Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 34

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	2(Q1,5)	1(Q19)		1(Q26)			6
2	Polynomials	20	1(Q4)			1(Q28)			4
3	Pair of Linear Equations in Two Variables		1(Q2)		1(Q25)				3
4	Quadratic Equations		1(Q3)				1(Q32)		6
5	Arithmetic Progression					1(Q29)		1(Q36)	7
6	Triangles	15	2(Q6,9)		1(Q21)		1(Q33)		9
7	Circles		1(Q13)		1(Q23)	1(Q30)			6
8	Coordinate Geometry	6	2(Q8,11)					1(Q37)	6
9	Introduction to Trigonometry	12	2(Q7,10)	1(Q20)	1(Q22)	1(Q27)			8
10	Some Applications of Trigonometry							1(Q38)	4
11	Areas Related to Circles	10	3(Q12,14,16)		1(Q24)				5
12	Surface Areas and Volumes						1(Q34)		5
13	Statistics	11	2(Q15,18)			1(Q31)	1(Q35)		10
14	Probability		1(Q17)						1
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based/integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. The number 313 – 310 is divisible by

(a) 2 and 3 (b) 3 and 10 (c) 2, 3 and 10 (d) 2, 3 and 13

2. which of the following is true if following pair of linear equations has unique solution?

$3x - 2y = -8$

$(2m - 5)x + 7y - 6 = 0$

(a) $m = \frac{11}{4}$ (b) $m = -\frac{11}{4}$ (c) $m \neq -\frac{11}{4}$ (d) $m \neq \frac{11}{4}$

3. If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, then k = ?

(a) 1 or 4 (b) –1 or 4 (c) 1 or – 4 (d) –1 or – 4

4. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

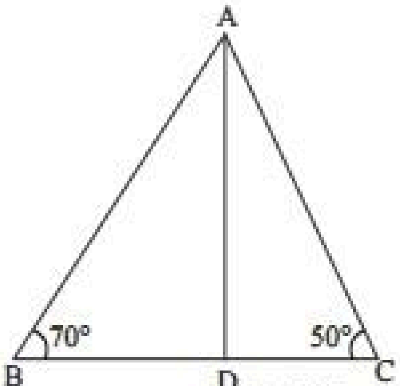
(a) $m, m+3$ (b) $-m, m+3$ (c) $m, -(m+3)$ (d) $-m, -(m+3)$

5. Which of the following statement(s) is/are always true?

(a) The sum of two distinct irrational numbers is rational. (b) The rationalising factor of a number is unique.

(c) Every irrational number is a surd. (d) None of these

6. In ΔABC, $\frac{AB}{AC} = \frac{BD}{DC}$, ∠B = 70° and ∠C = 50°. Then, ∠BAD = _______.



(a) 30° (b) 40° (c) 50° (d) 45°

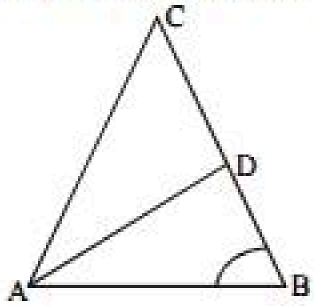
7. If $x = a\,(\text{cosec}\,\theta + \cot\theta)$ and $y = \frac{b(1 - \cos\theta)}{\sin\theta}$, then xy =

(a) $\frac{a^2 + b^2}{a^2 - b^2}$ (b) $a^2 - b^2$ (c) ab (d) $\frac{a}{b}$

8. If $\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(–6, 5) and B(–2, 3), then what is the value of 'a'?

(a) –4 (b) –12 (c) 12 (d) –6

9. In the figure, ABC is a triangle in which AD bisects $\angle A$, AC = BC, $\angle B = 72°$ and CD = 1cm. Length of BD (in cm) is



(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{3}+1}{2}$

10. If $\cot\theta = \left(\frac{15}{8}\right)$, then evaluate $\frac{(2+2\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)\,(2-2\cos\theta)}$

(a) 1 (b) $\frac{225}{64}$ (c) $\frac{156}{7}$ (d) –1

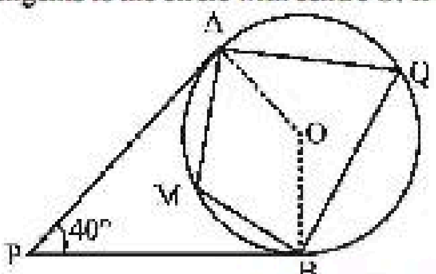
11. The points (a, b), (a^1, b^1) and $(a - a^1, b - b^1)$ are collinear if

(a) $ab = a^1b^1$ (b) $ab^1 = a^1b$ (c) a = b (d) $a^1 = b^1$

12. If the sum of the circumferences of two circles with diameters d_1 and d_2 is equal to the circumference of a circle of diameter d, then

(a) $d_1^2 + d_2^2 = d^2$ (b) $d_1 + d_2 = d$ (c) $d_1 + d_2 > d$ (d) $d_1 + d_2 < d$

13. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 40°$, find $\angle AQB$ and $\angle AMB$.



(a) $\angle AQB = 70°$, $\angle AMB = 110°$ (b) $\angle AQB = 110°$, $\angle AMB = 70°$

(c) $\angle AQB = 100°$, $\angle AMB = 50°$ (d) $\angle AQB = 60°$, $\angle AMB = 40°$

14. Find area of minor segment made by a chord which subtends right-angle at the centre of a circle of radius 10 cm.

(a) 24.5 cm^2 (b) 25.5 cm^2 (c) 24.5 cm^2 (d) 28.5 cm^2

15. There are three sections A, B and C in class X with 25, 40 and 35 students respectively. The average marks obtained by section A, B and C are 70%, 65% and 50% respectively. Find the average marks of entire class X.

(a) 59% (b) 56% (c) 63% (d) 61%

16. The diameter of a garden roller is 1.4 m, and 2m long. How much area will it cover in 5 revolutions.

(a) 44 m^2 (b) 140 m^2 (c) 440 m^2 (d) 220 m^2

17. Two dice are thrown at a time, then find the probability that the difference of the numbers shown on the dice is 1.

(a) $\frac{3}{16}$ (b) $\frac{5}{18}$ (c) $\frac{7}{36}$ (d) $\frac{7}{18}$

18. Calculate the mean of the following frequency distribution:

C. I.	0–80	80–160	160–240	240–320	320–400
Frequency	22	35	44	25	24

(a) 195.5 (b) 198.8 (c) 196.8 (d) 195

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion:** $n^2 - n$ is divisible by 2 for every positive integer.

Reason: $\sqrt{2}$ is not a rational number.

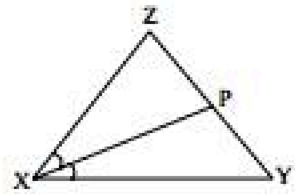
20. **Assertion:** In a right angled triangle, if $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $\tan\theta = \sqrt{3}$.

Reason: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. In a ΔXYZ, if the internal bisector of $\angle X$ meets YZ at 'P', then prove $\frac{XY + XZ}{XZ} = \frac{YZ}{PZ}$

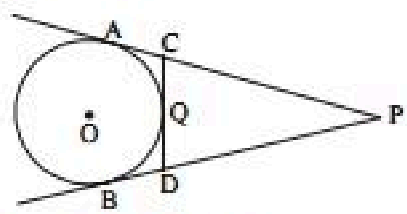


22. If $\sec\theta.\sin\theta = 0$, then find the value of θ.

OR

If $5\tan\theta = 4$, then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$

23. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD.



24. Find the radius of the circle inscribed in a square of side 10cm.

25. Solve the following pair of equations : $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ and $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

OR

Solve the system of equations : $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ and $\frac{3}{x} + \frac{2}{y} = 0$ and hence find 'p' for which $y = px - 4$.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Prove that $\sqrt{5}$ is irrational.

27. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$. Hence, find the values of $\cos\theta$ and $\sin\theta$.

OR

If $3\cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

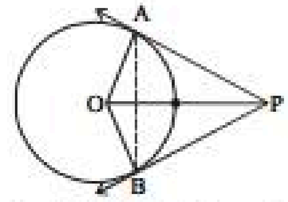
28. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

29. The first three terms of an AP respectively are $3y - 1$, $3y + 5$ and $5y + 1$. Then y equals:

OR

If k, $2k - 1$ and $2k + 1$ are three consecutive terms of an AP the value of k is

30. In the adjoining figure, PA and PB are tangents to a circle with centre O. If OP is equal to the diameter of the circle, prove that ΔABP is an equilateral triangle.



31. If the median for the following frequency distribution is 28.5, find the values of x and y :

Class Interval	Frequencies
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5
Total	60

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

33. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

OR

If the corresponding sides of two triangles are proportional (i.e., in the same ratio), then prove that their corresponding angles are equal and hence the two triangles are similar.

34. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

OR

Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

35. The median of the following data is 525. Find the values of x and y if the total frequency is 100.

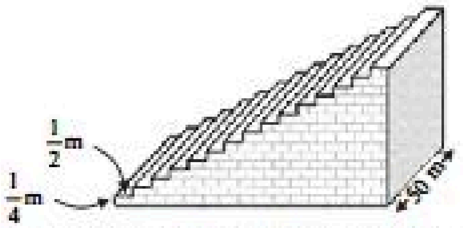
Class Interval	0–100	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	900–1000
Frequency	2	5	x	12	17	20	y	9	7	4

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. **Case - Study 1:** Read the following passage and answer the questions given below.

Students of class X were given a task to observe the application of arithmetic progression for the construction of staiyers at a football ground.



In the figure a small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}m$ and a tread of $\frac{1}{2}m$

Then, answer the following questions

(i) Find volume of the concrete for the first step of terrace.

(ii) Find volume of the concrete for the 8th step of terrace.

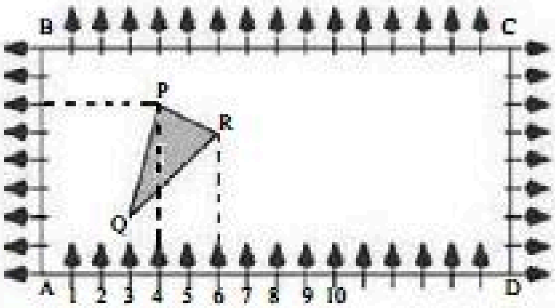
(iii) Find the common difference of A.P formed by the volume of steps of terrace.

OR

Find total volume of concrete required to build the terrace.

37. Case - Study 2: Read the following passage and answer the questions given below.

Class X students of a secondary school in Krishnagar have been allotted a rectangular plot of a land for gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the fig. The students are to sow seeds of flowering plants on the remaining area of the plot.



Considering A as origin, answer following questions.

(i) Considering A as the origin, what are the coordinates of A?

(ii) What are the coordinates of P?

(iii) What are the coordinates of R?

OR

What are the coordinates of D?

38. Case - Study 3: Read the following passage and answer the questions given below.

Two pillars of equal height are one either side of a road, which is 100 m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars, then

(i) Find distance of the point from a pillar making an angle of 60° is

(ii) Find distance of the point from a pillar making an angle of 30° is

(iii) Find the height of each pillar is

OR

Find the sum of distance of point from both tops.

Solution

SAMPLE PAPER-4

1. **(d)** $3^{13} - 3^{10} = 3^{10}(33 - 1) = 3^{10}(26) = 2 \times 13 \times 3^{10}$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

2. **(c)** For a pair of linear equations having unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{3}{2m-5} \neq \frac{-2}{7}$$

or $-4m + 10 \neq 21$

or $-4m \neq 11$

or $m \neq -\frac{11}{4}$

3. **(a)** Since the roots are equal, we have $D = 0$.

$\therefore$ $4(k+2)^2 - 36k = 0 \Rightarrow (k+2)^2 - 9k = 0$

$\therefore$ $k^2 - 5k + 4 = 0 \Rightarrow k^2 - 4k - k + 4 = 0$

$\Rightarrow$ $k(k-4) - (k-4) = 0$

$\Rightarrow$ $(k-4)(k-1) = 0 \Rightarrow k = 4$ or $k = 1$.

4. **(b)** $x^2 - (m+3)x + mx - m(m+3) = 0$

$\Rightarrow$ $x[x - (m+3)] + m[x - (m+3)] = 0$

$\Rightarrow$ $(x+m)[x - (m+3)] = 0$

$x + m = 0$ $\quad$ $x - (m+3) = 0$

$x = -m$ $\quad$ $x = m + 3$

5. **(d)**

6. **(a)** In ABC, $\angle A = 180° - (70° + 50°) = 60°$.

$\frac{BD}{DC} = \frac{AB}{AC}$. It means AD is the bisector of DA.

$\therefore$ $\angle BAD = \frac{1}{2} \times 60° = 30°$

7. **(c)** We have, $x = a\,(\text{cosec}\,\theta + \cot\theta)$

$\Rightarrow$ $\frac{x}{a} = (\text{cosec}\,\theta + \cot\theta)$ ...(1)

and $y = b\left(\frac{1-\cos\theta}{\sin\theta}\right) \Rightarrow \frac{y}{b} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow$ $\frac{y}{b} = \text{cosec}\,\theta - \cot\theta$ ...(2)

$\Rightarrow$ $\frac{x}{a} \times \frac{y}{b} = (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta)$

$\Rightarrow$ $\frac{xy}{ab} = (\text{cosec}^2\theta - \cot^2\theta)$ $\quad \therefore$ $xy = ab$

8. **(b)** Coordinates of mid-point are given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Here, coordinates of mid-point are $\left(\frac{a}{3}, 4\right)$

So, $\frac{a}{3} = \frac{-6-2}{2}$

$\therefore$ $a = -12$

9. **(c)** Let $BD = x$ cm

Since AC = BC, therefore ΔABC is an isoscele triangle.

$\Rightarrow$ $\angle B = \angle CAB = 72°$

Since AD bisects $\angle A$

$\therefore$ $\angle DAB = 36°$ so, In ΔADB, $\angle ADB = 72°$

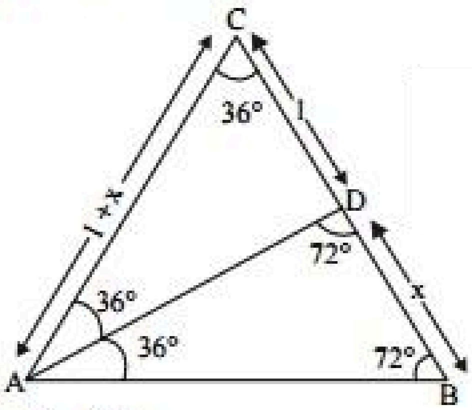
$\Rightarrow$ ADB is an isoscele triangle

$\therefore$ AB = AD = 1cm

$\Rightarrow$ AB = 1 cm

Similarly, ADC is also an isoscele triangle.

$\therefore$ AD = CD $\Rightarrow$ AD = 1 cm



Now $\frac{AC}{AB} = \frac{CD}{BD}$

$\Rightarrow$ $\frac{1+x}{1} = \frac{1}{x} \Rightarrow x + x^2 - 1 = 0$

$\Rightarrow$ $x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$BD = \frac{\sqrt{5} - 1}{2}$

10. **(b)** $\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)} = \frac{2(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2)(1 - \cos\theta)}$

$= \frac{2(1 - \sin^2\theta)}{2(1 - \cos^2\theta)} = \frac{2\cos^2\theta}{2\sin^2\theta} = \cot^2\theta = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$

11. **(b)** $\frac{a - (a - a^1)}{b - (b - b^1)} = \frac{a^1}{b^1}$

12. **(b)** $\pi d_1 + \pi d_2 = \pi d \Rightarrow d_1 + d_2 = d$

13. **(a)** Since, OA $\perp$ PA and OB $\perp$ PB,

$\therefore$ In quadrilateral AOBP,

$\Rightarrow$ $40° + 90° + 90° + \angle AOB = 360°$

$\Rightarrow$ $\angle AOB = 140°$

Also, $\angle AQB = \frac{1}{2}$ of $\angle AOB = 70°$ and $\angle AMB = \frac{1}{2}$

of reflex $\angle AOB = \frac{1}{2} \times (360° - 140°) = \frac{1}{2} \times 220 = 110°$

[$\because$ The angle subtended by an arc at the centre is double the angle subtended by the arc at any point on the remaining part of the cirlce.]

14. **(d)** Let AB be the chord of circle such that $\angle AOB = 90°$

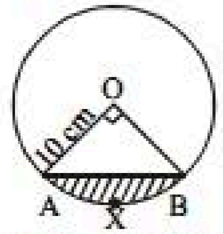
Let OA = 10 cm

$\therefore$ $AB = 10\sqrt{2}$ cm

Area of minor segment A X B

= Area of the sector AOB – Area of ΔAOB

$= \frac{90°}{360°} \times \pi(10)^2 - \frac{1}{2} \times 10 \times 10$



$= 25\,p - 50 = 25 \times 3.14 - 50 = 78.5 - 50 = 28.5 \text{ cm}^2.$

15. **(d)** $n_1 = 25 \quad \overline{x_1} = 70\%$

$n_2 = 40 \quad \overline{x_2} = 65\%$

$n_3 = 35 \quad \overline{x_3} = 50\%$

$$\overline{x} = \frac{n_1\overline{x_1} + n_2\overline{x_2} + n_3\overline{x_3}}{n_1 + n_2 + n_3}$$

$$= \frac{(25 \times 70) + (40 \times 65) + (35 \times 50)}{25 + 40 + 35}$$

$$= \frac{1750 + 2600 + 1750}{100} = \frac{6100}{100} = 61\%$$

16. **(a)** $r = \frac{1.4}{2} = 0.7$ m and $h = 2$ m

Area covered = C.S.A × number of revolutions

$= 2\pi rh \times 5 = 10\pi rh$

$\Rightarrow \quad 10\left(\frac{22}{7}\right)(0.7)(2) = 44 \text{ m}^2$

17. **(b)** $n(S) = 6 \times 6 = 36$

E= {(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4), (5,6), (6,5)}

$n(E) = 10$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

18. **(c)** Let a = 200

C.I.	f_i	x_i	$d_i = x_i - 200$	f_id_i
0 – 80	22	40	– 160	– 3520
80 – 160	35	120	– 80	– 2800
160 – 240	44	200	0	0
240 – 320	25	280	+ 80	+ 2000
320 – 400	24	360	+ 160	+ 3840
	150			– 480

$$\overline{x} = a + \frac{1}{n}\sum_{i=1}^{5} f_i d_i = 200 + \frac{1}{150}(-480)$$

$= 200 - 3.2 = 196.8$

19. **(b)** Put n = 1 or 2.

20. **(a)** Both assertion and reason are correct and reason is the correct explanation of the assertion.

$\tan\theta = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$.

21. The internal bisector of $\angle X$ meets YZ at P

$\frac{XY}{XZ} = \frac{YP}{PZ}$ [1 Mark]

Add 1 on both the sides ; we get

$\Rightarrow \frac{XY}{XZ} + 1 = \frac{YP}{PZ} + 1 \Rightarrow \frac{XY + XZ}{XZ} = \frac{YP + PZ}{PZ} = \frac{YZ}{PZ}$

[1 Mark]

22. $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1}$

$= \operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$ [1 Mark]

$= \operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1 - \sin\theta} - \frac{\sin\theta}{1 + \sin\theta}\right]$

$= \frac{1}{\sin\theta}\left[\frac{(1 + \sin\theta) - (1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}\right]$

$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1 - \sin^2\theta}\right]$

$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$ [1 Mark]

= RHS

OR

$5\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5} = \frac{\text{Perpendicular}}{\text{Base}}$

$(\text{Hypotenuse})^2 = (\text{Perp})^2 + (\text{Base})^2 = (4)^2 + (5)^2 = 41$

Hypotenuse = $\sqrt{41}$ [½ Mark]

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{4}{\sqrt{41}}$;

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{\sqrt{41}}$ [½ Mark]

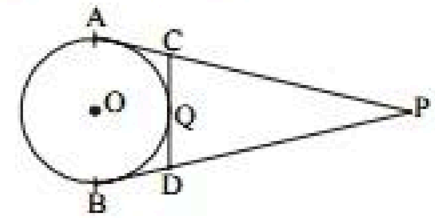
Consider $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{5 \times \frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}}$

$$= \frac{\frac{20}{\sqrt{41}} - \frac{15}{\sqrt{41}}}{\frac{20}{\sqrt{41}} + \frac{10}{\sqrt{41}}} = \frac{\frac{20 - 15}{\sqrt{41}}}{\frac{20 + 10}{\sqrt{41}}} = \frac{\frac{5}{\sqrt{41}}}{\frac{30}{\sqrt{41}}} = \frac{5}{\sqrt{41}} \times \frac{\sqrt{41}}{30} = \frac{1}{6}$$

[1 Mark]

23. **Given :** PA and PB are tangents to the circle from an external point P. CD is another tangent at Q.

PA = 12 cm, QC = QD = 3 cm



To find: PC + PD [½ Mark]

Proof : PA = PC + AC

12 = PC + 3

[$\because$ QC = AC = 3 cm, tangents from external point to a circle are equal in length]

PC = 9 cm ...(i) [½ Mark]

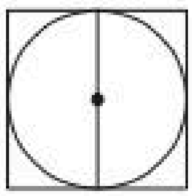
Similarly, BD = QD = 3 cm

and PB = PA = 12 cm [½ Mark]

PB = PD + BD $\Rightarrow$ 12 = PD + 3 $\Rightarrow$ PD = 9 cm

Now, PC + PD = 9 + 9 = 18 cm [½ Mark]

24. $r = \frac{10}{2}$ cm = 5 cm [2 Marks]



25. Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

Then,

$2X + 3Y = 2$

$4X - 9Y = -1$

By elimination method

$$\begin{array}{l} 4X - 9Y = -1 \\ 6X + 9Y = 6 \\ \underline{+ \quad + \quad +} \\ 10X = 5 \end{array}$$

[1 Mark]

$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$ [½ Mark]

Now, $3Y = 2 - 2X = 2 - 2\left(\frac{1}{2}\right) = 1 = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$

[½ Mark]

Thus $x = 4, y = 9$ is the solution.

OR

$\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ .....(i)

$\frac{3}{x} + \frac{2}{y} = 0$ .....(ii)

Multiplying (i) by 3 and equation (ii) by 2 we get,

$\frac{6}{x} + \frac{6}{3y} = \frac{3}{6}$ .....(iii)

$\frac{6}{x} + \frac{4}{y} = 0$ .....(iv)

Subtract (iii) and (iv), we get,

$-\frac{2}{y} = \frac{3}{6} \Rightarrow \frac{-2}{y} = \frac{1}{2} \Rightarrow y = -4$ [1 Mark]

Putting the value of $y = -4$ in (i) we get,

$\frac{2}{x} + \frac{2}{3(-4)} = \frac{1}{6} \Rightarrow \frac{2}{x} - \frac{2}{12} = \frac{1}{6}$

$\Rightarrow \frac{2}{x} = \frac{1}{6} + \frac{1}{6} \Rightarrow \frac{2}{x} = \frac{2}{6} \Rightarrow x = 6$ [½ Mark]

Given equation is $y = px - 4$

Putting the value of $x = 6$ and $y = -4$ we get $-4 = p \times 6 - 4$

$\Rightarrow -4 + 4 = 6p \Rightarrow p = 0$ [½ Mark]

26. Let if possible $\sqrt{5} = \frac{p}{q}$, where p and q are co-prime.

$\therefore \quad 5 \times q^2 = p^2$ ....(i)

$\Rightarrow$ 5 divides $p \Rightarrow p = 5 \times p_1$; p_1 is an integer. ......(ii)

[½ Mark]

From (i) and (ii), we get :

$5 \times q^2 = (5 \times p_1)^2 = 5^2 \times p_1^2 \Rightarrow q^2 = 5 \times p_1^2 \Rightarrow$ 5 divides q

[½ Mark]

$\Rightarrow \quad q = 5 \times q_1$; q_1 is an integer .....(iii)

[½ Mark]

From (ii) and (iii), we find 5 a common factor of p and q. It contradicts that p and q are co-prime.

Hence, $\sqrt{5}$ is an irrational number. [½ Mark]

27. $\sec\theta + \tan\theta = p$

since $\sec^2\theta - \tan^2\theta = 1 \Rightarrow (\sec\theta + \tan\theta) \times (\sec\theta - \tan\theta) = 1$

[½ Mark]

$\Rightarrow \quad p \times (\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ [½ Mark]

By elimination method

$\sec\theta + \tan\theta = p$ ...(i)

$\sec\theta - \tan\theta = \frac{1}{p}$ ...(ii)

$\underline{+ \quad + \quad +}$

$2\sec\theta = p + \frac{1}{p} \Rightarrow 2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$

[½ Mark]

$\because \sec\theta = \frac{1}{\cos\theta} \Rightarrow \cos\theta = \frac{2p}{p^2+1}$ [½ Mark]

Subtract equation (ii) from (i)

$\sec\theta + \tan\theta = p$

$\sec\theta - \tan\theta = \frac{1}{p}$ [½ Mark]

$\underline{- \quad + \quad -}$

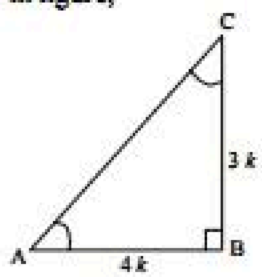
$2\tan\theta = p - \frac{1}{p} \Rightarrow \tan\theta = \frac{p^2-1}{2p}$ [½ Mark]

Now, $\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \tan\theta \cdot \cos\theta$

$= \frac{p^2-1}{2p} \times \frac{2p}{p^2+1} = \frac{p^2-1}{p^2+1}$ [½ Mark]

OR

In figure,



$3 \cot A = 4$ (given) $\Rightarrow \cot A = \frac{4}{3}$

$\Rightarrow \left(\frac{\text{Base}}{\text{Perpendicular}}\right) = \frac{AB}{BC} = \frac{4}{3}$ [½ Mark]

Let $AB = 4k$ and $BC = 3k$

In right angled ΔABC,

$AC^2 = BC^2 + AB^2$ (By pythagoras theorem)

$\Rightarrow AC = \sqrt{(4k)^2 + (3k)^2} = \pm 5k$

$\Rightarrow AC = 5k$ ($\because$ side cannot negative) [½ Mark]

Then, $\sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$ and

$\tan A = \frac{BC}{AB} = \frac{3}{4}$ [½ Mark]

Now, LHS $= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{7}{25}$ [½ Mark]

RHS $= \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

Therefore, LHS = RHS,

$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ [1 Mark]

28. If α and β are the zeroes of $2x^2 - 3x + 1$, then

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \quad \alpha + \beta = \frac{3}{2}$

and $\quad \alpha\beta = \frac{c}{a}$

$\Rightarrow \quad \alpha\beta = \frac{1}{2}$ [1 Mark]

New quadratic polynomial whose zeroes are 3α and 3β is:

x^2 – (Sum of the roots)x + Product of the roots

$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$

$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$

$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$ [1 Mark]

$= x^2 - \frac{9}{2}x + \frac{9}{2}$

$= \frac{1}{2}(2x^2 - 9x + 9)$

Hence, required quadratic polynomial is

$\frac{1}{2}(2x^2 - 9x + 9)$. [1 Mark]

29. $a = 5, l = 45$

Let d = common difference

$S_n = 400$

$l = a + (n - l)d$

$(n - l)d = 45 - 5 = 40$ ...(i)

$S_n = 400$

$\frac{n}{2}[2a + (n-1)d] = 400$

$n[10 + 40] = 800$

$n = 16$

$d = \frac{40}{n-1} = \frac{40}{15} = \frac{8}{3}$

OR

$a_7 = \frac{1}{9}, a_9 = \frac{1}{7}$

$\therefore \quad a + 6d = \frac{1}{9}$ [By term formula]

$a + 8d = \frac{1}{7}$

$-\quad -\quad -$

$-2d = \frac{1}{9} - \frac{1}{7}$

$-2d = \frac{-2}{63}$

$d = \frac{1}{63}$

$a + 6d = \frac{1}{9}$

$a + \frac{2}{21} = \frac{1}{9}$

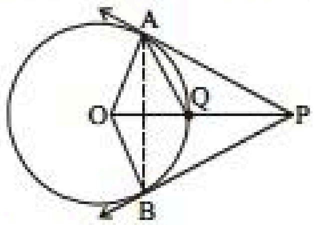
$a = \frac{1}{9} - \frac{2}{21}$

$a = \frac{7-6}{63} = \frac{1}{63}$

$a_{63} = a + 62d$

$a_{63} = \frac{63}{63} = 1$

30. Let OP meet the circle at Q. Join AQ. As OP is equal to the diameter of the circle and OQ is radius,so OQ = QP i.e. Q is mid-point of OP. Since PA is tangent to the circle at A and OA is its radius, OA ⊥ AP i.e. ∠OAP = 90°. In right triangle OAP, Q is mid-point of hypotenuse,



$\therefore$ AQ = OQ = QP

Also OA = OQ (radii of same circle)

$\Rightarrow$ OA = OQ = AQ $\Rightarrow$ ΔOAQ is equilateral

$\Rightarrow$ ∠AOQ = 60° $\Rightarrow$ ∠AOP = 60°. [1 Mark]

In ΔOAP, ∠OPA + ∠AOP + ∠OAP = 180°

$\Rightarrow$ ∠OPA + 60° + 90° = 180° $\Rightarrow$ ∠OPA = 30°

$\Rightarrow$ ∠APB = 60° ($\therefore$ OP is bisector of ∠APB) [½ Mark]

Also PA = PB $\Rightarrow$ ∠PAB = ∠PBA.

In ΔPAB, ∠PAB + ∠PBA + ∠APB = 180° [½ Mark]

$\Rightarrow$ 2 ∠PAB + 60° = 180° $\Rightarrow$ ∠PAB = 60°

$\Rightarrow$ Triangle ABP is equilateral. [1 Mark]

31.

Class	f	c.f.
0 – 10	5	5
10 – 20	x	$x+5$
20 – 30	20	$x+25$
30 – 40	15	$x+40$
40 – 50	y	$x+y+40$
50 – 60	5	$x+y+45$
	$\Sigma f = 60$	

[1 Mark]

From table, $N = 60 = x + y + 45 \Rightarrow x + y = 60 - 45 = 15$ ... (i) [½ Mark]

Since, Median = 28.5

Median class = 20 – 30

Median = $l + \dfrac{\left(\dfrac{N}{2} - c.f.\right)}{f} \times h$ [½ Mark]

$\Rightarrow 28.5 = 20 + \dfrac{[30-(x+5)]}{20} \times 10 \Rightarrow 8.5 = \dfrac{25-x}{2}$

$\Rightarrow 25 - x = 17 \Rightarrow x = 25 - 17 = 8$ [½ Mark]

From (i), $y = 15 - 8 = 7$ [½ Mark]

32. Let the policeman catches the thief after t minutes.

$\because$ Uniform speed of the thief = 100 metres/minute

So, distance covered by thief in (t + 1) minutes = 100 (t + 1) metres [1 Mark]

$\therefore$ Distance covered by policeman in t minutes

= Sum of t terms of an AP whose first term 100 and common difference 10 [1 Mark]

$= \dfrac{t}{2}[2 \times 100 + (t-1) \times 10]$

$= t(5t + 95)$

$= 5t^2 + 95t$ [2 Marks]

If the policeman catches the thief, then :

$5t^2 + 95t = 100(t+1)$

$5t^2 - 5t - 100 = 0$

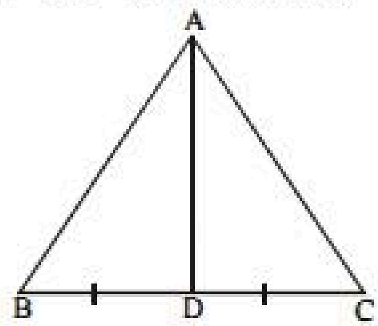
$\therefore \quad t^2 - t - 20 = 0$

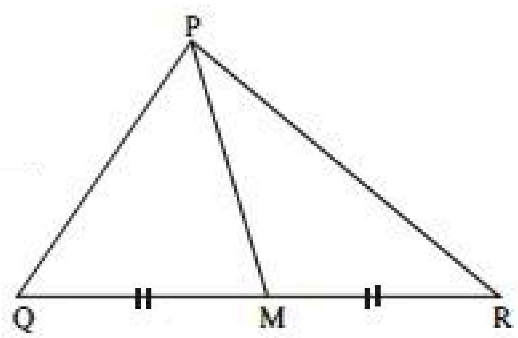
So, $t = -4$ and $t = 5$

Thus, t = 5 minutes (t > 0) [1 Mark]

Hence, the policeman catches the thief after 5 minutes.

33. **Proof:** $BC = 2BD$ (AD is the median)





[1 Mark]

and $QR = 2QM$ (PM is the median)

Given, $\dfrac{AB}{PQ} = \dfrac{AD}{PM} = \dfrac{BC}{QR}$

$\Rightarrow \dfrac{AB}{PQ} = \dfrac{AD}{PM} = \dfrac{2BD}{2QM}$ [1 Mark]

In triangles ABD and PQM,

$\dfrac{AB}{PQ} = \dfrac{AD}{PM} = \dfrac{BD}{QM}$ [1 Mark]

$\therefore \quad \Delta ABD \sim \Delta PQM$ (SSS Similarity)

$\Rightarrow \quad \angle B = \angle Q$ (By CPST) [1 Mark]

In ΔABC and ΔPQR,

$\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

and $\angle B = \angle Q$ (SAS Similarity)

$\therefore \quad \Delta ABC \sim \Delta PQR$ [1 Mark]

OR

Given : Two ΔABC and ΔDEF such that

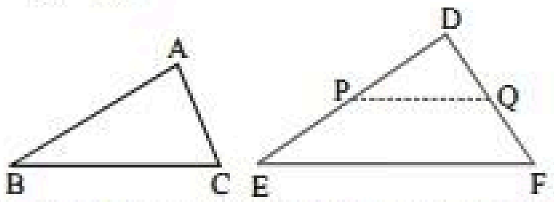
$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on DF such that $DP = AB$ and $DQ = AC$. Join PQ. [1Mark]

Proof : In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (By construction)



Therefore, by converse of basic proportionality theorem, $PQ \parallel EF$

So $\angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$ (corresponding angles)

Hence by AA similarity, $\Delta DPQ \sim \Delta DEF$ ...(i) [1 Mark]

Hence the corresponding sides of similar ΔDPQ and ΔDEF are proportional.

i.e., $\frac{DP}{DE} = \frac{PQ}{EF} \Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ $(\because DP = AB)$

But, $\frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{PQ}{EF} = \frac{BC}{EF} \Rightarrow PQ = BC$ ...(ii)

Now, in ΔABC and ΔDPQ [1 Mark]

$AB = DP$ (By Construction)

$AC = DQ$ (By Construction)

$BC = PQ$ [From (ii)]

So by SSS congruence rule

$\Delta ABC \cong \Delta DPQ$ ...(iii) [1 Mark]

From (i) and (iii)

$\Delta ABC \sim \Delta DPQ \sim \Delta DEF$ [1 Mark]

34. Let the radius of spherical marble = 0.7 cm [1 Mark]

Volume of 1 marble $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3 \text{ cm}^3$ [1 Mark]

Volume of 150 marble $= 200\pi(0.7)^3 \text{ cm}^3$ [1 Mark]

Let h be the rise in the height of water

$\therefore$ Volume of water raised = Volume of 150 marbles [1 Mark]

So, $\pi \times 7^2 \times h = 200\pi(0.7)^3 \Rightarrow h = \frac{200 \times 7 \times 7 \times 7}{7 \times 7 \times 10 \times 10 \times 10}$

$\Rightarrow h = 1.4$ cm [2 Marks]

OR

Let the radius of hemisphere = r

Now, volume of hemisphere $= \frac{2}{3}\pi r^3$ [1 Mark]

Surface area of hemisphere $= 3\pi r^2$ [1 Mark]

A.T.Q, volume of hemisphere = surface area of hemisphere

$\Rightarrow \frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units [1 Mark]

So, the required diameter of hemisphere = 2r = 9 units.

35.

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	$N = 100$	

Hence, $76 + x + y = 100$ [1 Mark]

$\Rightarrow x + y = 100 - 76 = 24$

Given, Median = 525,

$\Rightarrow$ Median class = 500 – 600 [1 Mark]

Here, $l = 500$

$h = 100$

$c.f. = 36 + x$

$f = 20$

Now, Median $= l + \frac{\frac{n}{2} - c.f.}{f} \times h$ [1 Mark]

$$525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$25 = (50 - 36 - x)5$ [1 Mark]

$(14 - x) = 5$

$x = 14 - 5 = 9$

Substituting the value of x in equation (i),

$y = 24 - 9 = 15$ [1 Mark]

36. (i) Volume of concrete required for the first step of terrace $= \frac{1}{4} \times \frac{1}{2} \times 50 = 6.25 \text{ m}^3$ [1 Mark]

(ii) Volume of concrete required for the 8th| step of terrace $= 8 \times \frac{1}{4} \times \frac{1}{2} \times 50 = 50 \text{ m}^3$ [1 Mark]

(iii) The A.P. formed by the volume of concrete required to build the first step, second step, third step,.........,fifteenth step

$$= \frac{1}{4}\times\frac{1}{2}\times 50, \left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \ldots\ldots, \left(15\times\frac{1}{4}\right)\times\frac{1}{2}\times 50$$

$$= \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8}, \ldots\ldots, 15\times\frac{50}{8}$$

$\therefore$ The common difference of A.P. formed by volume of steps of terrace

$$= 2\times\frac{50}{8} - \frac{50}{8} = \frac{50}{8} = \frac{25}{4}$$ [2 Marks]

OR

Total volume of concrete required to build the terrace

$$= 1\times\frac{50}{8} + 2\times\frac{50}{8} + 3\times\frac{50}{8} + \ldots\ldots + 15\times\frac{50}{8}$$

$$= \frac{50}{8}[1+2+3+\ldots+15]$$

$$= \frac{50}{8}\times\frac{15}{2}[2\times 1 + (15-1)\times 1]$$

$$= \frac{50}{8}\times\frac{15}{2}\times 16 = 750 \text{ m}^3$$ [2 Marks]

37. (i) $(0, 0)$ [1 Mark]

(ii) $(4, 6)$ [1 Mark]

(iii) $(6, 5)$ [2 Marks]

OR

$(16, 0)$ [2 Marks]

38. (i) $x \tan 60° = (100 - x) \tan 30°$

$$x\sqrt{3} = (100 - x)\frac{1}{\sqrt{3}} \Rightarrow x = 25$$ [1 Mark]

$\therefore$ Distance of p from pillar making angle $60° = 25$m

(ii) Distance of p from pillar making angle $30°$

$= 100 - 25 = 75$m [1 Mark]

(iii) Height $= x \tan 60° = 25\sqrt{3}$ m. [2 Marks]

OR

$$\text{Sum} = \sqrt{(25\sqrt{3})^2 + (25)^2} + \sqrt{(25\sqrt{3})^2 + (75)^2}$$ [2 Marks]

$$= 50 + 50\sqrt{3} = 50\left(\sqrt{3+1}\right)\text{m}.$$