Fatigue Design of shaft

- Variable load fatigue design (static design TOF)

Soderberg eqn (Ductile) → $S_{yc} > S_{yt} > S_{ys}$

$$\frac{\sigma_m}{S_{yt}} K_t + \frac{\sigma_a}{\sigma_e} K_f \le \frac{1}{N}$$

Goodman's eqn (Brittle) → $S_{yc} > S_{ys} > (S_{yt} = S_{ut})$

$$\frac{\sigma_m}{S_{ut}} \cdot K_t + \frac{\sigma_a}{\sigma_e} K_f \le \frac{1}{N}$$

For shear soderberg

$$\frac{\tau_m}{S_{ys}} K_t + \frac{\tau_a}{\tau_e} K_f \le \frac{1}{N}$$

K_t = theoritical or static stress concentration factor

K_f = Actual or fatigue stress concentration factor

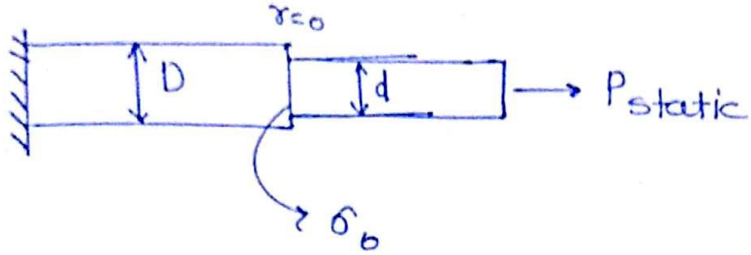
σ_m / τ_m = Mean stress

σ_a, τ_a = Variable stress or stress amplitude

σ_e, τ_e = Surface endurance limit of machine component under actual working condition

N = Factor of safety.

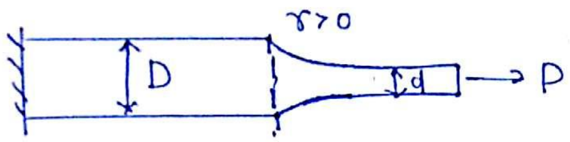
+ K_t :- Theoritical or static stress Concentration Factor:-



$$\sigma_0 = \frac{P}{\frac{\pi}{4}d^2} = \frac{P}{A_{min}}$$

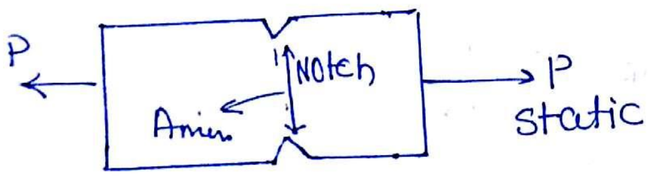
Nominal stress
(maxm stress.

$$\sigma_{act} = K_{t_1}\sigma_0 \qquad K_{t_1} > 1$$



$$\sigma_0 = \frac{P}{\frac{\pi}{4}d^2} = \frac{P}{A_{min}}$$

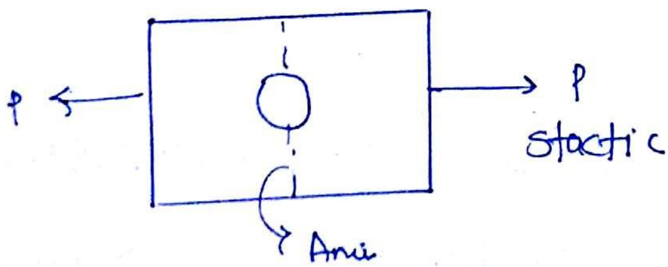
$$\sigma_{act} = K_{t_2}\sigma_0 \qquad K_{t_2} > 1$$



one notch.

$$\sigma_0 = \frac{P}{A_{min}}$$

$$\sigma_{act} = K_t\sigma_0 \qquad K_t > 1$$

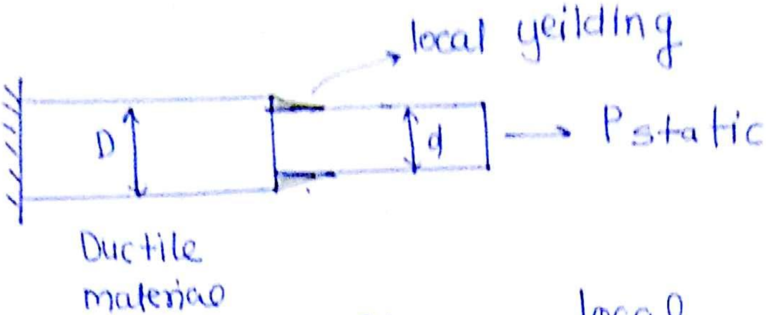


$$\sigma_0 = \frac{P}{A_{min}}$$

$$\sigma_{act} = K_t\sigma_0$$

** $\boxed{K_t = 3}$

*



$r = 0$ $\xrightarrow{\text{local yeilding}}$ $r > 0$

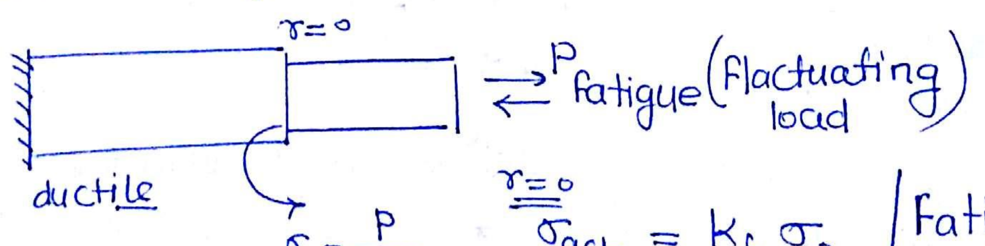
$K_t \uparrow\uparrow$ $\quad$ $K_t \downarrow\downarrow$

Conclusion :- (for above case)

① The effect of stress concentration factor under static loading is not serious for Ductile material because the Geometry Near the discontinuity is rearranged by the phinomena of local yeilding hence K_t can be neglected ($K_t = 1$)

② The effect of K_t under static loading is more serious for brittle material because they does not permit any yelding hence K_t can not be neglected for brittle materials.

K_f :- Fatigue Concentration Factor :-



$\sigma_0 = \dfrac{P}{\frac{\pi}{4} d^2}$

$r = 0$

$\sigma_{act} = K_f \, \sigma_0$

$K_f \uparrow\uparrow$

Fatigue ⇒ No-yeilding

fatigue Fail ⇒ Fracture (Crack)

The effect of k_f under fatigue loading is more seriours for both ductile & brittle material

Hence k_f can not be neglected.

* σ_m, τ_m (Mean stress):-

$$\boxed{\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}}$$ ($\sigma_{max}, \sigma_{min}$ with sign)

$$\boxed{\tau_m = \frac{\tau_{max} + \tau_{min}}{2}}$$ (with sign)

tension '+'

Compression '-'

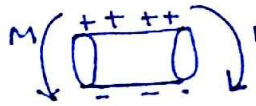
* σ_a, τ_a

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$ (with sign)

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2}$$ (with sign)

* σ_e Surface endurance limit of machine under actual working condition:—

Lab For any Material under standard Cond^n



(i) std size

(ii) std surface finish

(iii) std load

$\sigma_e^* =$ std endurance limit

$\sigma_{e_{MS}}^* = 0.5\, S_{ut}$

$\sigma_{e_{CI}}^* = 0.4\, S_{ut}$

$\sigma_{e_{Cast\ Alloy}}^* = 0.3\, S_{ut}$

$\sigma_b = K_a \cdot K_b \cdot K_c \cdot\cdot \sigma_e^*$

↓

under actual condition

K_a - Size factor → Size ↑ → Ka ↓ → σ_e ↓

K_b - Surface finish → Roughness ↑ → K_b ↓ → σ_e ↓

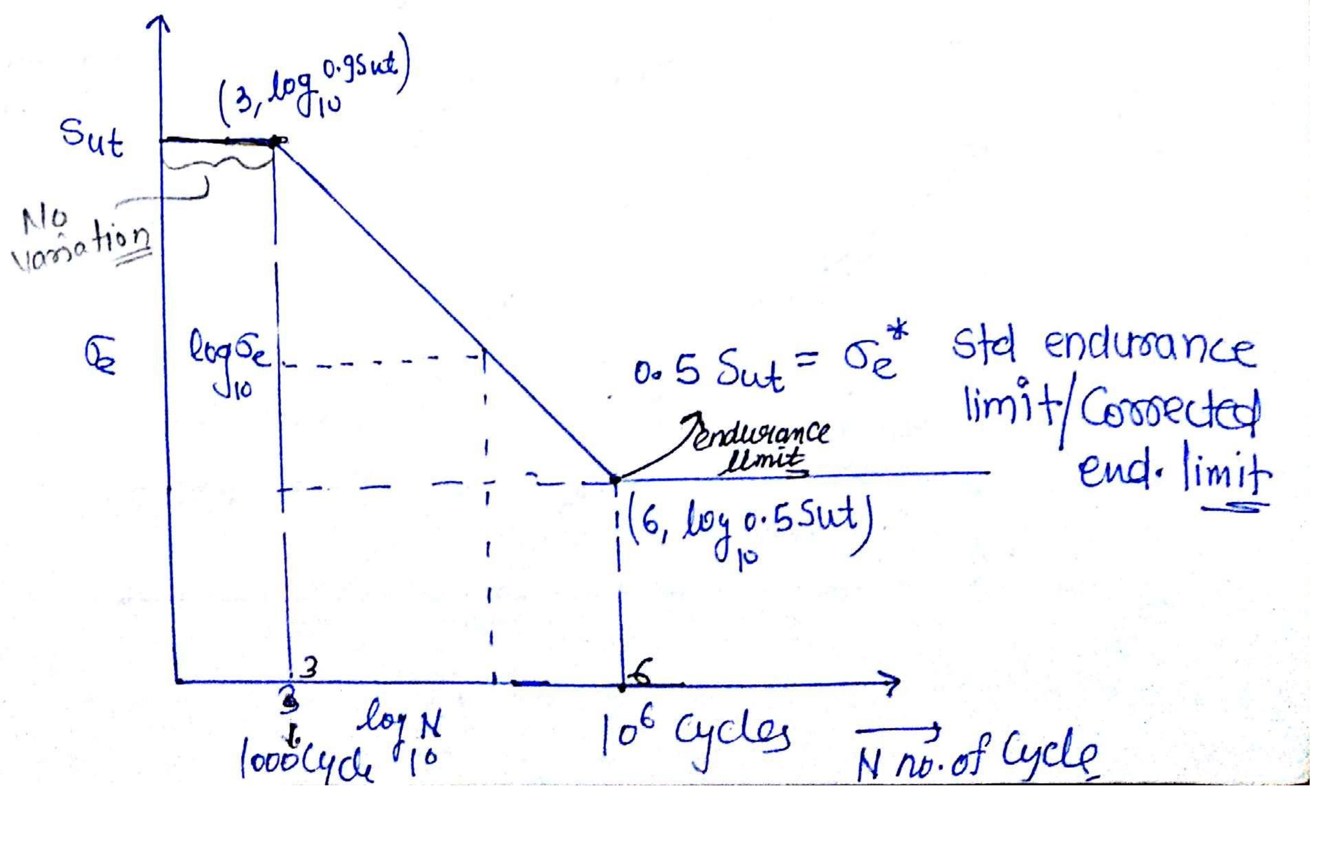
K_c - load factor ⇒ if bending ⇒ $K_c = 1$

twisting ⇒ $K_c = 0.58$

Axial load ⇒ $K_c = 0.74$

σ_e^* = it is not a property of material
it depends upon std condition.

* S-N Curve:- (Mild steel)
[M.S.]



$$\left(y - \log_{10} 0.9 S_{ut}\right) = \frac{\log_{10} .5 S_{ut} - \log_{10} 0.9 S_{ut}}{6-3} (x-3)$$

*

Soderberg line $\quad \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{\sigma_e} \le \frac{1}{N}$ {Most safe}

Goodman line $\quad \frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{\sigma_e} \le \frac{1}{N}$

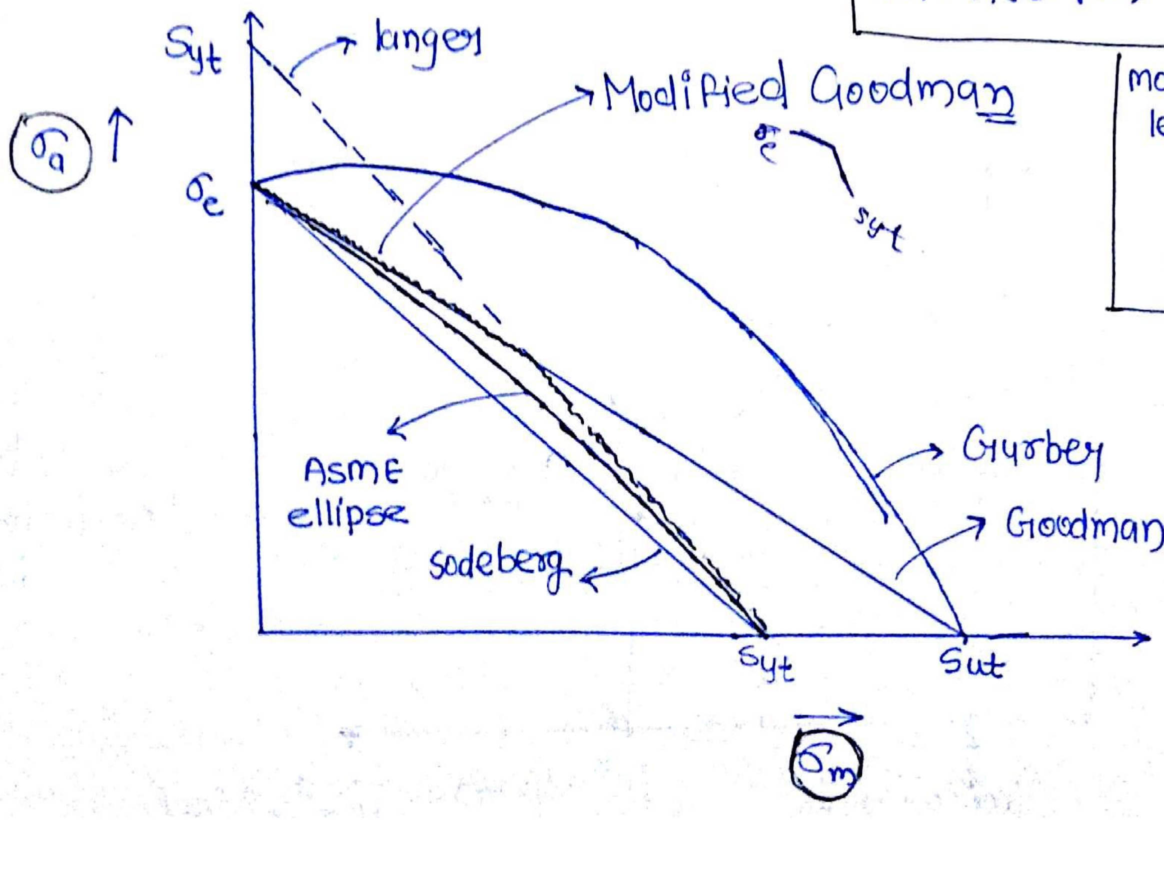
* langer line $\quad \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \le \frac{1}{N}$

Gurber's (parabola) $\quad \left(\frac{\sigma_m \cdot N}{S_{ut}}\right)^2 + \frac{\sigma_a \cdot N}{\sigma_e} \le 1$

ASME Ellipse $\quad \left(\frac{\sigma_m}{S_{yt}}\right)^2 + \left(\frac{\sigma_a}{\sigma_e}\right)^2 \le \frac{1}{N^2}$

$\boxed{\text{dia}\uparrow \rightarrow \text{Area}\uparrow \rightarrow \sigma_m \downarrow, \sigma_a \downarrow}$

more dia less N



Modified Goodman.

Safe Result of.

$$\left\{\begin{array}{c} \dfrac{\sigma_m}{S_{ut}} + \dfrac{\sigma_a}{\sigma_e} \le \dfrac{1}{N} \\ \oplus \\ \dfrac{\sigma_m}{S_{yt}} + \dfrac{\sigma_a}{S_{yt}} \le \dfrac{1}{N} \end{array}\right.$$

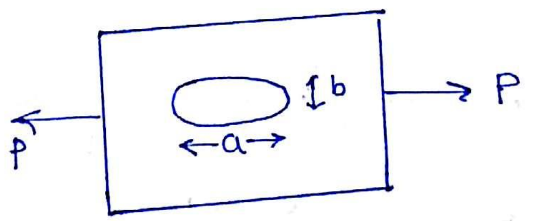
Consider less $\underline{N}$

Calculation of K_t :-

① chart method (experiment method) <u>99% used</u>

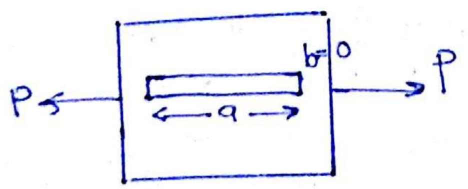
② Formula Method :-

Don't use k_t unless not mention in question



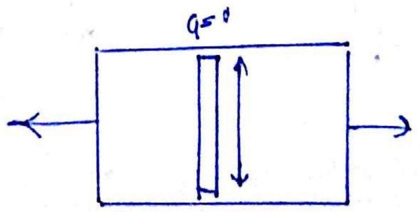
$$\boxed{k_t = 1 + \frac{2b}{a}}$$

*



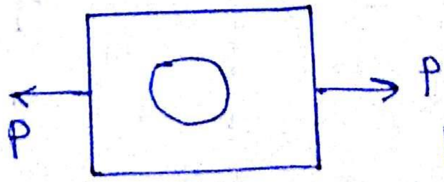
$k_t = 1$

*



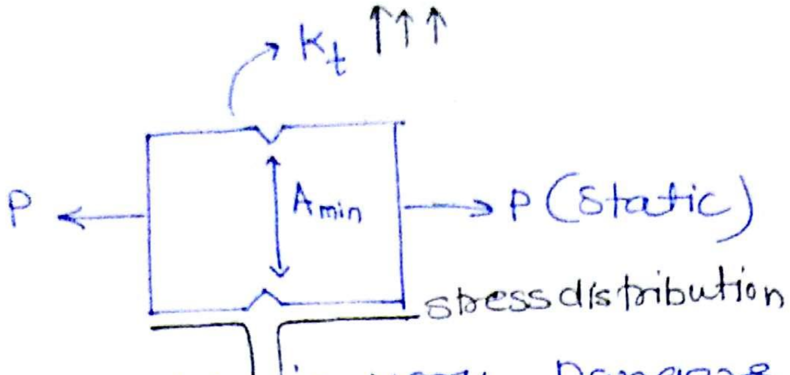
$k_t \rightarrow \infty$

*



$k_t = 3$

*



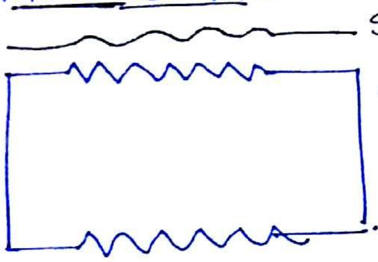
$$\sigma_0 = \frac{P}{A_{min}}$$

$$\sigma_{act} = K_t \sigma_0$$

one notch is very Dangers under static loading for axial load.

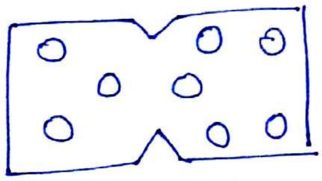
Method of Reducing K_t

(1) Provide more Notch:-



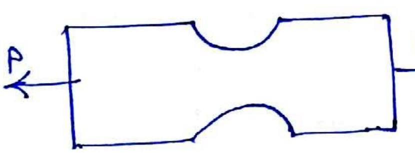
$K_t \downarrow$

(2) Drilling holes



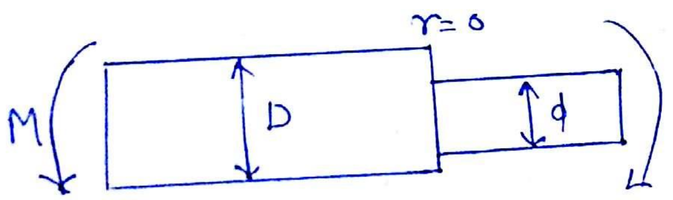
$K_t \downarrow$

(3) Removal of material



$K_t \downarrow$

*

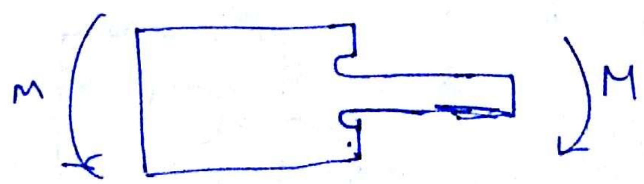


in bending $K_t \uparrow\uparrow\uparrow$

Very dangerous in bending

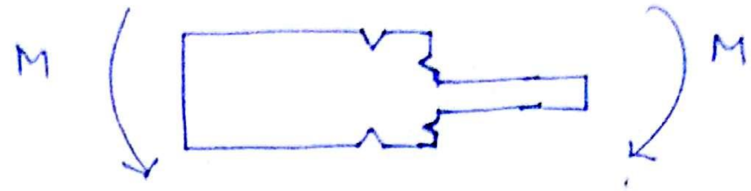
Method of Reducing 'K_t'

(1) Under Cutting

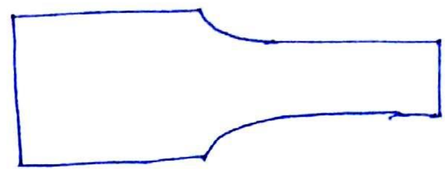


$K \downarrow$

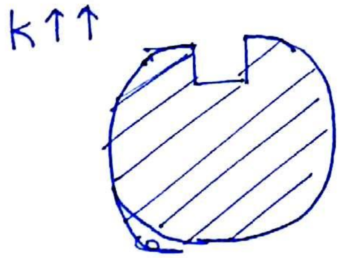
(ii) Providing Notches. $K_t \downarrow$



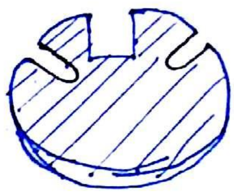
(iii) Providing radius of Curvature:-



* Imp
* key way is very dangerous in twisting ($K_t \uparrow$)



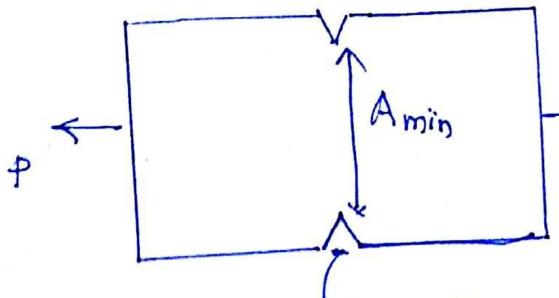
To Reduce K_t Drilling holes



K ↓ decrease

Notch Sensitivity :-

Static loading

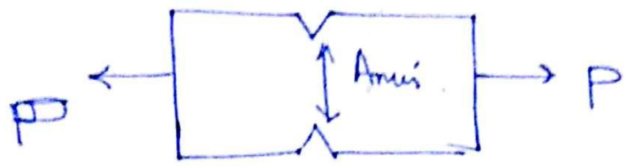


$$\sigma_o = \frac{P}{A_{min}}$$

$$\sigma_{static} = K_t \sigma_o$$

Increase in static ~~load~~ stress over Nominal stress = $K_t \sigma_o - \sigma_o$

Fatigue loading / Actual loading



$$\sigma_0 = P/A_{min}$$

$$\sigma_{act} = K_f \cdot \sigma_0$$

Increase in Actual stress over nominal stress $= K_f \sigma_0 - \sigma_0$

$$q = \frac{\text{Increase in Actual stress over nominal}}{\text{Increase in static stress over nominal}}$$

$$\cancel{q =} \frac{K_f \sigma_0 - \sigma_0}{K_t \sigma_0 - \sigma_0}$$

Stress Ratio $= \dfrac{\sigma_{min}}{\sigma_{max}}$

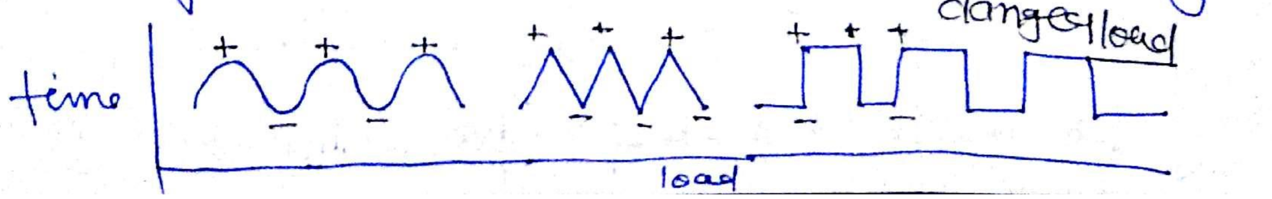
$$\boxed{q = \frac{K_f - 1}{K_t - 1}} \qquad 0 \le q \le 1$$

if $q = 0 \Rightarrow K_f = 1 \Rightarrow$ Notch is not sensitive

if $q = 1 \Rightarrow K_f = K_t \Rightarrow$ Notch is Fully sensitive

* Notch is more dangerous in static loading as compare to Fatigue loading.

*



Q.1.3

10 cm $\longrightarrow$ 95 MPa

$$\frac{95}{9.5} \times 10 = 100 \text{ MPa}$$

Q.11

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{\sigma_e} \leq \frac{1}{N}$$

$$\cancel{\sigma_m = \frac{-40+160}{2} = 100 \text{ KN}}\ \cancel{60 \text{ KN}}$$

$$\cancel{\sigma_a = \frac{160-(-40)}{2} = 100 \text{ KN}}$$

$$\cancel{\frac{60 \times 10^3}{420} + \frac{100}{240} \leq \left(\frac{1}{N}\right)} \quad \frac{\pi}{4}(39^2$$

$$\sigma_{max} = \frac{160 \times 10^3}{\frac{\pi}{4}(30)^2} = 226.35 \text{ MPa}$$

$$\sigma_{min} = \frac{40 \times 10^3}{\frac{\pi}{4}(30)^2} = -56.58 \text{ MPa}$$

$$\sigma_m = \frac{226.35 - 56.58}{2} = 84.88 \text{ MPa}$$

$$\sigma_a = \frac{226.35 - (-56.58)}{2} = 141.46 \text{ MPa}$$

$$\frac{84.84}{420} + \frac{141.46}{240} = \frac{1}{N} \Rightarrow N = 1.26$$

1.9

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} \le \frac{1}{N}$$

spherical $\sigma_H = \sigma_\ell = \frac{PD}{4t}$

$$\sigma_{max} = \frac{8 \times 200}{4} = 400\ MPa$$

$$\sigma_{min} = \frac{4 \times 200}{4} = 200\ MPa$$

$$\sigma_m = 300\ MPa \qquad \sigma_a = 100\ MPa$$

$$\frac{300}{800} + \frac{100}{400} \le \frac{1}{N}$$

$$\underline{N = 1.6} \quad \text{Ans}$$

if Modified Goodman

$$\frac{300}{600} + \frac{100}{600} = \frac{1}{N}$$

$$N = 1.5$$

then ans $= 1.5$ Ans by Modified Goodman

Q.1.13

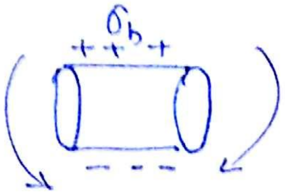
$\sigma_{max} =$

$$\frac{60\times10^3}{240} + \frac{80\times10^3}{160} \leq \frac{\pi}{2\times4}(d)^2$$

Area

$A = 1000\ mm^2$

(14)

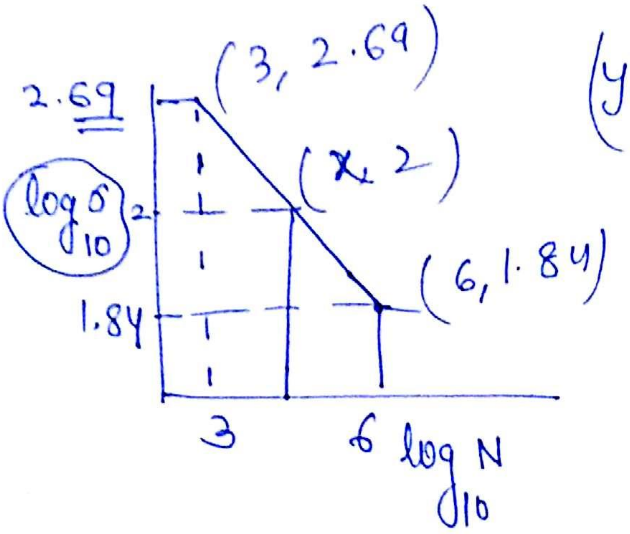


$(\sigma_b)_{max} = 100$ MPa

$(\sigma_b)_{min} = -100$ MPa

P.g 206

Q.2.10



$$(y - 1.84) = \frac{2.69-1.84}{3}(x-6)$$

$$2-1.84 = \frac{2.69-1.84}{3}(x-6)$$

$x = 5.45$

$\log_{10} N = 5.4501$

$N = 281914$