}}{2^{\frac{-35}{6}}\times5^{\frac{3}{5}}}$$

$$=\frac{2^{\frac{1}{6}}\times5^{\frac{3}{5}}\times2^{-6}}{2^{\frac{-35}{6}}\times5^{\frac{3}{5}}}=2^{\left(\frac{1}{6}-6+\frac{35}{6}\right)}\times5^{\frac{3}{5}-\frac{3}{5}}$$

$$=2^{\left(\frac{1-36+35}{6}\right)}\times5^0$$

$$=2^{\frac{36-36}{6}}\times5^0$$

$$=2^0\times5^0=1\times1=1$$

Second term $=\dfrac{\left(10^3\right)^2\div\left(10^{3^2}\right)}{\left(10^2\right)^3\div10^{2^3}}=\dfrac{10^{2\times3}\div10^9}{10^{2\times3}\div10^8}$

$$=\frac{10^6\div10^9}{10^6\div10^8}=\frac{10^{(6-9)}}{10^{(6-8)}}$$

$$=\frac{10^{-3}}{10^{-2}}=10^{(-3+2)}=10^{-1}=\frac{1}{10}$$

$\therefore$ **Expression**

$$=\frac{\sqrt[6]{2}\left[(625)^{\frac{3}{5}}\times(1024)^{\frac{-6}{5}}\div(25)^{\frac{3}{5}}\right]^{\frac{1}{2}}}{\left(\sqrt[3]{128}\right)^{\frac{-5}{2}}\times(125)^{\frac{1}{5}}}+\frac{\left(10^3\right)^2\div\left(10^{3^2}\right)}{\left(10^2\right)^3\div10^{2^3}}$$

$$=1+\frac{1}{10}=\frac{10+1}{10}=1.1$$

48. (2) $10^{0.48}=x,\ 10^{0.7}=y$

and $x^z=y^2$

$\therefore\ \left(10^{0.48}\right)^z=\left(10^{0.7}\right)^2$

$\Rightarrow\ 10^{0.48z}=10^{1.4}$

$\Rightarrow\ 0.48z=1.4$

$\Rightarrow\ z=\dfrac{1.4}{0.48}=\dfrac{140}{48}=\dfrac{35}{12}=2\dfrac{11}{12}$

49. (1) **Expression** $=(27)^{\frac{-2}{3}}+\left[\left(2^{\frac{-2}{3}}\right)^{\frac{-5}{3}}\right]^{\frac{-9}{10}}$

$$=(3^3)^{\frac{-2}{3}}+\left(2^{\frac{-2}{3}\times\frac{-5}{3}}\right)^{\frac{-9}{10}}=3^{3\times\frac{-2}{3}}+2^{\frac{-2}{3}\times\frac{-5}{3}\times\frac{-9}{10}}$$

$$=3^{-2}+2^{-1}=\frac{1}{3^2}+\frac{1}{2}=\frac{1}{9}+\frac{1}{2}=\frac{2+9}{18}=\frac{11}{18}$$

50. (2) Expression $= \dfrac{(0.3)^{\frac{1}{3}}\left(\frac{1}{27}\right)^{\frac{1}{4}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}}}{(0.9)^{\frac{2}{3}}(3)^{-\frac{1}{2}}(243)^{-\frac{1}{4}}}$

$$= \frac{(0.3)^{\frac{1}{3}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}} \times (3)^{\frac{1}{2}}(243)^{\frac{1}{4}}}{(0.9)^{\frac{2}{3}} \times (27)^{\frac{1}{4}}}$$

$$= \frac{(0.3)^{\frac{1}{3}}(3^2)^{\frac{1}{6}}(0.9)^{2\times\frac{2}{3}} \times (3)^{\frac{1}{2}}(3^5)^{\frac{1}{4}}}{(0.9)^{\frac{2}{3}} \times (3^3)^{\frac{1}{4}}}$$

$$= \frac{(0.3)^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times (0.9)^{\frac{4}{3}} \times 3^{\frac{1}{2}} \times 3^{\frac{5}{4}}}{(0.9)^{\frac{2}{3}} \times 3^{\frac{3}{4}}}$$

$$= (0.3)^{\frac{1}{3}} \times (0.9)^{\frac{4}{3}-\frac{2}{3}} \times 3^{\frac{1}{3}+\frac{1}{2}+\frac{5}{4}-\frac{3}{4}}$$

$$= (0.3)^{\frac{1}{3}} \times (0.9)^{\frac{2}{3}} \times 3^{\frac{4+6+15-9}{12}}$$

$$= (0.3)^{\frac{1}{3}} \times (0.3 \times 3)^{\frac{2}{3}} \times 3^{\frac{16}{12}}$$

$$= (0.3)^{\frac{1}{3}} \times (0.3)^{\frac{2}{3}} \times (3)^{\frac{2}{3}} \times (3)^{\frac{4}{3}}$$

$$= (0.3)^{\frac{1}{3}+\frac{2}{3}} \times 3^{\frac{2}{3}+\frac{4}{3}}$$

$$= (0.3)^{\frac{1+2}{3}} \times 3^{\frac{2+4}{3}} = 0.3 \times 3^2$$

$= 0.3 \times 9 = 2.7$

51. (3) First term $= \sqrt{\dfrac{(12.12)^2 - (8.12)^2}{(0.25)^2 + (0.25)(19.99)}}$

$$= \sqrt{\frac{(12.12 + 8.12)(12.12 - 8.12)}{0.25(0.25 + 19.99)}}$$

$$\left[\because (a^2 - b^2) = (a+b)(a-b)\right]$$

$$= \sqrt{\frac{20.24 \times 4}{0.25 \times 20.24}} = \sqrt{\frac{4}{0.25}} = \sqrt{\frac{4 \times 100}{25}}$$

$$= \sqrt{\frac{400}{25}} = \sqrt{16} = 4$$

Second term $= \dfrac{\left[\left(8^{\frac{-3}{4}}\right)^{\frac{5}{2}}\right]^{\frac{8}{15}} \times 16^{\frac{3}{4}}}{\sqrt[3]{\left[\left\{(128)^{-5}\right\}^{\frac{3}{7}}\right]^{\frac{-1}{5}}}}$

$$= \frac{\left[\left\{(2^3)^{\frac{-3}{4}}\right\}^{\frac{5}{2}}\right]^{\frac{8}{15}} \times (2^4)^{\frac{3}{4}}}{\sqrt[3]{\left[\left\{(2^7)^{-5}\right\}^{\frac{3}{7}}\right]^{\frac{-1}{5}}}} \quad \left[\because 8 = 2^3, 16 = 2^4, 128 = 2^7\right]$$

$$= \frac{2^{3\times\frac{-3}{4}\times\frac{5}{2}\times\frac{8}{15}} \times 2^{4\times\frac{3}{4}}}{2^{7\times(-5)\times\frac{3}{7}\times\left(-\frac{1}{5}\right)\times\frac{1}{3}}}$$

$$\left[\because \left[\left\{(a^m)^n\right\}^p\right]^q = a^{mnpq} \text{ and } \sqrt[3]{x} = (x)^{\frac{1}{3}}\right]$$

$$= \frac{2^{-3} \times 2^3}{2} = \frac{2^{-3+3}}{2} = \frac{2^0}{2} = \frac{1}{2}$$

∴ Expression = First term + second term

$$= 4 + \frac{1}{2} = 4\frac{1}{2}$$

52. (1) Expression $= \dfrac{2^n \times 2^4 - 2 \times 2^n}{2 \times 2^n \times 2^3} + \dfrac{1}{2^3}$

$$= \frac{2^n\left(2^4 - 2\right)}{2^n\left(2 \times 2^3\right)} + \frac{1}{2^3}$$

$$= \frac{16-2}{16} + \frac{1}{8} = \frac{7}{8} + \frac{1}{8} = 1$$

53. (4) Expression

$$=\left[\frac{(1.331)^{-1}+(1.331)^{-2}+(1.331)^{-3}+....+(1.331)^{-6}}{(1.331)^{-2}+(1.331)^{-3}+(1.331)^{-4}+...+(1.331)^{-7}}\right]^{\frac{1}{3}}+1.1$$

$$=\left[\frac{\{(1.331)^{-1}+(1.331)^{-2}+...+(1.331)^{-6}\}}{(1.331)^{-1}\{(1.331)^{-1}+(1.331)^{-2}+...+(1.331)^{-6}\}}\right]^{\frac{1}{3}}+1.1$$

$$=\left[\frac{1}{(1.331)^{-1}}\right]^{\frac{1}{3}}+1.1=(1.331)^{\frac{1}{3}}+1.1$$

$$=(1.1\times1.1\times1.1)^{\frac{1}{3}}+1.1=\{(1.1)^3\}^{\frac{1}{3}}+1.1$$

$$=(1.1)^{3\times\frac{1}{3}}+1.1=1.1+1.1=1$$

54. (2) $\left[\frac{1.2.4+2.4.8+3.6.12+......}{1.3.9.+2.6.18+3.9.27+......}\right]^{\frac{1}{3}}$

$$=\left[\frac{8(1^3+2^3+3^3+......)}{27(1^3+2^3+3^3+......)}\right]^{\frac{1}{3}}=\left(\frac{8}{27}\right)^{\frac{1}{3}}=\frac{2}{3}$$

55. (1) First term $=\frac{1}{\sqrt{11-2\sqrt{30}}}$

$$=\sqrt{\frac{1}{11-2\sqrt{30}}}=\sqrt{\frac{1\times(11+2\sqrt{30})}{(11-2\sqrt{30})(11+2\sqrt{30})}}$$

(Rationalising the denominator)

$$=\sqrt{\frac{(11+2\sqrt{30})}{(11)^2-(2\sqrt{30})^2}}$$

($\because$ $(a+b)(a-b)=a^2-b^2$)

$$=\sqrt{\frac{11+2\sqrt{30}}{121-120}}=\sqrt{\frac{(11+2\sqrt{30})}{1}}$$

$$=\sqrt{(11+2\sqrt{30})}=\sqrt{5+6+2\times\sqrt{5}\times\sqrt{6}}$$

$$=\sqrt{(\sqrt{5})^2+(\sqrt{6})^2+2\sqrt{5\times6}}=\sqrt{(\sqrt{5}+\sqrt{6})^2}$$

$[a^2+b^2+2ab=(a+b)^2]$

$$=(\sqrt{5}+\sqrt{6})$$

Second term $=\frac{3}{\sqrt{7-2\sqrt{10}}}$

$$=\sqrt{\frac{9}{(7-2\sqrt{10})}}$$

$$=\sqrt{\frac{9\times(7+2\sqrt{10})}{(7-2\sqrt{10})(7+2\sqrt{10})}}$$

$$=\sqrt{\frac{9\times(7+2\sqrt{10})}{49-40}}=\sqrt{\frac{9\times(7+2\sqrt{10})}{9}}$$

$$=\sqrt{7+2\sqrt{10}}=\sqrt{5+2+2\sqrt{10}}$$

$$=\sqrt{\sqrt{(5)^2}+(\sqrt{2})^2+2\sqrt{5}\times\sqrt{2}}$$

$$=\sqrt{(\sqrt{5}+\sqrt{2})^2}=(\sqrt{5}+\sqrt{2})$$

Similarly, third term $=\frac{4}{\sqrt{8+4\sqrt{3}}}$

$$=\sqrt{\frac{16}{(8+4\sqrt{3})}}=\sqrt{\frac{16\times(8-4\sqrt{3})}{(8+4\sqrt{3})(8-4\sqrt{3})}}$$

$$=\sqrt{\frac{16\times(8-4\sqrt{3})}{64-48}}=\sqrt{\frac{16(8-4\sqrt{3})}{16}}$$

$$=\sqrt{(8-4\sqrt{3})}=\sqrt{(\sqrt{6})^2+(\sqrt{2})^2-2\times\sqrt{6}\times\sqrt{2}}$$

$$=\sqrt{(\sqrt{6}-\sqrt{2})^2}=(\sqrt{6}-\sqrt{2})$$

$\therefore$ Expression

$$=\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{\sqrt{7-2\sqrt{10}}}-\frac{4}{\sqrt{8+4\sqrt{3}}}$$

$$=(\sqrt{5}+\sqrt{6})-(\sqrt{5}+\sqrt{2})-(\sqrt{6}-\sqrt{2})$$

$$=\sqrt{5}+\sqrt{6}-\sqrt{5}-\sqrt{2}-\sqrt{6}+\sqrt{2}=0$$

56. (4) $\frac{4\sqrt{3}}{2-\sqrt{2}}-\frac{30}{4\sqrt{3}-\sqrt{18}}-\frac{\sqrt{18}}{3-2\sqrt{3}}$

Rationalising the denominators by correspon ing conjugates

$$=\frac{4\sqrt{3}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$$

$$-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{\left(4\sqrt{3}-\sqrt{18}\right)\left(4\sqrt{3}+\sqrt{18}\right)}-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)}$$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{(2)^2-\left(\sqrt{2}\right)^2}-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{\left(4\sqrt{3}\right)^2-\left(\sqrt{18}\right)^2}-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{(3)^2-\left(2\sqrt{3}\right)^2}$$

$[(a-b)(a+b)=a^2-b^2]$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{4-2}-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{48-18}-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{9-12}$$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{2}-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{30}-\frac{3\sqrt{2}\left(3+2\sqrt{3}\right)}{-3}$$

$$=2\sqrt{3}\left(2+\sqrt{2}\right)-\left(4\sqrt{3}+\sqrt{18}\right)+\sqrt{2}\left(3+2\sqrt{3}\right)$$

$$=4\sqrt{3}+2\sqrt{6}-4\sqrt{3}-3\sqrt{2}+3\sqrt{2}+2\sqrt{6}$$

$$=2\sqrt{6}+2\sqrt{6}=4\sqrt{6}$$

57. (3) First term $=\left(28-10\sqrt{3}\right)^{\frac{1}{2}}$

$$=\left(25+3-10\sqrt{3}\right)^{\frac{1}{2}}$$

$$=\left[(5)^2+\left(\sqrt{3}\right)^2-2\times5\times\sqrt{3}\right]^{\frac{1}{2}}$$

$$=\left[\left(5-\sqrt{3}\right)^2\right]^{\frac{1}{2}}=\left(5-\sqrt{3}\right)^{2\times\frac{1}{2}}=\left(5-\sqrt{3}\right)$$

Second term $=\left(7+4\sqrt{3}\right)^{-\frac{1}{2}}=\left(4+3+4\sqrt{3}\right)^{-\frac{1}{2}}$

$$=\left[(2)^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}\right]^{-\frac{1}{2}}$$

$$=\left[\left(2\times\sqrt{3}\right)^2\right]^{-\frac{1}{2}}=\left(2+\sqrt{3}\right)^{2\times-\frac{1}{2}}$$

$$=\left(2\times\sqrt{3}\right)^{-1}=\frac{1}{\left(2+\sqrt{3}\right)}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

Third term $=\dfrac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$

$$=\frac{\sqrt{7}}{\sqrt{9+7+2\times3\times\sqrt{7}}-\sqrt{9+7-2\times3\times\sqrt{7}}}$$

$$=\frac{\sqrt{7}}{\sqrt{\left(3+\sqrt{7}\right)^2}-\sqrt{\left(3-\sqrt{7}\right)^2}}$$

$$=\frac{\sqrt{7}}{\left(3+\sqrt{7}\right)-\left(3-\sqrt{7}\right)}=\frac{\sqrt{7}}{3+\sqrt{7}-3+\sqrt{7}}$$

(Taking square root)

$$=\frac{\sqrt{7}}{2\sqrt{7}}=\frac{1}{2}$$

$\therefore$ Expression $=\left(28-10\sqrt{3}\right)^{\frac{1}{2}}-\left(7+4\sqrt{3}\right)^{-\frac{1}{2}}$

$$+\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$$

$$=5-\sqrt{3}-\left(2-\sqrt{3}\right)+\frac{1}{2}$$

$$=5-\sqrt{3}-2+\sqrt{3}+\frac{1}{2}=3+\frac{1}{2}=3\frac{1}{2}$$

58. (1) $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}=\dfrac{\sqrt{8-2\sqrt{7}}}{\sqrt{16+6\sqrt{7}}-4}$

(Multiplying numerator and denominator by $\sqrt{2}$)

$$=\frac{\sqrt{(1)^2+\left(\sqrt{7}\right)^2-2\times1\times\sqrt{7}}}{\sqrt{(3)^2+\left(\sqrt{7}\right)^2+2.3.\sqrt{7}}-4}$$

$\left\{\therefore 8=1+7=1^2+\left(\sqrt{7}\right)^2 \text{ and } 16=9+7=3^2+\left(\sqrt{7}\right)^2\right\}$

$$=\frac{\sqrt{\left(\sqrt{7}-1\right)^2}}{\sqrt{\left(\sqrt{7}+3\right)^2}-4}\qquad\left\{\because a^2+b^2-2ab=(a-b)^2\right\}$$

$$=\frac{\sqrt{7}-1}{\sqrt{7}+3-4}=\frac{\sqrt{7}-1}{\sqrt{7}-1}=1$$

59. (2) Expression

$$=\frac{26-15\sqrt{3}}{\left[5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right]^2}+\frac{\sqrt{10}+\sqrt{18}}{\sqrt{8}+\sqrt{\left(3-\sqrt{5}\right)}}$$

Now, $\sqrt{38+5\sqrt{3}} = \sqrt{(38+5\sqrt{3})\times\frac{2}{2}}$

$=\sqrt{\frac{76+2(5\sqrt{3})}{2}}$

$=\sqrt{\frac{75+1+2(5\sqrt{3})}{2}} = \sqrt{\frac{(5\sqrt{3})^2+(1)^2+2(5\sqrt{3})\times 1}{2}}$

$=\sqrt{\frac{(5\sqrt{3}+1)^2}{2}} = \frac{5\sqrt{3}+1}{\sqrt{2}}$

$\therefore \left(5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right)^2 = \left(5\sqrt{2}-\frac{5\sqrt{3}+1}{\sqrt{2}}\right)^2$

$=\left(\frac{10-5\sqrt{3}-1}{\sqrt{2}}\right)^2 = \frac{(9-5\sqrt{3})^2}{2}$

and $\sqrt{(3-\sqrt{5})} = \sqrt{(3-\sqrt{5})\times\frac{2}{2}}$

$=\sqrt{\frac{6-2\sqrt{5}}{2}} = \sqrt{\frac{5+1-2(\sqrt{5})\times 1}{2}}$

$=\sqrt{\frac{(\sqrt{5})^2+(1)^2-2\sqrt{5}}{2}}$

$=\sqrt{\frac{(\sqrt{5}-1)^2}{2}} = \frac{\sqrt{5}-1}{\sqrt{2}}$

$\therefore \frac{26-15\sqrt{3}}{\left(5\sqrt{2}-\frac{5\sqrt{3}+1}{\sqrt{2}}\right)^2} + \frac{\sqrt{2}(\sqrt{5}+3)}{\sqrt{8}+\frac{\sqrt{5}-1}{\sqrt{2}}}$

$=\frac{2(26-15\sqrt{3})}{(9-5\sqrt{3})^2} + \frac{\sqrt{2}\sqrt{2}(\sqrt{5}+3)}{4+\sqrt{5}-1}$

$=\frac{52-30\sqrt{3}}{156-90\sqrt{3}} + \frac{2(3+\sqrt{5})}{(3+\sqrt{5})}$

$=\frac{52-30\sqrt{3}}{3(52-30\sqrt{3})} + 2 = \frac{1}{3}+2 = 2\frac{1}{3}$

60. (3) First term $=(28+10\sqrt{3})^{\frac{1}{2}}$

$=(28+2\times5\times\sqrt{3})^{\frac{1}{2}} = (28+2\times\sqrt{5}\times\sqrt{3})^{\frac{1}{2}}$

$=(25+3+2\times\sqrt{5}\times\sqrt{3})^{\frac{1}{2}}$

$=\left[(5)^2+(\sqrt{3})^2+2\times\sqrt{5}\times\sqrt{3}\right]^{\frac{1}{2}}$

$=\left[(5+\sqrt{3})^2\right]^{\frac{1}{2}} = 5+\sqrt{3}$

Second term $=(7-4\sqrt{3})^{-\frac{1}{2}} = (7-2\times2\times\sqrt{3})^{-\frac{1}{2}}$

$=(4+3-2\times2\times\sqrt{3})^{-\frac{1}{2}}$

$=\left[(2)^2+(\sqrt{3})^2-2\times2\times\sqrt{3}\right]^{-\frac{1}{2}}$

$=\left[(2-\sqrt{3})^2\right]^{-\frac{1}{2}} = (2-\sqrt{3})^{2\times-\frac{1}{2}}$

$=(2-\sqrt{3})^{-1} = \frac{1}{(2-\sqrt{3})}$

$=\frac{1\times(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})}{4-3} = 2+\sqrt{3}$

$\therefore$ Expression

$=(5+\sqrt{3})-(2+\sqrt{3}) = 5+\sqrt{3}-2-\sqrt{3} = 3$

61. (2) Expression $=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{(2)^2+(\sqrt{3})^2+2\times2\times\sqrt{3}}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{(2+\sqrt{3})^2}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8(2+\sqrt{3})}}$

$$= \sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{19+8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{16+3+2\times4\times\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{(4)^2+\left(\sqrt{3}\right)^2+2\times4\times\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$$

$$= \sqrt{-\sqrt{3}+4+\sqrt{3}} = \sqrt{4} = 2$$

62. (3) Expression $= \sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}$

$$= \sqrt{6-4\sqrt{3}+\sqrt{16-2\times2\sqrt{3}\times2}}$$

$$= \sqrt{6-4\sqrt{3}+\sqrt{12+4-2\times2\sqrt{3}\times2}}$$

$$= \sqrt{6-4\sqrt{3}+\sqrt{\left(2\sqrt{3}\right)^2+(2)^2-2\times2\sqrt{3}\times2}}$$

$$= \sqrt{6-4\sqrt{3}+\sqrt{\left(2\sqrt{3}-2\right)^2}}$$

$$= \sqrt{6-4\sqrt{3}+2\sqrt{3}-2}$$

$$= \sqrt{4-2\sqrt{3}} = \sqrt{3+1-2\times\sqrt{3}\times1}$$

$$= \sqrt{\left(\sqrt{3}\right)^2+(1)^2-2\times\sqrt{3}\times1}$$

$$= \sqrt{\left(\sqrt{3}-1\right)^2} = \sqrt{3}-1$$

63. (2) Expression $= \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

On rationalising the denominator,

$$= \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \times \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$= \frac{1+x+1-x+2\sqrt{1+x}.\sqrt{1-x}}{(1+x)-(1-x)} = \frac{1+\sqrt{1-x^2}}{x}$$

Putting $x = \dfrac{\sqrt{3}}{2}$

$$\therefore \text{Expression} = \frac{1+\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1+\sqrt{1-\frac{3}{4}}}{\frac{\sqrt{3}}{2}} = \frac{1+\sqrt{\frac{4-3}{4}}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \frac{3}{2}\times\frac{2}{\sqrt{3}} = \sqrt{3}$$

64. (3) Expression

$$= \left[\frac{1\times3\times9+1\times2\times3\times2\times9\times2+1\times3\times3\times3\times9\times3+......}{1\times5\times25+1\times2\times5\times2\times25\times2+1\times3\times5\times3\times25\times3+......}\right]^{\frac{1}{3}}$$

$$= \left[\frac{1\times3\times9+1\times3\times9\times2^3+1\times3\times9\times3^3+......}{1\times5\times25+1\times5\times25\times2^3+1\times5\times25\times3^3+.....}\right]^{\frac{1}{3}}$$

$$= \left[\frac{1\times3\times9\times(1+2^3+3^3+......)}{1\times5\times25\times(1+2^3+3^3+.....)}\right]^{\frac{1}{3}}$$

$$= \left[\frac{3\times9}{5\times25}\right]^{\frac{1}{3}} = \left[\left(\frac{3}{5}\right)^3\right]^{\frac{1}{3}} = \frac{3}{5}$$

65. (2) Expression $= (-1)^{(-1)^{(-1)......}}$

We know that

$$(-1)^{(-1)} = \frac{1}{(-1)} = -1$$

$$\therefore (-1)^{(-1)^{(-1)}} = \left[(-1)^{(-1)}\right]^{-1}$$

$$= (-1)^{-1} = -1$$

Clearly, $(-1)^{(-1)^{(-1)......}} = -1$

66. (4) Expression $= \dfrac{(625)^{6.25} \times (25)^{2.6}}{(625)^{6.75} \times (5)^{1.2}}$

$$= \frac{(5^4)^{6.25} \times (5^2)^{2.6}}{(5^4)^{6.75} \times (5)^{1.2}} = \frac{(5)^{4\times6.25} \times (5)^{2\times2.6}}{(5)^{4\times6.75} \times (5)^{1.2}}$$

$$= \frac{(5)^{25} \times (5)^{5.2}}{(5)^{27} \times (5)^{1.2}} = \frac{(5)^{25+5.2}}{(5)^{27+1.2}} = \frac{(5)^{30.2}}{(5)^{28.2}}$$

$= (5)^{30.2-28.2} = (5)^2 = 25$

67. (1) Expression $= (16)^{0.16} \times (2)^{0.36}$

$= (2^4)^{0.16} \times (2)^{0.36}$ $= (2)^{4\times0.16} \times (2)^{0.36}$

$= (2)^{0.64} \times (2)^{0.36}$ $= (2)^{0.64+0.36}$

$= 2^1 = 2$

68. (2) Expression $= \dfrac{2+\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

[Mulitplying N^r and D^r by $(2+\sqrt{3})$]

$$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} \qquad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{4+3+2\times2\sqrt{3}}{4-3} = 7+4\sqrt{3}$$

69. (2) Expression $= \sqrt{\dfrac{19+8\sqrt{3}}{7-4\sqrt{3}}}$

$$= \sqrt{\frac{19+2\times4\times\sqrt{3}}{7-2\times2\times\sqrt{3}}}$$

$$= \sqrt{\frac{16+3+2\times4\times\sqrt{3}}{4+3-2\times2\times\sqrt{3}}}$$

$$= \sqrt{\frac{(4)^2+(\sqrt{3})^2+2\times4\times\sqrt{3}}{(2)^2+(\sqrt{3})^2-2\times2\times\sqrt{3}}}$$

$$= \sqrt{\frac{(4+\sqrt{3})^2}{(2-\sqrt{3})^2}} \qquad [\because (a\pm b)^2 = a^2+b^2 \pm 2ab]$$

$$= \frac{4+\sqrt{3}}{2-\sqrt{3}}$$

Rationalising the denominator,

$$= \frac{(4+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{8+2\sqrt{3}+4\sqrt{3}+3}{4-3}$$

$= 11+6\sqrt{3}$

70. (3) First part $= \sqrt{\dfrac{(2.4)^6+9(5.76)+6(2.4)^4}{(2.4)^4+6(5.76)+9}}$

$$= \sqrt{\frac{(2.4)^6+9(2.4)^2+6(2.4)^4}{(2.4)^4+6(2.4)^2+9}}$$

$$= \sqrt{\frac{(2.4)^2\left[(2.4)^4+9+6(2.4)^2\right]}{(2.4)^4+6(2.4)^2+9}}$$

$= \sqrt{(2.4)^2} = 2.4$

Second part $= \dfrac{\left[(3^{-2})^{-5}\right]^{\frac{1}{5}} + \left[(4^{-3})^{-6}\right]^{\frac{1}{6}} - (3^{-4})^{\frac{-1}{2}}}{\left[(2^{-3})^{-4}\right]^{\frac{1}{4}}}$

$$= \frac{(3)^{(-2)\times(-5)\times\left(\frac{1}{5}\right)} + (4)^{(-3)\times(-6)\times\frac{1}{6}} - (3)^{(-4)\times\left(-\frac{1}{2}\right)}}{(2)^{(-3)\times(-4)\times\left(\frac{1}{4}\right)}}$$

$$\left[\because \left[(a^x)^y\right]^z = a^{xyz}\right]$$

$$= \frac{3^2+4^3-3^2}{2^3} = \frac{9+64-9}{8} = \frac{64}{8} = 8$$

$\therefore$ Expression $= 2.4 + 8 = 10.4$

71. (1) First part $= \sqrt{11-2\sqrt{30}} = \sqrt{11-2\sqrt{6\times5}}$

$= \sqrt{11-2\times\sqrt{6}\times\sqrt{5}} = \sqrt{6+5-2\times\sqrt{6}\times\sqrt{5}}$

$= \sqrt{\left(\sqrt{6}\right)^2+\left(\sqrt{5}\right)^2-2\times\sqrt{6}\times\sqrt{5}}$

$= \sqrt{\left(\sqrt{6}-\sqrt{5}\right)^2} = \sqrt{6}-\sqrt{5}$

Second part $= \sqrt{7-2\sqrt{10}}$

$= \sqrt{7-2\sqrt{5\times2}} = \sqrt{7-2\sqrt{5}\times\sqrt{2}}$

$= \sqrt{5+2-2\sqrt{5}\times\sqrt{2}}$

$= \sqrt{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\times\sqrt{5}\times\sqrt{2}}$

$= \sqrt{\left(\sqrt{5}-\sqrt{2}\right)^2} = \sqrt{5}-\sqrt{2}$

Third part $= \dfrac{4}{\sqrt{6}+\sqrt{2}}$

Rationalising the denominator,

$= \dfrac{4}{\left(\sqrt{6}+\sqrt{2}\right)}\times\dfrac{\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}-\sqrt{2}\right)}$

$= \dfrac{4\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2} = \dfrac{4\left(\sqrt{6}-\sqrt{2}\right)}{6-2} = \sqrt{6}-\sqrt{2}$

$\therefore$ Expression $= \left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{5}-\sqrt{2}\right)-\left(\sqrt{6}-\sqrt{2}\right)$

$= \sqrt{6}-\sqrt{5}+\sqrt{5}-\sqrt{2}-\sqrt{6}+\sqrt{2} = 0$

72. (3) $\left(\dfrac{3}{5}\right)^3\left(\dfrac{3}{5}\right)^{-6} = \left(\dfrac{3}{5}\right)^{2x-1}$

$\Rightarrow \left(\dfrac{3}{5}\right)^3\left(\dfrac{3}{5}\right)^{-3}\left(\dfrac{3}{5}\right)^{-3} = \left(\dfrac{3}{5}\right)^{2x-1}$

$\Rightarrow \left(\dfrac{3}{5}\right)^0\left(\dfrac{3}{5}\right)^{-3} = \left(\dfrac{3}{5}\right)^{2x-1}$

$\Rightarrow 2x-1 = -3$

$\Rightarrow 2x = -3+1 = -2$

$\Rightarrow x = -1$

73. (4) Expression $= \sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}$

$= \sqrt{10+2\times\sqrt{2}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$

$= \sqrt{2+3+5+2\times\sqrt{2}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$

$= \sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{2}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$

$= \sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2} = \sqrt{2}+\sqrt{3}+\sqrt{5}$

$[(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc]$

74. (2) $\dfrac{\sqrt{7}-2}{\sqrt{7}+2} = \dfrac{\sqrt{7}-2}{\sqrt{7}+2}\times\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$

(Rationalising the denominator)

$= \dfrac{\left(\sqrt{7}-2\right)^2}{7-4} = \dfrac{7+4-4\sqrt{7}}{3}$

$= \dfrac{11}{3}-\dfrac{4\sqrt{7}}{3}$

$\therefore \dfrac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

$\Rightarrow \dfrac{11}{3}-\dfrac{4}{3}\sqrt{7} = a\sqrt{7}+b$

Clearly,

$a = -\dfrac{4}{3}$ and $b = \dfrac{11}{3}$

75. (1) Expression =

$\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\dfrac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{2}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$\times\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

[Rationalising the respective denominators]

$= \dfrac{3\sqrt{2}\left(\sqrt{6}-\sqrt{3}\right)}{6-3}-\dfrac{4\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{6-2}+\dfrac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{3-2}$

$= \sqrt{2}\left(\sqrt{6}-\sqrt{3}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)+\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$

$=\sqrt{12}-\sqrt{6}-\sqrt{18}+\sqrt{6}+\sqrt{18}-\sqrt{12}=0$

76. (2) $x+\frac{1}{4}\sqrt{x}+a^2=\left(\sqrt{x}\right)^2+2.\sqrt{x}.\frac{1}{8}+(a)^2$

Clearly $a=\frac{1}{8}$.

Then, expression $=\left(\sqrt{x}+\frac{1}{8}\right)^2$

77. (2) Given $x=\frac{\sqrt{3}}{2}$

$\frac{\sqrt{1+x}}{1+\sqrt{1+x}}\times\frac{1-\sqrt{1+x}}{1-\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}\times\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}$

$=\frac{\sqrt{1+x}-1-x}{1-1-x}+\frac{\sqrt{1-x}+1-x}{1-1+x}$

$=\frac{\sqrt{1-x}+1-x}{x}-\frac{\sqrt{1+x}-1-x}{x}$

$=\frac{\sqrt{1-x}+1-x-\sqrt{1+x}+1+x}{x}$

$=\frac{2+\sqrt{1-x}-\sqrt{1+x}}{x}=\frac{2+\sqrt{1-\frac{\sqrt{3}}{2}}-\sqrt{1+\frac{\sqrt{3}}{2}}}{\frac{\sqrt{3}}{2}}$

$=\frac{2+\sqrt{\frac{2-\sqrt{3}}{2}}-\sqrt{\frac{2+\sqrt{3}}{2}}}{\frac{\sqrt{3}}{2}}$

$=\frac{2+\frac{\sqrt{4-2\sqrt{3}}}{2}-\frac{\sqrt{4+2\sqrt{3}}}{2}}{\frac{\sqrt{3}}{2}}$

$=\frac{4+\sqrt{3}-1-\sqrt{3}-1}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

78. (1) $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3}=\frac{5+3+2\sqrt{15}}{2}$

$=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$

$\therefore y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=4-\sqrt{15}$

$\therefore x+y=4+\sqrt{15}+4-\sqrt{15}=8$

79. (2) $\sqrt{2}=1.414$ (Given)

Now, $\frac{\sqrt{2}-1}{\sqrt{2}+1}=\frac{\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}$

$=\frac{\left(\sqrt{2}-1\right)^2}{2-1}=\left(\sqrt{2}-1\right)^2$

$\therefore \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}=\sqrt{\left(\sqrt{2}-1\right)^2}=\sqrt{2}-1$

$=1.414-1=0.414$

80. (3) $(0.5)^2=0.25$ $\sqrt{0.49}=0.7$

$\sqrt[3]{0.008}=0.2$ $0.23=0.23$

$\therefore \sqrt{0.49}>(0.5)^2>0.23>\sqrt[3]{0.008}$

81. (3) $\sqrt{\frac{\sqrt{36}-\sqrt{24}+\sqrt{24}-\sqrt{16}}{5+\sqrt{24}}}$

$=\sqrt{\frac{6-4}{5+\sqrt{24}}}=\sqrt{\frac{2}{5+\sqrt{24}}}=\sqrt{\frac{2}{5+\sqrt{6\times4}}}$

$=\sqrt{\frac{2}{5+2\sqrt{6}}}=\sqrt{\frac{2}{5+2\sqrt{6}}\times\frac{5-2\sqrt{6}}{5-2\sqrt{6}}}$

$=\sqrt{\frac{2\left(5-2\sqrt{6}\right)}{25-24}}=\sqrt{2\left(5-2\sqrt{6}\right)}$

$=\sqrt{2\left[\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{3}\sqrt{2}\right]}$

$=\sqrt{2\left(\sqrt{3}-\sqrt{2}\right)^2}=\sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)=\sqrt{6}-2$

82. (1) $\sqrt[3]{4},\sqrt{2},\sqrt[6]{3},\sqrt[4]{5}$

LCM of 3, 2, 6, 4 = 12

$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{4}{12}}=\left(4^4\right)^{\frac{1}{12}}=(256)^{\frac{1}{12}}$

$\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{6}{12}}=\left(2^6\right)^{\frac{1}{12}}=(64)^{\frac{1}{12}}$

$\sqrt[6]{3}=(3)^{\frac{1}{6}}=(3)^{\frac{2}{12}}=\left(3^2\right)^{\frac{1}{12}}=(9)^{\frac{1}{12}}$

$\sqrt[4]{5}=(5)^{\frac{1}{4}}=(5)^{\frac{3}{12}}=\left(5^3\right)^{\frac{1}{12}}=(125)^{\frac{1}{12}}$

$\therefore (256)^{\frac{1}{12}} > (125)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (9)^{\frac{1}{12}}$

or, $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

83. (2) Expression $= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2 - (\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{3+2+2\sqrt{6}-3-2+2\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{4\sqrt{6}}{3-2} = 4\sqrt{6}$$

84. (4) $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} = \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}$

[Rationalising the denominator]

$$= \frac{3\sqrt{12}-3\sqrt{6}}{6-3} = \frac{3(2\sqrt{3}-\sqrt{6})}{3} = 2\sqrt{3}-\sqrt{6}$$

Similarly,

$$\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} = \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2}$$

$$= \sqrt{3}(\sqrt{6}-\sqrt{2}) = \sqrt{18}-\sqrt{6} = 3\sqrt{2}-\sqrt{6}$$

$$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{\sqrt{18}-\sqrt{12}}{3-2} = 3\sqrt{2}-2\sqrt{3}$$

$\therefore$ Expression

$$= 2\sqrt{3}-\sqrt{6}-3\sqrt{2}+\sqrt{6}+3\sqrt{2}-2\sqrt{3} = 0$$

85. (3) $\sqrt{12}+\sqrt{18} = \sqrt{3\times2\times2}+\sqrt{2\times3\times3}$

$= 2\sqrt{3}+3\sqrt{2}$

$\therefore$ Required difference $= 2\sqrt{3}+3\sqrt{2}-2\sqrt{3}-2\sqrt{2} = \sqrt{2}$

86. (3) LCM of the orders of the surds = LCM of 2,3,5 and 7 = 210

$5^{\frac{1}{2}} = 5^{\frac{105}{210}} = (5^{105})^{\frac{1}{210}}$

$4^{\frac{1}{3}} = 4^{\frac{70}{210}} = (4^{70})^{\frac{1}{210}}$

$2^{\frac{1}{5}} = 2^{\frac{42}{210}} = (2^{42})^{\frac{1}{210}}$

$3^{\frac{1}{7}} = 3^{\frac{30}{210}} = (3^{30})^{\frac{1}{210}}$

$\therefore$ The largest number $= 5^{\frac{1}{2}} = \sqrt{5}$

Quicker Approach

5 is the largest radicand and its order is smallest.

$\therefore$ Largest number $= \sqrt{5}$

87. (1) Expression $= \frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$

$$= \frac{\sqrt{7}}{\sqrt{9+7+2\times3\times\sqrt{7}}-\sqrt{9+7-2\times3\times\sqrt{7}}}$$

$$= \frac{\sqrt{7}}{(3+\sqrt{7})-(3-\sqrt{7})} = \frac{\sqrt{7}}{3+\sqrt{7}-3+\sqrt{7}} = \frac{1}{2}$$

88. (2) $\sqrt{7}-\sqrt{5} = \frac{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})}{\sqrt{7}+\sqrt{5}} = \frac{2}{\sqrt{7}+\sqrt{5}}$

Similarly,

$\sqrt{5}-\sqrt{3} = \frac{2}{\sqrt{5}+\sqrt{3}}$

$\sqrt{9}-\sqrt{7} = \frac{2}{\sqrt{9}+\sqrt{7}}$

$\sqrt{11}-\sqrt{9} = \frac{2}{\sqrt{11}+\sqrt{9}}$

$\therefore$ Largest number $= \sqrt{5}-\sqrt{3}$ because its denominator is the smallest.

89. (3) Let $x = \sqrt{7\sqrt{7\sqrt{7\sqrt{7....}}}}$

On squaring both sides,

$x^2 = 7x$

$\Rightarrow x^2 - 7x = 0$

$\Rightarrow x(x-7) = 0 \Rightarrow x = 7$

$\therefore 7 = (7^3)^{y-1} = 7^{3y-3}$

$\Rightarrow 3y - 3 = 1 \Rightarrow 3y = 4$

$\Rightarrow y = \frac{4}{3}$

90. (2) Expression $= \frac{1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}$

$$= \frac{2^{\frac{1}{3}}-1}{\left(2^{\frac{1}{3}}-1\right)\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}+1\right)} = \frac{2^{\frac{1}{3}}-1}{\left(2^{\frac{1}{3}}\right)^3-1}$$

$= 2^{\frac{1}{3}} - 1 = \sqrt[3]{2} - 1$

$[\because (a-b)(a^2+ab+b^2) = a^3 - b^3]$

91. (4) $\frac{1}{\sqrt{2}+\sqrt{1}} = \frac{1}{\sqrt{2}+\sqrt{1}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \sqrt{2} - 1$

$\therefore$ Expression

$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + ...$

$+ \sqrt{99} - \sqrt{98} + \sqrt{100} - \sqrt{99}$

$= \sqrt{100} - 1 = 10 - 1 = 9$

92. (3) $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$

$= \frac{(\sqrt{5}-2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$= \frac{5+4-4\sqrt{5}}{5-4} = 9 - 4\sqrt{5}$

$\therefore \frac{1}{x} = 9 + 4\sqrt{5}$

$\therefore x^4 + x^{-4} = x^4 + \frac{1}{x^4}$

$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$= \left[\left(x + \frac{1}{x}\right)^2 - 2\right]^2 - 2$

$= \left[\left(9 + 4\sqrt{5} + 9 - 4\sqrt{5}\right)^2 - 2\right]^2 - 2$

$= [(18)^2 - 2]^2 - 2$

$= (322)^2 - 2 = 103682$

whole number

Note : It is not required to find the product.

93. (1) $x + \frac{1}{x} = 3$

On squaring,

$\left(x + \frac{1}{x}\right)^2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$

Again, $\left(x + \frac{1}{x}\right)^3 = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 3 \times 3 = 18$

$\therefore \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$

$= 7 \times 18$

$\Rightarrow x^5 + \frac{1}{x^5} + \left(x + \frac{1}{x}\right) = 126$

$\Rightarrow x^5 + \frac{1}{x^5} = 126 - 3 = 123$

94. (1) $\sqrt{6} \times \sqrt{15} = x\sqrt{10}$

$\Rightarrow \sqrt{2 \times 3} \times \sqrt{3 \times 5} = x\sqrt{10}$

$\Rightarrow \sqrt{2} \times \sqrt{5} \times 3 = x\sqrt{10}$

$\Rightarrow 3\sqrt{10} = x\sqrt{10}$

$\Rightarrow x = 3$

95. (2) $x = \frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow x = \frac{2\sqrt{3 \times 8}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3} \times \sqrt{8}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x}{\sqrt{8}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x+\sqrt{8}}{x-\sqrt{8}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}}$

(By componendo and dividendo)

$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Again,

$\frac{x}{\sqrt{12}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x+\sqrt{12}}{x-\sqrt{12}}$

$= \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}}$

$$= \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$\therefore \frac{x+\sqrt{8}}{x-\sqrt{8}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}-\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}} = 2$$

96. (2) $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$$

$$= \frac{4+4\sqrt{3}+3}{4-3}$$

$$= 7+4\sqrt{3}$$

$$\therefore b = \frac{2-\sqrt{3}}{2+\sqrt{3}} = 7-4\sqrt{3}$$

$$\therefore a+b = 7+4\sqrt{3}+7-4\sqrt{3} = 14$$

$$ab = \left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)$$

$= 49 - 48 = 1$

$\therefore a^2 + b^2 + ab = (a+b)^2 - ab$

$= (14)^2 - 1 = 196 - 1 = 195$

97. (4) $x = \frac{2\sqrt{3}\times\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\Rightarrow \frac{x}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{2}}{x-\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

(By componendo and dividendo)

Similarly,

$$\frac{x}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}}$$

$$= \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$\therefore$ Expression

$$= \frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{3}}{x-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}-\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}} = 2$$

98. (2) $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{3-2} = 3+2-2\sqrt{6} = 5-2\sqrt{6}$$

$$\therefore b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5+2\sqrt{6}$$

$\Rightarrow a + b = 10;$

$ab = \left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right) = 25 - 24 = 1$

$$\therefore \frac{a^2}{b} + \frac{b^2}{a} = \frac{a^3+b^3}{ab} = \frac{(a+b)^3 - 3ab\,(a+b)}{ab}$$

$= 10^3 - 3 \times 10 = 1000 - 30 = 970$

99. (4) $2\sqrt{x} = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2 - \left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$$

$$= \frac{4.\sqrt{5}.\sqrt{3}}{5-3} = 2\sqrt{15}$$

$$\therefore 2\sqrt{x} = 2\sqrt{15} \Rightarrow x = 15$$

100. (2) $x = 2+\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\therefore \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2$$

$$= 2+\sqrt{3}+2-\sqrt{3}+2$$

$$= 6$$

$$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$$

101. (4) Expression

$$= \frac{3+\sqrt{6}}{5\sqrt{3} - 2\sqrt{12} - \sqrt{32} + \sqrt{50}}$$

$$= \frac{3+\sqrt{6}}{5\sqrt{3} - 4\sqrt{3} - 4\sqrt{2} + 5\sqrt{2}}$$

$$= \frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}} = \sqrt{3} = 1.732$$

102. (3) $x = \dfrac{\sqrt{3}}{2}$

$$\therefore \sqrt{1+x} = \sqrt{1+\frac{\sqrt{3}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2}} = \sqrt{\frac{4+2\sqrt{3}}{4}}$$

$$= \sqrt{\frac{(\sqrt{3}+1)^2}{4}} = \frac{\sqrt{3}+1}{2}$$

$$\therefore \sqrt{1-x} = \frac{\sqrt{3}-1}{2}$$

$$\therefore \sqrt{1+x} + \sqrt{1-x}$$

$$= \frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}$$

$$= \frac{\sqrt{3}+1+\sqrt{3}-1}{2} = \sqrt{3}$$

103. (3) Expression $= \dfrac{(81)^{3.6} \times (9)^{2.7}}{(81)^{4.2} \times 3}$

$$= \frac{(3^4)^{3.6} \times (3^2)^{2.7}}{(3^4)^{4.2} \times 3} = \frac{3^{14.4} \times 3^{5.4}}{3^{16.8} \times 3}$$

$[\because (a^m)^n = a^{mn};\ a^m \times a^n = a^{m+n};\ a^m \div a^n = a^{m-n}]$

$$= \frac{3^{14.4+5.4}}{3^{16.8+1}} = \frac{3^{19.8}}{3^{17.8}} = 3^{19.8-17.8} = 3^2 = 9$$

104. (4) Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

On squaring both sides,

$$x^2 = 6 + \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x-3) + 2(x-3) = 0$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = 3$ and $x \neq -2$ because numbers are positive.

105. (3) $(0.1)^2 = 0.01$

$$\sqrt{0.0121} = \sqrt{0.11 \times 0.11} = 0.11$$

$$\sqrt{0.0004} = 0.02$$