

Class-X Session 2022-23
Subject - Mathematics (Standard)
Sample Question Paper - 40
With Solution

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			Mcq	A/R					
1	Real Number	6	1(Q1, 3)		1(Q21, 24)				6
2	Polynomials	20	1(Q4)		1(Q23)	1(Q28)			6
3	Pair of Linear Equations in Two Variables					1(Q26)		1(Q36)	7
4	Quadratic Equations								0
5	Arithmetic Progression				1(Q27)		1(Q38)		7
6	Triangles	15	4(Q2, 3, 7, 9)				1(Q32)		9
7	Circles			1(Q19)		1(Q30)			6
8	Coordinate Geometry	6	4(Q12, 14, 15, 17)		1(Q22)				6
9	Introduction to Trigonometry	12					1(Q33)	1(Q37)	9
10	Some Applications of Trigonometry					1(Q29)			3
11	Areas Related to Circles	10	1(Q10)			1(Q31)			4
12	Surface Areas and Volumes			1(Q13)				1(Q34)	6
13	Statistics	11	1(Q11)				1(Q35)		6
14	Probability			2(Q16, 18)	1(Q20)	1(Q25)			5
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

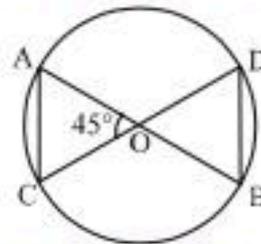
General Instructions

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 case based/integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.

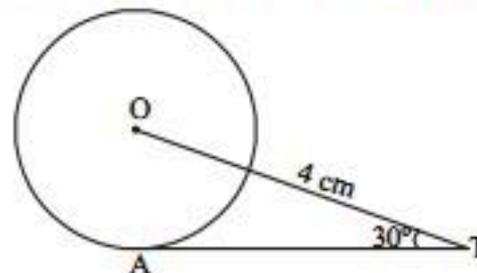
SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. If $\text{LCM}(x, 18) = 36$ and $\text{HCF}(x, 18) = 2$ then x is
 (a) 2 (b) 3 (c) 4 (d) 5
2. $\triangle ABC \sim \triangle PQR$. If AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively and $AB^2 : PQ^2 = 4 : 9$, then $AM : PN =$
 (a) 16 : 81 (b) 4 : 9 (c) 3 : 2 (d) 2 : 3
3. The ratio of LCM and HCF of the least composite and the least prime numbers is
 (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
4. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are
 (a) $m, m + 3$ (b) $-m, m + 3$ (c) $m, -(m + 3)$ (d) $-m, -(m + 3)$
5. In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are
 (a) congruent but not similar (b) similar but not congruent
 (c) neither congruent nor similar (d) congruent as well as similar
6. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A, is
 (a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm
7. If in fig. O is the point of intersection of two chords AB and CD such that $OB = OD$, then triangles OAC and ODB are
 (a) equilateral but not similar
 (b) isosceles but not similar
 (c) equilateral and similar
 (d) isosceles and similar



8. In figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then, AT is equal to



9. If $\triangle ABC \sim \triangle DFE$, $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, which of the following is true?
 (a) $DE = 12$ cm, $\angle F = 50^\circ$ (b) $DE = 12$ cm, $\angle F = 100^\circ$ (c) $EF = 12$ cm, $\angle D = 100^\circ$ (d) $EF = 12$ cm, $\angle D = 30^\circ$

10. Given below is the picture of the Olympic rings made by taking five congruent circles of radius 1 cm each, intersecting in such a way that the chord formed by joining the point of intersection of two circles is also of length 1 cm. Total area of all the dotted regions assuming the thickness of the rings to be negligible is



- (a) $4\left(\frac{\pi}{12} - \frac{\sqrt{3}}{4}\right) \text{ cm}^2$ (b) $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{ cm}^2$ (c) $4\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{ cm}^2$ (d) $8\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{ cm}^2$

11. The mean of the following frequency table is 50.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Which of the following is correct?

Consider the following.

- (I) $f_1 - f_2 = 4$ (II) $f_1 + f_2 = 52$ (III) $f_1 = 24$ (IV) $f_2 = 28$
 (a) I, II and III (b) II and IV (c) I and II (d) I and III

12. In what ratio is the line segment joining the points (3, 5) & (-4, 2) divided by y-axis?

- (a) 3 : 2 (b) 3 : 4 (c) 2 : 3 (d) 4 : 3

13. A copper wire when bent in the form of an equilateral triangle has area $121\sqrt{3} \text{ cm}^2$. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

- (a) 345.5 cm^2 (b) 346.5 cm^2 (c) 342.5 cm^2 (d) 340.25 cm^2

14. A circle passes through the vertices of a triangle ABC. If the vertices are A(-2, 5), B(-2, -3), C(2, -3), then the centre of the circle is

- (a) (0, 0) (b) (0, 1) (c) (-2, 1) (d) (0, -3)

15. If $\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(-6, 5) and B(-2, 3), then what is the value of 'a'?

- (a) -4 (b) -12 (c) 12 (d) -6

16. An unbiased die is rolled twice. Find the probability of getting the sum of two numbers as a prime

- (a) $\frac{3}{5}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{4}{5}$

17. If the mid point of the line joining (3, 4) and (k, 7) is (x, y) and $2x + 2y + 1 = 0$. Find the value of k.

- (a) 10 (b) -15 (c) 15 (d) -10

18. Two fair dice are thrown. Find the probability that both dice show different numbers.

- (a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{32}{36}$ (d) $\frac{29}{36}$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. Assertion: If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason: $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

20. **Assertion :** If a die is thrown, the probability of getting a number less than 3 and greater than 2 is zero.

Reason : Probability of an impossible event is zero.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the LCM of 66 & 486 by the Prime factorisation method. Hence find their HCF.
22. If the point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), find p. Also find the length of AB.

OR

Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x-axis. Also find the coordinates of the point of division.

23. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$, then find the value of a .
24. When 2^{256} is divided by 17 then find the remainder.
25. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

OR

A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble and a green marble at random from the jar is $\frac{1}{3}$ and $\frac{4}{9}$ respectively. How many white marbles does the jar contain?

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Find the value of a so that the point (3, a) lies on the line represented by $2x - 3y = 5$.

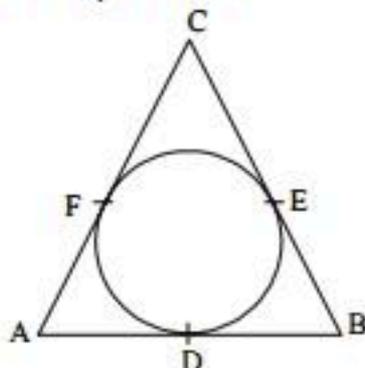
OR

Solve the following pair of linear equations by substitution method:

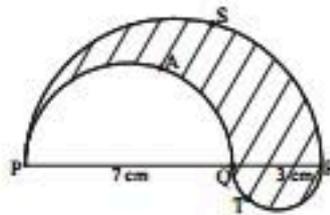
$$3x + 2y - 7 = 0$$

$$4x + y - 6 = 0$$

27. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP.
28. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.
29. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45° . If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$].
30. In fig., a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF.

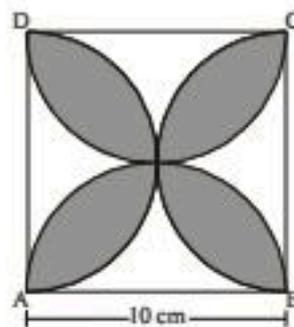


31. In fig., PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. [Use $\pi = 3.14$]



OR

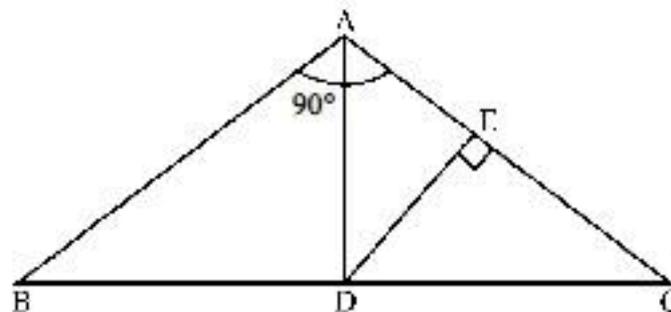
In figure, $ABCD$ is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. Find area of the shaded region.



SECTION-D

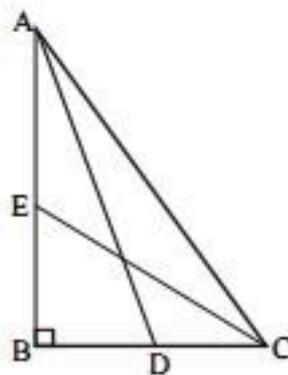
This section comprises of long answer-type questions (LA) of 5 marks each.

32. In fig. $\angle BAC = 90^\circ$, AD is its bisector. If $DE \perp AC$, prove that $DE \times (AB + AC) = AB \times AC$.



OR

1. In the figure, ABC is a right triangle, right angled at B . AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE .



33. If $\sin \theta = \frac{c}{\sqrt{c^2 + d^2}}$ and $d > 0$, find the value of $\cos \theta$ and $\tan \theta$.
34. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice × cream. The ice × cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
35. 3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in ₹)	Number of families
1000 - 1500	24
1500 - 2000	40
2000 - 2500	33
2500 - 3000	28
3000 - 3500	30
3500 - 4000	22
4000 - 4500	16
4500 - 5000	7

OR

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. **Case - Study 1:** Read the following passage and answer the questions given below.

A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while 1/4 mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

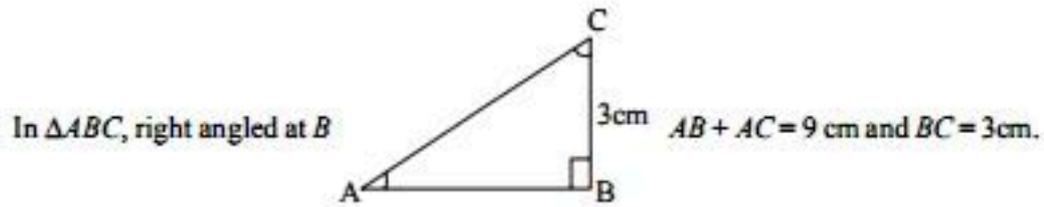
Type of Question	Marks given for correct answer	Marks deducted for wrong answer
True/False	1	0.25

- (i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?
 (ii) How many questions did he guess?
 (iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

37. **Case - Study 2:** Read the following passage and answer the questions given below.



- (i) What is the value of $\cot C$?
- (ii) What is the value of $\sec C$?
- (iii) Find the value of $\sin^2 C + \cos^2 C$.

OR

$$1 + \tan^2 C$$

38. **Case - Study 3:** Read the following passage and answer the questions given below.

Following two given series are in A.P.

2, 4, 6, 8

3, 6, 9, 12

First series contains 30 terms, while the second series contains 20 terms. Both of the above given series contain some terms, which are common to both of them.

- (i) What is the last term of both the above given A.P. ?
- (ii) What is the sum of both the above given A.P. ?
- (iii) Find the no. of terms identical to both the above given A.P.

OR

Find the 15th term from the last term of both the above A.P.

Solution

SAMPLE PAPER-9

1. (c) $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$36 \times 2 = 18 \times x$$

$$72 = 18x$$

$$4 = x$$

$$x = 4$$

2. (d) Ratio of altitudes = Ratio of sides for similar triangles

$$\text{So } AM : PN = AB : PQ = 2 : 3$$

3. (b) Least composite number is 4 and the least prime number is 2. $\text{LCM}(4, 2) : \text{HCF}(4, 2) = 4 : 2 = 2 : 1$

4. (b) $x^2 - (m+3)x + mx - m(m+3) = 0$

$$\Rightarrow x[x - (m+3)] + m[x - (m+3)] = 0$$

$$\Rightarrow (x+m)[x - (m+3)] = 0$$

$$\therefore x+m=0$$

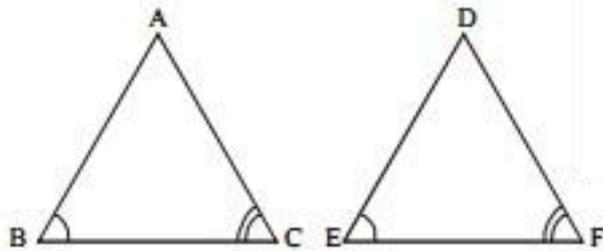
$$x - (m+3) = 0$$

$$x = -m$$

$$x = m+3$$

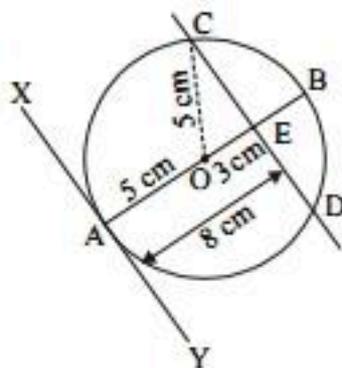
5. (b) In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$

$$\triangle ABC \sim \triangle DEF$$



Also, $\triangle ABC$ and $\triangle DEF$ do not satisfy any rule of congruency, (SAS, ASA, SSS), so both are not congruent.

6. (d)



Now, in figure

$$\angle OAY = 90^\circ$$

[Tangent at any point of a circle is perpendicular to the radius through the point of contact]

$$\angle OAY + \angle OED = 180^\circ$$

$$\Rightarrow \angle OED = 90^\circ$$

Also, $AE = 8 \text{ cm}$.

Join OC

Now, in right angled $\triangle OEC$,

$$OC^2 = OE^2 + EC^2$$

$$\Rightarrow EC^2 = OC^2 - OE^2$$

[by Pythagoras theorem]

$$= 5^2 - 3^2 = 25 - 9 = 16$$

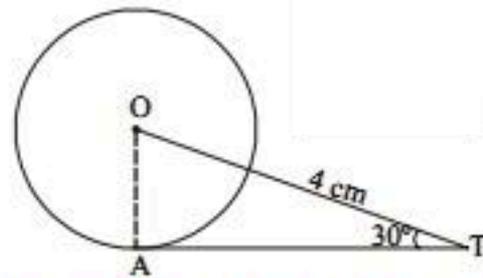
$$\Rightarrow EC = 4 \text{ cm}$$

Hence, length of chord $CD = 2CE$

$$= 2 \times 4 = 8 \text{ cm}$$

7. (d) isosceles and similar

8. (c) First join OA.



Then the tangent at any point of a circle is \perp to the radius through the point of contact.

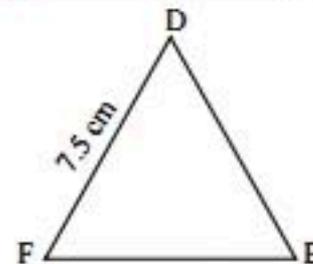
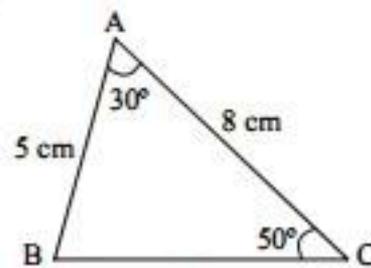
$$\therefore \angle OAT = 90^\circ$$

$$\text{In } \triangle OAT, \cos 30^\circ = \frac{AT}{OT}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4}$$

$$\Rightarrow AT = 2\sqrt{3} \text{ cm}$$

9. (b) Given, $\triangle ABC \sim \triangle DFE$ then $\angle A = \angle D = 30^\circ$, $\angle C = \angle E = 50^\circ$



$$\angle B = \angle F = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$$

Also, $AB = 5 \text{ cm}$, $AC = 8 \text{ cm}$
and $DF = 7.5 \text{ cm}$

$$\text{Therefore } \frac{AB}{DF} = \frac{AC}{DE}$$

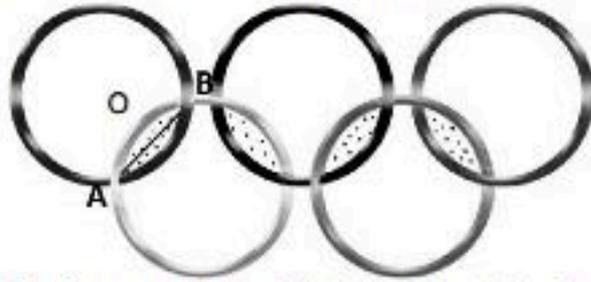
[By property of similar triangles]

$$\Rightarrow \frac{5}{7.5} = \frac{8}{DE}$$

$$DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

So, $DE = 12 \text{ cm}$, $\angle F = 100^\circ$

10. (d)



Let O be the centre of the circle $OA = OB = AB = 1 \text{ cm}$.
So ΔOAB is an equilateral triangle and $\therefore \angle AOB = 60^\circ$

Required area = $8 \times$ Area of one segment with $r = 1 \text{ cm}$, $\theta = 60^\circ$

Area of sector OAB - Area of an equilateral ΔOAB ,

$$= 8 \times \left(\frac{60}{360} \times \pi \times 1^2 - \frac{\sqrt{3}}{4} \times 1^2 \right)$$

$$= 8 \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) \text{ cm}^2$$

11. (c)

Class	f_i	x_i	$f_i x_i$
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i x_i = 3480 + 30f_1 + 70f_2$

We have $\Sigma f_i = 120$

$$\Rightarrow 68 + f_1 + f_2 = 120 \Rightarrow f_1 + f_2 = 52 \dots (i)$$

Now, mean = 50

$$\Rightarrow 50 = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 50 = \frac{3480 + 30f_1 + 70f_2}{120}$$

$$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2 \Rightarrow 30f_1 + 70f_2 = 2520 \dots (ii)$$

Solving (i) and (ii) we get; $f_1 = 28$, $f_2 = 24$

12. (b) Let the required ratio be $K : 1$

\therefore The coordinates of the required point on the y-axis is

$$x = \frac{K(-4) + 3(1)}{K+1}; y = \frac{K(2) + 5(1)}{K+1}$$

Since, it lies on y-axis

\therefore Its x-coordinates = 0

$$\therefore \frac{-4K + 3}{K+1} = 0 \Rightarrow -4K + 3 = 0$$

$$\Rightarrow K = \frac{3}{4}$$

$$\Rightarrow \text{Required ratio} = \frac{3}{4} : 1$$

\therefore ratio = 3 : 4

13. (b) Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 121\sqrt{3}$$

$$\Rightarrow a^2 = 484$$

$$\Rightarrow a = 22 \text{ cm}$$

Perimeter of equilateral $\Delta = 3a$

$$= 3(22)$$

$$= 66 \text{ cm}$$

Since the wire is bent into the form of Q circle, So perimeter

of circle = 66 cm

$$\Rightarrow 2\pi r = 66$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 66 \Rightarrow r = 66 \times \frac{1}{2} \times \frac{7}{22}$$

$$\Rightarrow r = 10.5 \text{ cm}$$

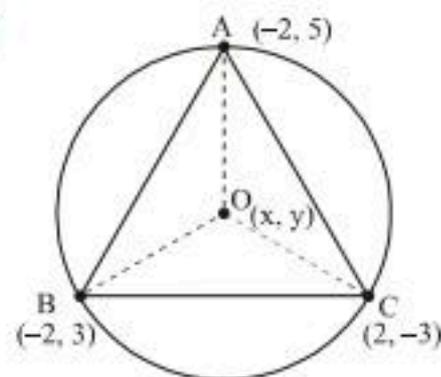
So Area enclosed by circle = πr^2

$$= \frac{22}{7} \times 10.5 \times 10.5$$

$$= 22 \times 1.5 \times 10.5$$

$$= 346.5 \text{ cm}^2$$

14. (b)



Let $O(x, y)$ is the centre of the given circle.

Join OA , OB & OC .

$$\therefore OA = OB = OC$$

$$\therefore OA^2 = OB^2$$

$$\Rightarrow \sqrt{(x+2)^2 + (y-5)^2} = \sqrt{(x+2)^2 + (y+3)^2}$$

$$\Rightarrow x^2 + 4 + 4x + y^2 + 25 - 10y = x^2 + 4 + 4x + y^2 + 9 + 6x$$

$$\Rightarrow 16y = 16 \Rightarrow y = 1$$

$$\text{Again: } OB^2 = OC^2$$

$$\Rightarrow \sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x-2)^2 + (y+3)^2}$$

$$\Rightarrow x^2 + 4 + 4x + (y+3)^2 = x^2 + 4 - 4x + (y+3)^2$$

$$\Rightarrow 8x = 0 \Rightarrow x = 0$$

\therefore centre of the circle is (0, 1).

15. (b) Coordinates of mid-point are given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Here, coordinates of mid-point are $\left(\frac{a}{3}, 4 \right)$

$$\text{So, } \frac{a}{3} = \frac{-6-2}{2}$$

$$\therefore a = -12$$

16. (b) The sum of the two numbers lies between 2 and 12.

So the primes are 2, 3, 5, 7, 11.

No. of ways for getting 2 = (1, 1) = 1

No. of ways of getting 3 = (1, 2), (2, 1) = 2

No. of ways of getting 5 = (1, 4), (4, 1), (2, 3), (3, 2) = 4

No. of ways of getting 7

= (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3) = 6

No. of ways of getting 11 = (5, 6), (6, 5) = 2

No. of favourable ways = 1 + 2 + 4 + 6 + 2 = 15

No. of exhaustive ways = 6 × 6 = 36

Probability of the sum as a prime

$$= \frac{15}{36} = \frac{5}{12}$$

17. (b) Since (x, y) is midpoint of (3, 4) and (k, 7)

$$\therefore x = \frac{3+k}{2} \text{ and } y = \frac{4+7}{2}$$

Also $2x + 2y + 1 = 0$ putting values we get

$$3 + k + 4 + 7 + 1 = 0$$

$$\Rightarrow k + 15 = 0 \Rightarrow k = -15$$

18. (b) $S = \{(1, 1), \dots, (1, 6), (2, 1), \dots, (2, 6), (3, 1), \dots, (3, 6), (4, 1), \dots, (4, 6), (5, 1), \dots, (5, 6), (6, 1), \dots, (6, 6)\}$
 $n(S) = 36$

Let E be the event that both dice show different numbers.

$E = \{(1, 2), (1, 3), \dots, (1, 6), (2, 1), (2, 3), (2, 4), \dots, (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)\}$

$$n(E) = 30$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

19. (a) $(OA)^2 = (AB)^2 + (OB)^2$

$$AB = \sqrt{25-9} = 4 \text{ cm.}$$

Both Assertion and Reason are correct.

Also, Reason is the correct explanation of the Assertion.

20. (a) Both statements are correct. Event given in Assertion is an impossible event.

21. The Prime factorisation of 66 & 486 gives

$$66 = 2 \times 3 \times 11$$

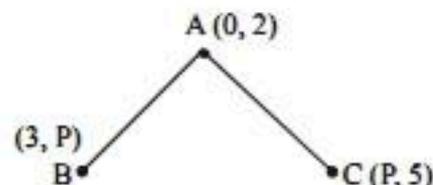
$$486 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^5 \quad [\frac{1}{2} \text{ Mark}]$$

\therefore The LCM of these two integer is : $2 \times 3^5 \times 11 = 5346$
 [1/2 Mark]

$$\text{HCF}(66, 486) = \frac{66 \times 486}{\text{LCM}(66, 486)} = \frac{66 \times 486}{5346} = 6$$

[1 Mark]

- 22.



Given that : $AB = AC$

By distance formula :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad [1 \text{ Mark}]$$

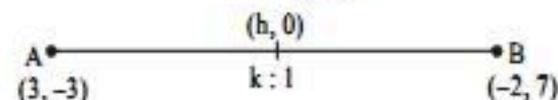
$$\sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$$

Squaring both sides

$$9 + p^2 + 4 - 4p = p^2 + 9$$

$$\Rightarrow 4 - 4p = 0 \Rightarrow p = 1 \quad [1 \text{ Mark}]$$

OR



Let $(h, 0)$ be point on x-axis which divides AB in $k : 1$.
By section formula :

$$h = \frac{-2k+3}{k+1}, 0 = \frac{7k-3}{k+1} \Rightarrow k = \frac{3}{7} \quad [1 \text{ Mark}]$$

$$\text{Now, } h = \frac{-2\left(\frac{3}{7}\right)+3}{\frac{3}{7}+1} = \frac{-6+21}{10} = \frac{3}{2}$$

point on x-axis = $\left(\frac{3}{2}, 0\right)$ and ratio is $3 : 7$. [1 Mark]

23. Given, α and $\frac{2}{\alpha}$ are the zeroes of $x^2 + 4x + 2a$.

Now

$$\text{Product of the zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} \quad [1 \text{ Mark}]$$

$$\alpha \times \frac{2}{\alpha} = \frac{2a}{1}$$

$$2 = 2a$$

$$\therefore a = 1 \quad [1 \text{ Mark}]$$

24. When 2^{256} is divided by 17 then, $\frac{2^{256}}{17} = \frac{2^{256}}{2^4+1} = \frac{(2^4)^{64}}{(2^4+1)}$

[1/2 Mark]

By remainder theorem when $f(x)$ is divided by $x+a$ the remainder = $f(-a)$ [1/2 Mark]

Here $f(x) = (2^4)^{64}$ and $x = 2^4$ and $a = 1$ [1/2 Mark]

$$\therefore \text{Remainder} = f(-1) = (-1)^{64} = 1 \quad [1/2 \text{ Mark}]$$

25. $P(\text{winning}) = 0.08$

Total tickets sold = 6000

Let the number of tickets she bought be x , then probability

$$\text{of winning} = \frac{x}{6000} \quad [1 \text{ Mark}]$$

$$\Rightarrow \frac{x}{6000} = 0.08 \Rightarrow x = 6000 \times 0.08 \Rightarrow x = 480 \quad [1 \text{ Mark}]$$

Thus the girl bought 480 tickets.

OR

Let there be x blue, y green and z white marbles in the jar.

$$\text{Then, } x + y + z = 54 \quad \dots(i) \quad [1/2 \text{ Mark}]$$

$$\therefore P(\text{selecting a blue marble}) = \frac{x}{54} \Rightarrow \frac{x}{54} = \frac{1}{3} \Rightarrow x = 18$$

[1/2 Mark]

$$\text{Similarly, } P(\text{selecting a green marble}) = \frac{y}{54}$$

$$\Rightarrow \frac{y}{54} = \frac{4}{9} \Rightarrow y = 24 \quad [1/2 \text{ Mark}]$$

Substituting the values of x and y in (i), $z = 12$

Hence, the jar contains 12 white marbles. [1/2 Mark]

26. Since, $(3, a)$ lies on the line $2x - 3y = 5$

So, $2 \times 3 - 3a = 5$ [1 Mark]

$$\Rightarrow 6 - 3a = 5$$

$$= 6 - 5 = 3a$$

$$\Rightarrow a = \frac{1}{3}$$

[1 Mark]

OR

$$3x + 2y - 7 = 0 \quad \dots(i)$$

$$4x + y - 6 = 0 \quad \dots(ii)$$

From (ii), $y = 6 - 4x$ [1/2 Mark]

Value of y put in eqn. (i)

$$3x + 2y - 7 = 0$$

$$\Rightarrow 3x + 2(6 - 4x) - 7 = 0$$

$$\Rightarrow 3x + 12 - 8x - 7 = 0$$

$$\Rightarrow 5x = 5$$

$$\therefore x = 1$$

[1 Mark]

Substitute the value of x in eq(ii) to get value of y ,

$$4x + y - 6 = 0$$

[1/2 Mark]

$$\Rightarrow 4(1) + y - 6 = 0$$

$$\Rightarrow 4 + y - 6 = 0$$

$$\Rightarrow y - 2 = 0$$

$$\therefore y = 2$$

Hence, values of x and y are 1 and 2. [1 Mark]

27. Given : $a_5 + a_9 = 30$

$$a_{25} = 3a_8$$

$$\text{Now, } a + 4d + a + 8d = 30 \Rightarrow 2a + 12d = 30$$

$$\Rightarrow a + 6d = 15 \quad \dots(i)$$

[1/2 Mark]

$$\text{and, } a + 24d = 3a + 21d \Rightarrow 2a - 3d = 0 \quad \dots(ii)$$

[1/2 Mark]

From eqs. (i) and (ii)

$$2a + 12d = 30$$

$$-2a - 3d = 0$$

$$\hline 15d = 30 \Rightarrow d = 2$$

[1/2 Mark]

$$\text{Now, put } d = 2 \text{ in eq. (i) : } a + 12 = 15 \Rightarrow a = 3$$

Required A.P. = 3, 5, 7,

[1/2 Mark]

$$\therefore 5 = 210 \times 5 + 55y \Rightarrow y = \frac{-1045}{55} = (-19) \quad [1 \text{ Mark}]$$

28. Let $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

$$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$

[1 Mark]

$$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$$

[1 Mark]

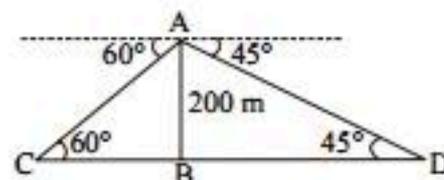
$$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$$

Put $P(x) = 0$

$$\therefore \text{Zeroes are, } x = \frac{2}{\sqrt{3}}, 2\sqrt{3}$$

[1 Mark]

- 29.



[1 Mark]

Let AB is the height of light house = 200m
Two ships are at points C and D on either side of AB (light house)

In $\triangle ABC$,

$$\tan 60^\circ = \frac{AB}{BC} \quad [1 \text{ Mark}]$$

$$\Rightarrow BC = \frac{200\sqrt{3}}{3} = \frac{200 \times 1.73}{3} = 115.33 \text{ m}$$

In $\triangle ABD$,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$\Rightarrow BD = 200$$

$$\begin{aligned} \text{Distance between both ships} &= BC + BD \\ &= 115.33 + 200 \\ &= 315.33 \text{ m} \end{aligned}$$

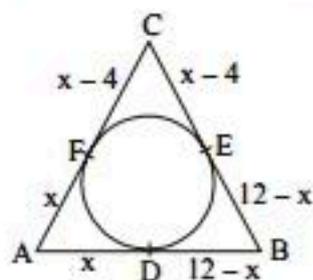
[1 Mark]

30. LET AD = X

$$\therefore AF = AD = x$$

Then,

$$\begin{aligned} DB &= AB - AD \\ &= 12 - x \quad (\because AB = 12 \text{ cm}) \end{aligned}$$



$$\therefore BC = 8 \text{ cm}$$

$$\begin{aligned} \text{So, } CE &= BC - BE \\ &= 8 - (12 - x) \\ &= x - 4 \end{aligned}$$

$$\Rightarrow CF = CE = x - 4 \quad [1 \text{ Mark}]$$

[\because tangents drawn on a circle from an external point are equal in lengths]

$$\text{Now, } AC = 10 \text{ cm}$$

$$AF + CF = 10$$

$$x + x - 4 = 10$$

$$2x - 4 = 10$$

$$2x = 14$$

$$x = 7 \text{ cm}$$

$$\therefore AD = x = 7 \text{ cm} \quad [1 \text{ Mark}]$$

$$\begin{aligned} \text{Now, } BE &= 12 - x \\ &= 12 - 7 = 5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{And, } CF &= x - 4 \\ &= 7 - 4 = 3 \text{ cm} \end{aligned} \quad [1 \text{ Mark}]$$

Hence, the lengths of AD, BE and CF are 7 cm, 5 cm and 3 cm respectively.

31. Perimeter of shaded region = Perimeter (QTR + QAP + PSR) [1 Mark]

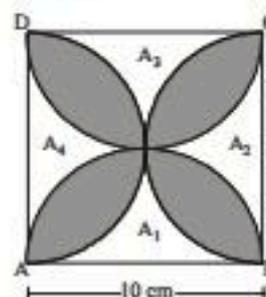
$$= \pi \left[5 + \frac{3}{2} + \frac{7}{2} \right] = \pi \left[\frac{20}{2} \right] = 10\pi = 31.4 \text{ cm} \quad [2 \text{ Marks}]$$

OR

We mark the area of unshaded regions as A_1, A_2, A_3 and A_4 as shown in the figure.

Area of the shaded region

$$= \text{Area of the square } ABCD - \text{Area of the unshaded portion } (A_1 + A_2 + A_3 + A_4).$$



Area of unshaded region $A_1 + A_3$

$$= \text{Area of square } ABCD - \text{Area of semi circle on BC as diameter} - \text{Area of semi-circle on AD as diameter.}$$

$$= 10^2 - \frac{\pi \cdot 5^2}{2} - \frac{\pi \cdot 5^2}{2} = 100 - 25\pi. \quad [1 \text{ Mark}]$$

Similarly, area of unshaded region $A_2 + A_4 = \text{Area of square } ABCD - \text{Area of the semi-circles on diameter AB and DC.}$

$$\text{So, Area } A_2 + A_4 = 10^2 - 2 \times \frac{\pi \times 5^2}{2} = 100 - 25\pi.$$

$$\text{So, Area } A_1 + A_2 + A_3 + A_4 = 200 - 50\pi.$$

Area of the shaded region = Area of square ABCD - Area ($A_1 + A_2 + A_3 + A_4$) [1 Mark]

$$= 10^2 - (200 - 50\pi)$$

$$= (100 - 200 + 50\pi) \text{ sq. cm.}$$

$$= 50\pi - 100 = 50(\pi - 2) \text{ sq. cm.}$$

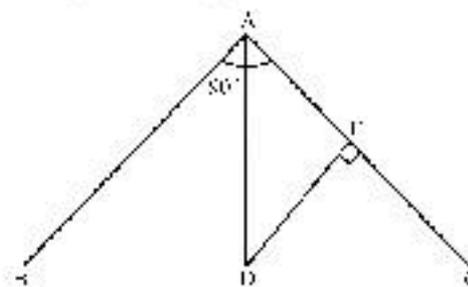
$$= 50(3.14 - 2) = 50 \times 1.14 = 57 \text{ cm}^2. \quad [1 \text{ Mark}]$$

32. It is give that AD is the bisector of $\angle A$ of $\triangle ABC$.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{AB}{AC} + 1 = \frac{BD}{DC} + 1 \quad [\text{Adding 1 on both sides}]$$

[1 Mark]

$$\Rightarrow \frac{AB + AC}{AC} = \frac{BD + DC}{DC} \quad [1/2 \text{ Mark}]$$



$$\Rightarrow \frac{AB + AC}{AC} = \frac{BC}{DC} \quad \dots (1) \quad [1/2 \text{ Mark}]$$

In \triangle 's CDE and CBA, we have

$$\angle DCE = \angle BCA = \angle C \quad [\text{Common}]$$

$\angle BAC = \angle DEC$ [Each equal to 90°]
 So, by AA-criterion of similarity, we have [1 Mark]

$$\Delta CDE \sim \Delta CBA \Rightarrow \frac{CD}{CB} = \frac{DE}{BA} \Rightarrow \frac{AB}{DE} = \frac{BC}{DC} \quad \dots (2)$$

[1 Mark]

From (1) and (2), we have $\frac{AB+AC}{AC} = \frac{AB}{DE}$
 $\Rightarrow DE \times (AB+AC) = AB \times AC$ [1 Mark]

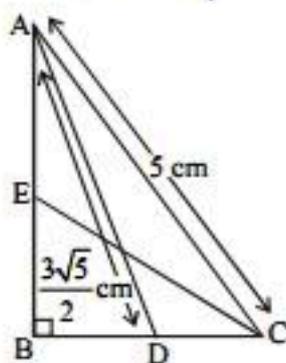
OR

In ΔABC , $\angle B = 90^\circ$ and AD and CE are two medians.
 $\therefore AC^2 = AB^2 + BC^2 = (5)^2 = 25 \quad \dots (i)$

(By Pythagoras theorem)
 In ΔABD , $AD^2 = AB^2 + BD^2$

$$\Rightarrow \left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\Rightarrow \frac{45}{4} = AB^2 + \frac{BC^2}{4} \quad \dots (ii)$$



In ΔEBC , $CE^2 = BC^2 + \frac{AB^2}{4} \quad \dots (iii)$ [2 Marks]

Subtracting equation (ii) from equation (i),
 $\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$
 $\Rightarrow BC^2 = \frac{55}{3} \quad \dots (iv)$ [1 Mark]

$AB^2 + \frac{55}{12} = \frac{45}{4}$ From equation (ii),
 $\Rightarrow AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$ [1 Mark]

$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4}$$

From equation (iii),
 $= \frac{240}{12} = 20$ [1 Mark]

$\therefore CE = 2\sqrt{5}$ cm.

33. Given $\sin \theta = \frac{c}{\sqrt{c^2 + d^2}}$
 $\cos^2 \theta = 1 - \sin^2 \theta$ [1 Mark]

$$= 1 - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)^2$$

$$= 1 - \frac{c^2}{c^2 + d^2}$$

$$= \frac{c^2 + d^2 - c^2}{c^2 + d^2}$$

$$= \frac{d^2}{c^2 + d^2}$$

[2 Marks]

$$\cos \theta = \frac{d}{\sqrt{c^2 + d^2}}$$

We know that, $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$= \frac{\frac{c}{\sqrt{c^2 + d^2}}}{\frac{d}{\sqrt{c^2 + d^2}}}$$

$$\tan \theta = \frac{c}{d}$$

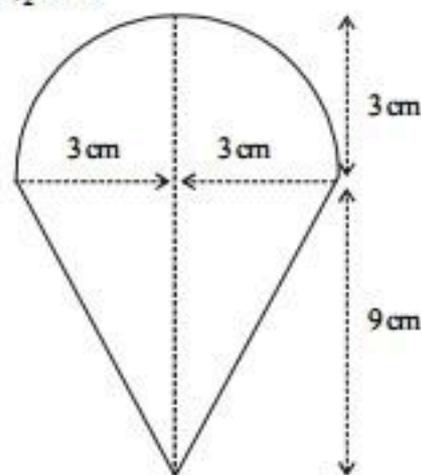
[2 Marks]

34. Volume of one ice-cream cone as shown in figure

$$= \frac{1}{3} \pi \times (3)^2 \times 9 + \frac{2}{3} \pi (3)^3 \text{ cm}^3 = 45 \pi \text{ cm}^3$$

Volume of the ice-cream in the cylindrical container (of height 15 cm and diameter 12 cm) [2 Marks]

$$= \pi \times (6)^2 \times 15$$



[2 Marks]

Let the number of cones made be n .

Then, $n \times 45 \pi = \pi \times (6)^2 \times 15$
 $\Rightarrow 45n = 36 \times 15 \Rightarrow n = 12$ [1 Mark]

35. The class 1500 – 2000 has the maximum frequency, therefore, this the modal class.

Here $l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$ and $f_2 = 33$

Now, let us substitute these values in the formula

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

[1 Mark]

$$= 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500$$

[1 Mark]

$$= 1500 + \frac{16}{23} \times 500 = 1500 + 347.83 = 1847.83$$

∴ Modal monthly expenditure = ₹ 1847.83
Let the assumed mean be $A = 3250$ and $h = 500$.

Exp. (in ₹)	f_i	x_i	$d_i = x_i - A = x_i - 3250$	$u_i = \frac{x_i - A}{h} = \frac{x_i - 3250}{500}$	$f_i u_i$
1000-1500	24	1250	-2000	-4	-96
1500-2000	40	1750	-1500	-3	-120
2000-2500	33	2250	-1000	-2	-66
2500-3000	28	2750	-500	-1	-28
3000-3500	30	3250	0	0	0
3500-4000	22	3750	500	1	22
4000-4500	16	4250	1000	2	32
4500-5000	7	4750	1500	3	21
	$N = \sum f_i = 200$				$\sum f_i u_i = -235$

We have, $N = 200$, $A = 3250$, $h = 500$ [2 Marks]
and $\sum f_i u_i = -235$

$$\text{Mean} = A + h \left(\frac{1}{N} \sum f_i u_i \right) = 3250 + 500 \times \frac{-235}{200}$$

$$= 3250 - 587.5 = 2662.5 \quad [1 \text{ Mark}]$$

Hence, the average expenditure is ₹ 2662.50.

OR

The class 40 - 50 has the maximum frequency, therefore, this is the modal class.

Here $l = 40$, $h = 10$, $f_1 = 20$, $f_0 = 12$ and $f_2 = 11$ [2 Marks]

Now, let us substitute these values in the formula

Mode =

$$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 40 + \left(\frac{20 - 12}{40 - 12 - 11} \right) \times 10$$

[2 Marks]

$$= 40 + \frac{8}{17} \times 10 = 40 + 4.7 = 44.7 \quad [1 \text{ Mark}]$$

Hence, mode = 44.7 cars

36. (i) No. of correct questions are 96 [1 Mark]

(ii) He guessed 24 questions. [1 Mark]

(iii) Marks = $80 - \frac{1}{4}$ of 40 = 70

OR

Here, $x + y = 120$... (i)

$$x - \frac{1}{4}y = 95 \quad \dots \text{(ii)} \quad [2 \text{ Marks}]$$

On solving (i) & (ii) $x = 100$

37. (i) $\cot C = \frac{BC}{AB} = \frac{3}{4}$ [1 Mark]

(ii) $\sec C = \frac{AC}{BC} = \frac{5}{3}$ [1 Mark]

(iii) $\sin C = \frac{4}{5}$, $\cos C = \frac{3}{5}$
L.H.S = $\sin^2 C + \cos^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2$

$$= \frac{16+9}{25} = 1 = \text{R.H.S} \quad [2 \text{ Marks}]$$

OR

$$\tan C = \frac{4}{3}$$

$$1 + \tan^2 C = 1 + \frac{16}{9} = \frac{25}{9} \quad [2 \text{ Marks}]$$

38 (i) 2, 4, 6, 8,
Last term, $t_{30} = 2 + (30 - 1)2 = 2 + 2(29) = 60$

3, 6, 9, 12,

Last term, $t_{20} = 3 + (20 - 1)3 = 3 + 57 = 60$ [1 Mark]

(ii) For 2, 4, 6, 8,

$$S_{30} = \frac{30}{2} (2 + 60) = 930$$

For 3, 6, 9, 12,

$$S_{20} = \frac{20}{2} (3 + 60) = 630 \quad [1 \text{ Mark}]$$

(iii) Let m^{th} term of the first series is common with the n^{th} term of the second series.

$$\begin{aligned} t_m &= t_n \\ 2 + (m - 1)2 &= 3 + (n - 1)3 \\ 2 + 2m - 2 &= 3 + 3n - 3 \Rightarrow 2m = 3n \end{aligned}$$

$$\frac{m}{3} = \frac{n}{2} = k \text{ (let) } m = 3k, n = 2k$$

$$1 \leq m \leq 30 \quad | \quad 1 \leq n \leq 20$$

$$1 \leq 3k \leq 30 \quad | \quad 1 \leq 2k \leq 20$$

$$\frac{1}{3} \leq k \leq 10 \quad | \quad \frac{1}{2} \leq k \leq 10$$

Hence, $k = 1, 2, 3, \dots, 10$. For each value of k , we get one identical term.

Thus, number of identical terms = 10. [2 Marks]

OR

As per part (i), the last term of both A.Ps is 60 and $x = 15$,
For first A.P $d = -2$, $a = 60$

$$a_{15} = 60 + (15 - 1)y - z,$$

$$= 60 + 14 \times -2 = 60 - 28$$

$$a_{15} = 32$$

Similar for second A.P $d = -3$

$$a_{15} = 60 + (15 - 1) \times -3 = 60 - 42$$

$$= 18$$

[2 Marks]