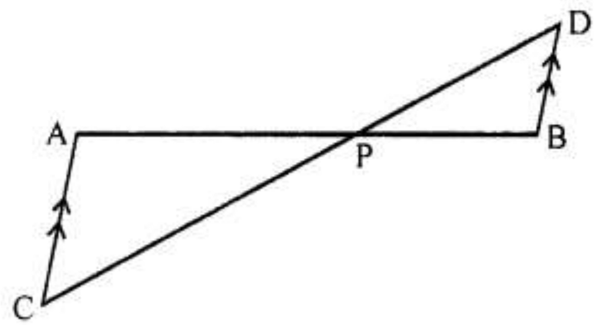
Similarity (With Applications to Maps & Models)

Exercise 15A

Question 1.
In the figure, given below, straight lines AB and CD intersect at P; and AC // BD. Prove that:

(i) ΔAPC and ΔBPD are similar.
(ii) If BD = 2.4 cm AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm; find the lengths of PA and PC.



Solution:

(i)

In ΔAPC and ΔBPD,
∠APC = ∠BPD .....(vertically opposite angles)
∠ACP = ∠BDP .....(alternate angles since AC||BD)
∴ ΔAPC ~ ΔBPD ....(AA criterion for similarity)

(ii)

In ΔAPC and ΔBPD,
∠APC = ∠BPD .....(vertically opposite angles)
∠ACP = ∠BDP .....(alternate angles since AC||BD)
∴ ΔAPC ~ ΔBPD ....(AA criterion for similarity)

So, $\frac{PA}{PB} = \frac{PC}{PD} = \frac{AC}{BD}$

$\Rightarrow \frac{PA}{3.2} = \frac{PC}{4} = \frac{3.6}{2.4}$

So, $\frac{PA}{3.2} = \frac{3.6}{2.4}$ and $\frac{PC}{4} = \frac{3.6}{2.4}$

$\Rightarrow PA = \frac{3.6 \times 3.2}{2.4} = 4.8 \text{ cm}$

and $PC = \frac{3.6 \times 4}{2.4} = 6 \text{ cm}$

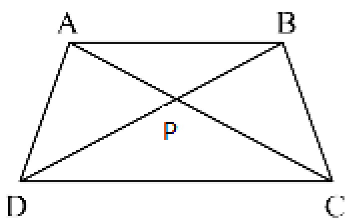
Hence, $PA = 4.8$ cm and $PC = 6$ cm.

Question 2.
In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that:

(i) ΔAPB is similar to ΔCPD
(ii) PA × PD = PB × PC

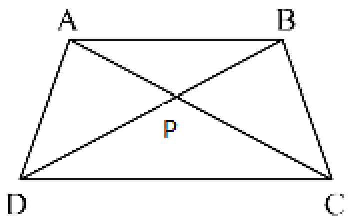
Solution:

(i)



In ΔAPB and ΔCPD,
∠APB = ∠CPD .....(vertically opposite angles)
∠ABP = ∠CDP .....(alternate angles since AB||DC)
∴ ΔAPB ~ ΔCPD ....(AA criterion for similarity)

(ii)



In ΔAPB and ΔCPD,

$\angle APB = \angle CPD$ .....(vertically opposite angles)

$\angle ABP = \angle CDP$ .....(alternate angles since AB||DC)

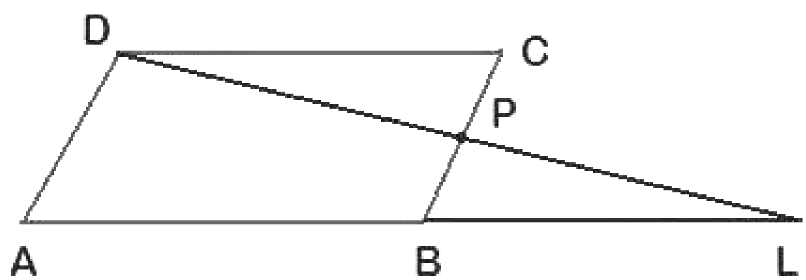
$\therefore \Delta APB \sim \Delta CPD$ ....(AA criterion for similarity)

$\Rightarrow \frac{PA}{PC} = \frac{PB}{PD}$ .....(Since corresponding sides of similar triangles are equal.)

$\Rightarrow PA \times PD = PB \times PC$

Question 3.
P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that:
(i) DP: PL = DC: BL.
(ii) DL: DP=AL: DC.

Solution:
(i)



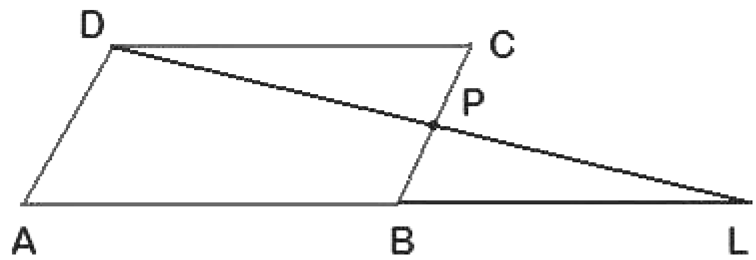
Since AD||BC, that is, AD||BP,
by the Basic Proportionality theorem, we get

$\frac{DL}{DP} = \frac{AL}{AB}$

Since ABCD is a parallelogram, AB = DC.

So, $\frac{DL}{DP} = \frac{AL}{DC}$.

(ii)



Since AD||BC, that is, AD||BP,
by the Basic Proportionality theorem, we get

$$\frac{DP}{PL} = \frac{AB}{BL}$$

Since ABCD is a parallelogram, AB = DC.

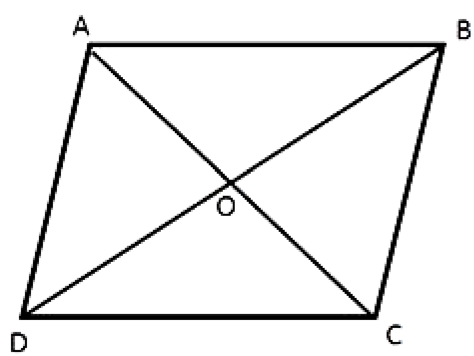
So, $\frac{DP}{PL} = \frac{DC}{BL}$.

Question 4.
In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O. If AO = 2CO and BO=2DO; show that:

(i) ΔAOB is similar to ΔCOD.
(ii) OA × OD – OB × OC.

Solution:

(i)



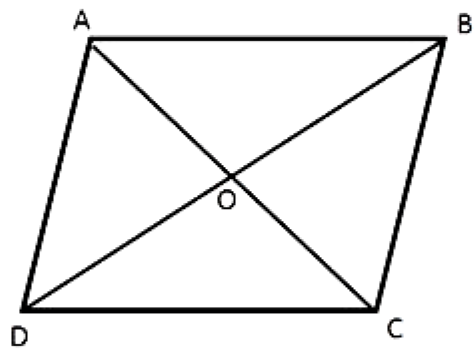
Since AO = 2CO and BO = 2DO,

$$\frac{AO}{CO} = \frac{2}{1} = \frac{BO}{DO}$$

Also, $\angle AOB = \angle DOC$ ....(vertically opposite angles)

So, $\Delta AOB \sim \Delta COD$ ....(SAS criterion for similarity)

(ii)



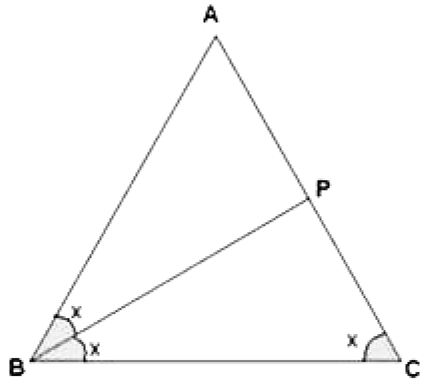
Since AO = 2CO and BO = 2DO,

$$\frac{AO}{CO} = \frac{2}{1} = \frac{BO}{DO}$$

So, $OA \times OD = OB \times OC$.

Question 5.
In ΔABC, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that:
(i) CB: BA=CP: PA
(ii) AB × BC = BP × CA

Solution:
(i)



In ΔABC,

$\angle ABC = 2\angle ACB$

Let $\angle ACB = x$

$\Rightarrow \angle ABC = 2\angle ACB = 2x$

Given BP is bisector of $\angle ABC$.

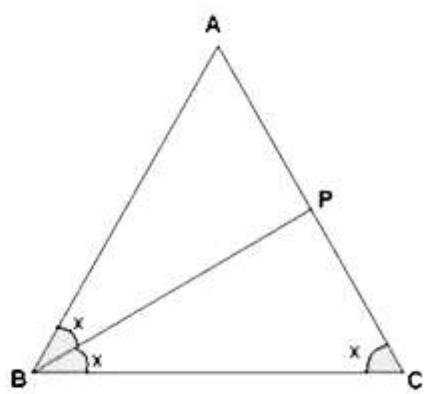
Hence $\angle ABP = \angle PBC = x$.

Using the angle bisector theorem,

that is, the bisector of an angle divides the side opposite to it in the ratio of other two sides.

Hence, $CB : BA = CP : PA$.

(ii)



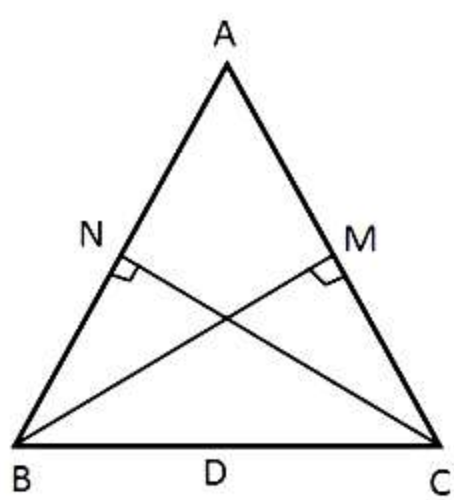
In ΔABC,
$\angle ABC = 2\angle ACB$
Let $\angle ACB = x$
$\Rightarrow \angle ABC = 2\angle ACB = 2x$
Given BP is bisector of $\angle ABC$.
Hence $\angle ABP = \angle PBC = x$.
Using the angle bisector theorem,
that is, the bisector of an angle divides the side opposite to it
in the ratio of other two sides.
Hence, $CB : BA = CP : PA$.
Consider ΔABC and ΔAPB,
$\angle ABC = \angle APB$ ....[Exterior angle property]
$\angle BCP = \angle ABP$ ....[Given]
$\therefore \Delta ABC \sim \Delta APB$ [AA criterion for Similarity]
$\frac{CA}{AB} = \frac{BC}{BP}$ ....(Corresponding sides of similar triangles are proportional.)
$\Rightarrow AB \times BC = BP \times CA$

Question 6.

In ΔABC; BM ⊥ AC and CN ⊥ AB; show that:

$\frac{\mathbf{AB}}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$

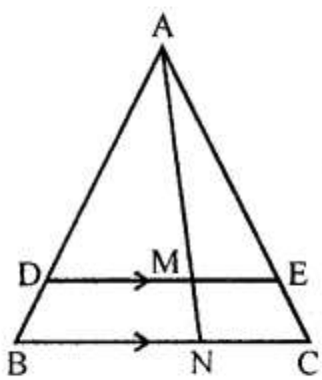
Solution:



In ΔABM and ΔACN,
$\angle AMB = \angle ANC$ ....($BM \perp AC$ and $CN \perp AB$)
$\angle BAM = \angle CAN$ .....(common angle)
$\Rightarrow \Delta ABM \sim \Delta ACN$ ....(AA criterion for Similarity)
$\Rightarrow \frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$

Question 7.
In the given figure, DE//BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.

(i) Write all possible pairs of similar triangles.
(ii) Find lengths of ME and DM.



Solution:
(i)
In ΔAME and ΔANC,
$\angle AME = \angle ANC$ ....(Since DE||BC that is, ME||NC.)
$\angle MAE = \angle NAC$ .....(common angle)
$\Rightarrow \Delta AME \sim \Delta ANC$ ....(AA criterion for Similarity)

In ΔADM and ΔABN,
$\angle ADM = \angle ABN$ ....(Since DE||BC that is, DM||BN.)
$\angle DAM = \angle BAN$ ....(common angle)
$\Rightarrow \Delta ADM \sim \Delta ABN$ ....(AA criterion for Similarity)

In ΔADE and ΔABC,
$\angle ADE = \angle ABC$ ....(Since DE||BC that is, ME||NC.)
$\angle AED = \angle ACB$ ....(Since DE||BC.)
$\Rightarrow \Delta ADE \sim \Delta ABC$ ....(AA criterion for Similarity)

(ii)

In ΔAME and ΔANC,

$\angle AME = \angle ANC$ ....(Since DE||BC that is, ME||NC.)

$\angle MAE = \angle NAC$ .....(common angle)

$\Rightarrow \Delta AME \sim \Delta ANC$ ....(AA criterion for Similarity)

$\Rightarrow \frac{ME}{NC} = \frac{AE}{AC}$

$\Rightarrow \frac{ME}{6} = \frac{15}{24}$

$\Rightarrow ME = 3.75$ cm

In ΔADE and ΔABN,

$\angle ADE = \angle ABC$ ....(Since DE||BC that is, ME||NC.)

$\angle AED = \angle ACB$ ....(Since DE||BC.)

$\Rightarrow \Delta ADE \sim \Delta ABC$ ....(AA criterion for Similarity)

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{15}{24}$ .....(i)

In ΔADM and ΔABN,

$\angle ADM = \angle ABN$ ....(Since DE||BC that is, DM||BN.)

$\angle DAM = \angle BAN$ ....(common angle)

$\Rightarrow \Delta ADM \sim \Delta ABN$ ....(AA criterion for Similarity)

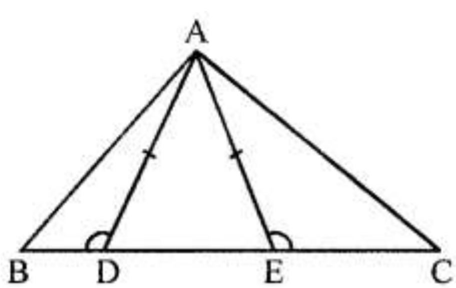
$\Rightarrow \frac{DM}{BN} = \frac{AD}{AB} = \frac{15}{24}$ ....(from (i))

$\Rightarrow \frac{DM}{24} = \frac{15}{24}$

$\Rightarrow DM = 15$ cm

Question 8.

In the given figure, AD =AE and $AD^2 = BD \times EC$
Prove that: triangles ABD and CAE are similar.



Solution:

In ΔABD and ΔCAE,

$\angle ADE = \angle AED$ ....(Angles opposite equal sides are equal.)

So, $\angle ADB = \angle AEC$

.....(Since $\angle ADB + \angle ADE = 180^\circ$ and $\angle AEC + \angle AED = 180^\circ$)

Also, $AD^2 = BD \times EC$

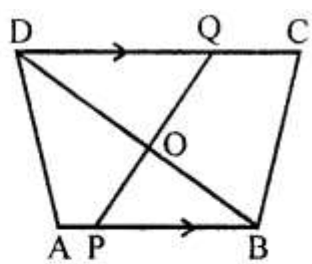
$\Rightarrow \frac{AD}{BD} = \frac{EC}{AD}$

$\Rightarrow \frac{AD}{BD} = \frac{EC}{AE}$

$\Rightarrow \Delta ABD \sim \Delta CAE$ ....(SAS criterion for Similarity)

Question 9.

In the given figure, AB // DC, BO = 6 cm and DQ = 8 cm; find: BP × DO.



Solution:

In ΔDOQ and ΔBOP,

$\angle QDO = \angle PBO$ ....(Since AB||DC that is, PB||DQ.)

So, $\angle DOQ = \angle BOP$ ....(vertically opposite angles)

$\Rightarrow \Delta DOQ \sim \Delta BOP$ ....(AA criterion for Similarity)

$\Rightarrow \frac{DO}{BO} = \frac{DQ}{BP}$

$\Rightarrow \frac{DO}{6} = \frac{8}{BP}$

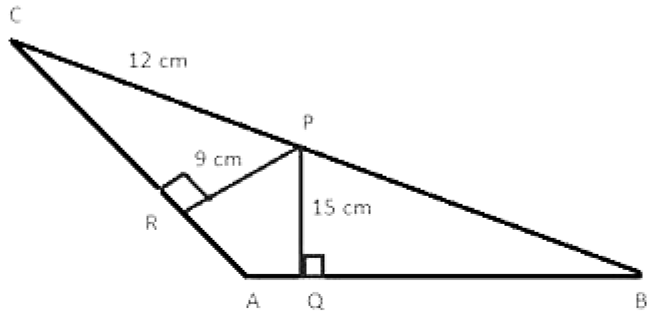
$\Rightarrow BP \times DO = 48\ cm^2$

Question 10.

Angle BAC of triangle ABC is obtuse and AB =AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and

PR=9 cm; find the length of PB

Solution:



In ΔABC,
$AC = AB$ ....(Given)
$\Rightarrow \angle ABC = \angle ACB$ ....(Angles opposite equal sides are equal.)
In ΔPRC and ΔPQB,
$\angle ABC = \angle ACB$
$\angle PRC = \angle PQB$ ....(Both are right angles.)
$\Rightarrow \Delta PRC \sim \Delta PQB$ .....(AA criterion for Similarity)
$\Rightarrow \frac{PR}{PQ} = \frac{RC}{QB} = \frac{PC}{PB}$
$\Rightarrow \frac{PR}{PQ} = \frac{PC}{PB}$
$\Rightarrow \frac{9}{15} = \frac{12}{PB}$
$\Rightarrow PB = 20$ cm

Question 11.
State, true or false:

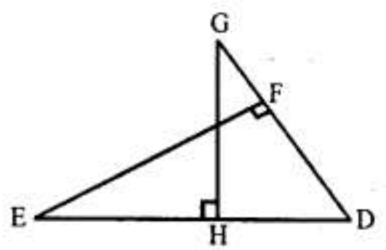
(i) Two similar polygons are necessarily congruent.
(ii) Two congruent polygons are necessarily similar.
(iii) All equiangular triangles are similar.
(iv) All isosceles triangles are similar.
(v) Two isosceles-right triangles are similar.
(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
(vii) The diagonals of a trapezium, divide each other into proportional segments.

Solution:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) True

Question 12.

Given = ∠GHE = ∠ DFE = 90°, DH = 8, DF = 12, DG = 3x + 1 and DE = 4x + 2.



Find; the lengths of segments DG and DE.

Solution:

In ΔDHG and ΔDFE,

$\angle GHD = \angle DFE = 90°$

$\angle D = \angle D \quad$ (Common)

$\therefore \Delta DHG \sim \Delta DFE$

$\Rightarrow \frac{DH}{DF} = \frac{DG}{DE}$

$\Rightarrow \frac{8}{12} = \frac{3x-1}{4x+2}$

$\Rightarrow 32x + 16 = 36x - 12$

$\Rightarrow 28 = 4x$

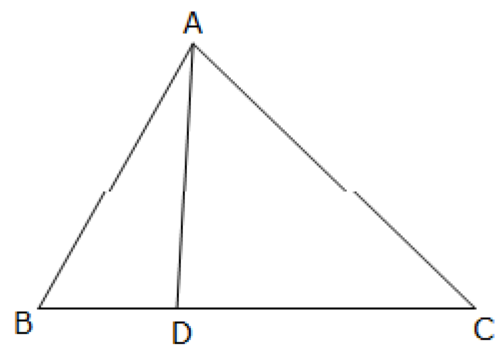
$\Rightarrow x = 7$

$\therefore DG = 3 \times 7 - 1 = 20$

$DE = 4 \times 7 + 2 = 30$

Question 13.
D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that $CA^2 = CB \times CD$.

Solution:



In ΔADC and ΔBAC,
$\angle ADC = \angle BAC$ (Given)
$\angle ACD = \angle ACB$ (Common)
$\therefore \Delta ADC \sim \Delta BAC$
$\therefore \dfrac{CA}{CB} = \dfrac{CD}{CA}$
Hence, $CA^2 = CB \times CD$

Question 14.
In the given figure, ΔABC and ΔAMP are right angled at B and M respectively. Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Prove that ΔABC ~ ΔAMP
(ii) Find AB and BC.

Solution:

(i) In ΔABC and ΔAMP,
$\angle BAC = \angle PAM$ [Common]
$\angle ABC = \angle PMA$ [Each = 90°]
$\Delta ABC \sim \Delta AMP$ [AA Similarity]
(ii)
$AM = \sqrt{AP^2 - PM^2} = \sqrt{15^2 - 12^2} = 11$
Since $\Delta ABC - \Delta AMP$,

$$\frac{AB}{AM}=\frac{BC}{PM}=\frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{AM}=\frac{BC}{PM}=\frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{11}=\frac{BC}{12}=\frac{10}{15}$$

From this we can write,

$$\frac{AB}{11}=\frac{10}{15}$$

$$\Rightarrow AB = \frac{10\times 11}{15} = 7.33$$

$$\frac{BC}{12}=\frac{10}{15}$$

$$\Rightarrow BC = 8 \text{ cm}$$

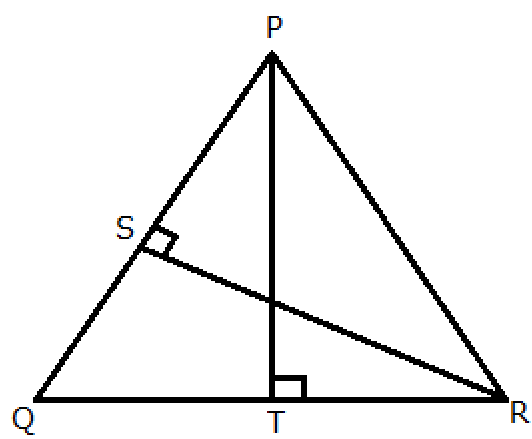
Question 15.

Given : RS and PT are altitudes of A PQR prove that:

(i)ΔPQT ~ ΔQRS,

(ii) PQ × QS = RQ × QT.

Solution:



(i)

In ΔPQT and ΔQRS,

$\angle PTQ = \angle RSQ = 90°$ (Given)

$\angle PQT = \angle RQS$ (Common)

$\Delta PQT \sim \Delta RQS$ (By AA similarity)

(ii)

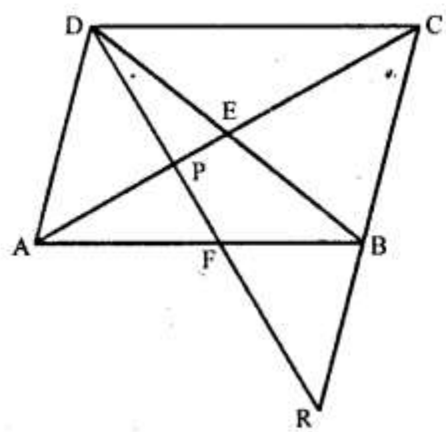
Since, triangles PQT and RQS are similar.

$\therefore \frac{PQ}{RQ} = \frac{QT}{QS}$

$\Rightarrow PQ \times QS = RQ \times QT$

Question 16.

Given : ABCD is a rhombus, DPR and CBR are straight lines



Prove that: DP × CR = DC × PR.

Solution:

In ΔDPA and ΔRPC,

$\angle DPA = \angle RPC$ (Vertically opposite angles)

$\angle PAD = \angle PCR$ (Alternate angles)

$\Delta DPA \sim \Delta RPC$

$\therefore \frac{DP}{PR} = \frac{AD}{CR}$

$\frac{DP}{PR} = \frac{DC}{CR}$ (AD = DC, as ABCD is rhombus)

Hence, DP × CR = DC × PR

Question 17.
Given: FB = FD, AE ⊥ FD and FC ⊥ AD. Prove : $\frac{FB}{AD} = \frac{BC}{ED}$

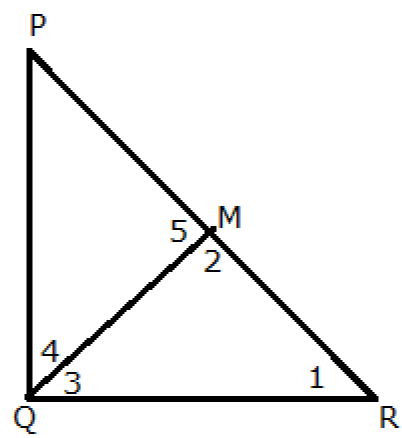
Solution:

Given, FB = FD
$\therefore \angle FDB = \angle FBD \quad \ldots (1)$
In ΔAED and ΔFCB,
$\angle AED = \angle FCB = 90°$
$\angle ADE = \angle FBC$ [Using (1)]
$\Delta AED \sim \Delta FCB$ [By AA similarity]
$\therefore \frac{AD}{FB} = \frac{ED}{BC}$
$\frac{FB}{AD} = \frac{BC}{ED}$

Question 18.
In Δ PQR, ∠ Q = 90° and QM is perpendicular to PR, Prove that:

(i) $PQ^2 = PM \times PR$
(ii) $QR^2 = PR \times MR$
(iii) $PQ^2 + QR^2 = PR^2$

Solution:



(i) In Δ PQM and Δ PQR,

$\angle PMQ = \angle PQR = 90^o$

$\angle QPM = \angle RPQ$ (Common)

$\therefore \Delta PQM \sim \Delta PRQ$ (By AA similarity)

$$\therefore \frac{PQ}{PR} = \frac{PM}{PQ}$$

$\Rightarrow PQ^2 = PM \times PR$

(ii) In Δ QMR and Δ PQR,

$\angle QMR = \angle PQR = 90^o$

$\angle QRM = \angle QRP$ (Common)

$\therefore \Delta QMR \sim \Delta PQR$ (By AA similarity)

$$\therefore \frac{QR}{PR} = \frac{MR}{QR}$$

$\Rightarrow QR^2 = PR \times MR$

(iii) Adding the relations obtained in (i) and (ii), we get,

$$\begin{aligned} PQ^2 + QR^2 &= PM \times PR + PR \times MR \\ &= PR(PM + MR) \\ &= PR \times PR \\ &= PR^2 \end{aligned}$$

Question 19.

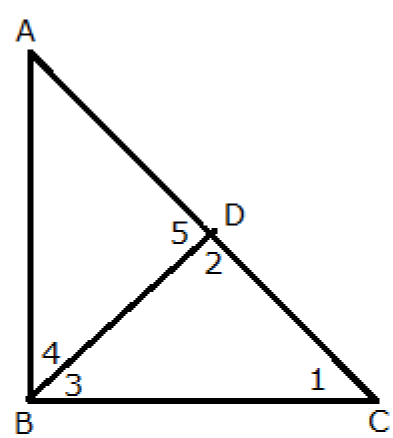
In Δ ABC, ∠ B = 90° and BD × AC.

(i) If CD = 10 cm and BD = 8 cm; find AD.

(ii) If AC = 18 cm and AD = 6 cm; find BD.

(iii) If AC = 9 cm, AB = 7 cm; find AD.

Solution:



(i) In Δ CDB,

$\angle 1 + \angle 2 + \angle 3 = 180^o$

$\angle 1 + \angle 3 = 90^o$ ..... (1)(Since, $\angle 2 = 90^o$)

$\angle 3 + \angle 4 = 90^o$ .....(2) (Since, $\angle ABC = 90^o$)

From (1) and (2),

$\angle 1 + \angle 3 = \angle 3 + \angle 4$

$\angle 1 = \angle 4$

Also, $\angle 2 = \angle 5 = 90^o$

$\therefore \Delta CDB \sim \Delta BDA$ (By AA similarity)

$\Rightarrow \frac{CD}{BD} = \frac{BD}{AD}$

$\Rightarrow BD^2 = AD \times CD$

$\Rightarrow (8)^2 = AD \times 10$

$\Rightarrow AD = 6.4$

Hence, AD = 6.4 cm

(ii) Also, by similarity, we have :

$\frac{BD}{DA} = \frac{CD}{BD}$

$BD^2 = 6 \times (18 - 6)$

$BD^2 = 72$

Hence, BD = 8.5 cm

(iii)

Clearly, $\Delta ADB \sim \Delta ABC$

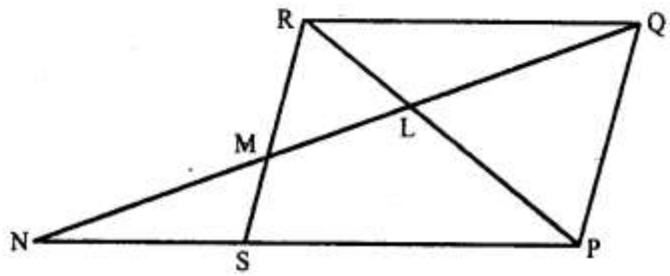
$\therefore \frac{AD}{AB} = \frac{AB}{AC}$

$AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5\frac{4}{9}$

Hence, $AD = 5\frac{4}{9}$ cm

Question 20.

In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM.

Solution:

In ΔRLQ and ΔPLN,

$\angle RLQ = \angle PLN$ (Vertically opposite angles)

$\angle LRQ = \angle LPN$ (Alternate angles)

$\Delta RLQ \sim \Delta PLN$ (AA similarity)

$\therefore \frac{RL}{LP} = \frac{RQ}{PN}$

$\frac{2}{3} = \frac{10}{PN}$

$PN = 15$ cm

In ΔRLM and ΔPLQ,

$\angle RLM = \angle PLQ$ (Vertically opposite angles)

$\angle LRM = \angle LPQ$ (Alternate angles)

$\Delta RLM \sim \Delta PLQ$ (AA similarity)

$\therefore \frac{RM}{PQ} = \frac{RL}{LP}$

$\frac{RM}{16} = \frac{2}{3}$

$RM = \frac{32}{3} = 10\frac{2}{3}$ cm

Question 21.
In quadrilateral ABCD, diagonals AC and BD intersect at point E. Such that

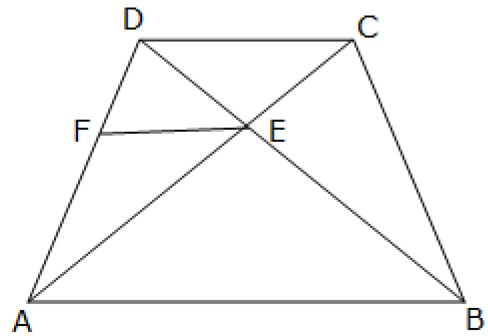
AE : EC = BE :'ED.

Show that ABCD is a parallelogram

Solution:

Given, AE: EC = BE: ED

Draw EF || AB



In Δ ABD, EF || AB

Using Basic Proportionality theorem,

$$\frac{DF}{FA} = \frac{DE}{EB}$$

But, given $\frac{DE}{EB} = \frac{CE}{EA}$

$$\therefore \frac{DF}{FA} = \frac{CE}{EA}$$

Thus, in Δ DCA, E and F are points on CA and DA respectively such that $\frac{DF}{FA} = \frac{CE}{EA}$

Thus, by converse of Basic proportionality theorem, FE || DC.

But, FE || AB.

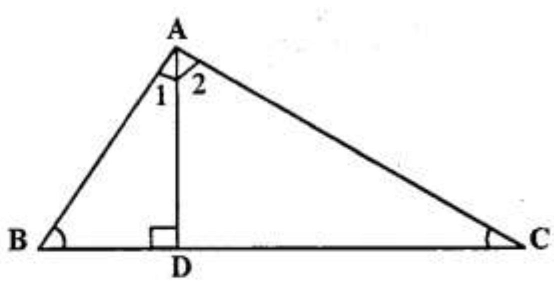
Hence, AB || DC.

Thus, ABCD is a trapezium.

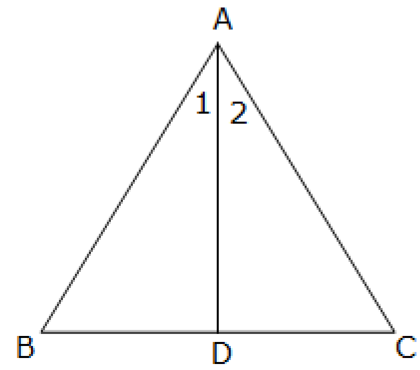
Question 22.

In Δ ABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$.

Show that angle BAC = 90°



Solution:



Given, $AD^2 = BD \times DC$

$\frac{AD}{DC} = \frac{BD}{AD}$

$\angle ADB = \angle ADC = 90^\circ$

$\therefore \Delta DBA \sim \Delta DAC$ (SAS similarity)

So, these two triangles will be equiangular.

$\therefore \angle 1 = \angle C$ and $\angle 2 = \angle B$

$\angle 1 + \angle 2 = \angle B + \angle C$

$\angle A = \angle B + \angle C$

By angle sum property,

$\angle A + \angle B + \angle C = 180^\circ$

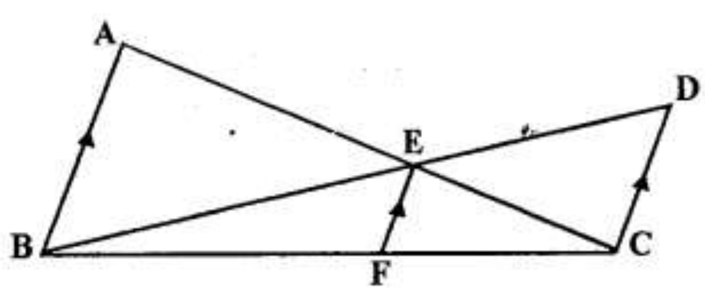
$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = \angle BAC = 90^\circ$

Question 23.

In the given figure AB // EF // DC; AB ~ 67.5 cm. DC = 40.5 cm and AE = 52.5 cm.



(i) Name the three pairs of similar triangles.
(ii) Find the lengths of EC and EF.

Solution:

(i) The three pair of similar triangles are:
Δ BEF and Δ BDC
Δ CEF and Δ CAB
Δ ABE and Δ CDE
(ii) Since, Δ ABE and Δ CDE are similar,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{67.5}{40.5} = \frac{52.5}{CE}$$

CE = 31.5 cm

Since, Δ CEF and Δ CAB are similar,

$$\frac{CE}{CA} = \frac{EF}{AB}$$

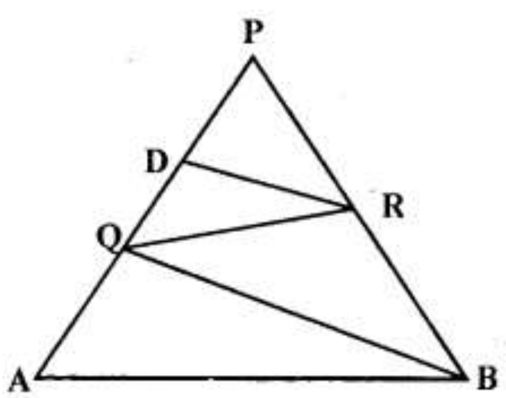
$$\frac{31.5}{52.5+31.5} = \frac{EF}{67.5}$$

$$\frac{31.5}{84} = \frac{EF}{67.5}$$

$$EF = \frac{2126.25}{84}$$

$$EF = \frac{405}{16} = 25\frac{5}{16} \text{ cm}$$

Question 24.
In the given figure, QR is parallel to AB and DR is parallel to QB.



Prove that— $PQ^2 = PD \times PA$.

Solution:

Given, QR is parallel to AB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PQ}{PA} = \frac{PR}{PB} \quad \dots (1)$$

Also, DR is parallel to QB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PD}{PQ} = \frac{PR}{PB} \quad \dots (2)$$

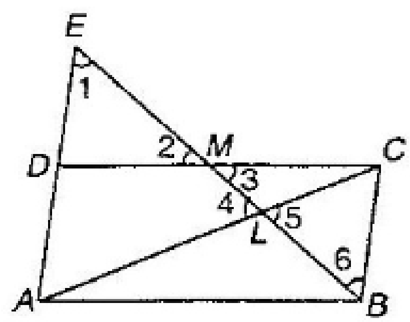
From (1) and (2), we get,

$$\frac{PQ}{PA} = \frac{PD}{PQ}$$

$$PQ^2 = PD \times PA$$

Question 25.
Through the mid-point M of the side CD o£. a parallelogram ABCD, the line BM is drawn ' intersecting diagonal AC in L and AD produced in E.
Prove that : EL = 2 BL.

Solution:



$\angle 1 = \angle 6$ (Alternate interior angles)
$\angle 2 = \angle 3$ (Vertically opposite angles)
DM = MC (M is the mid-point of CD)
$\therefore \Delta DEM \cong \Delta CBM$ (AAS congruence criterion)
So, DE = BC (Corresponding parts of congruent triangles)
Also, AD = BC (Opposite sides of a parallelogram)
$\Rightarrow$ AE = AD + DE = 2BC
Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$
$\therefore \Delta ELA \sim \Delta BLC$ (AA similarity)

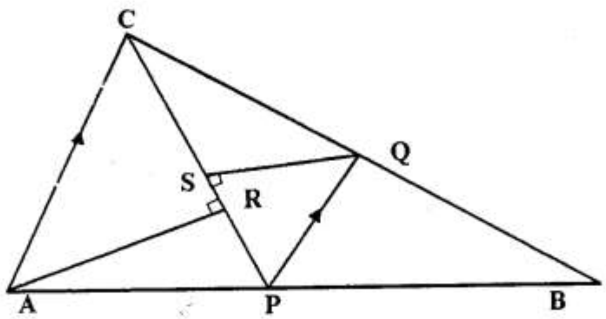
$$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2$$

$$\Rightarrow EL = 2BL$$

Question 26.

In the figure given below P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.



(i) Calculate the ratio PQ : AC, giving reason for your answer.
(ii) In triangle ARC, ∠ ARC = 90° and in triangle PQS, ∠ PSQ = 90°. Given QS = 6 cm, calculate the length of AR. [1999]

Solution:

(i) Given, AP: PB = 4: 3.
Since, PQ || AC. Using Basic Proportionality theorem,

$$\frac{AP}{PB} = \frac{CQ}{QB}$$

$$\Rightarrow \frac{CQ}{QB} = \frac{4}{3}$$

$$\Rightarrow \frac{BQ}{BC} = \frac{3}{7} \quad \ldots (1)$$

Now, ∠ PQB = ∠ ACB (Corresponding angles)
∠ QPB = ∠ CAB (Corresponding angles)

$\therefore \Delta PBQ \sim \Delta ABC$ (AA similarity)

$$\Rightarrow \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\Rightarrow \frac{PQ}{AC} = \frac{3}{7} \quad \text{[Using (1)]}$$

(ii) ∠ ARC = ∠ QSP = 90°
∠ ACR = ∠ SPQ (Alternate angles)

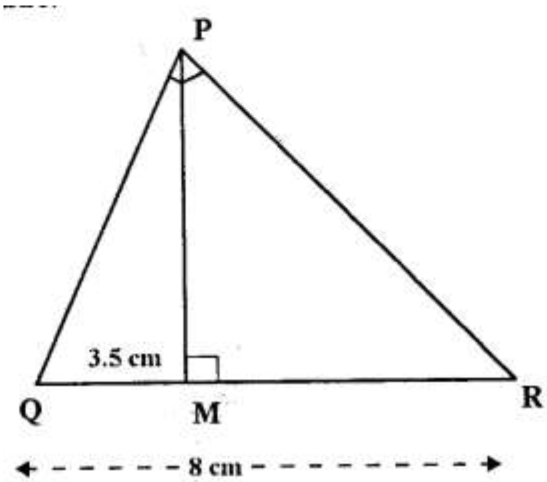
$\therefore \Delta ARC \sim \Delta QSP$ (AA similarity)

$$\Rightarrow \frac{AR}{QS} = \frac{AC}{PQ}$$

$$\Rightarrow \frac{AR}{QS} = \frac{7}{3}$$

$$\Rightarrow AR = \frac{7 \times 6}{3} = 14 \text{ cm}$$

Question 27.
In the right angled triangle QPR, PM is an altitude.



Given that QR = 8 cm and MQ = 3.5 cm. Calculate, the value of PR., [2000]
Given— In right angled Δ QPR, ∠ P = 90° PM ⊥ QR, QR = 8 cm, MQ = 3.5 cm
Calculate— PR

Solution:

We have:

$\angle QPR = \angle PMR = 90^0$

$\angle PRQ = \angle PRM$ (Common)

$\Delta PQR \sim \Delta MPR$ (AA similarity)

$\therefore \frac{QR}{PR} = \frac{PR}{MR}$

$PR^2 = 8 \times 4.5 = 36$

$PR = 6$ cm

Question 28.
In the figure given below, the medians BD and CE of a triangle ABC meet at G.
Prove that—

(i) Δ EGD ~ Δ CGB
(ii) BG = 2 GD from (i) above.

Solution:

(i) Since, BD and CE are medians.
AD = DC
AE = BE
Hence, by converse of Basic Proportionality theorem,
ED || BC
In Δ EGD and Δ CGB,

$\angle DEG = \angle GCB$ (Alternate angles)

$\angle EGD = \angle BGC$ (Vertically opposite angles)

$\Delta EGD \sim \Delta CGB$ (AA similarity)

(ii) Since, $\Delta EGD \sim \Delta CGB$

$$\frac{GD}{GB} = \frac{ED}{BC} \quad \ldots (1)$$

In Δ AED and Δ ABC,

$\angle AED = \angle ABC$ (Corresponding angles)

$\angle EAD = \angle BAC$ (Common)

$\Delta EAD \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{ED}{BC} = \frac{AE}{AB} = \frac{1}{2} \quad \text{(Since, E is the mid-point of AB)}$$

$$\Rightarrow \frac{ED}{BC} = \frac{1}{2}$$

From (1),

$$\frac{GD}{GB} = \frac{1}{2}$$

GB = 2GD

Exercise 15B

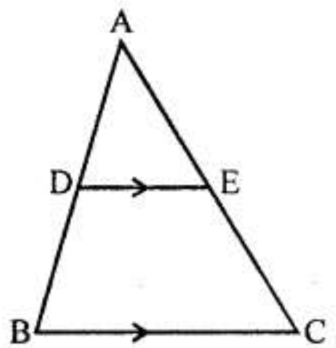
Question 1.
In the following figure, point D divides AB in the ratio 3:5. Find:

(i) $\frac{AE}{EC}$ (ii) $\frac{AD}{AB}$

(iii) $\frac{AE}{AC}$

Also, if:
(iv) DE = 2.4 cm, find the length of BC.

(v) BC = 4.8 cm, find the length of DE.



Solution:

(i).

Given that $\frac{AD}{DB} = \frac{3}{5}$.

So, $\frac{AD}{AB} = \frac{3}{8}$.

In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ ....(Since DE||BC, so the angles are corresponding angles.)

$\angle A = \angle A$ ....(Common angle)

$\therefore \Delta ADE \sim \Delta ABC$ ...(AA criterion for Similarity)

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{AE}{AC} = \frac{3}{8}$

(ii)

Given that $\frac{AD}{DB} = \frac{3}{5}$.

So, $\frac{AD}{AB} = \frac{3}{8}$.

(iii)

Given that $\frac{AD}{DB} = \frac{3}{5}$.

So, $\frac{AD}{AB} = \frac{3}{8}$.

In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ ....(Since DE||BC, so the angles are corresponding angles.)

$\angle A = \angle A$ ....(Common angle)

$\therefore \Delta ADE \sim \Delta ABC$ ...(AA criterion for Similarity)

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{AE}{AC} = \frac{3}{8}$

(iv)

Given that $\frac{AD}{DB} = \frac{3}{5}$.

So, $\frac{AD}{AB} = \frac{3}{8}$.

In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ ....(Since DE||BC, so the angles are corresponding angles.)

$\angle A = \angle A$ ....(Common angle)

$\therefore \Delta ADE \sim \Delta ABC$ ...(AA criterion for Similarity)

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3}{8} = \frac{2.4}{BC}$

$\Rightarrow BC = 6.4$ cm

(v)

Given that $\frac{AD}{DB} = \frac{3}{5}$.

So, $\frac{AD}{AB} = \frac{3}{8}$.

In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ ....(Since DE||BC, so the angles are corresponding angles.)

$\angle A = \angle A$ ....(Common angle)

$\therefore \Delta ADE \sim \Delta ABC$ ...(AA criterion for Similarity)

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3}{8} = \frac{DE}{4.8}$

$\Rightarrow DE = 1.8$ cm

Question 2.

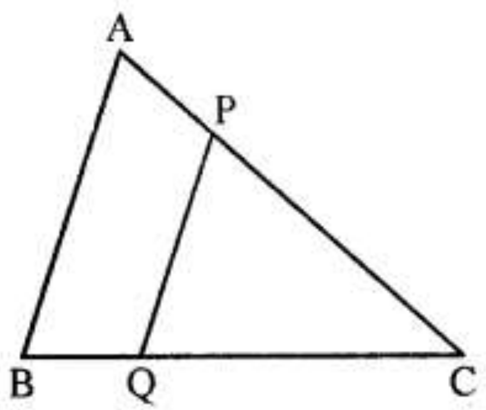
In the given figure, PQ//AB;

CQ = 4.8 cm QB = 3.6 cm and AB = 6.3 cm. Find:

(i) $\frac{CP}{PA}$

(ii) PQ

(iii) If AP=x, then the value of AC in terms of x.



Solution:

(i)

In ΔCPQ and ΔCAB,

$\angle PCQ = \angle ACB$ ....(Since PQ||AB, so the angles are corresponding angles.)

$\angle C = \angle C$ ....(Common angle)

$\therefore \Delta CPQ \sim \Delta CAB$ ...(AA criterion for Similarity)

$$\Rightarrow \frac{CP}{CA} = \frac{CQ}{CB}$$

$$\Rightarrow \frac{CP}{CA} = \frac{4.8}{8.4} = \frac{4}{7}$$

So, $\frac{CP}{PA} = \frac{4}{3}$.

(ii)

In ΔCPQ and ΔCAB,

$\angle PCQ = \angle ACB$ ....(Since PQ||AB, so the angles are corresponding angles.)

$\angle C = \angle C$ ....(Common angle)

$\therefore \Delta CPQ \sim \Delta CAB$ ...(AA criterion for Similarity)

$$\Rightarrow \frac{PQ}{AB} = \frac{CQ}{CB}$$

$$\Rightarrow \frac{PQ}{6.3} = \frac{4.8}{8.4}$$

$\Rightarrow PQ = 3.6$ cm

(iii)

In ΔCPQ and ΔCAB,

$\angle PCQ = \angle ACB$ ....(Since PQ||AB, so the angles are corresponding angles.)

$\angle C = \angle C$ ....(Common angle)

$\therefore \Delta CPQ \sim \Delta CAB$ ...(AA criterion for Similarity)

$$\Rightarrow \frac{CP}{AC} = \frac{CQ}{CB}$$

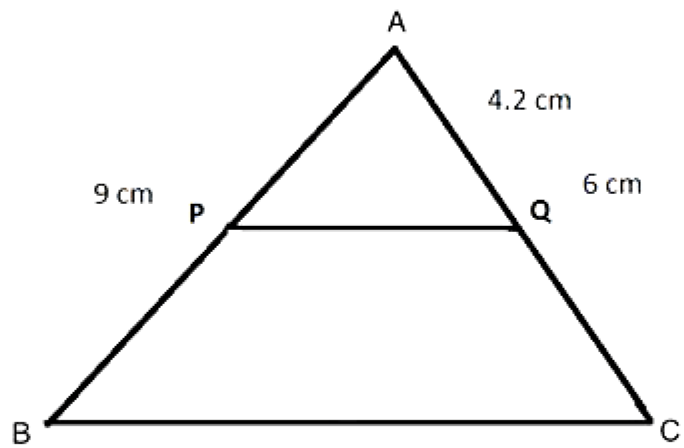
$$\Rightarrow \frac{CP}{AC} = \frac{4.8}{8.4} = \frac{4}{7}$$

So, if AC is 7 parts, and CP is 4 parts, then PA is 3 parts.

Thus, $AC = \frac{7}{3}PA = \frac{7}{3}x$.

Question 3.
A line PQ is drawn parallel tp the side BC of AABC which cuts side AB at P and side AC at Q. If AB = 9.0 cm, CA=6.0 cm and AQ = 4.2 cm, find the length of AP.

Solution:



In ΔAPQ and ΔABC,

$\angle ACQ = \angle ABC$ ....(Since PQ||BC, so the angles are corresponding angles.)

$\angle PAQ = \angle BAC$ ....(Common angle)

$\therefore \Delta APQ \sim \Delta ABC$ ...(AA criterion for Similarity)

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

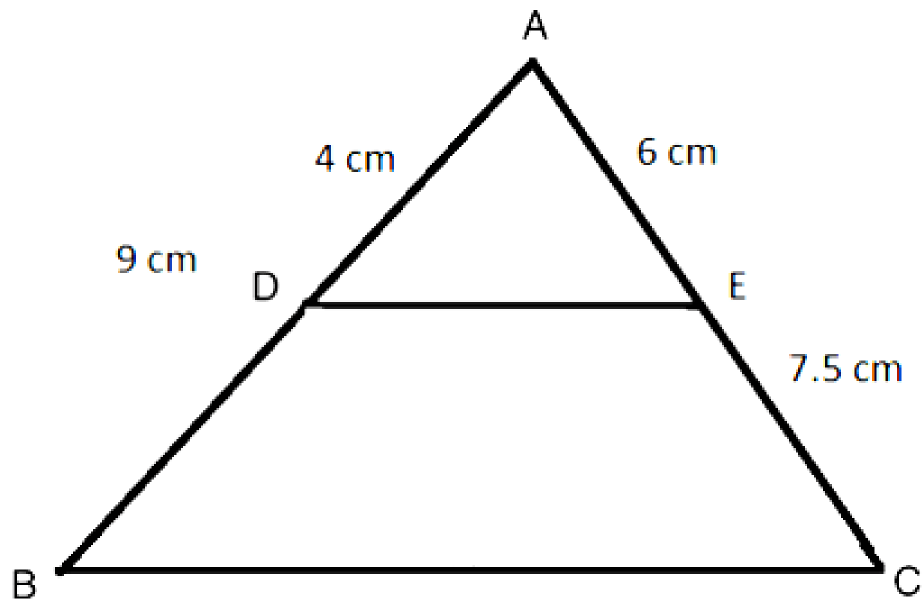
$\Rightarrow \frac{AP}{9} = \frac{4.2}{6}$

$\Rightarrow AP = 6.3$ cm

Question 4.
In ΔABC, D and E are the points on sides AB and AC respectively.
Find whether DE // BC, if:
(i) AB=9 cm, AD=4 cm, AE=6 cm and EC = 7.5 cm.
(ii) AB=63 cm, EC=11.0 cm, AD=0.8 cm and AE = 1.6 cm.

Solution:

(i).



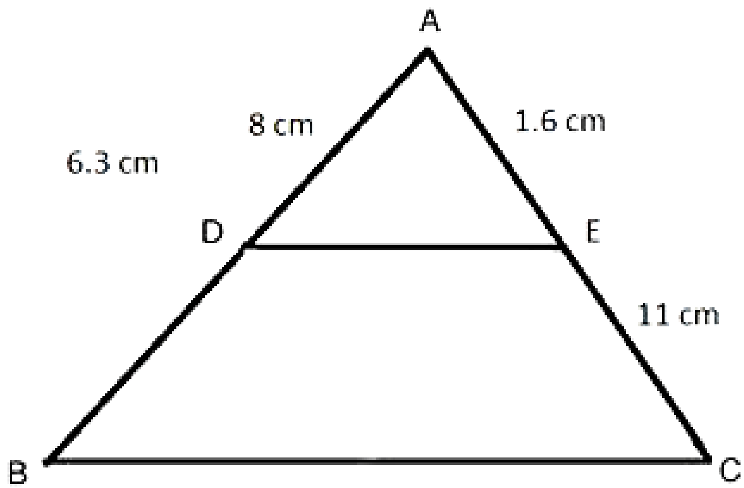
In ΔADE and ΔABC,

$\frac{AE}{EC} = \frac{6}{7.5} = \frac{4}{5}$

$\frac{AD}{BD} = \frac{4}{5}$ ....(Since AB = 9 cm and AD = 4 cm)

So, $\frac{AE}{EC} = \frac{AD}{BD}$.

$\therefore$ DE || BC ....(By the Converse of Mid-point theorem) (ii).



In ΔADE and ΔABC,

$$\frac{AE}{EC} = \frac{1.6}{11} = \frac{0.8}{5.5}$$

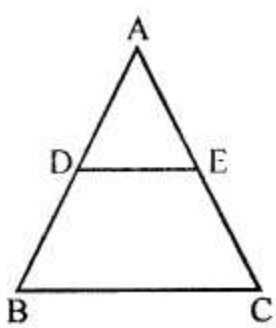
$$\frac{AD}{BD} = \frac{0.8}{6.3-8} = \frac{0.8}{5.5}$$

So, $\frac{AE}{EC} = \frac{AD}{BD}$.

$\therefore$ DE || BC ....(By the Converse of Mid-point theorem)

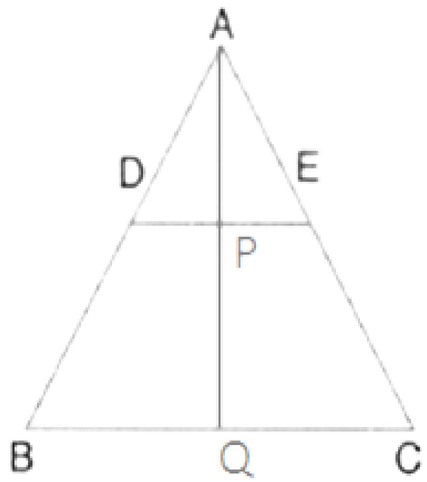
Question 5.

In the given figure, ΔABC ~ ΔADE. If AE: EC = 4 :7 and DE = 6.6 cm, find BC. If ‘x’ be the length of the perpendicular fromA to DE, find the length of perpendicular from



A to DE find the length of perpendicular from A to BC in terms of ‘x’.

Solution:



Given that $\Delta ABC \sim \Delta ADE$.

$\angle ABC = \angle ADE$ and $\angle ACB = \angle AED$

So, $DE || BC$

Also, $\frac{AB}{AD} = \frac{AC}{AE} = \frac{11}{4}$. ....$\left(\text{Since } \frac{AE}{EC} = \frac{4}{7}\right)$

In ΔADP and ΔABQ,

$\angle ADP = \angle ABQ$ ...(Since $DP||BQ$.)

$\angle APD = \angle AQB$ ...(Since $DP||BQ$.)

So, $\Delta ADP \sim \Delta ABQ$ ...(AA Criterion for Similarity)

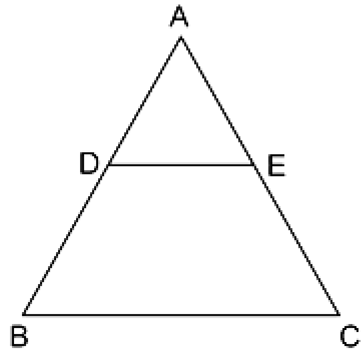
$\Rightarrow \frac{AD}{AB} = \frac{AP}{AQ}$

$\Rightarrow \frac{4}{11} = \frac{x}{AQ}$

$\Rightarrow AQ = \frac{11}{4}x$

Question 6.
A line segment DE is drawn parallel to base BC of AABC which cuts AB at point D and AC at point E. If AB = 5 BD and EC=3.2 cm, find the length of AE.

Solution:



Since DE || BC, $\Delta ADE \sim \Delta ABC$

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AB - BD}{BD} = \frac{AE}{EC}$$

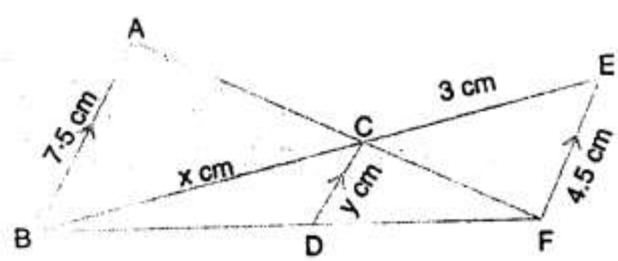
$$\Rightarrow \frac{5BD - BD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{4BD}{BD} = \frac{AE}{3.2}$$

$$\Rightarrow AE = 4 \times 3.2 = 12.8 \text{ cm}$$

Question 7.

In the figure, given below, AB, Cd and EF are parallel lines. Given AB = 7.5 cm, DC =y cm, EF=4.5 cm, BC=x cm and CE=3 cm, calculate the values of x and y.



Solution:

In ΔBEF, $DC||EF$.

$$\Rightarrow \frac{BD}{DF} = \frac{BC}{CE}$$

$$\Rightarrow \frac{BD}{DF} = \frac{x}{3}$$

So, $BD = x$ and $DF = 3$.

In ΔAFB, $DC||AB$.

$$\Rightarrow \frac{FD}{CD} = \frac{FB}{AB}$$

$$\Rightarrow \frac{FD}{CD} = \frac{FD + DB}{AB}$$

$$\Rightarrow \frac{3}{y} = \frac{x+3}{7.5} \quad \ldots(i)$$

In ΔBFE, $DC||EF$.

$$\Rightarrow \frac{BC}{CD} = \frac{BE}{EF}$$

$$\Rightarrow \frac{BC}{CD} = \frac{BC + CE}{EF}$$

$$\Rightarrow \frac{x}{y} = \frac{x+3}{4.5}$$

$$\Rightarrow y = \frac{4.5x}{x+3} \quad \ldots(ii)$$

Substituting (ii) in (i), we get

$$\frac{3}{\frac{4.5x}{x+3}} = \frac{x+3}{7.5}$$

$$\Rightarrow \frac{3x+9}{4.5x} = \frac{x+3}{7.5}$$

$$\Rightarrow 22.5x + 67.5 = 4.5x^2 + 13.5x$$

$$\Rightarrow 4.5x^2 + 13.5x - 22.5x - 67.5 = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow (x-5)(x+3) = 0$$

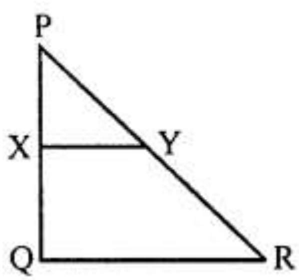
So, $x = 5$ and $x = -3$.

Since side of a triangle cannot be negative, $x = 5$.
Substituting this value in (ii), we get

$$y = \frac{4.5(5)}{x+3} = 2.8125$$

Hence, $x = 5$ and $y = 2.8125$

Question 8.
In the figure, given below, PQR is a right- angle triangle right angled at Q. XY is parallel to QR, PQ = 6 cm, P Y=4 cm and PX : XQ = 1:2. Calculate the lengths of PR and QR.



Solution:

Given that $\frac{PX}{XQ} = \frac{1}{2}$ and $XY || QR$.

So, $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$.

Since PY = 4 cm, YR = 8 cm.

Hence, PR = 12 cm.

Since ΔPQR is a right-angled triangle.

By Pythagoras theorem,

$QR^2 = PR^2 - PQ^2$

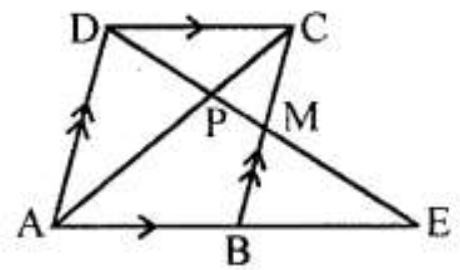
$\Rightarrow QR^2 = 12^2 - 6^2$

$\Rightarrow QR^2 = 144 - 36$

$\Rightarrow QR^2 = 108$

$\Rightarrow QR = 10.39$ cm

Question 9.
In the following figure, M is mid-point of BC of a parallelogram ABCD. DM intersects the diagonal AC at P and AB produced at E. Prove that PE = 2PD.



Solution:

In ΔBME and ΔDMC,

$\angle BME = \angle CMD$ ...(vertically opposite angles)

$\angle MCD = \angle MBE$ ...(alternate angles)

$BM = BC$ ...(M is the mid-point of BC)

So, $\Delta BME \cong \Delta DMC$ ...(AAS congruence criterion)

$\Rightarrow BE = DC = AB$

In ΔDCP and ΔEPA,

$\angle DPC = \angle EPA$ ...(vertically opposite angles)

$\angle CDP = \angle AEP$ ...(alternate angles)

$\Delta DCP \sim \Delta EAP$ ...(AA criterion for Similarity)

$$\Rightarrow \frac{DC}{EA} = \frac{CP}{AP} = \frac{PD}{EP}$$

$$\Rightarrow \frac{DC}{EA} = \frac{PD}{PE}$$

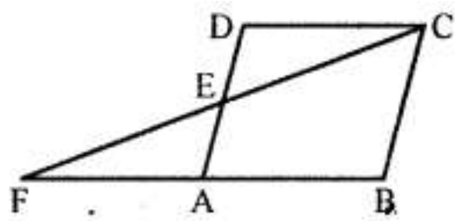
$$\Rightarrow \frac{EA}{DC} = \frac{PE}{PD}$$

$$\Rightarrow \frac{PE}{PD} = \frac{AB + EA}{DC}$$

$$\Rightarrow \frac{PE}{PD} = \frac{2DC}{DC}$$

$\Rightarrow PE = 2PD$

Question 10.
The given figure shows a parallelogram ABCD. E is a point in AD and CE produced meets BA produced at point F. If AE=4 cm, AF = 8 cm and AB = 12 cm, find the perimeter of the parallelogram ABCD.



Solution:

AF = 8 cm and AB = 12 cm
So, FB = 20 cm.
In ΔDEC and ΔEAF,
$\angle DEC = \angle EAF$ ...(vertically opposite angles)
$\angle EDC = \angle EAF$ ...(alternate angles)
So, $\Delta DEC \sim \Delta AEF$ ...(AA criterion for Similarity)

$\Rightarrow \frac{DE}{AE} = \frac{EC}{EF} = \frac{DC}{AF}$

$\Rightarrow \frac{DE}{AE} = \frac{DC}{AF}$

$\Rightarrow \frac{DE}{AE} = \frac{AB}{AF}$

$\Rightarrow \frac{DE}{4} = \frac{12}{8}$

$\Rightarrow DE = 6$ cm
So, AD = AE + ED = 4 + 6 = 10 cm
Perimeter of the parallelogram ABCD
= AB + BC + CD + AD
= 12 + 10 + 12 + 10
= 44 cm

Exercise 15C

Question 1.
(i) The ratio between the corresponding sides of two similar triangles is 2 is to 5. Find the ratio between the areas of these triangles.

(ii) Areas of two similar triangles are 98 sq. cm and 128 sq. cm. Find the ratio between the lengths of their corresponding sides.

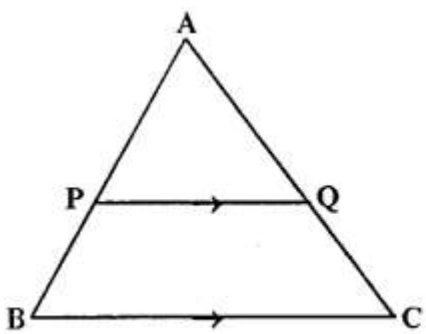
Solution:

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

(i) Required ratio = $\frac{2^2}{5^2} = \frac{4}{25}$

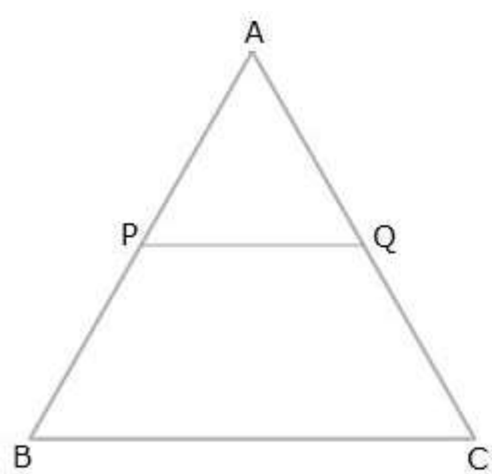
(ii) Required ratio = $\sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \frac{7}{8}$

Question 2.
A line PQ is drawn parallel to the base BC, of Δ ABC which meets sides AB and AC at points P and Q respectively. If AP = $\frac{1}{3}$ PB; find the value of:



(i) $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta APQ}$, (ii) $\frac{Area\ of\ \Delta APQ}{Area\ of\ trapezium\ PBCQ}$

Solution:



(i) $AP = \frac{1}{3} PB \Rightarrow \frac{AP}{PB} = \frac{1}{3}$

In ΔAPQ and ΔABC,

As $PQ \parallel BC$, corresponding angles are equal

$\angle APQ = \angle ABC$

$\angle AQP = \angle ACB$

$\Delta APQ \sim \Delta ABC$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ} = \frac{AB^2}{AP^2}$$

$$= \frac{4^2}{1^2} = 16 : 1$$

$$\left(\frac{AP}{PB} = \frac{1}{3} \Rightarrow \frac{AB}{AP} = \frac{4}{1}\right)$$

$$\frac{\text{Area of } \Delta APQ}{\text{Area of trapezium } PBCQ}$$

(ii) $= \dfrac{\text{Area of } \Delta APQ}{\text{Area of } \Delta ABC - \text{Area of } \Delta APQ}$

$$= \frac{1}{16 - 1} = 1 : 15$$

Question 3.

The perimeters of two similar triangles are 30 cm and 24cm. If one side of first triangle is 12cm, determine the corresponding side of the second triangle.

Solution:

Let $\Delta ABC \sim \Delta DEF$

Then, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{AB + BC + AC}{DE + EF + DF}$

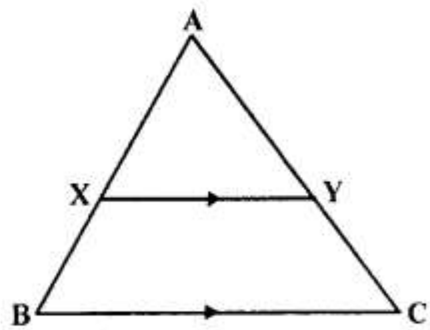
$$= \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE}$$

$$\Rightarrow \frac{30}{24} = \frac{12}{DE}$$

$\Rightarrow DE = 9.6$ cm

Question 4.
In the given figure AX : XB = 3 : 5



Find :
(i) the length of BC, if length of XY is 18 cm.
(ii) ratio between the areas of trapezium XBCY and triangle ABC.

Solution:

Given, $\frac{AX}{XB} = \frac{3}{5} \Rightarrow \frac{AX}{AB} = \frac{3}{8} \quad \ldots (1)$

(i)

In ΔAXY and ΔABC,

As $XY \parallel BC$, corresponding angles are equal

$\angle AXY = \angle ABC$

$\angle AYX = \angle ACB$

$\Delta AXY \sim \Delta ABC$

$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$

$\Rightarrow \frac{3}{8} = \frac{18}{BC}$

$\Rightarrow BC = 48$ cm

(ii)

$$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{AX^2}{AB^2} = \frac{9}{64}$$

$$\frac{\text{Area of } \Delta ABC - \text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{64-9}{64} = \frac{55}{64}$$

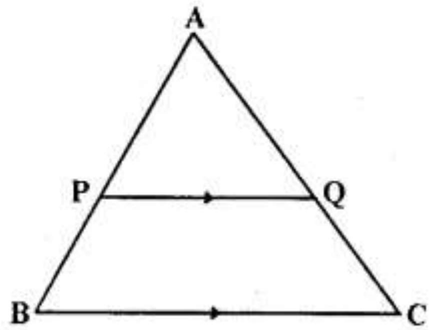
$$\frac{\text{Area of trapezium XBCY}}{\text{Area of } \Delta ABC} = \frac{55}{64}$$

Question 5.

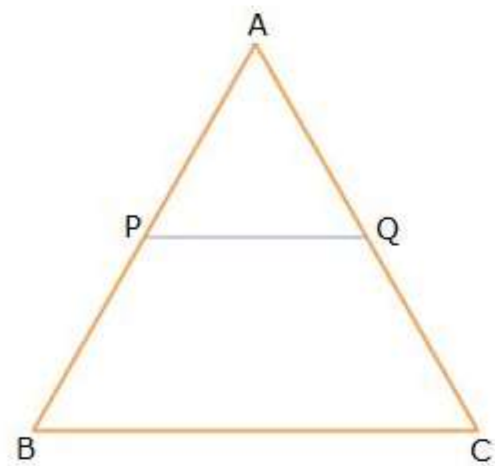
ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ || BC and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

Given— In Δ ABC, PQ || BC in such away that area APQ = area PQCB

To Find— The ratio ol' BP : AB



Solution:



From the given information, we have:

$$ar(\Delta APQ) = \frac{1}{2} ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{2}$$

$$\Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2}$$

$$\Rightarrow \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

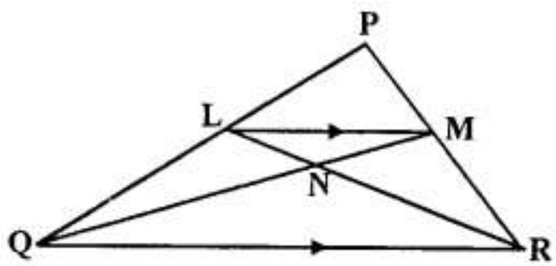
$$\Rightarrow \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

$\Rightarrow 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$

Question 6.
In the given triangle PQR, LM is parallel to QR and PM : MR = 3 : 4



Calculate the value of ratio:

(i) $\frac{PL}{PQ}$ **and then** $\frac{LM}{QR}$ **(ii)** $\frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR}$

(iii) $\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta LQN}$

Solution:

(i)
In ΔPLM and ΔPQR,
As LM || QR, corresponding angles are equal
$\angle PLM = \angle PQR$
$\angle PML = \angle PRQ$
$\Delta PLM \sim \Delta PQR$

$\Rightarrow \frac{PM}{PR} = \frac{LM}{QR}$

$\Rightarrow \frac{3}{7} = \frac{LM}{QR} \quad \left(\because \frac{PM}{MR} = \frac{3}{4} \Rightarrow \frac{PM}{PR} = \frac{3}{7}\right)$

Also, by using Basic Proportionality theorem, we have:

$\frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4}$

$\Rightarrow \frac{LQ}{PL} = \frac{4}{3}$

$\Rightarrow 1 + \frac{LQ}{PL} = 1 + \frac{4}{3}$

$\Rightarrow \frac{PL + LQ}{PL} = \frac{3+4}{3}$

$\Rightarrow \frac{PQ}{PL} = \frac{7}{3}$

$\Rightarrow \frac{PL}{PQ} = \frac{3}{7}$

(ii) Since Δ LMN and Δ MNR have common vertex at M and their bases LN and NR are along the same straight line

$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR} = \frac{LN}{NR}$

Now, in ΔLNM and ΔRNQ,

$\angle NLM = \angle NRQ$ (Alternate angles)

$\angle LMN = \angle NQR$ (Alternate angles)

$\Delta LNM \sim \Delta RNQ$ (AA similarity)

$\therefore \frac{MN}{QN} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$

$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR} = \frac{LN}{NR} = \frac{3}{7}$

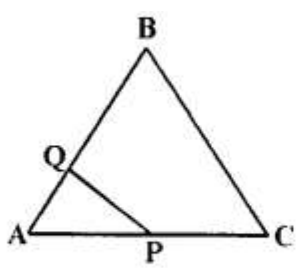
(iii) Since Δ LQM and Δ LQN have common vertex at L and their bases QM and QN are along the same straight line

$\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta LQN} = \frac{QM}{QN} = \frac{10}{7}$

$\left(\because \frac{MN}{QN} = \frac{3}{7} \Rightarrow \frac{QM}{QN} = \frac{10}{7}\right)$

Question 7.
The given diagram shows two isosceles triangles which are similar also. In (he given diagram, PQ and BC are not parallel:

PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ.



Calculate—
(i) the length of AP
(ii) the ratio of the areas of triangle APQ and triangle ABC.

Solution:

(i)

Given, $\Delta AQP \sim \Delta ACB$

$$\Rightarrow \frac{AQ}{AC} = \frac{AP}{AB}$$

$$\Rightarrow \frac{3}{4+AP} = \frac{AP}{3+12}$$

$$\Rightarrow AP^2 + 4AP - 45 = 0$$

$$\Rightarrow (AP+9)(AP-5) = 0$$

$$\Rightarrow AP = 5 \text{ units} \qquad \text{(as length cannot be negative)}$$

(ii)

Since, $\Delta AQP \sim \Delta ACB$

$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ACB)} = \frac{PQ^2}{BC^2}$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{BC^2} \qquad (PQ = AP)$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left(\frac{5}{15}\right)^2 = \frac{1}{9}$$

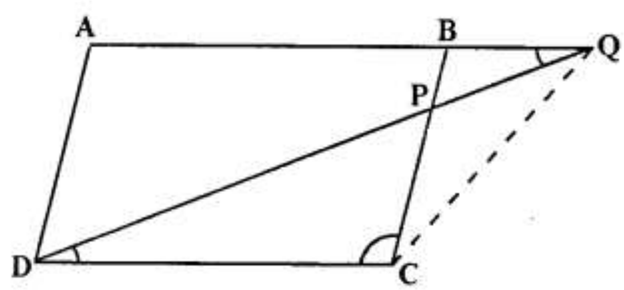
Question 8.

In the figure, given below, ABCD is a parallelogram. P is a point on BC such that BP : PC =1:2. DP produced meets AB produced at Q. Given the area of triangle CPQ = 20 cm2.

Calculate—

(i) area of triangle CDP

(ii) area of parallelogram ABCD [1996]



Solution:

(i) In ΔBPQ and ΔCPD

$\angle BPQ = \angle CPD$ (Vertically opposite angles)

$\angle BQP = \angle PDC$ (Alternate angles)

$\Delta BPQ \sim \Delta CPD$ (AA similarity)

$\therefore \frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{CD} = \frac{1}{2}$ $\left(\because \frac{BP}{PC} = \frac{1}{2}\right)$

Also, $\frac{ar(\Delta BPQ)}{ar(\Delta CPD)} = \left(\frac{BP}{PC}\right)^2$

$\Rightarrow \frac{10}{ar(\Delta CPD)} = \frac{1}{4}$ $\left[ar(\Delta BPQ) = \frac{1}{2} \times ar(\Delta CPQ), ar(\Delta CPQ) = 20\right]$

$\Rightarrow ar(\Delta CPD) = 40 \text{ cm}^2$

(ii) In ΔBQP and ΔAQD

As BP || AD, corresponding angles are equal

$\angle QBP = \angle QAD$

$\angle BQP = \angle AQD$ (Common)

$\Delta BQP \sim \Delta AQD$ (AA similarity)

$\therefore \frac{AQ}{BQ} = \frac{QD}{QP} = \frac{AD}{BP} = 3$ $\left(\because \frac{BP}{PC} = \frac{PQ}{PD} = \frac{1}{2} \Rightarrow \frac{PQ}{QD} = \frac{1}{3}\right)$

Also, $\frac{ar(\Delta AQD)}{ar(\Delta BQP)} = \left(\frac{AQ}{BQ}\right)^2$

$\Rightarrow \frac{ar(\Delta AQD)}{10} = 9$

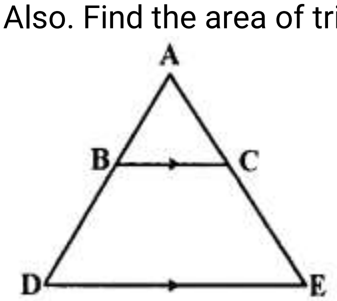
$\Rightarrow ar(\Delta AQD) = 90 \text{ cm}^2$

$\therefore ar(ADPB) = ar(\Delta AQD) - ar(\Delta BQP) = 90 \text{ cm}^2 - 10 \text{ cm}^2 = 80 \text{ cm}^2$

$ar(ABCD) = ar(\Delta CDP) + ar(ADPB) = 40 \text{ cm}^2 + 80 \text{ cm}^2 = 120 \text{ cm}^2$

Question 9.

In the given figure. BC is parallel to DE. Area of triangle ABC = 25 cm^2.
Area of trapezium BCED = 24 cm^2 and DE = 14 cm. Calculate the length of BC.
Also. Find the area of triangle BCD.



Solution:

In ΔABC and ΔADE,

As $BC \parallel DE$, corresponding angles are equal

$\angle ABC = \angle ADE$

$\angle ACB = \angle AED$

$\Delta ABC \sim \Delta ADE$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{49} = \frac{BC^2}{14^2} \qquad (ar(\Delta ADE) = ar(\Delta ABC) + ar(\text{trapezium BCED}))$$

$BC^2 = 100$

$BC = 10$ cm

In trapeziumBCED,

$$\text{Area} = \frac{1}{2}(\text{Sum of parallel sides}) \times h$$

Given : Area of trapezium BCED $= 24\ cm^2, BC = 10cm, DE = 14cm$

$$\therefore 24 = \frac{1}{2}(10 + 14) \times h$$

$$\Rightarrow h = \frac{48}{(10 + 14)}$$

$$\Rightarrow h = \frac{48}{24}$$

$\Rightarrow h = 2$

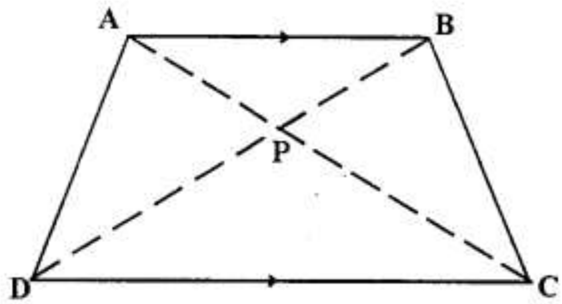
$$\text{Area of } \Delta BCD = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times h$$

$$= \frac{1}{2} \times 10 \times 2$$

$\therefore$ Area of $\Delta BCD = 10\ cm^2$

Question 10.
The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3 : 5.



Find:
(i) Δ APB : Δ CPB
(ii) Δ DPC : Δ APB
(iii) Δ ADP : Δ APB
(iv) Δ APB : Δ ADB

Solution:

(i) Since Δ APB and Δ CPB have common vertex at B and their bases AP and PC are along the same straight line

$\therefore \frac{ar(\Delta APB)}{ar(\Delta CPB)} = \frac{AP}{PC} = \frac{3}{5}$

(ii) Since Δ DPC and Δ BPA are similar

$\therefore \frac{ar(\Delta DPC)}{ar(\Delta BPA)} = \left(\frac{PC}{AP}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

(iii) Since Δ ADP and Δ APB have common vertex at A and their bases DP and PB are along the same straight line

$\therefore \frac{ar(\Delta ADP)}{ar(\Delta APB)} = \frac{DP}{PB} = \frac{5}{3}$

$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5}\right)$

(iv) Since Δ APB and Δ ADB have common vertex at A and their bases BP and BD are along the same straight line

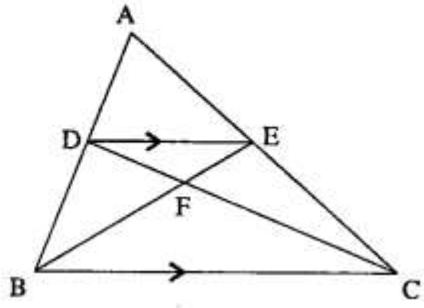
$\therefore \frac{ar(\Delta APB)}{ar(\Delta ADB)} = \frac{PB}{BD} = \frac{3}{8}$

$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \Rightarrow \frac{BP}{BD} = \frac{3}{8}\right)$

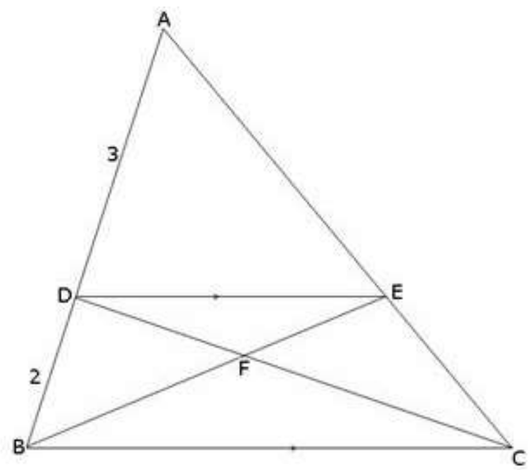
Question 11.
In the given figure, ARC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

(i) Determine the ratios $\frac{AD}{AB}, \frac{DE}{BC}$.

(ii) Prove that ΔDEF is similar to ΔCBF. Hence, find $\frac{EF}{FB}$.
(iii) What is the ratio of the areas of ΔDEF and ΔBFC?



Solution:



(i) Given, DE || BC and $\frac{AD}{DB} = \frac{3}{2}$

In ΔADE and ΔABC,

$\angle A = \angle A$(Corresponding Angles)

$\angle ADE = \angle ABC$(Corresponding Angles)

$\therefore$ ΔADE ~ ΔABC (By AA- similarity)

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$ ....(1)

Now $\frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{3}{3+2} = \frac{3}{5}$

Using (1), we get $\frac{AD}{AB} = \frac{3}{5} = \frac{DE}{BC}$. ....(2)

(ii) In Δ DEF and Δ CBF,

$\angle$FDE = $\angle$FCB(Alternate Angle)

$\angle$DFE = $\angle$BFC(Vertically Opposite Angle)

$\therefore \Delta$ DEF $\sim \Delta$ CBF(By AA- similarity)

$$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5} \text{ using (2)}$$

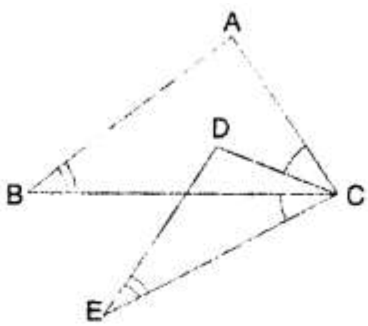
$$\frac{EF}{FB} = \frac{3}{5}.$$

(iii) Since the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, therefore

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25}.$$

Question 12.

In the given figure, ∠B = ∠E, ∠ACD = ∠BCE, AB=10.4 cm and DE=7.8 cm. Find the ratio between areas of the ΔABC and ΔDEC.



Solution:

$$\text{Given, } \angle ACD = \angle BCE$$

$$\angle ACD + \angle BCD = \angle BCE + \angle BCD$$

$$\angle ACB = \angle DCE$$

$$\text{Also, given } \angle B = \angle E$$

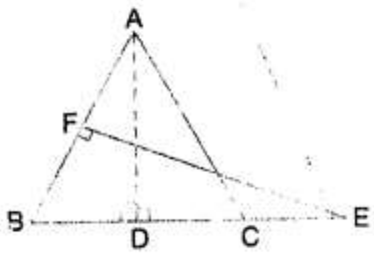
$$\therefore \Delta ABC \sim \Delta DEC$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEC)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{10.4}{7.8}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Question 13.

Triangle ABC is an isosceles triangle in which AB = AC = 13 cm and BC = 10 cm. AD is perpendicular to BC. If CE = 8 cm and EF ⊥ AB, find:

(i) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta FEB}$ (ii) $\frac{\text{area of } \Delta FEB}{\text{area of } \Delta ABC}$



Solution:

(i) AB = AC(Given)

$\therefore \angle FBE = \angle ACD$

$\angle BFE = \angle ADC$

$\Delta EFB \sim \Delta ADC$ (AA similarity)

$$\therefore \frac{ar(\Delta ADC)}{ar(\Delta EFB)} = \left(\frac{AC}{BE}\right)^2$$

$$= \left(\frac{AC}{BC + CE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2 = \frac{169}{324} \quad \ldots (1)$$

(ii) Similarly, it can be proved that $\Delta ADB \sim \Delta EFB$

$$\therefore \frac{ar(\Delta ADB)}{ar(\Delta EFB)} = \left(\frac{AB}{BE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2$$

$$= \frac{169}{324} \quad \ldots (2)$$

From (1) and (2),

$$\frac{ar(\Delta ABC)}{ar(\Delta EFB)} = \frac{169 + 169}{324} = \frac{338}{324} = \frac{169}{162}$$

$\therefore ar(\Delta EFB) : ar(\Delta ABC) = 162 : 169$

Exercise 15D

Question 1.
A triangle ABC has been enlarged by scale factor m = 2.5 to the triangle A' B' C'.
Calculate:

(i) the length of AB, if A’ B’ = 6 cm.
(ii) the length of C’ A’ if CA = 4 cm.

Solution:
(i)
Given that ABC is a triangle that has been enlarged
by scale factor m = 2.5 to the triangle A'B'C'.
A'B' = 6 cm
So, AB(2.5) = A'B'
$\Rightarrow$ AB(2.5) = 6
$\Rightarrow$ AB = 2.4 cm

(ii)
Given that ABC is a triangle that has been enlarged
by scale factor m = 2.5 to the triangle A'B'C'.
A'B' = 6 cm
So, AB(2.5) = A'B'
$\Rightarrow$ AB(2.5) = 6
$\Rightarrow$ AB = 2.4 cm
If CA = 4 cm.
So, CA(2.5) = C'A'
$\Rightarrow$ (4)(2.5) = C'A'
$\Rightarrow$ C'A' = 10 cm

Question 2.
A triangle LMN has been reduced by scale factor 0.8 to the triangle L’ M’ N’. Calculate:
(i) the length of M’ N’, if MN = 8 cm.
(ii) the length of LM, if L’ M’ = 5.4 cm.

Solution:
(i)
Given that LMN is a triangle that has been reduced
by scale factor m = 0.8 to the triangle L'M'N'.
MN = 6 cm
So, MN(0.8) = M'N'
$\Rightarrow$ (8)(0.8) = M'N'
$\Rightarrow$ M'N' = 6.4 cm

(ii)
Given that LMN is a triangle that has been reduced by scale factor m = 0.8 to the triangle L'M'N'.
L'M' = 5.4 cm
So, LM(0.8) = L'M'
$\Rightarrow$ LM(0.8) = L'M'
$\Rightarrow$ LM(0.8) = 5.4
$\Rightarrow$ LM = 6.75 cm

Question 3.
A triangle ABC is enlarged, about the point O as centre of enlargement, and the scale factor is 3. Find:

(i) A' B', if AB = 4 cm.
(ii) BC, if B' C' = 15 cm.
(iii) OA, if OA'= 6 cm.
(iv) OC', if OC = 21 cm.

Also, state the value of:

(a) $\frac{OB'}{OB}$ (b) $\frac{C'A'}{CA}$

Solution:

(i)
Given that ABC is enlarged and the scale factor m = 3 to the triangle A'B'C'.
AB = 4 cm
So, AB(3) = A'B'
$\Rightarrow$ (4)(3) = A'B'
$\Rightarrow$ A'B' = 12 cm

(ii)
Given that ABC is enlarged and the scale factor m = 3 to the triangle A'B'C'.
B'C' = 15 cm
So, BC(3) = B'C'
$\Rightarrow$ BC(3) = 15
$\Rightarrow$ BC = 5 cm

(iii)

Given that ABC is enlarged and the scale factor m = 3 to the triangle A'B'C'.

OA' = 6 cm

So, OA(3) = OA'

$\Rightarrow$ OA(3) = 6

$\Rightarrow$ OA = 2 cm

(iv)

Given that triangle ABC is enlarged and the scale factor is m = 3 to the triangle A'B'C'.
OC = 21 cm
So, (OC)3 = OC'
i.e. 21 x 3 = OC'
i.e. OC' = 63 cm

The ratio of the lengths of two corresponding sides of two similar triangles.

(a) Given that ABC is enlarged and the scale factor m = 3 to the triangle A'B'C'.

$$\Rightarrow \frac{OB'}{OB} = 3$$

(b) Given that ABC is enlarged and the scale factor m = 3 to the triangle A'B'C'.

$$\Rightarrow \frac{C'A'}{CA} = 3$$

Question 4.
A model of an aeroplane is made to a scale of 1:400. Calculate:
(i) the length, in cm, of the model; if the length of the aeroplane is 40 m.
(ii) the length, in m, of the aeroplane, if length of its model is 16 cm.

Solution:

The ratio of the lengths of two corresponding sides of two similar triangles.

A model of an aeroplane is made to a scale of 1:400.

So, the length of the model = $\frac{1}{400} \times 4000 = 10$ cm

(ii)

The ratio of the lengths of two corresponding sides of two similar triangles.

A model of an aeroplane is made to a scale of 1:400.

So, the length of the aeroplane = $400 \times \frac{16}{100} = 64$ m

Question 5.
The dimensions of the model of a multistory building are 1.2 m × 75 cm × 2 m. If the scale factor is 1:30; find the actual dimensions of the building.

Solution:

The ratio of the lengths of two corresponding sides of two similar triangles.
The scale factor is 1:30.

The actual dimensions of the building = $\frac{30}{1}$(dimensions of the model of the building)

$\Rightarrow$ The actual dimensions of the building = $\frac{30}{1}\left(1.2 \times \frac{75}{100} \times 2\right)$

$\Rightarrow$ The actual dimensions of the building = $36 \text{ m} \times 22.5 \text{ m} \times 60 \text{ m}$

Question 6.
On a map drawn to a scale of 1: 2,50,000; a triangular plot of land has the following measurements : AB = 3 cm, BC = 4 cm and angle ABC = 90°.
Calculate:
(i) the actual lengths of AB and BC in km.
(ii) the area of the plot in sq. km.

Solution:

The ratio of the lengths of two corresponding sides of two similar triangles.
The scale factor is 1:2,50,000.

The length of AB on the map = $\frac{1}{2,50,000}$(the actual length of AB)

$\Rightarrow 3 = \frac{1}{2,50,000}$(the actual length of AB)

$\Rightarrow$ the actual length of AB = $3 \times 2,50,000$

$\Rightarrow$ the actual length of AB = $7,50,000 = 7.5$ km

The length of BC on the map = $\frac{1}{2,50,000}$(the actual length of BC)

$\Rightarrow 4 = \frac{1}{2,50,000}$(the actual length of BC)

$\Rightarrow$ the actual length of BC = $4 \times 2,50,000$

$\Rightarrow$ the actual length of BC = $1,00,000 = 10$ km

(ii)

The area of the plot in sq. km

$= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 7.5 \times 10$

$= 37.5$ sq. km

Question 7.

A model of a ship of made to a scale 1 : 300

(i) The length of the model of ship is 2 m. Calculate the lengths of the ship.

(ii) The area of the deck ship is 180,000 m^2. Calculate the area of the deck of the model.

(iii) The volume of the model in 6.5 m^3. Calculate the volume of the ship. (2016)

Solution:

i. Scale factor $k = \frac{1}{300}$

Length of the model of the ship $= k \times$ Length of the ship

$\Rightarrow 2 = \frac{1}{300} \times$ Length of the ship

$\Rightarrow$ Length of the ship $= 600$ m

ii. Area of the deck of the model $= k^2 \times$ Area of the deck of the ship

$\Rightarrow$ Area of the deck of the model $= \left(\frac{1}{300}\right)^2 \times 180,000$

$= \frac{1}{90000} \times 180,000$

$= 2 \ m^2$

iii. Volume of the model $= k^3 \times$ Volume of the ship

$\Rightarrow 6.5 = \left(\frac{1}{300}\right)^3 \times$ Volume of the ship

$\Rightarrow$ Volume of the ship $= 6.5 \times 27000000 = 17,55,00,000 \ m^3$

Question 7(old).

A model of ship is made to a scale of 1: 200.

(i) The length of the model is 4 m; calculate the length of the ship.

(ii) The area of the deck of the ship is 160000 m^2; find the area of the deck of the model.

(iii) The volume of the model is 200 litres; calculate the volume of the ship in m^3.

Solution:

Scale factor = k = $\frac{1}{200}$

(i) Length of model = k $\times$ actual length of the ship

$\Rightarrow$Actual length of the ship = 4 $\times$ 200 = 800 m

(ii) Area of the deck of the model = k^2 $\times$ area of the deck of the ship

$= \left(\frac{1}{200}\right)^2 \times 160000\ m^2 = 4\ m^2$

(iii) Volume of the model = k^3 $\times$ volume of the ship

Volume of the ship

$= \frac{1}{k^3} \times 200$ litres

$= (200)^3 \times 200$ litres

$= 1600000000$ litres

$= 1600000\ m^3$

Question 8.

An aeroplane is 30 in long and its model is 15 cm long. If the total outer surface area of the model is 150 cm^2, find the cost of painting the outer surface of the aeroplane at the rate of ₹ 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

Solution:

15cm represents = 30 m

1cm represents $\frac{30}{15} = 2m$

1 cm^2 represents 2m $\times$ 2m = 4 m^2

Surface area of the model = 150 cm^2

Actual surface area of aeroplane = 150 $\times$ 2 $\times$ 2 m^2 = 600 m^2

50 m^2 is left out for windows

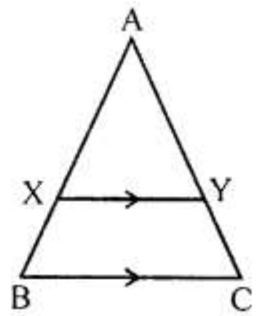
Area to be painted = 600 - 50 =50 m^2

Cost of painting per m^2 = Rs. 120

Cost of painting 550 m^2 = 120 $\times$ 550 = Rs. 66000

Exercise 15E

Question 1.
In the following figure, XY is parallel to BC, AX = 9 cm, XB = 4.5cm and BC = 18 cm.



Find:

(i) $\frac{AY}{YC}$ (ii) $\frac{YC}{AC}$ (iii) XY

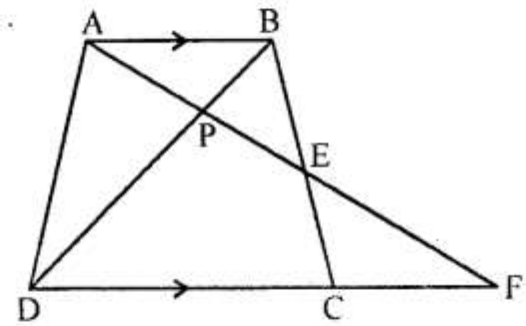
Solution:

(i)
Given that XY||BC.
So, $\Delta AXY \sim \Delta ABC$.
$\Rightarrow \frac{AX}{AB} = \frac{AY}{AC}$
$\Rightarrow \frac{9}{13.5} = \frac{AY}{AC}$
$\Rightarrow \frac{AY}{YC} = \frac{2}{1}$

(ii)
Given that XY||BC.
So, $\Delta AXY \sim \Delta ABC$.
$\Rightarrow \frac{AX}{AB} = \frac{AY}{AC}$
$\Rightarrow \frac{9}{13.5} = \frac{AY}{AC}$
$\Rightarrow \frac{YC}{AC} = \frac{4.5}{13.5} = \frac{1}{3}$

Question 2.
In the following figure, ABCD to a trapezium with AB//DC. If AB = 9 cm, DC = 18 cm, CF= 13.5 cm, AP=6 cm and BE = 15 cm.

Calculate:
(i) EC
(ii) AF
(iii) PE



Solution:
(i)
In ΔAEB and ΔFEC,
$\angle AEB = \angle FEC$ ...(vertically opposite angles)
$\angle BAE = \angle CFE$ ...(Since AB||DC.)
$\Delta AEB \sim \Delta FEC$ ....(AA criterion for Similarity)
$\Rightarrow \frac{AE}{FE} = \frac{BE}{CE} = \frac{AB}{FC}$
$\Rightarrow \frac{15}{CE} = \frac{9}{13.5}$
$\Rightarrow CE = 22.5$ cm

(ii)
In ΔAPB and ΔFPD,
$\angle APB = \angle FPD$ ...(vertically opposite angles)
$\angle BAP = \angle DFP$ ...(Since AB||DF.)
$\Delta APB \sim \Delta FPD$ ....(AA criterion for Similarity)
$\Rightarrow \frac{AP}{FP} = \frac{AB}{FD}$
$\Rightarrow \frac{6}{FP} = \frac{9}{31.5}$
$\Rightarrow FP = 21$ cm
So, $AF = AP + PF = 6 + 21 = 27$ cm.

(iii)

In ΔAPB and ΔFPD,

$\angle APB = \angle FPD$ ...(vertically opposite angles)

$\angle BAP = \angle DFP$ ...(Since AB||DF.)

$\Delta APB \sim \Delta FPD$ ....(AA criterion for Similarity)

$\Rightarrow \frac{AP}{FP} = \frac{AB}{FD}$

$\Rightarrow \frac{6}{FP} = \frac{9}{31.5}$

$\Rightarrow FP = 21$ cm

So, AF = AP + PF = 6 + 21 = 27 cm.

In ΔAEB and ΔFEC,

$\angle AEB = \angle FEC$ ...(vertically opposite angles)

$\angle BAE = \angle CFE$ ...(Since AB||DC.)

$\Delta AEB \sim \Delta FEC$ ....(AA criterion for Similarity)

$\Rightarrow \frac{AE}{FE} = \frac{BE}{CE} = \frac{AB}{FC}$

$\Rightarrow \frac{AE}{FE} = \frac{9}{13.5}$

$\frac{AF - EF}{FE} = \frac{9}{13.5}$

$\Rightarrow \frac{AF}{EF} - 1 = \frac{9}{13.5}$

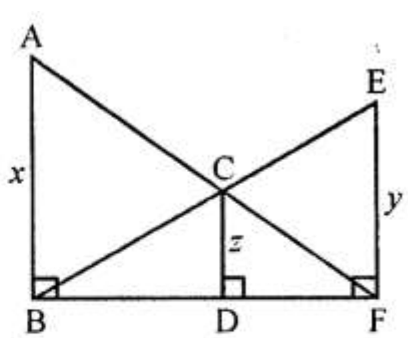
$\Rightarrow \frac{27}{EF} = \frac{9}{13.5} + 1 = \frac{22.5}{13.5}$

$\Rightarrow EF = \frac{27 \times 13.5}{22.5} = 16.2$ cm

Now, PE = PF − EF = 21 − 16.2 = 4.8 cm

Question 3.

In the following figure, AB, CD and EF are perpendicular to the straight line BDF.



If AB = x and CD = z unit and EF = y unit, prove that : $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

Solution:

In ΔFDC and ΔFBA,

$\angle FDC = \angle FBA$ ...(Since DC||AB)

$\angle DFC = \angle BFA$ ...(common angle)

$\Delta FDC \sim \Delta FBA$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{DC}{AB} = \frac{DF}{BF}$$

$$\Rightarrow \frac{z}{x} = \frac{DF}{BF} \quad ....(i)$$

In ΔBDC and ΔBFE,

$\angle BDC = \angle BFE$ ...(Since DC||FE)

$\angle DBC = \angle FBE$ ...(common angle)

$\Delta BDC \sim \Delta BFE$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{BD}{BF} = \frac{DC}{EF}$$

$$\Rightarrow \frac{BD}{BF} = \frac{z}{y} \quad ....(ii)$$

Adding (i) and (ii), we get

$$\frac{BD}{BF} + \frac{DF}{BF} = \frac{z}{y} + \frac{z}{x}$$

$$\Rightarrow 1 = \frac{z}{y} + \frac{z}{x}$$

$$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$$

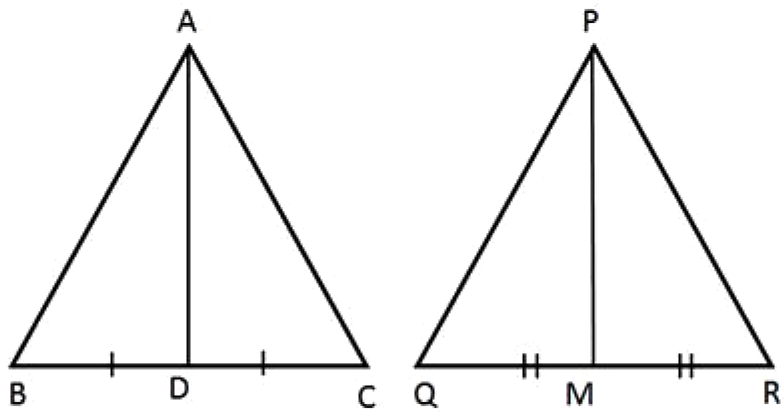
Hence proved.

Question 4.

Triangle ABC is similar to triangle PQR. If AD and PM are corresponding medians of the two triangles, prove that:

$\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution:



Given that $\Delta ABC \sim \Delta PQR$.

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

$\angle ABC = \angle PQR$, that is, $\angle ABD = \angle PQM$

Also, $\angle ADB = \angle PMQ$ ....(both are right angles)

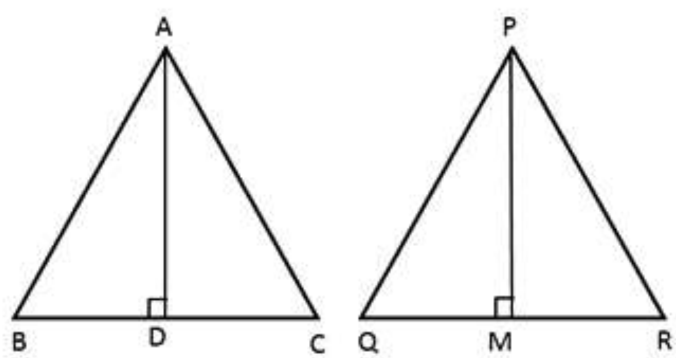
So, $\Delta ABD \sim \Delta PQM$ .....(AA criterion for Similarity)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

Question 5.

Triangle ABC is similar to triangle PQR. If AD and PM are altitudes of the two triangles, prove that: $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution:



Given that $\Delta ABC \sim \Delta PQR$.

$\angle ABC = \angle PQR$, that is, $\angle ABD = \angle PQM$

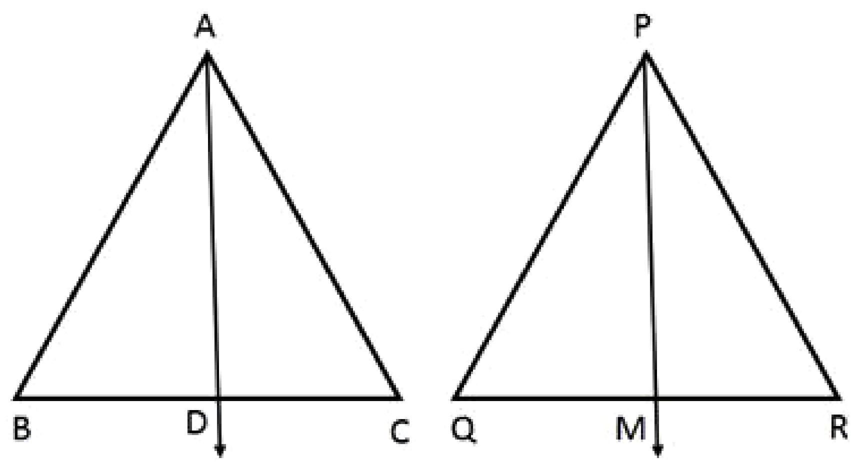
Also, $\angle ADB = \angle PMQ$ ....(both are right angles)

So, $\Delta ABD \sim \Delta PQM$ .....(AA criterion for Similarity)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

Question 6.
Triangle ABC is similar to triangle PQR. If bisector of angle BAC meets BC at point D and bisector of angle QPR meets QR at point M, prove that: $\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution:



Given that $\Delta ABC \sim \Delta PQR$.

$\Rightarrow \angle BAC = \angle QPR$

$\Rightarrow \frac{1}{2}\angle BAC = \frac{1}{2}\angle QPR$

$\Rightarrow \angle BAD = \angle QPM$

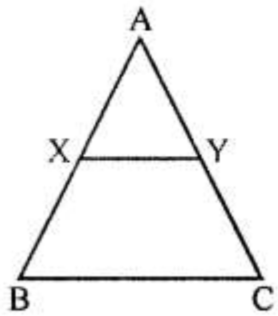
Also, $\angle ABC = \angle PQR$, that is, $\angle ABD = \angle PQM$

So, $\Delta ABD \sim \Delta PQM$ .....(AA criterion for Similarity)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

Question 7.

In the following figure, ∠AXY = ∠AYX. If $\frac{\mathbf{BX}}{\mathbf{AX}} = \frac{\mathbf{CY}}{\mathbf{AY}}$, show that triangle ABC is isosceles.



Solution:

Given that ∠AXY = ∠AYX.

So, AX = AY....(Sides opposite equal angles are equal.)

Also, $\frac{BX}{AX} = \frac{CY}{AY}$ .....(By the Basic Proportionality theorem)

So, BX = CY

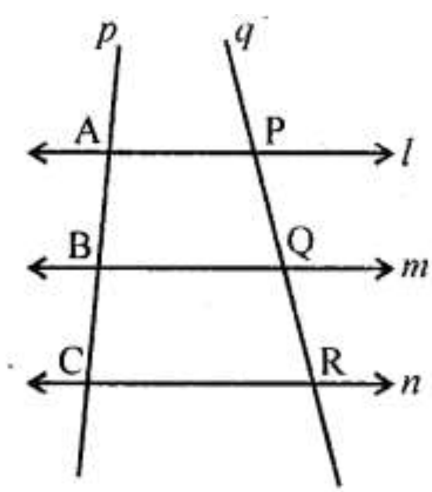
Thus, AX + BX = AY + CY

⇒ AB = AC

Hence, ΔABC is an isosceles triangle.

Question 8.

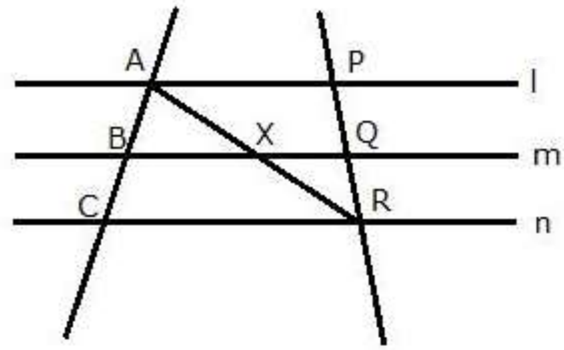
In the following diagram, lines l, m and n are parallel to each other. Two transversals p and q intersect the parallel lines at points A, B, C and P, Q, R as shown.



Prove that: $\frac{AB}{BC} = \frac{PQ}{QR}$

Solution:

Join AR.



In Δ ACR, BX || CR. By Basic Proportionality theorem,

$$\frac{AB}{BC} = \frac{AX}{XR} \quad \dots (1)$$

In Δ APR, XQ || AP. By Basic Proportionality theorem,

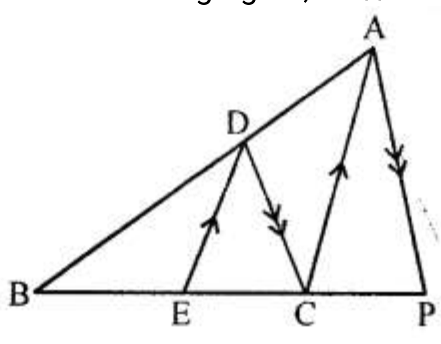
$$\frac{PQ}{QR} = \frac{AX}{XR} \quad \dots (2)$$

From (1) and (2), we get,

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

Question 9.

In the following figure, DE //AC and DC //AP. Prove that: $\frac{BE}{EC} = \frac{BC}{CP}$



Solution:

Since DE||AC,

$\frac{BE}{EC} = \frac{BD}{DA}$ .....(By the Basic Proportionality theorem)

Since DC||AP,

$\frac{BC}{CP} = \frac{BD}{DA}$ .....(By the Basic Proportionality theorem)

Hence, $\frac{BE}{EC} = \frac{BC}{CP}$.

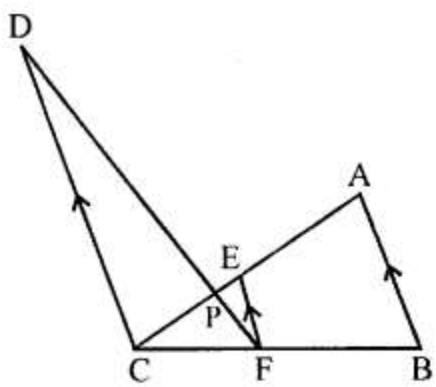
Question 10.

In the figure given below, AB//EF// CD. If AB = 22.5 cm, EP = 7.5 cm, PC = 15 cm and DC = 27 cm.

Calculate:

(i) EF

(ii) AC



Solution:

(i)

In ΔPCD and ΔPEF,

$\angle CPD = \angle EPF$ ....(vertically opposite angles)

$\angle DCE = \angle FEP$ ....(Since DC||EF.)

$\Delta PCD \sim \Delta PEF$ ....(AA criterion for Similarity)

$\Rightarrow \frac{27}{EF} = \frac{15}{7.5}$

$\Rightarrow EF = 13.5$ cm

(ii)

In ΔPCD and ΔPEF,

$\angle CPD = \angle EPF$ ....(vertically opposite angles)

$\angle DCE = \angle FEP$ ....(Since DC||EF.)

$\Delta PCD \sim \Delta PEF$ ....(AA criterion for Similarity)

$\Rightarrow \frac{27}{EF} = \frac{15}{7.5}$

$\Rightarrow EF = 13.5$ cm

Since EF||AB, $\Delta CEF \sim \Delta CAB$.

$\Rightarrow \frac{EC}{AC} = \frac{EF}{AB}$

$\Rightarrow \frac{22.5}{AC} = \frac{13.5}{22.5}$

$\Rightarrow AC = 37.5$ cm

Question 11.

In ΔABC, ∠ABC = ∠DAC. AB = 8 cm, AC = 4 cm, AD = 5 cm.

(i) Prove that ΔACD is similar to ΔBCA.
(ii) Find BC and CD.
(iii) Find area of ΔACD: area of ΔABC. (2014)

Solution:

(i)

In ΔACD and ΔBCA,

$\angle DAC = \angle ABC$ ....(given)

$\angle ACD = \angle BCA$ ....(common angles)

$\Delta ACD \sim \Delta BCA$ ....(AA criterion for Similarity)

(ii)

In ΔACD and ΔBCA,

$\angle DAC = \angle ABC$ ....(given)

$\angle ACD = \angle BCA$ ....(common angles)

$\Delta ACD \sim \Delta BCA$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{AB}$$

$$\Rightarrow \frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow \frac{4}{BC} = \frac{5}{8}$$

$$\Rightarrow BC = \frac{32}{5} = 6.4 \text{ cm}$$

$$\Rightarrow \frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow CD = \frac{20}{8} = 2.5 \text{ cm}$$

(iii)

In ΔACD and ΔBCA,

$\angle DAC = \angle ABC$ ....(given)

$\angle ACD = \angle BCA$ ....(common angles)

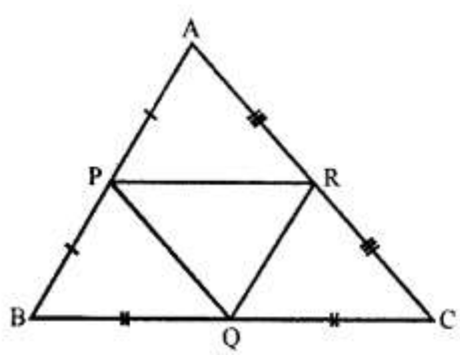
$\Delta ACD \sim \Delta BCA$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta ABC)} = \frac{5^2}{8^2} = \frac{25}{64}$$

Question 12.

In the given triangle P, Q and R are the midpoints of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.



Solution:

In Δ ABC, PR || BC. By Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AR}{RC}$$

Also, in Δ PAR and Δ ABC,

$\angle PAR = \angle BAC$ (Common)

$\angle APR = \angle ABC$ (Corresponding angles)

$\Delta PAR \sim \Delta BAC$ (AA similarity)

$$\frac{PR}{BC} = \frac{AP}{AB}$$

$$\frac{PR}{BC} = \frac{1}{2} \quad \text{(As P is the mid-point of AB)}$$

$$PR = \frac{1}{2}BC$$

Similarly, $PQ = \frac{1}{2}AC$

$$RQ = \frac{1}{2}AB$$

Thus, $\frac{PR}{BC} = \frac{PQ}{AC} = \frac{RQ}{AB}$

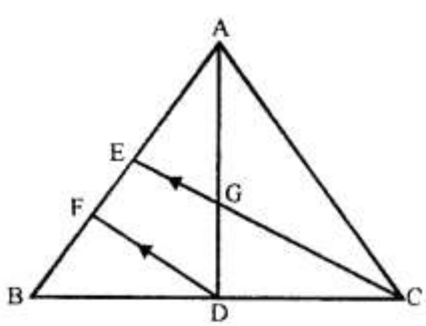
$\Rightarrow \Delta QRP \sim \Delta ABC$ (SSS similarity)

Question 13.

In the following figure, AD and CE are medians of Δ ABC. DF is drawn parallel to CE. Prove that:

(i) EF = FB;

(ii) AG : GD = 2 : 1



Solution:

(i)

In ΔBFD and ΔBEC,

$\angle BFD = \angle BEC$ (Corresponding angles)

$\angle FBD = \angle EBC$ (Common)

$\Delta BFD \sim \Delta BEC$ (AA similarity)

$\therefore \frac{BF}{BE} = \frac{BD}{BC}$

$\frac{BF}{BE} = \frac{1}{2}$ (As D is the mid-point of BC)

$BE = 2BF$

$BF = FE = 2BF$

Hence, $EF = FB$

(ii) In ΔAFD, $EG \parallel FD$. Using Basic Proportionality theorem,

$\frac{AE}{EF} = \frac{AG}{GD}$ ... (1)

Now, AE = EB (as E is the mid-point of AB)

AE = 2EF (Since, EF = FB, by (i))

From (1),

$\frac{AG}{GD} = \frac{2}{1}$

Hence, AG: GD = 2: 1.

Question 14.

The two similar triangles are equal in area. Prove that the triangles are congruent.

Solution:

Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$

Now $\frac{area(\Delta ABC)}{area(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Since area (ΔABC) = area (ΔPQR)

Therefore AB = PQ

BC = QR

AC = PR

So, respective sides of two similiar triangles are also of same length

So, $\Delta ABC \cong \Delta PQR$ (by SSS rule)

Question 15.
The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their:

(i) medians
(ii) perimeters
(iii) areas

Solution:

The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
(i) The ratio between the medians of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 3: 5
(ii) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 3: 5
(iii) The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.
∴ Required ratio = (3)2 : (5)2 = 9: 25

Question 16.
The ratio between the areas of two similar triangles is 16 : 25. Find the ratio between their:

(i) perimeters
(ii) altitudes
(iii) medians.

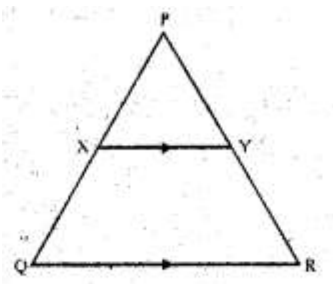
Solution:

The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.
So, the ratio between the sides of the two triangles = 4: 5
(i) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4: 5
(ii) The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4: 5
(iii) The ratio between the medians of two similar triangles is same as the ratio between their sides.
∴ Required ratio = 4: 5

Question 17.
The following figure shows a triangle PQR in which XY is parallel to QR. If PX: XQ = 1:3 and QR = 9 cm, find the length of XY.

Further, if the area of Δ PXY = x cm^2; find in terms of x, the area of :

(i) triangle PQR.
(ii) trapezium XQRY.



Solution:

In Δ PXY and Δ PQR, XY is parallel to QR, so corresponding angles are equal.

$\angle PXY = \angle PQR$

$\angle PYX = \angle PRQ$

Hence, $\Delta PXY \sim \Delta PQR$ (By AA similarity criterion)

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{QR} \quad (PX : XQ = 1:3 \Rightarrow PX : PQ = 1:4)$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{9}$$

$$\Rightarrow XY = 2.25 \text{ cm}$$

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{Ar(\Delta PXY)}{Ar(\Delta PQR)} = \left(\frac{PX}{PQ}\right)^2$$

$$\frac{x}{Ar(\Delta PQR)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$Ar(\Delta PQR) = 16x \text{ cm}^2$$

(ii) Ar (trapezium XQRY) = Ar (Δ PQR) - Ar (Δ PXY)

$= (16x - x) \text{ cm}^2$

$= 15x \text{ cm}^2$

Question 18.

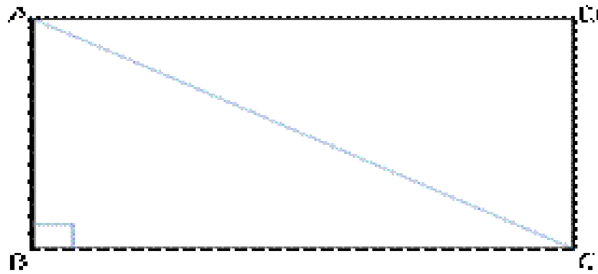
On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 24 cm, and BC = 32 cm. Calculate :

(i) The diagonal distance of the plot in kilometre
(ii) The area of the plot in sq. km.

Solution:

Scale :- 1 : 20000

1 cm represents 20000 cm= $\frac{20000}{1000\times 100}$ = 0.2 km



(i)

$AC^2 = AB^2 + BC^2$

$= 24^2 + 32^2$

= 576 + 1024 = 1600

AC = 40 cm

Actual length of diagonal = 40 × 0.2 km = 8 km

(ii)

1 cm represents 0.2 km

1 cm2 represents 0.2 × 0.2 km^2

The area of the rectangle ABCD = AB × BC

= 24 × 32 = 768 cm^2

Actual area of the plot = 0.2 × 0.2 × 768 km^2 = 30.72 km2

Question 19.

The dimensions of the model of a multi-storeyed building are lm by 60 cm by 1.20 m. If the scale factor is 1 : 50,. find the actual dimensions of the building. Also, find :

(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq cm.

(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is $90m^3$.

Solution:

The dimensions of the building are calculated as below.
Length = 1 × 50 m = 50 m
Breadth = 0.60 × 50 m = 30 m
Height = 1.20 × 50 m = 60 m
Thus, the actual dimensions of the building are 50 m × 30 m × 60 m.
(i)

Floor area of the room of the building $= 50 \times \left(\frac{50}{1}\right)^2 = 125000\ \text{cm}^2 = \frac{125000}{100 \times 100} = 12.5\ \text{m}^2$

(ii)
Volume of the model of the building

$$= 90 \times \left(\frac{1}{50}\right)^3 = 90 \times \left(\frac{1}{50}\right) \times \left(\frac{1}{50}\right) \times \left(\frac{1}{50}\right) = 90 \times \left(\frac{100 \times 100 \times 100}{50 \times 50 \times 50}\right)\ \text{cm}^3$$

$$= 720\ \text{cm}^3$$

Question 20.

In ΔABC, ∠ACB = 90° and CD ⊥ AB. Prove that : $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

Solution:

(i)
In ΔPQL and ΔRMP
$\angle LPQ = \angle QRP$ (Given)
$\angle RQP = \angle RPM$ (Given)
$\Delta PQL \sim \Delta RMP$ (AA similarity)
(ii)
As $\Delta PQL \sim \Delta RMP$ (Proved above)

$$\frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{RM}$$

$$\Rightarrow QL \times RM = PL \times PM$$

(iii)
$\angle LPQ = \angle QRP$ (Given)
$\angle Q = \angle Q$ (Common)
$\Delta PQL \sim \Delta RQP$ (AA similarity)

$$\Rightarrow \frac{PQ}{RQ} = \frac{QL}{QP} = \frac{PL}{PR}$$

$$\Rightarrow PQ^2 = QR \times QL$$

Question 21.
A triangle ABC with AB = 3 cm, BC = 6 cm and AC = 4 cm is enlarged to ΔDEF such that the longest side of ΔDEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of ΔDEF.

Solution:

Triangle ABC is enlarged to DEF. So, the two triangles will be similar.

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Longest side in $\Delta ABC = BC = 6$ cm

Corresponding longest side in $\Delta DEF = EF = 9$ cm

Scale factor $= \frac{EF}{BC} = \frac{9}{6} = \frac{3}{2} = 1.5$

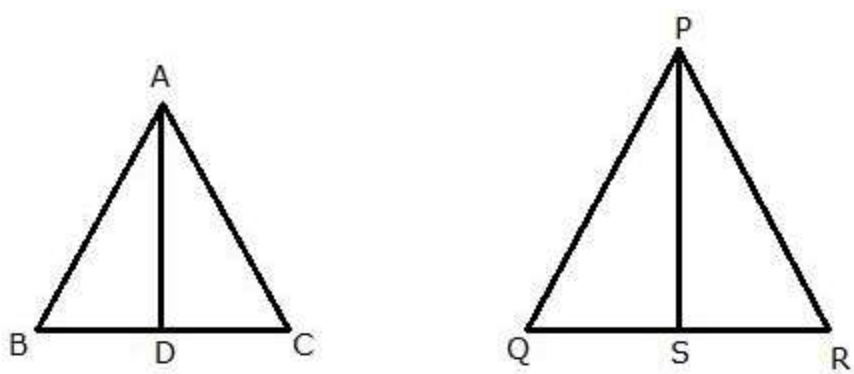
$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$

$DE = \frac{3}{2}AB = \frac{9}{2} = 4.5$ cm

$DF = \frac{3}{2}AC = \frac{12}{2} = 6$ cm

Question 22.
Two isosceles triangles have equal vertical angles. Show that the triangles are similar. If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

Solution:



Let ABC and PQR be two isosceles triangles.

Then, $\frac{AB}{AC} = \frac{1}{1}$ and $\frac{PQ}{PR} = \frac{1}{1}$

Also, $\angle A = \angle P$ (Given)

$\therefore \Delta ABC \sim \Delta PQR$ (SAS similarity)

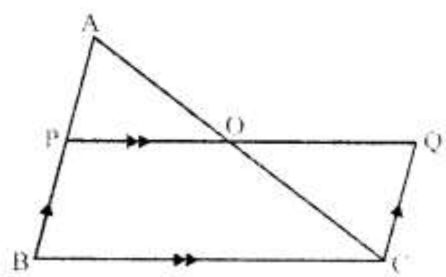
Let AD and PS be the altitude in the respective triangles.
We know that the ratio of areas of two similar triangles is equal to the square of their corresponding altitudes.

$$\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

$$\frac{16}{25} = \left(\frac{AD}{PS}\right)^2$$

$$\frac{AD}{PS} = \frac{4}{5}$$

Question 23.
In ΔABC, AP: PB = 2 :3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA.



Find:
(i) area ΔAPO: area ΔABC.
(ii) area ΔAPO: area ΔCQO.

Solution:

In triangle ABC, PO || BC. Using Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AO}{OC}$$

$$\Rightarrow \frac{AO}{OC} = \frac{2}{3} \quad \ldots (1)$$

(i)

$\angle PAO = \angle BAC$ (Common)

$\angle APO = \angle ABC$ (Corresponding angles)

$\Delta APO \sim \Delta ABC$ (AA similarity)

$$\therefore \frac{Ar(\Delta APO)}{Ar(\Delta ABC)} = \left(\frac{AO}{AC}\right)^2 = \left(\frac{2}{2+3}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

(ii)

$\angle POA = \angle COQ$ (Vertically opposite angles)

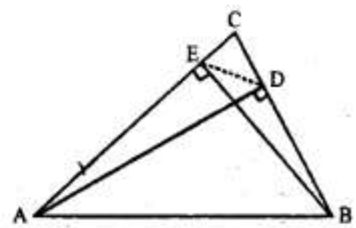
$\angle PAO = \angle QCO$ (Alternate angles)

$\Delta AOP \sim \Delta COQ$ (AA similarity)

$$\therefore \frac{Ar(\Delta AOP)}{Ar(\Delta COQ)} = \left(\frac{AO}{CO}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Question 24.

The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.



Show that:

(i) ΔADC – ΔBEC

(ii) CA × CE = CB × CD

(iii) Δ ABC – ΔDEC

(iv) CD × AB = CA × DE

Solution:

(i) $\angle ADC = \angle BEC = 90^o$

$\angle ACD = \angle BCE$ (Common)

$\Delta ADC \sim \Delta BEC$ (AA similarity)

(ii) From part (i),

$$\frac{AC}{BC} = \frac{CD}{EC} \quad \dots (1)$$

$\Rightarrow CA \times CE = CB \times CD$

(iii) In ΔABC and ΔDEC,

From (1),

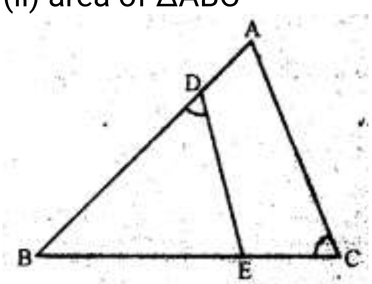
$$\frac{AC}{BC} = \frac{CD}{EC} \Rightarrow \frac{AC}{CD} = \frac{BC}{EC}$$

$\angle DCE = \angle BCA$ (Common)
$\Delta ABC \sim \Delta DEC$ (SAS similarity)
(iv) From part (iii),
$$\frac{AC}{DC} = \frac{AB}{DE}$$
$\Rightarrow CD \times AB = CA \times DE$

Question 25.
In the given figure, ABC is a triangle-with ∠EDB = ∠ACB.
Prove that ΔABC ~ ΔEBD.
If BE=6 cm, EC = 4 cm,

BD = 5 cm and area of
ΔBED = 9 cm^2. Calculate the

(i) length of AB
(ii) area of ΔABC



Solution:

In ΔABC and ΔEBD,
$\angle ACB = \angle EDB$ (given)
$\angle ABC = \angle EBD$ (common)
$\Delta ABC \sim \Delta EBD$ (by AA- similarity)

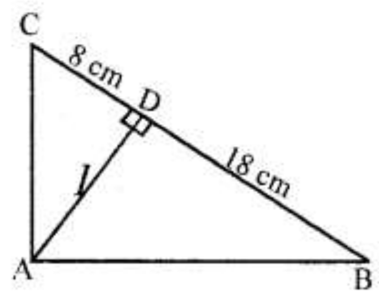
(i) We have, $\frac{AB}{BE} = \frac{BC}{BD} \Rightarrow AB = \frac{6 \times 10}{5} = 12$ cm

(ii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$\Rightarrow \text{Area of } \Delta ABC = \left(\frac{12}{6}\right)^2 \times 9 \text{ cm}^2$

$= 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2$

Question 26.
In the given figure, ABC is a right-angled triangle with ZBAC = 90°.
(i) Prove ΔADB ~ ΔCDA.
(ii) If BD = 18 cm, CD = 8 cm, find AD.
(iii) Find the ratio of the area of ΔADB is to area of ΔCDA.



Solution:

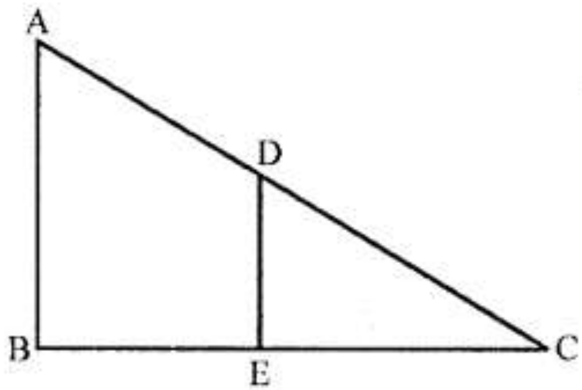
(i) Let $\angle CAD = x$
$\Rightarrow m\angle DAB = 90^\circ - x$
$\Rightarrow m\angle DBA = 180^\circ - (90^\circ + 90^\circ - x) = x$
$\Rightarrow \angle CAD = \angle DBA$ ....(1)
In $\triangle ADB$ and $\triangle CDA$,
$\angle ADB = \angle CDA$ ....[Each 90°]
$\angle ABD = \angle CAD$ ....[FRom (1)]
$\therefore \triangle ADB \sim \triangle CDA$ ....[By A.A.]

(ii) Since the corresponding sides of similar triangles are proportional, we have
$$\frac{BD}{AD} = \frac{AD}{CD}$$
$$\Rightarrow \frac{18}{AD} = \frac{AD}{8}$$
$$\Rightarrow AD^2 = 18 \times 8 = 144$$
$$\Rightarrow AD = 12\ cm$$

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
$$\Rightarrow \frac{Ar(\triangle ADB)}{Ar(\triangle CDA)} = \frac{AD^2}{CD^2} = \frac{12^2}{8^2} = \frac{144}{64} = \frac{9}{4} = 9:4$$

Question 27.
In the given figure, AB and DE are perpendicular to BC.

(i) Prove that ΔABC ~ ΔDEC
(ii) If AB = 6 cm: DE = 4 cm and AC = 15 cm. Calculate CD.
(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Solution:

(i)
In ΔABC and ΔDEC,
∠ABC = ∠DEC ....(both are right angles)
∠ACB = ∠DCE ....(common angles)
ΔABC ~ ΔDEC ....(AA criterion for Similarity)

(ii)
In ΔABC and ΔDEC,
∠ABC = ∠DEC ....(both are right angles)
∠ACB = ∠DCE ....(common angles)
ΔABC ~ ΔDEC ....(AA criterion for Similarity)

$\Rightarrow \frac{AB}{DE} = \frac{AC}{CD}$

$\Rightarrow \frac{6}{4} = \frac{15}{CD}$

$\Rightarrow CD = 10 \text{ cm}$

(iii)

In $\triangle ABC$ and $\triangle DEC$,

$\angle ABC = \angle DEC$.....(both are right angles)

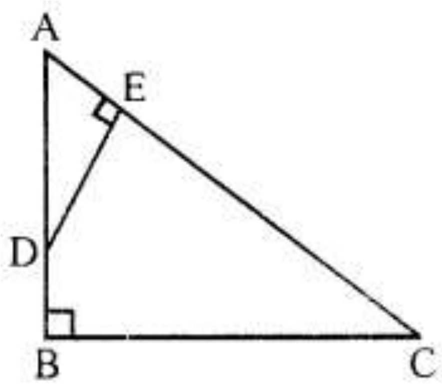
$\angle ACB = \angle DCE$.....(common angles)

$\triangle ACB \sim \triangle DEC$.....(AA criterion for Similarity)

$$\frac{ar(\triangle ABC)}{ar(\triangle DEC)} = \frac{AB^2}{DE^2} = \frac{6^2}{4^2} = \frac{36}{16}$$

$$\Rightarrow \frac{ar(\triangle ABC)}{ar(\triangle DEC)} = \frac{9}{4}$$

Question 28.
ABC is a right angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC. Prove that:



(i) ΔADE ~ ΔACB.
(ii) If AC = 13 cm, BC = 5 cm and AE=4 cm. Find DE and AD.
(iii) Find, area of ΔADE : area of quadrilateral BCED. (2015)

Solution:

(i)

In ΔADE and ΔACB,

$\angle AED = \angle ABC$ ....(both are right angles)

$\angle DAE = \angle CAB$ ....(common angles)

$\Delta ADE \sim \Delta ACB$ ....(AA criterion for Similarity)

(ii)

In ΔADE and ΔACB,

$\angle AED = \angle ABC$ ....(both are right angles)

$\angle DAE = \angle CAB$ ....(common angles)

$\Delta ADE \sim \Delta ACB$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{AE}{AB} = \frac{DE}{BC} = \frac{AD}{AC}$$

$$\Rightarrow \frac{4}{AB} = \frac{DE}{5} = \frac{AD}{13} \quad \text{....(i)}$$

In right ΔABC,

$\Rightarrow AB^2 + BC^2 = AC^2$

$\Rightarrow AB^2 + 5^2 = 13^2$

$\Rightarrow AB^2 = 144$

$\Rightarrow AB = 12$ cm

From (i), we get

$$\frac{4}{12} = \frac{DE}{5} = \frac{AD}{13}$$

So, $DE = \frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$ cm

$$\frac{AD}{13} = \frac{4}{12} \Rightarrow AD = \frac{52}{12} = 4\frac{1}{3} \text{ cm}$$

(iii)

We need to find the area of ΔADE and quadrilateral BCED.

Area of $\Delta ADE = \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 4 \times \frac{5}{3} = \frac{10}{3}$ cm^2

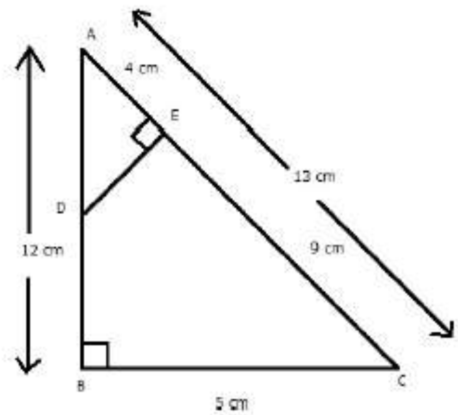
Area of quad.BCED = Area of ΔABC – Area of ΔADE

$$= \frac{1}{2} \times BC \times AB - \frac{10}{3}$$

$$= \frac{1}{2} \times 5 \times 12 - \frac{10}{3}$$

$$= 30 - \frac{10}{3}$$

$$= \frac{80}{3} \text{ cm}^2$$

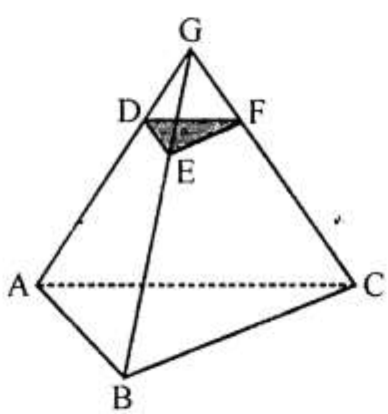


Thus ratio of areas of ΔADE to quadrilateral BCED$=\dfrac{\frac{10}{3}}{\frac{80}{3}} = \frac{1}{8}$

Question 29.

Given: AB // DE and BC // EF. Prove that:

(i) $\frac{AD}{DG} = \frac{CF}{FG}$
(ii) ΔDFG ~ ΔACG.



Solution:

(i) In Δ AGB, DE || AB , by Basic proportionality theorem,

$$\frac{GD}{DA} = \frac{GE}{EB} \text{ .... (1)}$$

In Δ GBC, EF || BC, by Basic proportionality theorem,

$$\frac{GE}{EB} = \frac{GF}{FC} \text{ .... (2)}$$

From (1) and (2), we get,

$$\frac{GD}{DA} = \frac{GF}{FC}$$

$$\frac{AD}{DG} = \frac{CF}{FG}$$

(ii)

From (i), we have:

$$\frac{AD}{DG} = \frac{CF}{FG}$$

$$\angle DGF = \angle AGC \quad \text{(Common)}$$

$$\therefore \Delta DFG \sim \Delta ACG \quad \text{(SAS similarity)}$$

Question 30.

i.

In ΔPQR and ΔSPR,

$\angle PSR = \angle QPR$ ... given

$\angle PRQ = \angle PRS$ ... common angle

$\Rightarrow \Delta PQR \sim \Delta SPR$ (AA Test)

ii. Find the lengths of QR and PS.

Since ΔPQR ~ ΔSPR ... from (i)

$$\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR} \text{ .... (a)}$$

$$\frac{QR}{PR} = \frac{PR}{SR} \quad \text{... from (a)}$$

$$\Rightarrow \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow QR = \frac{6 \times 6}{3} = 12cm$$

$$\frac{PQ}{SP} = \frac{PR}{SR} \quad \text{... from (a)}$$

$$\Rightarrow \frac{8}{SP} = \frac{6}{3}$$

$$\Rightarrow SP = \frac{8 \times 3}{6} = 4cm$$

iii.

$$\frac{\text{area of } \Delta PQR}{\text{area of } \Delta SPR} = \frac{PQ^2}{SP^2} = \frac{8^2}{4^2} = \frac{64}{16} = 4$$