## Probability

## MATHEMATICS

## OLYMPIAD EXCELLENCE BOOK

1.	What is the chance of throwing a number greater than 4 with an ordinary dice whose faces are numbered from 1 to 6?					
	(a) $\frac{1}{3}$	(b) $\frac{1}{6}$	(c) $\frac{1}{9}$	(d) $\frac{1}{8}$		
2.	Find the chance of thre	owing at least one ace in a	a simple throw with two di	ce.		
	(a) $\frac{1}{12}$	(b) $\frac{1}{3}$	(c) $\frac{1}{4}$	(d) $\frac{11}{36}$		
3.	Three coins are tossed	. What is the probability c	f getting 2 Tails and 1 hea	ad?		
	(a) $\frac{1}{4}$	(b) $\frac{3}{8}$	(c) $\frac{2}{3}$	(d) $\frac{1}{8}$		
4.		. What is the probability c	f getting 1 Tail and 2 Hea	ds?		
	(a) $\frac{3}{8}$	(b) 1	(c) $\frac{2}{3}$	(d) $\frac{3}{4}$		
5.	Three coins are tossed	. What is probability of ge	tting Neither 3 heads nor	3 tails?		
	(a) $\frac{1}{2}$	(b) $\frac{1}{3}$	(c) $\frac{2}{3}$	(d) $\frac{3}{4}$		
6.	Three coins are tossed	. What is the probability c	f getting Three heads?			
	(a) $\frac{1}{8}$	(b) $\frac{1}{4}$	(c) $\frac{1}{2}$	(d) $\frac{2}{3}$		
7.			0 passed the first, 50 pass 1 has failed in both the exa	sed the second and 30 passed both. Find aminations?		
	(a) $\frac{1}{5}$	(b) $\frac{1}{7}$	(c) $\frac{5}{7}$	(d) $\frac{5}{6}$		
8.	What is the probability	of throwing a number gro	eater than 2 with a fair dic	e?		
	(a) $\frac{2}{3}$	(b) $\frac{2}{5}$	(c) 1	(d) $\frac{3}{5}$		
9.	In rolling two dices, fin	d the probability that ther	e is at least one '6'.			
	(a) $\frac{11}{36}$	(b) $\frac{22}{36}$	(c) $\frac{15}{36}$	(d) $\frac{29}{36}$		

- **10.** Vijay throws three dice in a game. If it is known that he needs 15 or higher in this throw to win then find the chance of his winning the game.
  - (a)  $\frac{5}{54}$  (b)  $\frac{17}{216}$  (c)  $\frac{13}{216}$  (d)  $\frac{15}{216}$

11. In a horse race, there were 18 horses numbered 1 – 18. The probability that horse 1 would win is 1/6, that horse 2 would win is 1/10 and that horse 3 would win is 1/8. Assuming that a tie is impossible, find the chance that one of the three will win.

(a)  $\frac{47}{120}$  (b)  $\frac{119}{120}$  (c)  $\frac{11}{129}$  (d)  $\frac{1}{5}$ 

**12.** A bag contains 20 balls marked 1 to 20. One ball is drawn at random. Find the probability that it is marked with a number multiple of 5 or 7.

- (a)  $\frac{3}{10}$  (b)  $\frac{7}{10}$  (c)  $\frac{1}{11}$  (d)  $\frac{2}{3}$
- **13.** The probability that a student will pass in mathematics is 3/5 and the probability that he will pass in English is 1/3. If the probability that he will pass in both mathematics and English is 1/8, what is the probability that he will pass in at least one subject?
  - (a)  $\frac{97}{120}$  (b)  $\frac{87}{120}$  (c)  $\frac{53}{120}$  (d)  $\frac{120}{297}$
- **14.** Two fair dices are thrown. Given that the sum on the die is less than or equal to 4, find the probability that only one dice shows two.
  - (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c)  $\frac{2}{3}$  (d)  $\frac{1}{3}$
- 15. What is the chance that a leap year, selected at random, will contains 53 Sundays?

(a) 
$$\frac{2}{7}$$
 (b)  $\frac{3}{7}$  (c)  $\frac{1}{7}$  (d)  $\frac{5}{7}$ 

**16.** Out of all the 2- digit integers between 1 to 200, a 2–digit number has to be selected at random. What is the probability that selected number is not divisible by 7?

(a)  $\frac{11}{90}$  (b)  $\frac{33}{90}$  (c)  $\frac{55}{90}$  (d)  $\frac{164}{191}$ 

- **17.** Tom and Dick are running in the same race; the probability of their winning are 1/5 and <sup>1</sup>/<sub>2</sub> respectively. Find the probability that either of them will win the race.
  - (a)  $\frac{7}{10}$  (b)  $\frac{3}{10}$  (c)  $\frac{1}{5}$  (d)  $\frac{7}{9}$

**18.** Two dice are thrown. If the total on the faces of the two dices are 6, find the probability that there are two odd numbers on the faces.

(a)  $\frac{2}{5}$  (b)  $\frac{1}{5}$  (c)  $\frac{5}{9}$  (d)  $\frac{3}{5}$ 

**19.** A speaks the truth 3 out of 4 times, and B 5 out of 6 times. What is the probability that they will contradict each other in stating the same fact?

(a)  $\frac{2}{3}$  (b)  $\frac{1}{3}$  (c)  $\frac{5}{6}$  (d) None of these

**20.** In a bag, there are 12 black and 6 white balls. Two balls are chosen at random and first one is found to be black. The probability that the second one is also black is:

(a)  $\frac{11}{17}$  (b)  $\frac{12}{17}$  (c)  $\frac{13}{18}$  (d) None of these

21. Seven white balls and three black balls are randomly placed in a row. Find the probability that no two black balls are placed adjacent to each other.

(a)  $\frac{7}{15}$  (b)  $\frac{2}{15}$  (c)  $\frac{3}{7}$  (d)  $\frac{2}{7}$ 

**22.** Out of a pack of 52 cards, one card is lost; from the remainder of the pack, two cards are drawn and are found to be spades. Find the chance that the missing card is a spade.

- (a)  $\frac{11}{50}$  (b)  $\frac{11}{49}$  (c)  $\frac{10}{49}$  (d)  $\frac{10}{50}$
- **23.** The odds in favour of an event are 2:7. Find the probability of occurrence of this event.
  - (a)  $\frac{2}{9}$  (b)  $\frac{5}{12}$  (c)  $\frac{7}{12}$  (d)  $\frac{2}{5}$

**24.** A coin is tossed twice if the coin shows head it is tossed again but if it shows a tail then a die is tossed. If 8 possible outcomes are equally likely, find the probability that the die shows a number greater than 4, if it is known that the first throw of the coin results in a tail.

(a) 
$$\frac{1}{3}$$
 (b)  $\frac{2}{3}$  (c)  $\frac{2}{5}$  (d)  $\frac{4}{15}$ 

- **25.** In a class, 45% students read English, 30% read French and 20% read both English and French. One student is selected at random. Find the probability that he reads English, if it is known that he reads French.
  - (a)  $\frac{1}{3}$  (b)  $\frac{2}{3}$  (c)  $\frac{6}{6}$  (d) None of these

26. The probability that A can solve a problem is <sup>2</sup>/<sub>3</sub> and the probability that B can solve the same problem is <sup>3</sup>/<sub>5</sub>. Find the probability that at least one of A and B will be able to solve the problem.
(a) <sup>12</sup>/<sub>15</sub>
(b) <sup>13</sup>/<sub>15</sub>
(c) <sup>19</sup>/<sub>45</sub>
(d) None of these

- **27.** A four digit number is formed with the digits 1, 3, 4, 5 without repetition, Find the chance that the number is divisible
  - (a)  $\frac{3}{4}$  (b)  $\frac{1}{4}$  (c)  $\frac{9}{16}$  (d)  $\frac{1}{16}$

by 5.

- **28.** If one number is selected from the first 70 natural numbers, the probability that the number is a solution of  $x^2 + 2x > 4$  is
  - (a)  $\frac{69}{70}$  (b)  $\frac{1}{70}$  (c) 1 (d) 0

29. An urn contains 9 red balls and p green balls. If the probability of picking a red ball is thrice that of picking a green ball, then p is equal to \_\_\_\_\_.
(a) 6 (b) 7 (c) 2 (d) 3

**30.** A four - digit number is formed by using the digits 1, 2, 4, 8 and 9 without repetition. If one number is selected from those numbers, then what is the probability that it will be an odd number?

- (a)  $\frac{1}{5}$  (b)  $\frac{2}{5}$  (c)  $\frac{3}{5}$  (d)  $\frac{4}{5}$
- **31.** The probability that the month of April has exactly 5 Mondays is (a)  $\frac{4}{7}$  (b)  $\frac{5}{7}$  (c)  $\frac{3}{7}$

**32.** A basket contains 10 fruits of which 3 are rotten. If one fruit is drawn from the basket, then the probability that the fruit drawn is not rotten is\_\_\_\_\_.

(d)  $\frac{2}{7}$ 

(a) 
$$\frac{4}{5}$$
 (b)  $\frac{3}{5}$  (c)  $\frac{7}{10}$  (d)  $\frac{3}{10}$ 

- **33.** A bag contains 5 pens and 6 pencils. If a boy selects 2 articles from the bag, then what is the probability that the selected articles will be a pen and a pencil?
  - (a)  $\frac{2}{11}$  (b)  $\frac{3}{11}$  (c)  $\frac{6}{11}$  (d)  $\frac{5}{11}$

**34.** A three-digit number is to be formed using the digits 3, 4, 7, 8 and 2 without repetition, what is the probability that it is an odd number?

(a)  $\frac{2}{5}$  (b)  $\frac{1}{5}$  (c)  $\frac{4}{5}$  (d)  $\frac{3}{5}$ 

**35.** Two cards are drawn form a pack of cards one after another so that the first card is replaced before drawing the second card. What is the probability that the first card is an ace and the second is a number card?

(a)  $\frac{9}{169}$  (b)  $\frac{1}{52}$  (c)  $\frac{1}{4}$  (d)  $\frac{17}{52}$ 

**36.** A box contains 60 pens which are blue-inked or black-inked. If a pen is picked at random, the probability of picking a blue-inked pen is  $\frac{2}{5}$ , what is the number of blue-inked pens in the box? (a) 32 (b) 48 (c) 30 (d) 24

**37.** All the three face cards of spades are removed from a well - shuffled pack of 52 cards. A card is drawn at random from the remaining pack. Find the probability of getting a queen.

- (a)  $\frac{3}{52}$  (b)  $\frac{3}{49}$  (c)  $\frac{1}{26}$  (d)  $\frac{1}{52}$
- **38.** A fair coin is tossed thrice. Identify the probability of getting 3 tails as a fraction.
  - (a)  $\frac{1}{8}$  (b)  $\frac{3}{8}$  (c)  $\frac{7}{8}$  (d)  $\frac{1}{4}$

**39.** Set  $\{x: 5 \le x \le 22, x \text{ is an integer}\}$ . If an element from set *P* is picked at random, calculate the probability that it is a prime number.

(a)  $\frac{5}{18}$  (b)  $\frac{1}{3}$  (c)  $\frac{7}{9}$  (d)  $\frac{5}{6}$ 

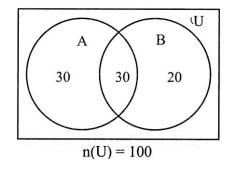
**40.** Two dice are rolled at once. What it the probability of getting an even number on the first die or a total of 8? (a)  $\frac{4}{9}$  (b)  $\frac{5}{9}$  (c)  $\frac{7}{9}$  (d)  $\frac{2}{9}$ 

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ANSWER - KEY						
<b>1.</b> A	<b>2</b> . D	<b>3.</b> B	<b>4.</b> A	<b>5.</b> D		
<b>6.</b> A	<b>7.</b> A	<b>8.</b> A	<b>9.</b> A	<b>10.</b> A		
<b>11.</b> A	<b>12.</b> A	<b>13.</b> A	<b>14.</b> D	<b>15.</b> A		
<b>16.</b> D	<b>17.</b> A	<b>18.</b> D	<b>19.</b> B	<b>20.</b> A		
<b>21.</b> A	<b>22.</b> A	<b>23.</b> A	<b>24.</b> A	<b>25.</b> B		
<b>26.</b> B	<b>27.</b> B	<b>28.</b> A	<b>29.</b> D	<b>30.</b> B		
<b>31</b> . D	<b>32.</b> C	<b>33.</b> C	<b>34</b> . A	<b>35</b> . A		
<b>36.</b> D	<b>37.</b> B	<b>38.</b> A	<b>39.</b> B	<b>40.</b> B		

## SOLUTIONS

- 1. (A) : 5 and 6 out of a sample space of 1, 2, 3, 4, 5 or 6=2/6=1/3
- (D): Event space is: (1 and 1) or (1 and 2) or (1 and 3) or (1 and 4) or (1 and 5) or (1 and 6) or (2 and 1) or (3 and 1) or (4 and 1) or (5 and 1) or (6 and 1)
  Total 11 out of 36 possibilities = 11/36
- **3.** (B): Event space is: T and T and H or T and H and T or H and T and T  $\Rightarrow 3 \times 1/8 = 3/8$ .
- **4.** (A): Event space is: T and H and H or H and H and T or H and T and  $H = 3 \times 1/8 = 3/8$ .
- 5. (D) Probability of 3 heads = 1/8 Also, Probability of 3 tails = 1/8. Required probability = 1 (1/8 + 1/8) = 6/8 = 3/4.
- **6.** (A): H and H and H  $\Rightarrow$  one event out of  $8 \Rightarrow 1/8$
- 7. (A) Let 'U' be universal set; U = 100 students



no. of failed students

 $= n(U) - n(A \cup B) = 100 - 80 = 20$ 

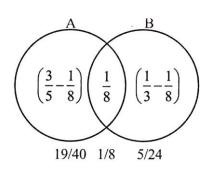
From the figure it, is evident that 80 students passed at least 1 exam. Thus, 20 failed both and the required probability is 20/100 = 1/5.

- **8.** (A) Favourable events are appearance of 3 or 4 or 5 or  $6 \Rightarrow$  Prob. = 4/6 = 2/3
- 9. (A) With a six on the first dice, there are 6 possibilities of outcomes that can appear on the other dice (6 & 1, 6 & 2, 6 & 3, 6 & 4, 6 & 5 and 6 & 6). At the same time with 6 on the second dice there are 5 more possibilities for outcomes on the first dice: (1 & 6, 2 & 6, 3 & 6, 4 & 6, 5 & 6). Also, the total outcomes are 36. Hence, the required probability is 11/36.

- 10. (A) Favourable events are appearance of 15 or 16 or 17 or 18.
  15 can be got as: 5 and 5 and 5 on the die (one way) or 6 and 5 and 4 on the die (six ways) or 6 and 6 and 3 on the die (3 ways) ⇒ Total 10 ways.
  16 can be got as: 6 and 6 and 4 on the die (3 ways) or 6 and 5 and 5 on the die (3 ways) ⇒ Total 6 ways.
  Similarly, 17 can be got in 3 ways and 18 can be got in one way. Thus the required probability is
- **11.** (A) 1/6 + 1/10 + 1/8 = 47/120

10+6+3+1)/216 = 20/216 = 5/54.

- **12.** (A) Positive outcomes are: 5, 7, 10, 14, 15 or 20 thus, 6/20 = 3/10.
- **13.** (A)



 $P(\text{pass}) = P(A \cup B)$ 

$$=\frac{18}{40}+\frac{1}{8}+\frac{5}{24}$$

We have: 19/40+1/8+5/24-97/120

**14.** (D) The possible outcomes are:

(1,1); (1,2); (2,1), (2,2); (3,1); (1,3). Out of six cases, in two cases there is exactly one '2' on only one of the dice. Thus, the correct answer is 2/6 = 1/3.

This question needs careful reading. The entire sample space of 36 events [(1, 1)....(12, 6)]; (2, 1)....(2, 6)....(6, 1) to (6, 6)] is not to be considered as the sample space is restricted to six events by the statement "given that.....or equal to 4" in the question.

- 15. (A): A leap year has 366 days which means 52 completed week's and 2 more days. The last two days can be (Sunday, Monday) or (Monday, Tuesday) or...... (Saturday, Sunday). In 2 cases out of 7, we have a 53<sup>rd</sup> Sunday. Thus, 2/7 is the required answer.

$$\therefore P = \frac{191 - 27}{191} \text{ (as total no of 2 digit integers = 191)}$$
$$= \frac{164}{191}$$

**17.** (A) 
$$\frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$

- **18.** (D) A total of six on two dice can be made in any of the following ways (1, 5), (5, 1), (2, 4), (4, 2) (3, 3). Out of this, (1, 5) (5, 1) and (3, 3) are favourable events.
- **19.** (B) They will contradict each other if: A is true and B is false or A is false and B is true.  $\therefore$  Prob. =  $(3/4) \times (1/6) + (1/4) \times (5/6) = 1/3$ .
- **20.** (A) 11/17 (if the first one is black, there will be 11 black balls left out of 17)
- **21.** (A) When you put the 7 balls with a gap between them in a row, you will have 8 spaces.
- **22.** (A) This problem has to be treated as if we are selecting the third card out of the 50 remaining cards, 11 of these are spades. Hence, Prob. =  $\frac{11}{50}$
- **23.** (A): Total number of outcomes = 2 + 7 = 9Favourable number of cases = 2

$$P(E) = \frac{2}{9}$$

**24.** (A): Here sample space is given as,

 $S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$  Let a be the event that the die shows a number greater than 4 and B be the event that the first throw of the coin results in a tail then,

 $A = \{T5, T6\}$ 

 $B = \{T1, T2, T3, T4, T5, T6\}$ 

:. Required probability

$$\Rightarrow \qquad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{8}}{\frac{6}{8}} = \frac{1}{3}$$

**25.** (B): Let A be the event of reading English and B is the event of reading French.

Then 
$$P(A) = \frac{45}{100} = \frac{9}{20}$$
,  $P(B) = \frac{30}{100} = \frac{3}{10}$   
and  $P(A \cap B) = \frac{20}{100} = \frac{1}{5}$   
 $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{2}{3}$ 

**26.** (B) 
$$P(A) = \frac{2}{3}, P(B) = \frac{3}{5}$$

Required probability =  $P(A \text{ or } B) = P(A \cup B)$  be the events of solving the problem by A or B

$$= P(A) + P(B) - P(A)P(B)$$
$$= \frac{2}{3} + \frac{3}{5} - \frac{2}{3} \cdot \frac{3}{5} = \frac{13}{15}$$

**27.** (B): Total possible number of 4 digits = 24, the number is divisible by 5 if unit digit itself is 5. Therefore we fix 5 at unit place and then remaining 3 places can be filled up in 6 Ways.

Hence, the required probability 
$$= \frac{6}{24} = \frac{1}{4}$$
.

**28.** (A): 
$$x^2 + 2x - 4 > 0$$

root  $\alpha, \beta = \frac{-2 \pm \sqrt{4 + 16}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$ or,  $\left\{ x - \left( -1 + \sqrt{5} \right) \right\} \left\{ x - \left( 1 - \sqrt{5} \right) \right\} > 0$  $\therefore \qquad x \in \left( -\infty, -1 - \sqrt{5} \right) \cup \left( -1 + \sqrt{5}, \infty \right)$ 

 $\therefore$  Within natural numbers, only 1 is not the solution

$$\therefore \qquad P = \frac{69}{70}$$

**29.** (D): Probability of picking a red ball  $= \frac{9}{9+p}$ 

Probability of picking a green ball  $= \frac{p}{9+p}$ 

$$\therefore \qquad = \frac{9}{9+p} = \frac{3p}{9+p}$$

$$\Rightarrow \qquad 3p=9 \Rightarrow p=3.$$

**30.** (B): Total number of outcomes  $= 5 \times 4 \times 3 \times 2 = 120$  the number of favourable cases  $= (2 \times 4 \times 3 \times 2) = 48$  (i.e., odd number)

The required probability  $=\frac{48}{120}=\frac{2}{5}$ .

**31.** (D): April has 30 days = 28 + 2 within 28 days, there will definitely be 4 Monday. In a week of 7 days, 5<sup>th</sup> Monday has to come out from 2 days.

$$\therefore \qquad P = \frac{2}{7}$$

- **32.** (C): Not Available
- **33.** (C): It will be a pen and a pencil in two ways:
  - 1. P (first pen, then pencil) =  $\frac{5}{11} \times \frac{6}{10}$
  - 2. P (first pencil, then pen) =  $\frac{6}{11} \times \frac{5}{10}$
  - $\therefore \text{ Required } P = \frac{5 \times 6}{110} + \frac{6 \times 5}{110} = \frac{6}{11}$
- **34.** (A): If a number ends with an odd number, then it is odd number, find the number of 3 digit odd numbers formed by using the digits 3, 4, 7, 8 and 2, find the total number of 3 digit numbers formed by using the digits 3. 4, 7, 8 and 2.

In the box notation, odd numbers can be formed in the following ways:

	Hundred	Ten	Unit	
Odd nos.	3 ways	4 ways	2 ways	$2 \times 3 \times 4$

Similarly,

Total nos. 3 ways	4 ways	5 ways	$5 \times 4 \times 3$
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**35.** (A): There are 4 ace cards and 36 numbered cards,

 $\therefore$  P (ace card and number card) = P (ace card) × P (number card).

**36.** (D): Let BLUE pens be 'a'

P (blue) 
$$= \frac{a}{60} = \frac{2}{5} \Longrightarrow a = 24$$

**37.** (B): Total number of cards = 52

3 face cards of spades are removed.

Then remaining cards = 49

$$\therefore$$
 P (a queen) =  $\frac{3}{49}$ 

**39.** (B): Prime numbers, 
$$x : x \in [5, 22]$$
  
are 5, 7, 11, 13, 17, 19 = six

total numbers = 18

$$\therefore \qquad P = \frac{6}{18} = \frac{1}{3}$$

**40.** (B) Not Available