Linear equations

Tip 1

- Linear equations is one of the foundation topics in the Quant section of CAT.
- Hence, fundamentals of this concept are useful in solving the questions of the other topics by assuming the unknown values as variables.
- Be careful of silly mistakes in this topic as that is how students generally lose marks here.
- Generally, the number of equations needed to solve the given problem is equal to the number of variables

Tip 2

- A linear equation is an equation which gives straight line when plotted on a graph.
- Linear equations can be of one variable or two variable or three variable.
- Let a, b, c and d are constants and x, y and z are variables. A general form of single variable linear equation is $ax+b = 0$.
- A general form of two variable linear equation is $ax+by = c$.
- A general form of three variable linear equation is $ax+by+cz = d$.

Tip 3

Equations with two variables:

- Consider two equations ax+by = c and mx+ny = p. Each of these equations represent two lines on the x-y co-ordinate plane. The solution of these equations is the point of intersection.

- If $\frac{a}{m} = \frac{b}{n} \neq \frac{c}{p}$ then the slope of the two equations is equal and so they are parallel to each other. Hence, no point of intersection occurs. Therefore no solution.

- If $\frac{a}{m} \neq \frac{b}{n}$ then the slope is different and so they intersect each other at a single point. Hence, it has a single solution.

- If $\frac{a}{m} = \frac{b}{n} = \frac{c}{p}$ then the two lines are same and they have infinite points common to each other. So, infinite solutions occurs

Tip 4

General Procedure to solve linear equations:

- Aggregate the constant terms and variable terms
- For equations with more than one variable, eliminate variables by substituting equations in their place.
- Hence, for two equations with two variables x and y, express y in terms of x and substitute this in the other equation.
- For Example, let $x+y = 14$ and $x+4y = 26$ then $x = 14-y$ (from equation 1) substituting this in equation 2, we get $14-y+4y = 26$. Hence, $y = 4$ and $x = 10$.

Tip 5

General Procedure to solve linear equations:

For equations of the form $ax+by = c$ and $mx+ny = p$, find the LCM of b and n. Multiply each equation with a constant to make the y term coefficient equal to the LCM. Then subtract equation 2 from equation 1.

Example:

Let $2x+3y = 13$ and $3x+4y = 18$ are the given equations (1) and (2).

- LCM of 3 and 4 is 12.
- Multiplying (1) by 4 and (2) by 3, we get $8x+12y = 52$ and $9x+12y = 54$.
- (2)-(1) gives $x=2$, $y=3$

Tip 6

- If the system of equations has *n* variables with *n-1* equations then the solution is indeterminate

- If system of equations has *n* variables with *n-1* equations with some additional conditions like the variables are integers then the solution may be determinate

- If system of equations has *n* variables with *n-1* equations then some combination of variables may be determinable.

- For example, if ax+by+cz = d and mx+ny+pz = q, if a, b, c are in Arithmetic progression and m, n and p are in AP then the sum x+y+z is determinable.

Tip 7

Equations with three variables:
Let the equations be $a_1x+b_1y+c_1z = d_1$, $a_2x+b_2y+c_2z = d_2$ and $a_3x+b_3y+c_3z = d_3$. Here we define the following matrices.

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix} D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

- If Determinant of $D \neq 0$, then the equations have a unique solution.
- If Determinant of $D = 0$, and at least one but not all of the determinants D_x, D_y or D_z is zero, then no solution exists.
- If Determinant of $D = 0$, and all the three of the determinants D_x, D_y and D_z are zero, then there are infinitely many solution exists.
- Determinant can be calculated by $D = a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$