VECTORS

Scalar

↓

Magnitude

Vector

Magnitude Direction

Scalar quantity
⇒ Work
⇒ Speed
⇒ Distance

Vector quantity
⇒ Force
⇒ Velocity
⇒ Displacement.

Representation of Vector

length = Magnitude

$|A| = 4$ unit (Magnitude)

$\vec{A} = 4$ unit south (Direction).

Types of Vector

1. Equal and unequal.
2. Parallel and antiparallel.
3. Collinear.
4. Concurrent.
5. Coplanar.
6. Zero
7. Unit.

⇒ Equal and unequal vector
For two vectors to be simillar $\vec{A}$ and $\vec{B}$ should have equal magnitude and have same direction.

$$\underbrace{\vec{v_1} = 5 m/sec \text{ East} \qquad \vec{v_2} = 5 m/sec \text{ West}}_{\text{Unequal vectors}}$$

Unequal vectors.

⇒ Parallel Vector
* Same direction
* $\theta = 0°$

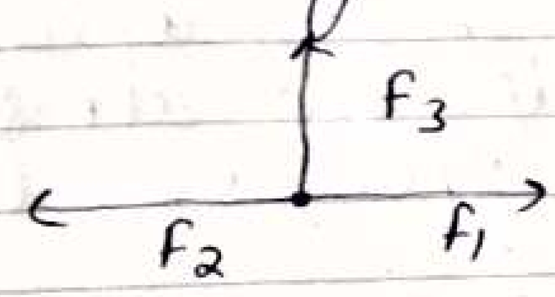
⇒ Antiparallel Vector
* Opposite direction
* $\theta = 180°$

⇒ Collinear.
In a same line.

⇒ Coplanar (In a single plane)

* 2 vectors are always coplanar.
* 3 vectors may be coplanar or may be not. (They may lie or may not lie on a semillar plane).

⇒ Concurrent Vector
* Forces (Acting at same point)



⇒ Zero vector
whose magnitude is 0. and direction is arbitary. (It can take any direction).

⇒ Unit Vector [($\hat{A}$)].

$\hat{x}$ in x-direction, $\hat{y}$ in y-direction

Magnitude = 1
* It gives direction

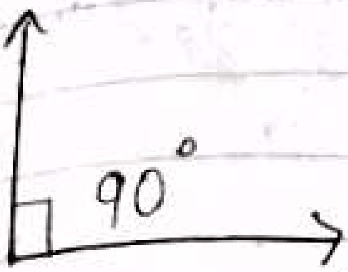
$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$

Parallel shift of vector.

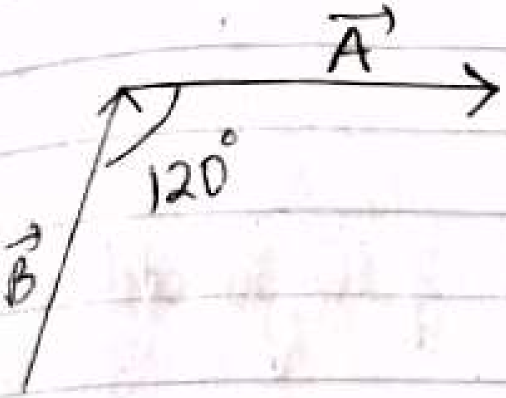
We can shift a vector, as it should be parallel shift and the shift should be on the same body

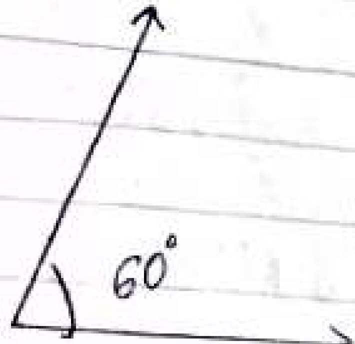
If a force of 10N is applied on a body, it can be shift parallel on the same body.

Angles between two vectors
(Tail to tail or head to head)

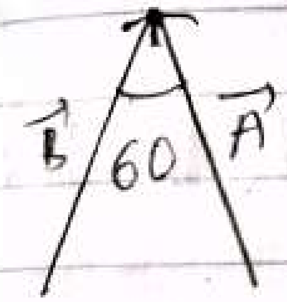
① 

$\theta = 90°$.

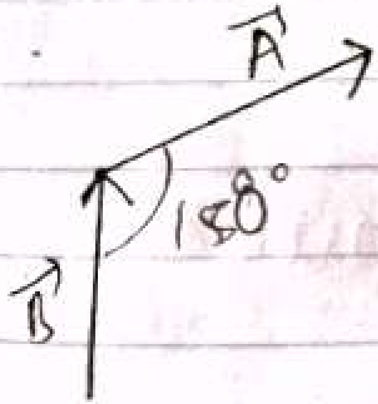
② 

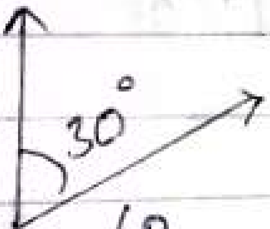


(Parallel shift)

③ 

$\theta = 60°$

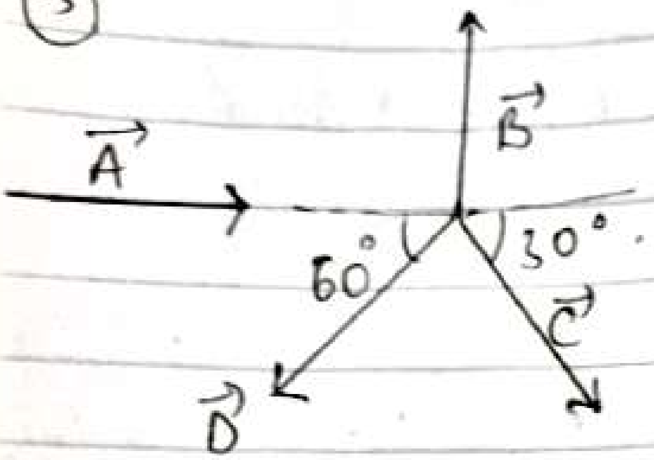
④ 

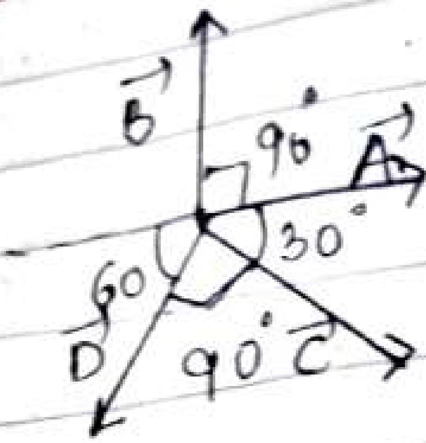


$\theta = 30°$

(Parallel shift

(5)





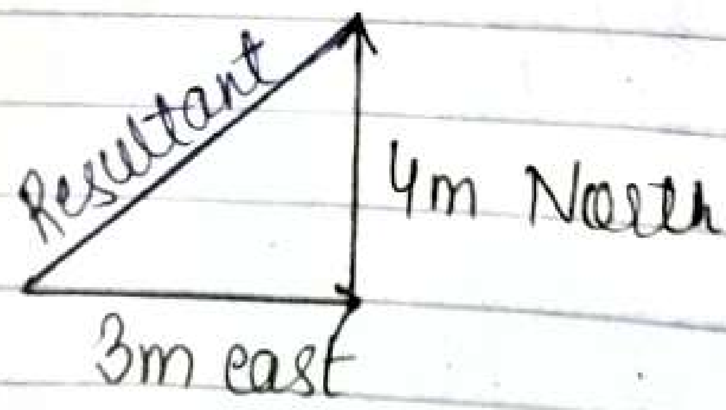
(Tail to Tail)

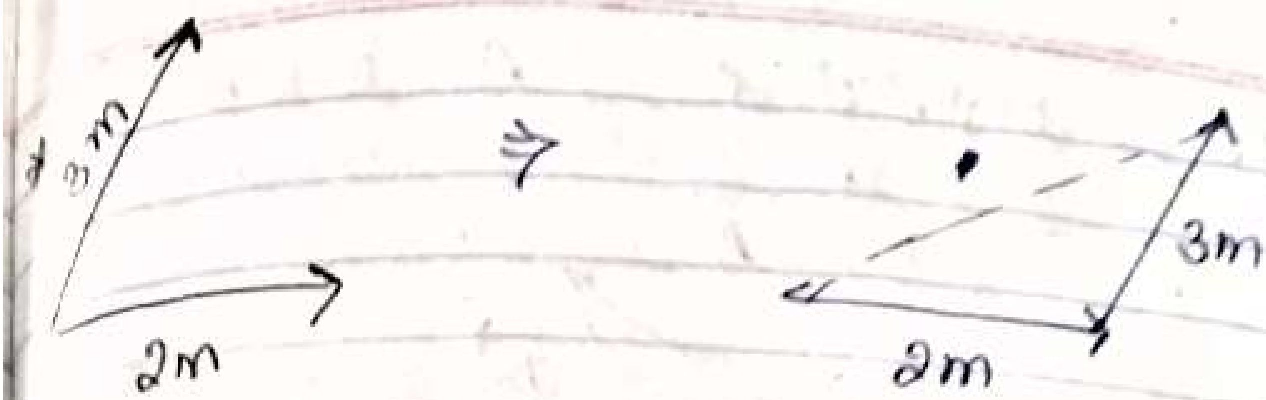
$(0 \leq \theta° \leq 180)$

Smaller angle will be preferred.

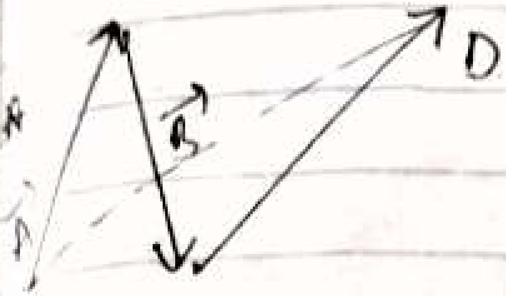
Addition AND SUBTRACTION.

(Head-Tail Method)

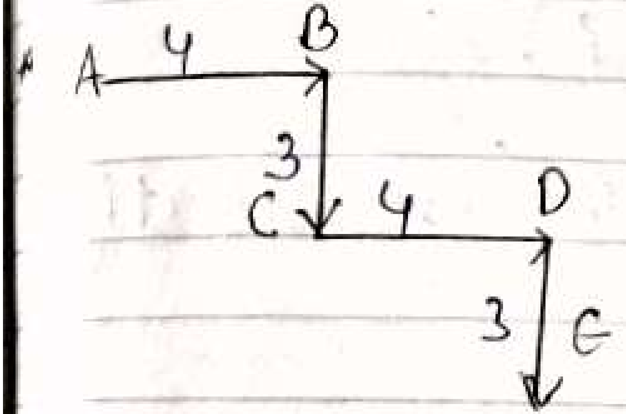




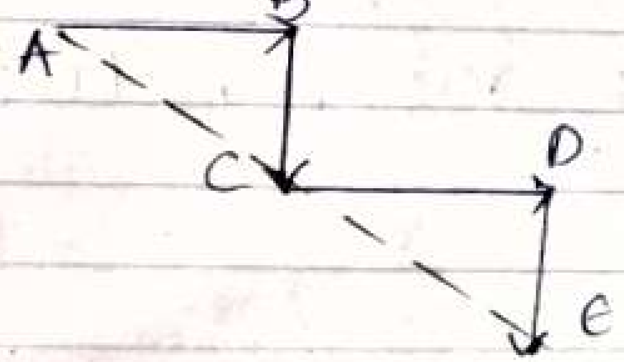
(Parallel shifting)



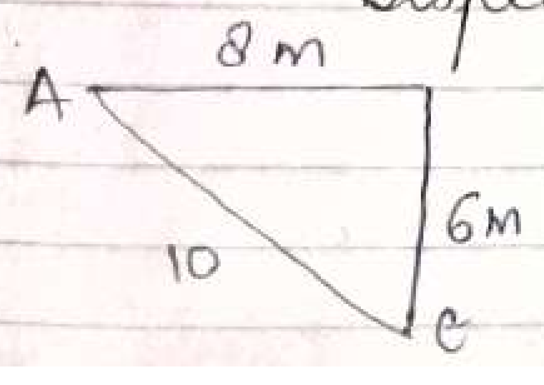
Resultant = Displacement = I^{st} vector tail + last vector head



Displacement = ?

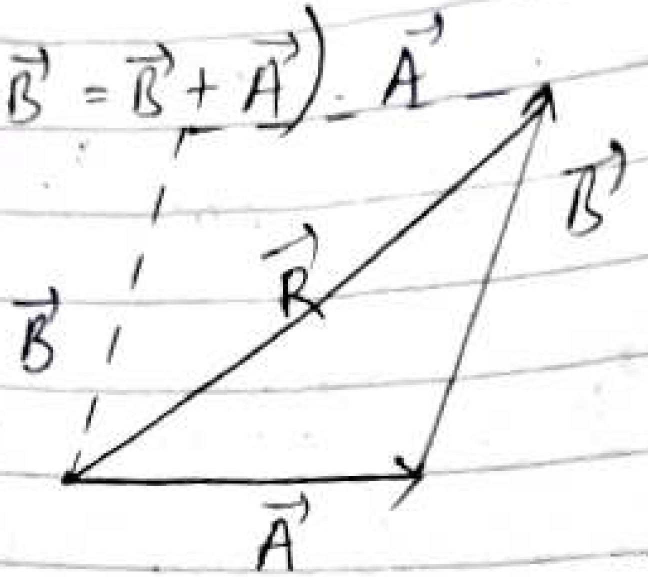


Displacement AE.



Is Vector Commutative $(\vec{A}+\vec{B}=\vec{B}+\vec{A})$

$\Rightarrow (\vec{A}+\vec{B}=\vec{B}+\vec{A})$



We know that,

$$\vec{A}+\vec{B}=\vec{R} \quad \text{—(i)}$$

$$\vec{B}+\vec{A}=\vec{R} \quad \text{—(ii)} \text{ (Parallel shifting)}$$

So, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Hence vectors are commutative

⇒ Vectors can not be added as all scalar quantities are added.

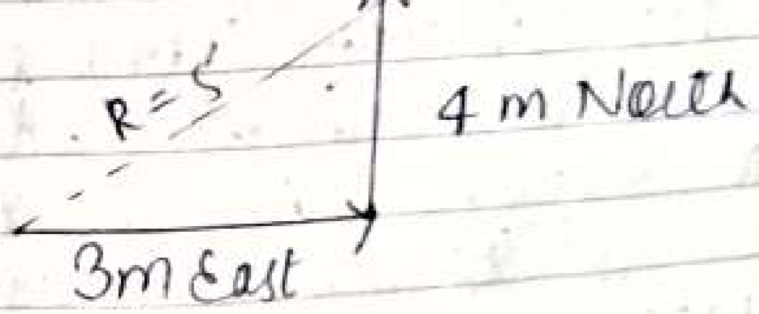
VECTOR ADDITION.

1. Head-Tail Method.
2. Parallelogram Law.
3. Triangle Law.

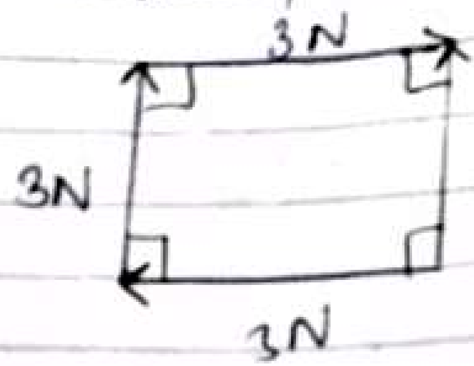
1. Head-tail method

Join tail of next vector with Head of previous vector

3m East + 4m North = R



⇒ Add 3 vectors
3N West, 3N North, 3N East



Resultant

$\vec{R} = 3\ [90^\circ]$

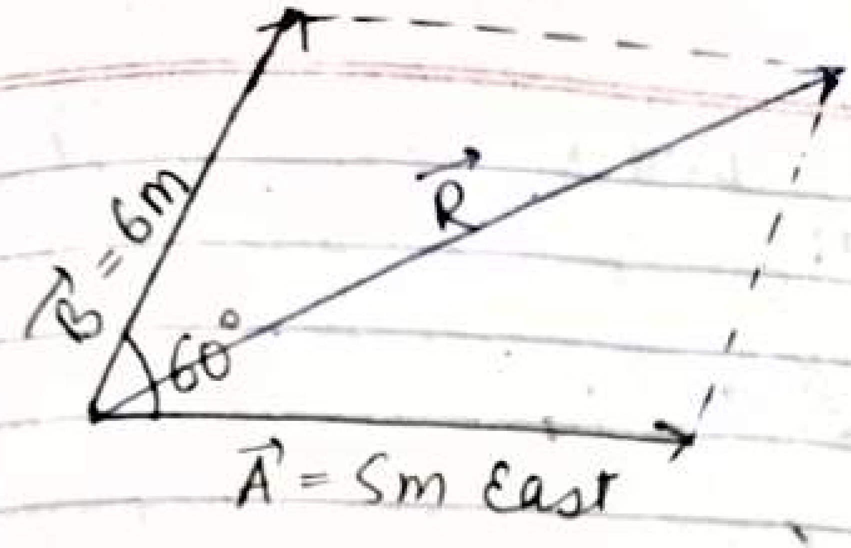
→ 5m east then 5m at 60° from east

This law fails here

<u>Parallelogram Law</u>

It allows us to add any kind of vector

⇒ Join two vectors from tail to tail as the two adjacent sides of parallelogram.., $\vec{A} = 5$ East,
$\vec{B}$, 6m, 60° from East.
(Imagine complete ‖gm)



$\vec{R}$ = diagonal of ||gm from common point.

$\vec{R} = \vec{A} + \vec{B}$.

$R^2 = A^2 + B^2 + 2AB\cos\theta$. (Magnitude)

θ = angle between 2 vectors

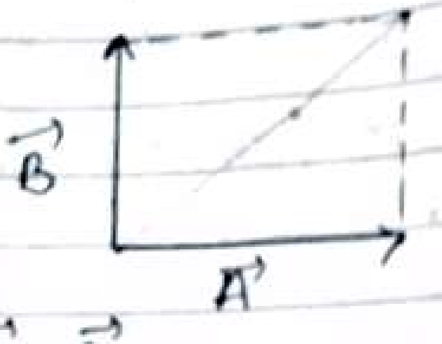
$R^2 = 5^2 + 6^2 + 2 \times 5 \times 6 \times \frac{1}{2}$

$R^2 = 25 + 36 + 30$.

$R = \pm 9$.

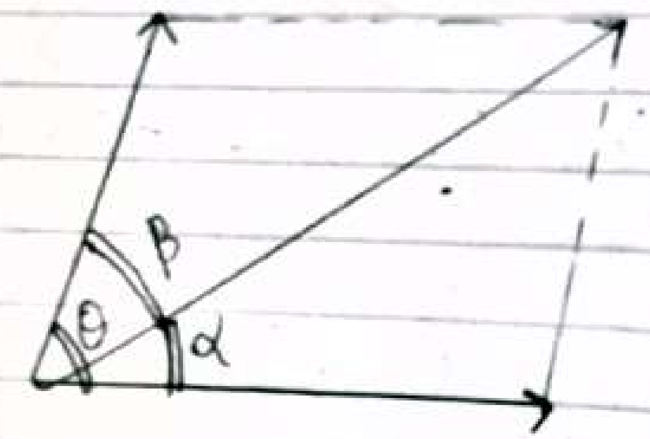
Quest- Add two vectors 6 units, 8 units at 90°

$\vec{A} = 6$, $\vec{B} = 8$, $\theta = 90°$



$\vec{R} = \vec{A} + \vec{B}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$
$R^2 = 36 + 64 + 2 \times 6 \times 8 \times 0$
$R = \pm 10$



$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}.$$

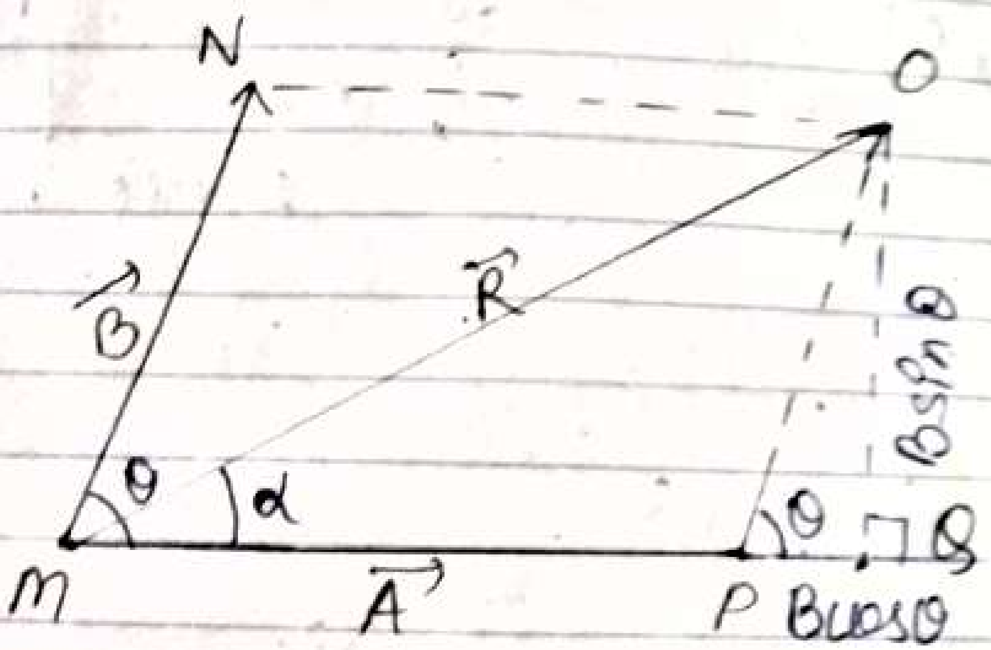
(Direction of resultant)

$\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$ $(\beta = \theta - \alpha)$

$\vec{R}$ direction is from vector $\vec{A}$ (α)

$\vec{R}$ direction from $\vec{B}$ (β)

Derive. $R^2 = A^2 + B^2 + 2AB\cos\theta$



Parallelogram's pair of opp sides is parallel and equal.

ΔPOQ

$\cos\theta = \dfrac{B}{H}$

$$\cos\theta = \frac{PQ}{PO}$$

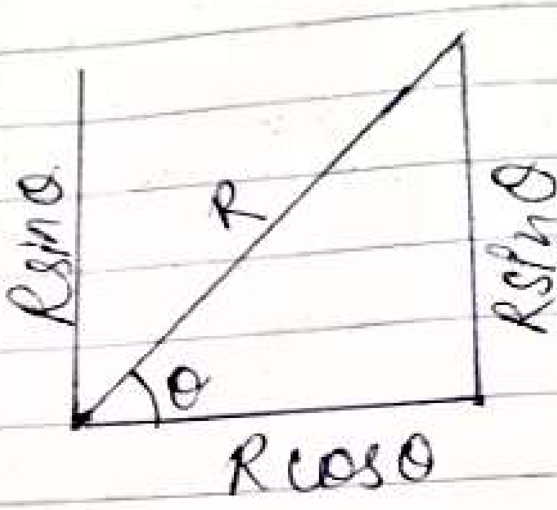
$$PQ = PO\cos\theta$$

$$\boxed{PQ = B\cos\theta}$$

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{OQ}{PO} = \frac{OQ}{B}$$

$$\boxed{OQ = B\sin\theta}$$



In ΔOQM

$$(OM)^2 = (OQ)^2 + (MQ)^2$$

$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$

$$R^2 = A^2 + B^2\sin^2\theta + B^2\cos^2\theta + 2AB\cos\theta$$

$$R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

Direction of Resultant

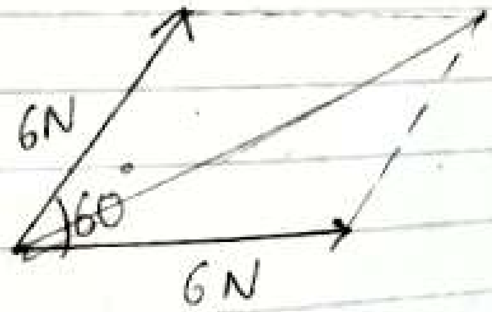
In ΔOQM

$\tan\alpha = \frac{P}{B}$

$\tan\alpha = \frac{OQ}{MQ} = \frac{B\sin\theta}{A+B\cos\theta}$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}}$$

Ques6- Two forces of magnitude 6N each at a point as shown. Find the resultant.



$R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = 36 + 36 + 2 \times 36 \cos 60^\circ$

$R^2 = 108$

$R = 6\sqrt{3}$

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\tan\alpha = \frac{1}{\sqrt{3}}, \alpha = 30^\circ$

If 2 vectors are equal in magnitude the resultant will pass through the angle between them.

Ques6- Two vectors of equal magnitude are added to give resultant, which is of same magnitude as the 2 vectors. Find the angle between them.

$R = A = B = x$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$x^2 = x^2 + x^2 + 2x^2\cos\theta$

$-x^2 = 2x^2\cos\theta$

$\cos\theta = \frac{-x^2}{2x^2} = \frac{-1}{2}$

$\cos\theta = \frac{-1}{2}$

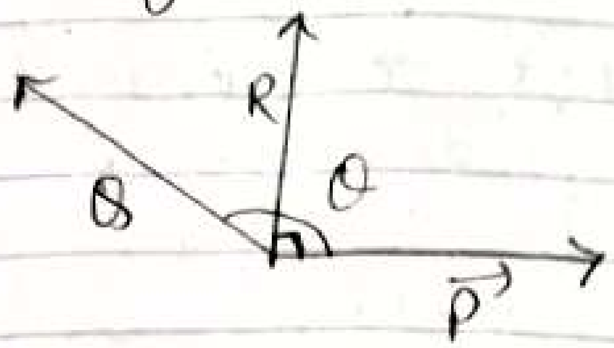
$\boxed{\theta = 120^\circ}$

Ques6- Two vectors P(smaller one) & Q as a sum of 18 and there resultant is 12. The resultant is

⊥ to smaller of two vector. Find the value of P & Q and angle between them.

$P+Q = 18$

$P' + Q' = 12$



$R^2 = A^2 + B^2 + 2AB\cos\theta$

$12^2 = P^2 + Q^2 + 2PQ\cos\theta$.

$12^2 = P^2 + Q^2 + 2P(-P)$

$12^2 = P^2 + Q^2 - 2P^2$.

$12^2 = Q^2 - P^2$

$(Q-P)(Q+P) = 144$

$Q-P(18) = 144$

$Q-P = 8$

$+ \; P+Q = 18$

$2Q = 26$, $\boxed{Q = 13}$.

$12^2 = 13^2 - P^2$

$\boxed{P = 5}$

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}.$$

$$\tan 90^\circ = \frac{Q\sin\theta}{P+Q\cos\theta}.$$

$P + Q\cos\theta = 0$

$Q\cos\theta = -P$.

$Q\cos\theta = -5$

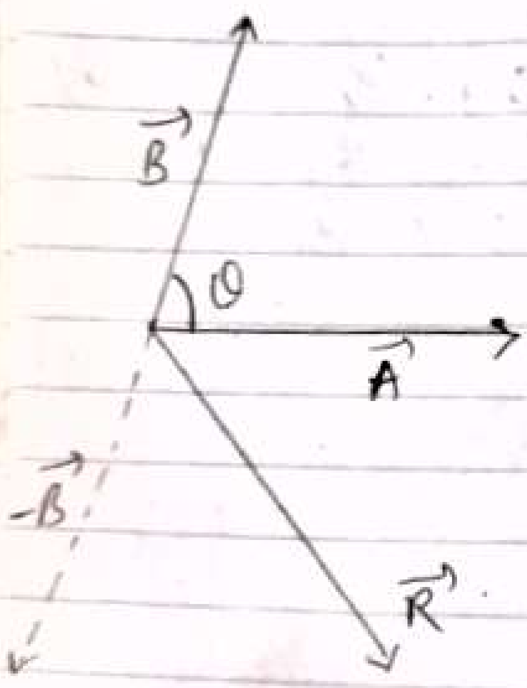
$$\boxed{\cos\theta = \frac{-5}{13}}.$$

Subtraction of Vector

(Means negative of vector (opp in direction

⇒ Vectors can only be added.

⇒ $\vec{A} - \vec{B}$ (×)
⇒ $\vec{A} + (-\vec{B})$ (✓)



$\vec{R} = \vec{A} + (-\vec{B})$

angle $= (180 - \theta)$

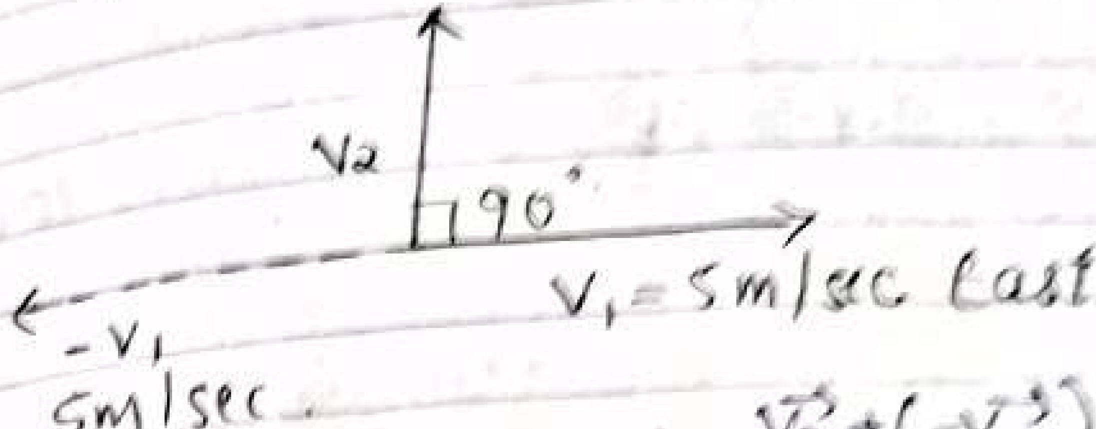
$R^2 = A^2 + B^2 + 2AB\cos\theta$.
$R^2 = A^2 + B^2 + 2AB\cos(180 - \theta)$
$R^2 = A^2 + B^2 + 2AB(-\cos\theta)$

$\cos(180 - \theta) = -\cos\theta$.

$$\boxed{R^2 = A^2 + B^2 - 2AB\cos\theta}$$

Quest- A car runs at 5m/sec East, takes a sharp turn to North and continues at 5m/sec. Find the change in velocity of car.

ΔV = change in velocity $\Rightarrow \vec{V_2} - \vec{V_1}$

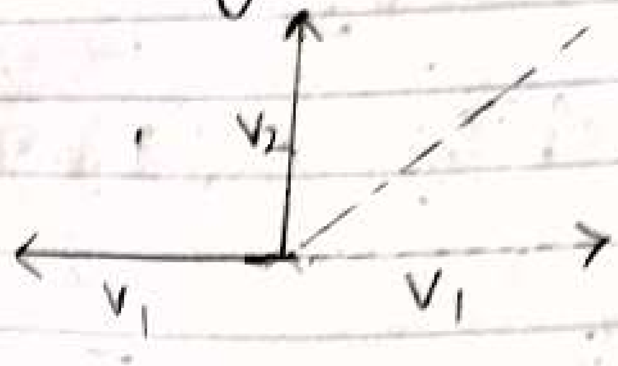


$\vec{V_2} + (-\vec{V_1})$

$R^2 = A^2 + B^2 - 2AB\cos 90°$

$[R = 5\sqrt{2}]$ North west.

Ques8 - A car running at 10m/sec (west) takes a sharp turn towards north and continues at 10m/sec. If it takes 2 sec in turning. find acc of car.



$$\vec{a} = \frac{\vec{V_2} - \vec{V_1}}{t}$$

$R^2 = A^2 + B^2 - 2AB\cos\theta$
$R^2 = 200 - 200\times 0$
$R = 10\sqrt{2}$ NE

$a = \frac{\Delta v}{t} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ NE m/sec^2.

Ques6- A plane moving with velocity v turns by 'θ' angle & its speed remains 'v'. find the change in velocity of plane.

$\vec{v_2} - \vec{v_1}$

Ans:- $(R)^2 = A^2 + B^2 - 2AB\cos\theta$
$(R)^2 = v^2 + v^2 - 2v\times v\cos\theta$.
$(R)^2 = 2v^2 - 2v^2\cos\theta$
$(R)^2 = 2v^2(1-\cos\theta)$
$(R)^2 = 2v^2 . 2\sin^2\theta/2$.

$(R)^2 = 4v^2\sin^2\theta/2$

$R = 2v\sin\theta/2$.

$\left\{ 1-\cos\theta = 2\sin^2\theta/2 \right\}$

Ques- The difference of 2 unit vectors is a unit vector. find the angle between 2 vectors.

$\vec{A} = 1, \vec{B} = 1, R = 1$

$R^2 = A^2 + B^2 - 2AB\cos\theta$

$1^2 = 1^2 + 1^2 - 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$

Ques- The sum and difference are equal in magnitude. Find the angle b/w vectors

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

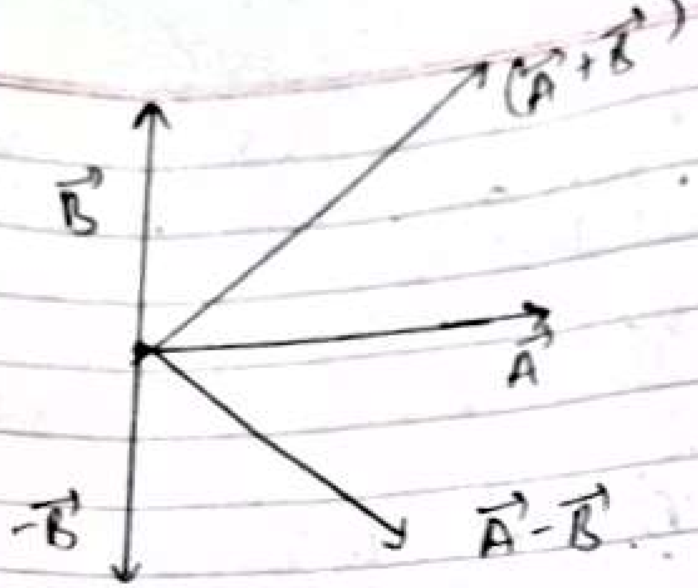
Let $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90°$



Ques6 - 3 vectors $\vec{A}+\vec{B}+\vec{C}=0$, if $|\vec{A}|=12$, $|B|=5$, $|C|=13$.
Find angle between $\vec{A}$ and $\vec{B}$.

$$|\vec{A}+\vec{B}|^2 = |(-C)|^2$$

$$A^2+B^2+2AB\cos\theta = C^2$$
$$144+25+120\cos\theta = 169$$
$$\cos\theta = 0$$
$$\theta = 90^\circ$$

⇒ We have 2 vectors 3 & 4, Their resultant cannot be

(a) 2
(b) 6
✓(c) 8
(d) 4

Max value of any vector

$$R = |A + B|$$

Min value of any vector

$$R = |A - B|$$

Multiplication of Vector

1. Scalar × Vector
2. Vector × Vector = Scalar
3. Vector × Vector = Vector

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$ Cartesian form

$3\vec{A} = 6\hat{i} - 3\hat{j} + 3\hat{k}$

Vector × Vector = Scalar

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{B} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{A} \cdot \vec{B} = C$ (3, 4, 5 . .)

$\Downarrow$ (dot product) $\downarrow$ Scalar

V × V = V

$\vec{A} \times \vec{B} = \vec{C}$ (vector product)

$\Downarrow$

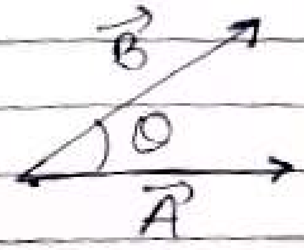
(cross product).

(22) * If the angle between A & B are greater than 90°, the dot product will be -ve
(Vectors ~~with~~ length will always be +ve)

Dot Product

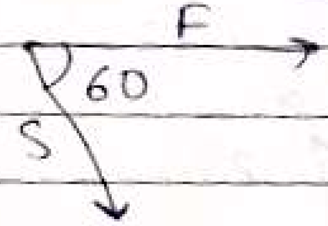
$\vec{A} \cdot \vec{B} = C$.

Work $= \vec{F} \cdot \vec{S}$



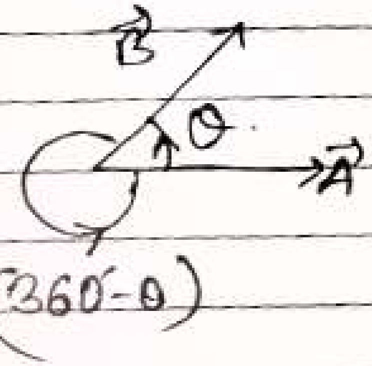
$\vec{A} \cdot \vec{B} = |A| \times |B| \times \cos\theta$.

$\vec{F} = \underline{10N}$ $\quad$ $\vec{S} = \underline{5m}$



Work $= \vec{F} \cdot \vec{S}$
Work $= 25J$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.



(A se B angle)
$\vec{A} \cdot \vec{B} = |A||B|\cos\theta$.

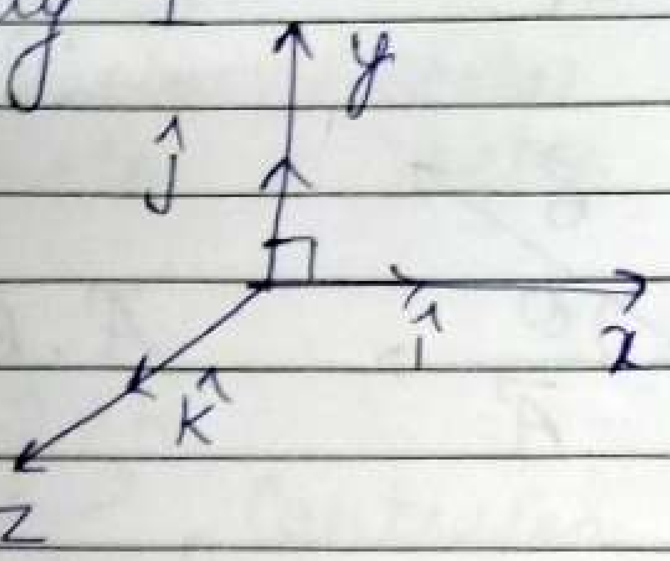
$\vec{B} \cdot \vec{A} = |B||A|\cos(360-\theta)$
(B se A angle)

$\cos(360-\theta) = \cos\theta$

* If $\vec{A} \perp \vec{B}$ $\quad$ $\theta = 90°$
$\vec{A} \cdot \vec{B} = |A||B|\cos 90°$
$\vec{A} \cdot \vec{B} = 0$

Orthogonal unit vectors

mutually $\perp$



$\hat{i}$ = whose mag is 1 and is in the direction of x.

similarly $\hat{j}$ & $\hat{k}$.

$\hat{i}.\hat{j} = 0$; $\hat{i}.\hat{i} = 1$

$\hat{i}.\hat{k} = 0$; $\hat{j}.\hat{j} = 1$

$\hat{j}.\hat{k} = 0$; $\hat{k}.\hat{k} = 1$

$\hat{i}.\hat{i} = |\hat{i}||\hat{i}| \cos 0°$

$\Rightarrow 1.$

Date ___/___/______ Page No.: ______

Quest- $\vec{A} = 2\hat{i} + 3\hat{j}$
$\vec{B} = 4\hat{i} + 5\hat{j}$

find $\vec{A}.\vec{B}$

$\Rightarrow (2\hat{i} + 3\hat{j}).(4\hat{i} + 5\hat{j})$

$\Rightarrow 8(1) + 15(1)$

$A.\vec{B} \Rightarrow 23$

$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$
$\Rightarrow 2 \times 4 + (3 \times 5) + 0 \times 0$
$\Rightarrow 23$

Quest- Find, if
$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$
$\vec{B} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{A}.\vec{B} = 2 - 1 + 3$
$\vec{A}.\vec{B} \Rightarrow 5$

Quest- If a vector $(2\hat{i} + 3\hat{j} + 8\hat{k})$ is $\perp$ to the vector $4\hat{i} - 4\hat{j} + a\hat{k}$, then the value of a is.

$\vec{A}.\vec{B} = 0 \quad (\perp)$

$0 = 8 - 12 + 8a$

$0 = -4 + 8a$

$4 = 8a$

$a = \frac{1}{2}$

Angle between two vectors

$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$

$$\boxed{\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}}$$

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{B} = \hat{i} - \hat{j}$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$

$\vec{A} \cdot \vec{B} = (2 + 1)$

$\Rightarrow 3$

$|A||B| = \sqrt{A_x^2 + A_y^2 + A_z^2}\ \sqrt{B_x^2 + B_y^2 + A_z^2}$

$\Rightarrow \sqrt{4+1+1}\ \sqrt{1+1}$

$\Rightarrow \sqrt{6}\ \sqrt{2}$

$\Rightarrow \sqrt{12}$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\boxed{\cos\theta = \frac{3}{2\sqrt{3}}}$$

Quest- $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$, find θ

$\vec{Q} = \hat{i} - \hat{j}$

$$\cos\theta = \frac{2-1}{\sqrt{6}\,\sqrt{2}}$$

$$\cos\theta \Rightarrow \frac{1}{2\sqrt{3}}.$$

Quest $\vec{R} = \hat{i} + \hat{j}$ (find θ).

$\vec{S} = \hat{i} - \hat{j}$.

$$\cos\theta = \frac{1-1}{\sqrt{2}\ \sqrt{2}} \Rightarrow \frac{0}{\sqrt{4}} \Rightarrow \frac{0}{2} = 0$$

Quest- Find the angle that $\vec{A} = \hat{i} + \hat{j}$ makes with x axes.

$\vec{A} = \hat{i} + \hat{j}$

$\vec{B} = \hat{i}$ (any vector along x-axes) can be $2\hat{i}, 0.5\hat{i}$ ... anything

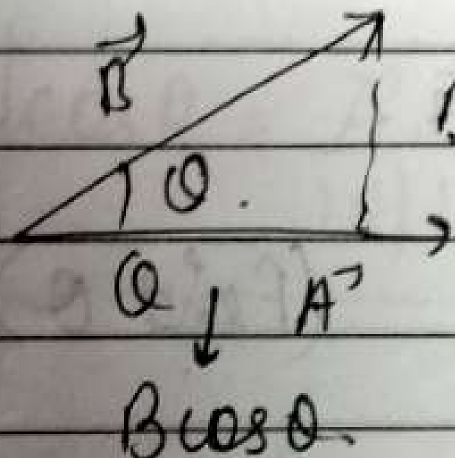
$\cos\theta = \frac{\vec{A}\,\vec{B}}{|A|\,|B|}$

$\cos\theta \Rightarrow \frac{1 * 1 \times 0}{\sqrt{2}\,\sqrt{1}}$

$\cos\theta \Rightarrow \frac{1}{\sqrt{2}}$

$\boxed{\theta = 45^\circ}$

$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta$



$\vec{B}$ projectection on $\vec{A}$

$B\cos\theta$ is along $\vec{A}$.

$\vec{A}\cdot\vec{B} = |\vec{A}|\,(|\vec{B}|\cos\theta)$

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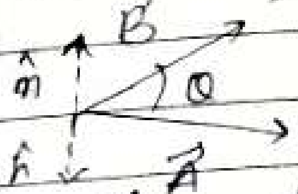
Cross / Vector Product

$\vec{A} \times \vec{B} = \vec{C}$ vector

$\vec{\text{Torque}}$ / moment of force $= \vec{F} \times$ displacement

$\vec{A} \times \vec{B} = \boxed{|A||B|\sin\theta.}\ \hat{n}$ (This is $\perp$ to $\vec{A}$ & $\vec{B}$)

$\hat{n}$ → direction

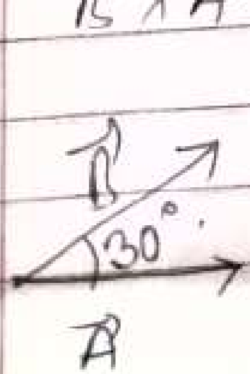


To give direction to vectors we use unit vector

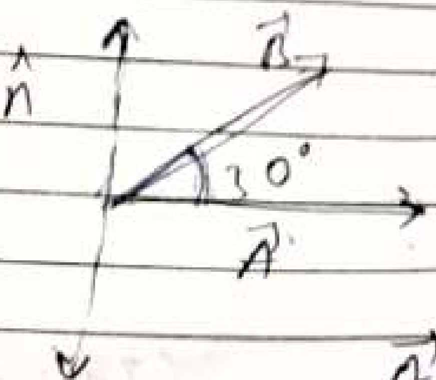
$\vec{C} \perp \vec{A}$
$\vec{C} \perp \vec{B}$.

Ques.⇒ $\vec{A} = 5$, $\vec{B} = 2$, $\theta = 30^\circ$ find $\vec{A} \times \vec{B} = |A||B|\sin\theta$
$\vec{B} \times \vec{A} =$

$\vec{A} \times \vec{B} = 5$
$\vec{B} \times \vec{A} = |B||A|\sin\theta. = 5$



$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$



(Right hand thumb rule)

$\vec{A} \times \vec{B}$ = while curling from $\vec{A}$ to $\vec{B}$ using R.H. Thumb rule the thumb is upward so the $\hat{n}$ will be upward.

$\vec{B} \times \vec{A}$ = while curling from $\vec{B}$ to $\vec{A}$ using R.H. Thumb rule the thumb will be downward

(as ~~[illegible]~~ we will take the smaller angle between the vectors).

$\vec{A} \times \vec{B}$ = 5 upwards
$\vec{B} \times \vec{A}$ = 5 downwards

so, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

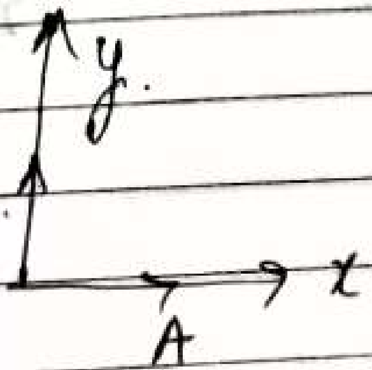
OR Screw Rule.

(Add both the vectors tail to tail; move the screw from $\vec{A}$ to $\vec{B}$, so the direction is upwards)

and move the screw from $\vec{B}$ to $\vec{A}$ so the screw will go downwards.

commutative rule is not valid for cross product.

Quest-



$\vec{A} \times \vec{B}$

what will be the direction of $\vec{A} \times \vec{B}$

upwards (outwards)

$\vec{B} \times \vec{A}$ (inwards) downwards.

Orthogonal unit vectors:

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

~~$\hat{k} \times \hat{i} =$~~

$\hat{i}$

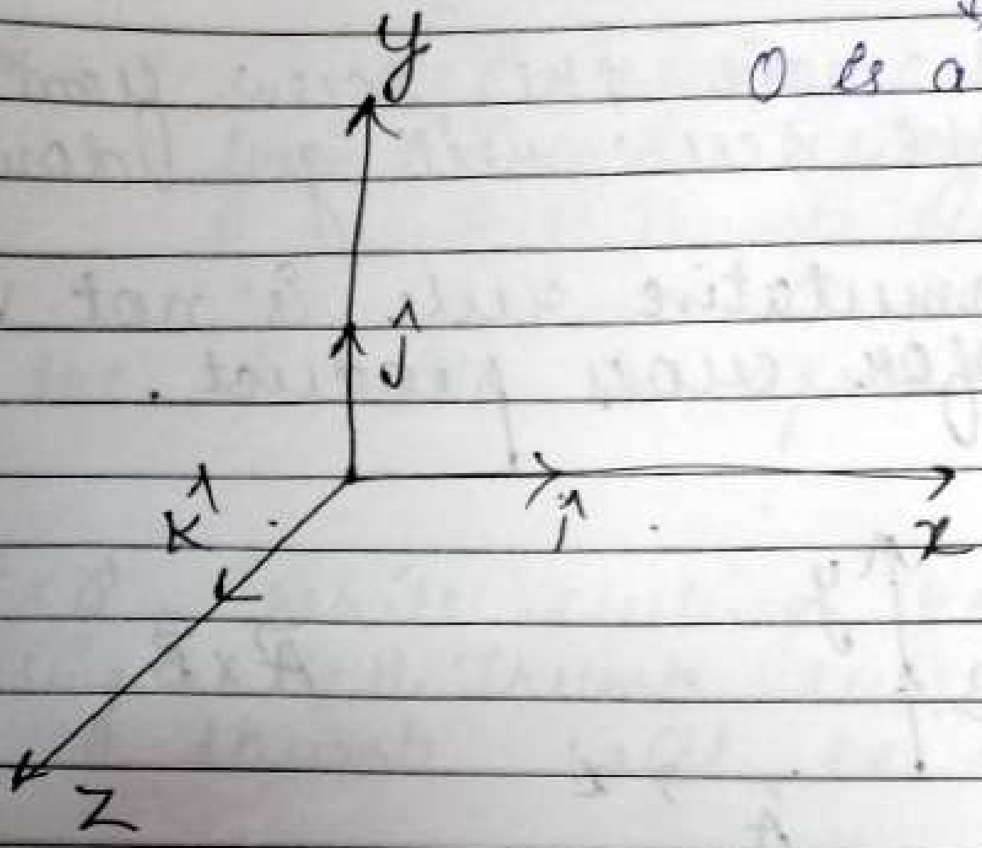
$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0 = \vec{0}$

$\hat{j} \times \hat{j} = |\hat{j}||\hat{j}| \sin 0 = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\Downarrow$ $\vec{0}$

0 is a vector



$\hat{i} \times \hat{j} = |\hat{i}||\hat{j}| \sin 90^\circ$

$\Rightarrow 1 \; \hat{n} \longrightarrow$ (unit vector outwards).

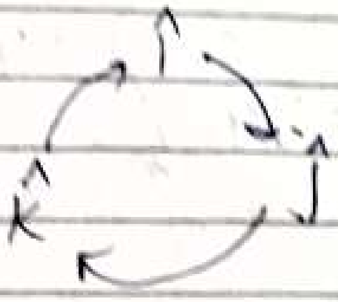
$\Rightarrow \hat{k}$

$\hat{k} \times \hat{i} = \hat{j}$

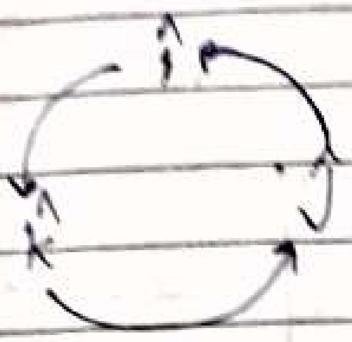
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$



Clockwise = +ve



anticlockwise = -ve

Quest $\vec{A} = 5\hat{i}$

$\vec{B} = 2\hat{k}$

$\vec{A} \times \vec{B} = 5\hat{i} \times 2\hat{k}$

$= -10\hat{j}$

$\vec{B} \times \vec{A} = 10\hat{j}$

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

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Ques6- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

$\vec{B} = 3\hat{i} + 2\hat{j} + 3\hat{k}$.

$\vec{A} \times \vec{B} = 6(0) + 4\hat{k} + 6(-\hat{j})$

$9(-\hat{k}) + 6(0) + 9\hat{i}$

$12\hat{j} + 8(-\hat{i}) + 12(0)$

$\vec{A} \times \vec{B} = \hat{i} + 6\hat{j} - 5\hat{k}$.

Short cuts.

$$\begin{array}{c|ccc|} & \hat{i} & \hat{j} & \hat{k} \\ A & 2 & 3 & 4 \\ B & 3 & 2 & 3 \end{array}$$

$\vec{A} \times \vec{B} = \hat{i}(9-8) - \hat{j}(6-12) + \hat{k}(4-9)$

$\vec{A} \times \vec{B} = 1\hat{i} - (-6)\hat{j} + (-5\hat{k})$

$$\left[\vec{A} \times \vec{B} = \hat{i} + 6\hat{j} - 5\hat{k}\right]$$

Ques6- Find the mag. of $\vec{A} \times \vec{B}$ if $A = 2\hat{i} + \hat{j} - \hat{k}$

and $B = 6\hat{i} + 3\hat{j} - 3\hat{k}$

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$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 6 & 3 & -3 \end{vmatrix}$$

$\Rightarrow \hat{i}(-3+3) - \hat{j}(-6+6) + \hat{k}(6-6)$

$\Rightarrow \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$

$\Rightarrow 0$

$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Mag 6 - If $\vec{A} \times \vec{B} = (3\hat{i} + 2\hat{j} + 4\hat{k})$

$|\vec{A} \times \vec{B}| = \sqrt{3^2 + 2^2 + 4^2}$ (Mag)

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* If $\vec{A} \times \vec{B} = 0$

Either $A = 0$ or $B = 0$

or

$|A||B| \sin\theta = 0$

$\sin\theta = 0$

$\theta = 0°$

$\vec{A} \cdot \vec{B} = 0$

$\theta = \perp$ or $90°$

$\vec{A} \times \vec{B} = 0$

$\theta = 0°$

Quest- $|\vec{A}| = 5$ (Mag)

$|B| = 6$.

$|\vec{A} \times \vec{B}| = 15$ (Mag)

find angle b/w A & B

$|\vec{A} \times \vec{B}| = |A||B| \sin\theta$.

$15 = 5 \times 6 \sin\theta$

$\sin\theta = \frac{1}{2}$, $\theta = 30°$ and $150°$

Find angle between $\vec{A}$ & $\vec{B}$.

$|\vec{A} \times \vec{B}| = |A \cdot B|$

$|A||B| \sin\theta = |A||B| \cos\theta$.

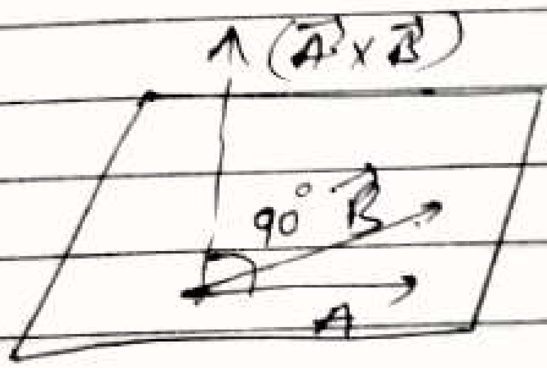
$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\boxed{\theta = 45^\circ}$ or $\frac{\pi}{4} \to 180'$

$A \cdot (\vec{A} \times \vec{B}) = ?$

$\vec{A} \cdot (\vec{A} \times \vec{B})$.



$\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$

$(\perp)$ to $(\vec{A} \times \vec{B})$.

If $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{C} = 0$

then $\vec{A}$ is $\parallel$ to

(a) $\vec{C}$ (c) $\vec{B} \times \vec{C}$

(b) $\vec{B}$ (d) $B \cdot C$

* Vector are which follows llgm law

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Unit Vectors

eg- current is not vector,

⇒ Magnitude = 1

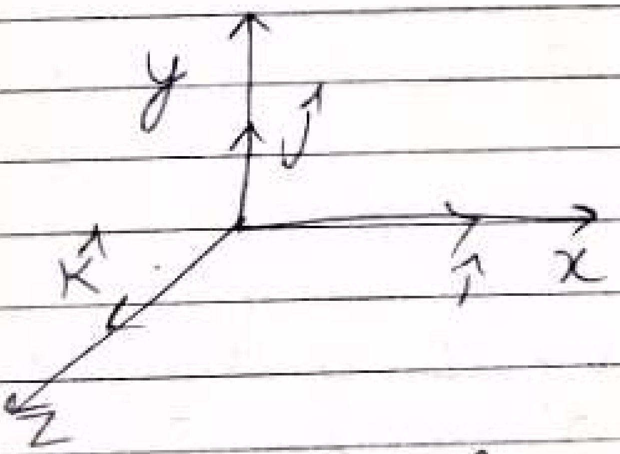
➤ It gives direction

$\vec{A}$ = Magnitude x direction

$\vec{A} = |A| \times \hat{A}$

$\hat{A} = \frac{\vec{A}}{|A|}$

Orthogonal unit vector



Ques- A force 10 N is in x direction. Represent it in vector form

$\vec{F} = 10\hat{i}$

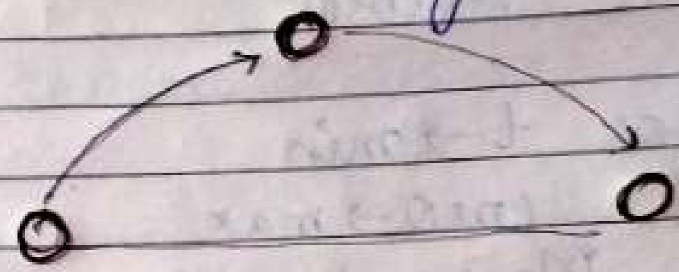
$\boxed{\vec{F} = 10\hat{i}}$

PROJECTILE MOTION

* Actual Meaning of projectile motion is motion under gravity.

* And in syllabus it is Motion in a plane (2-D)

eg:- When a ball is thrown upward with some angle.



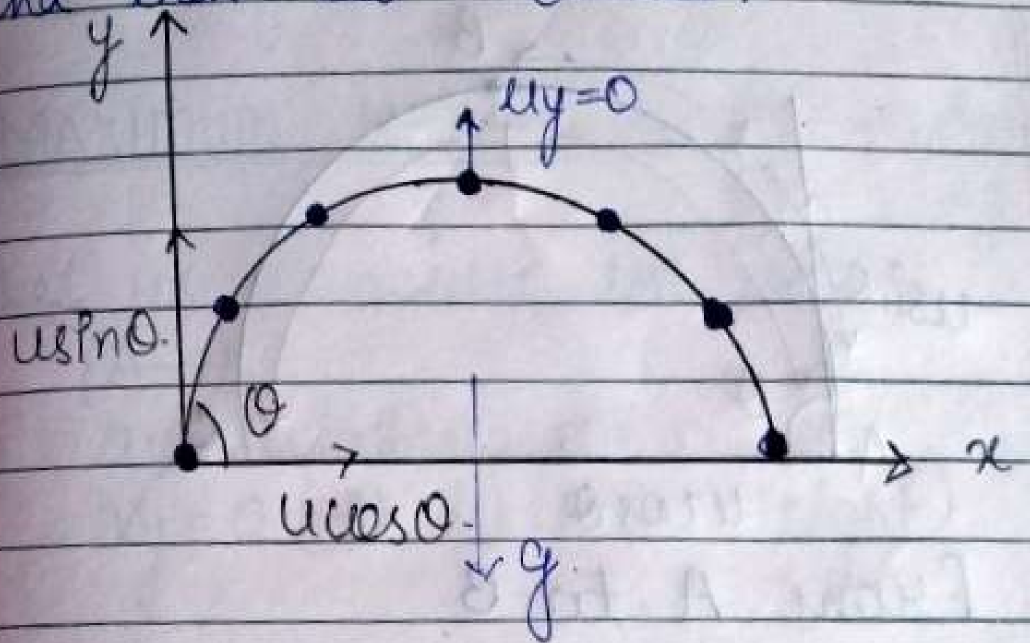
In a projectile Motion,
u is the initial velocity,
θ is the angle of projection,
the motion of an object is in 2 direction, so
Initial velocity in x direction is given by $u_x = u\cos\theta$,
Initial velocity of y direction is given by $u_y = u\sin\theta$

=> Acc. due to gravity in x-direction.
$a_x = 0$

=> Acc. due to gravity in y-direction

$a_y = -g$.

so, x direction velocity is always constant. whereas, velocity of y direction first increases, and at highest point it becomes 0 and then it increases.



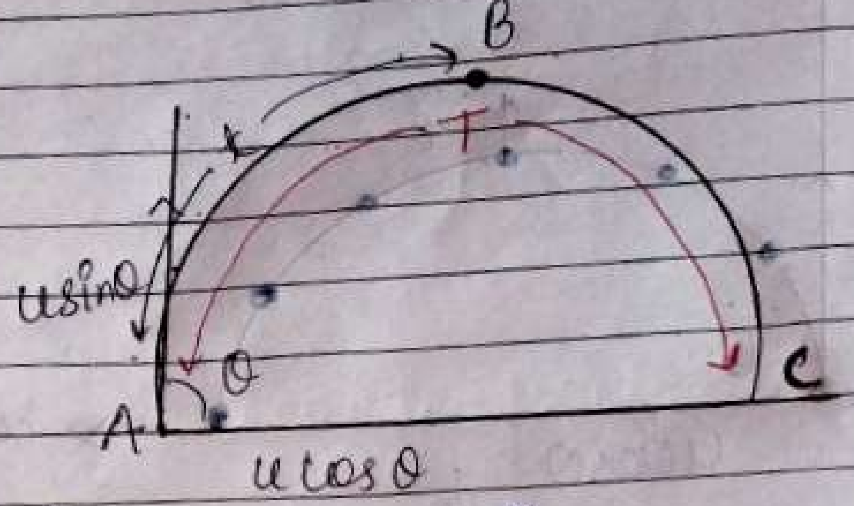
Assuming there is no air resistance, and no friction

In Projectile Motion

⇒ Time of flight is given by **T**
⇒ Max. Height is given by **H**
⇒ Horizontal Range is given by **R**

Time of flight $\left(T = \frac{2u\sin\theta}{g}\right)$

Let us consider motion in y-direction



From A to B.

$u_y = u\sin\theta$,
& we know $a_y = -g$, and

$v_y = 0$ (at B)

Let the object took time (t).

From first equation of motion

$$v = u + at$$
$$0 = u\sin\theta + (-g) \times t$$

$$\left[t = \frac{u\sin\theta}{g}\right] \text{ upto B}$$

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Total time = T

$T = 2t$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$ Time of flight

MAXIMUM HEIGHT $\left(H = \frac{u^2\sin^2\theta}{2g} \right)$

Let us consider the motion in y-direction $(A \to B)$.

$u_y = u\sin\theta$

$v_y = 0$ (At B) (Max Height)

$a_y = -g$

Displacement in Y-direction = H

Using 3rd equation of motion

$$v^2 = u^2 + 2as$$

$$0^2 = (u\sin\theta)^2 + 2(-g)H$$

$$2gH = u^2\sin^2\theta$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

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HORIZONTAL RANGE $\left(R = \frac{u^2 \sin 2\theta}{g}\right)$

Let us consider the motion in x-direction
$(A \rightarrow C)$

$u_x = u\cos\theta$
$a_x = 0$
$s_x = R$
$t = T$

Using second equation of motion

$$s = ut + \frac{1}{2}at^2$$

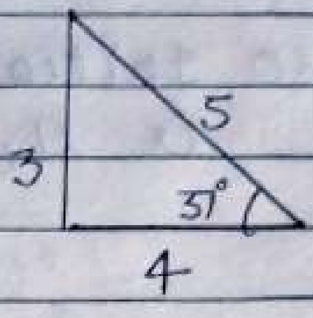
$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$\boxed{R = \frac{u^2 2\sin\theta\cos\theta}{g}}$$

OR

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Ques 1 - A ball is thrown with 5 m/sec at an angle of projection 37°. Find
(i) Time of flight,
(ii) Max. Height
(iii) Horizontal Range

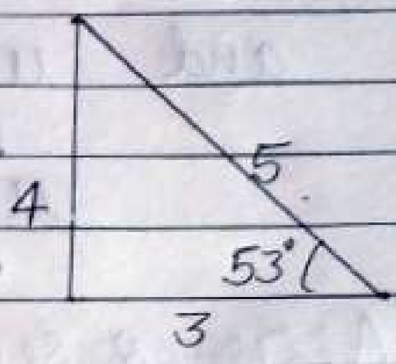


Ans (i) $T = \dfrac{2u\sin\theta}{g}$

$\theta \to 37^\circ$

$T = \dfrac{2 \times 5 \times \sin 37^\circ}{10}$

$T = \dfrac{3}{5} = 0.6$ sec.



(ii) $H = \dfrac{u^2\sin^2\theta}{2g}$

$H = \dfrac{25(\sin 37^\circ)^2}{2 \times 10}$

$H = \dfrac{9}{20}$

(iii) Range $= \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2 2\sin\theta\cos\theta}{g}$

$R = \dfrac{12}{5}$ m $= 2.4$ m

Ques6- A ball is thrown with 50m/sec at angle of projection 37°. find velocity vector and speed of particle after 2 sec of projection.

Ans6- so, Initial velocity in x direction is $u_x = u\cos\theta$

$= 50 \times \frac{4}{5}$

$u_x = 40\ m/sec$;

and $u_y = u\sin\theta$

$u_y = 30 m/sec$.

After 2 sec,

Velocity ~~speed~~ of particle in x-direction is $v_x = 40 m/sec$.

Velocity ~~speed~~ of particle in y-direction is v_y

$u_y = 30\ m/sec$.

$a_y = -10$

$t = 2sec$.

$v_y = u_y + at$

$v_y = 30 + (-10)2$

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$v_y = 30 - 20$

$v_y = 10\ m/sec.$

In vector form.

$$\vec{v} = 40\hat{j} + 10\hat{j}$$

speed $|v| = \sqrt{(40)^2 + (10)^2}$

$$= \sqrt{1600 + 100}$$

$$|v| = \sqrt{1700}$$

Ques6- For a projectile motion from ground to ground, $H = R$, find angle of projection

Ans:-

$$H = R$$

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\sin^2\theta}{2} = \sin 2\theta$$

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

$$\frac{\sin\theta}{2} = 2\cos\theta$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1} 4.$$

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Ques8- For a projectile Motion from ground to ground. If Max. Ht is $30\sqrt{3}$ m and Horizontal Range is 120m. Find initial velocity and angle of projection (u, θ)

Ans :-

$H = 30\sqrt{3}$
$R = 120$.

$$\frac{u^2 \sin^2\theta}{2g} = 30\sqrt{3} \quad \text{—— (i)}$$

$$\frac{u^2 \sin 2\theta}{g} = 120 \quad \text{or} \quad \frac{2u^2 \sin\theta\cos\theta}{g} = 120 \quad \text{—— (ii)}$$

Dividing eq (i) by (ii) we get

$$\frac{u^2 \sin^2\theta}{2g} \times \frac{g}{2u^2 \sin\theta\cos\theta} = \frac{30\sqrt{3}}{120}$$

$$\frac{\sin\theta}{4\cos\theta} = \frac{30\sqrt{3}}{120}$$

$$\tan\theta = \sqrt{3}.$$

$$\boxed{\theta = 60^\circ}$$

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$$\frac{u^2\sin^2\theta}{2g} = 30\sqrt{3}$$

$$\frac{u^2\sin^2 60^\circ}{2g} = 30\sqrt{3}$$

$$\frac{u^2\, 3}{2\times 10\times 4} = 30\sqrt{3}$$

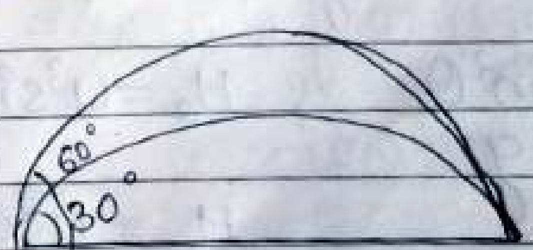
$$u^2 = \frac{80\times 30\sqrt{3}}{3}$$

$$u^2 = \frac{2400}{\sqrt{3}}$$

$$u = \sqrt{\frac{2400}{\sqrt{3}}}$$

RANGE : Range is same for two different angles of projection if u is same

θ°, '$90-\theta$', suppose angles are 30° and 60°.



PROOF

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}} \quad (\text{For } \theta)$$

For $(90-\theta)$

$$R = \frac{u^2 \sin 2(90-\theta)}{g}$$

$$R = \frac{u^2 \sin(180-2\theta)}{g}$$

$$\sin(180-\theta) = \sin\theta$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

For two angle of projection, Range is same, and vertical max Heights are different i.e. H_1 & H_2. Find relation between H_1, H_2 & R.

For θ, Height is H_1
For $(90-\theta)$, Height is H_2

$$H_1 = \frac{u^2 \sin^2\theta}{2g}, \quad H_2 = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$H_2 = \frac{u^2 \cos^2\theta}{2g}$$

$$u\sin\theta = \sqrt{2gH_1}$$

$$u\cos\theta = \sqrt{2gH_2}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 2 \sin\theta \cos\theta}{g}$$

$$= (u\sin\theta)(u\cos\theta) \times \frac{2}{g}$$

$$R = \sqrt{2gH_1}\,\sqrt{2gH_2} \times \frac{2}{g}$$

$$R = 2\sqrt{4H_1H_2}$$

$$R = 4\sqrt{H_1H_2}$$

When Range will be Maximum

$$R = \frac{u^2 \sin 2\theta}{g}$$

Range depends upon speed and θ

If $\theta \to 45^\circ$
$R \to max$.

When $\frac{dR}{d\theta} = 0$ (Range will be Maximum)

$$\frac{dR}{d\theta} = \frac{u^2 \cos 2\theta \times 2}{g}$$

$$\frac{u^2 \cos 2\theta \times 2}{g} = 0$$

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$$\cos 2\theta = 0$$
$$2\theta = 90^\circ$$
$$\theta = 45^\circ$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 90}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Max Range is $\frac{u^2}{g}$ when $\theta = 45^\circ$

Path of projectile motion is parabolic

⇒ condition of parabola

$$y^2 \propto x$$
$$y^2 = 4aX$$

When in an equation y & x are relatable that equation is called as equation of trajectory (Path)

time (t) is not Included.

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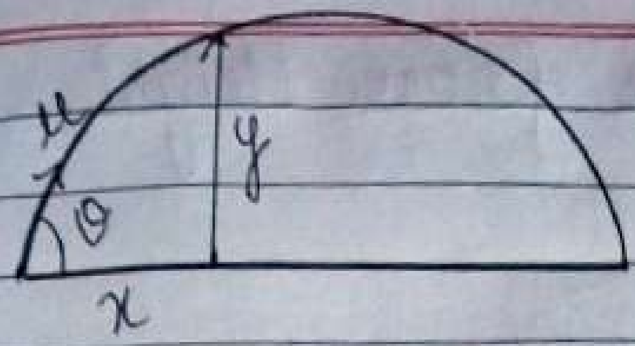


x

$u_x = u\cos\theta$

$a_x = 0$

t

$s = ut + \frac{1}{2}at^2$

$x = u\cos\theta\, t$

$t = \frac{x}{u\cos\theta}$

y

$u_y = u\sin\theta$

$a_y = -g$

t

$s = ut + \frac{1}{2}at^2$

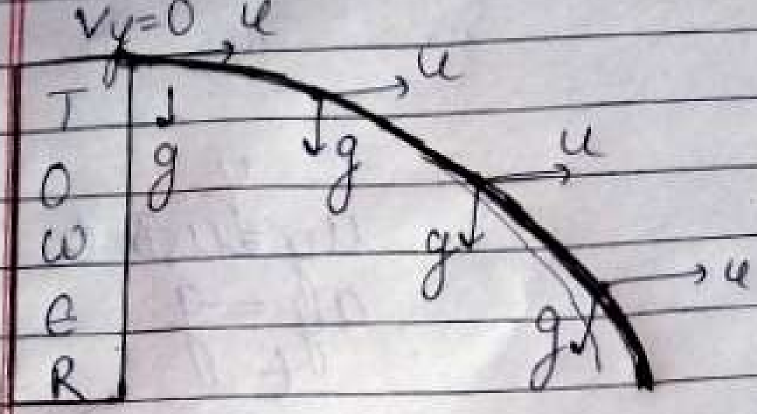
$y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\cdot\frac{x^2}{u^2\cos^2\theta}$$

$$\boxed{y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}}$$ (Parabola)

This is called Equation of Trajectory.

Projectile from Height.

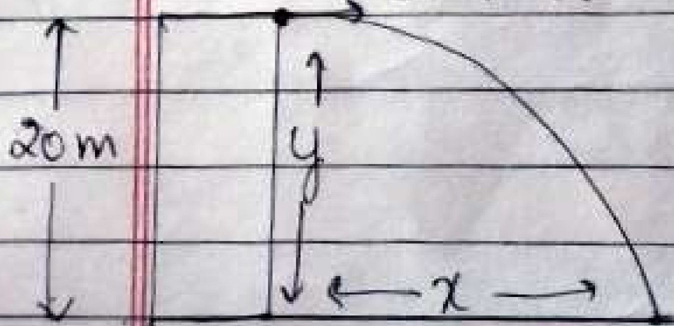


2-D motion

Velocity in x-direction remains constant whereas velocity in y direction increases (due to g)

Path → Parabolic path (projectile from height)

Ques- consider a building of 20m, from the top of building we threw a particle horizontally at the speed of 5m/sec. Find the time taken by particle to reach ground.



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Ans6- It is a 2-D motion (covers a distance in x & y direction).

Let us consider whole motion in y-direction.

$S_y = -20m$ (displacement)

$u_y = 0$, $a_y = -g = -10$

$s = ut + \frac{1}{2}at^2$

$-20 = 0 + \frac{1}{2}(-10)t^2$

$\boxed{t = 2\,sec}$

⇒ Find the Horizontal range

Let us consider motion in x-direction

$S_x = R$
$u_x = 5m/sec$
$a_x = 0$, $t = 2sec$

$s = ut + \frac{1}{2}at^2$

$R = 5 \times 2 + 0$
$\boxed{R = 10m}$

⇒ Find the velocity of ball when it hits the ground

$\vec{v} = \vec{v}_x + \vec{v}_y$

Y- direction

$u_y = 0$

$a_y = -10$

$v_y = ?$

$v = u + at$

$v = -20 \text{ m/sec}$

$\vec{v} = \vec{v}_x + \vec{v}_y$

$\boxed{\vec{v} = 5\hat{i} - 20\hat{j}}$

⇒ Speed

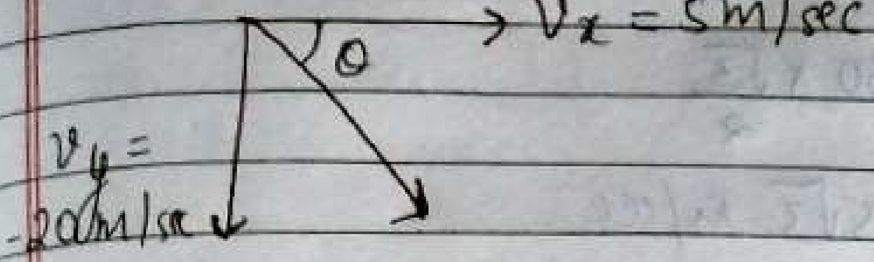
Speed = Mag. of velocity

Speed $= \sqrt{(5)^2 + (-20)^2}$

Speed $= \sqrt{25 + 400}$

Speed $= \sqrt{425}$

→ Find θ.



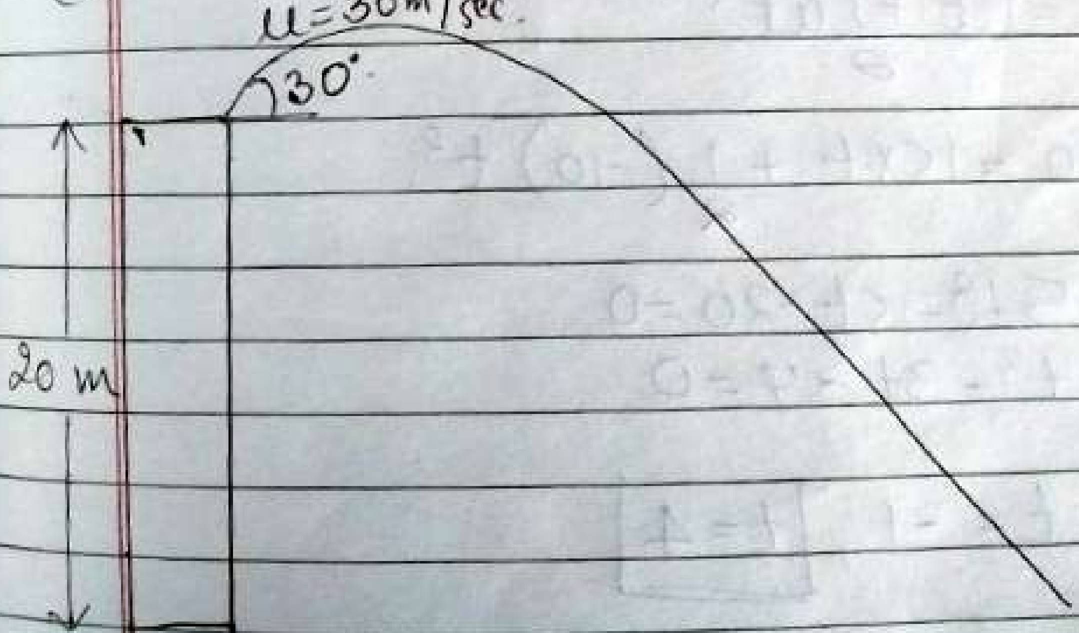
$$\tan\theta = \frac{v_y}{V_x}$$

$$\tan\theta = \frac{20}{5}$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1} 4$$

Ques - If a ball is thrown at 30° angle from a (20m) tower with initial speed 30m/sec.
(i) Find the time taken to reach ground.



In x-direction

$u_x = u\cos 30^\circ$

$u_x = 30 \times \frac{\sqrt{3}}{2}$

$u_x = 15\sqrt{3}$ m/sec.

In y-direction

$u_y = u\sin\theta$

$u_y = 15$ m/sec.

(i) Let us take motion in y-direction

$S_y = -20$ (shortest between initial and final position)

$a_y = -10$

$S = ut + \frac{1}{2}at^2$

$-20 = 15 \times t + \frac{1}{2}(-10)t^2$

$5t^2 - 15t - 20 = 0$

$t^2 - 3t - 4 = 0$

$t = -1, \boxed{t = 4}$

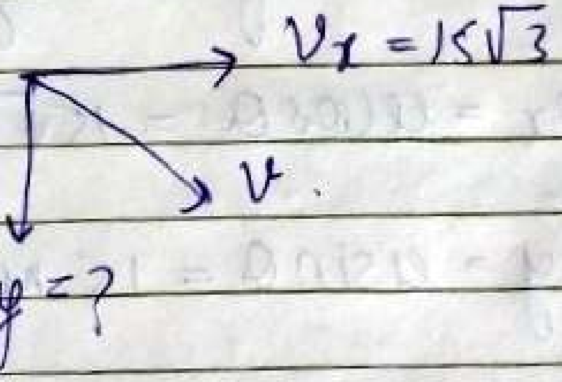
(ii) find Horizontal range

$u_x = 15\sqrt{3}$,
$a_x = 0$
$t = 4$ sec.
$S_x = R$

$S = ut + \frac{1}{2}at^2$

$\boxed{R = 60\sqrt{3}\ m}$

(iii) final velocity just before hitting grand



y-direction
$u_y = 15$
$a_y = -10$
$t = 4$ sec.
$v_y = ?$

$v = u + at$
$v = -25$ m/sec.

$\boxed{v_y = -25\ m/sec}$

Velocity $= 15\sqrt{3}\hat{i} - 25\hat{j}$.

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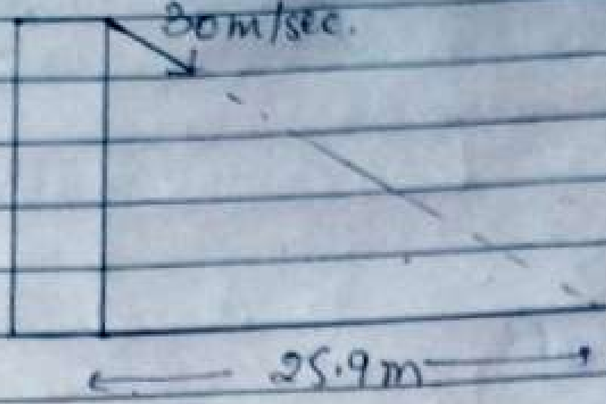
Ques:-



Find the height of tower?

Ans:- Component of velocity.

$$u_x = u\cos\theta = 15\sqrt{3} \text{ m/sec}$$

$$u_y = u\sin\theta = 15 \text{ m/sec}.$$

X-direction Motion

$u_x = 15\sqrt{3}$

$a = 0$

$S_x = 25.9$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$$t = \frac{25.9}{15\sqrt{3}}$$

Y-direction Motion

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$s = ut + \frac{1}{2}at^2$

$s = 15 \times \frac{2\sqrt{3}}{15\sqrt{3}} + \frac{1}{2}(-10) \times t^2$

EQUATION OF TRAJECTORY.

$\left\{ y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \right\}$

$y = x\tan\theta - \frac{gx^2 \sin\theta}{2u^2\cos^2\theta\sin\theta}$

$y = x\tan\theta - \frac{gx^2\tan\theta}{u^2 2\sin\theta\cos\theta}$

$y = x\tan\theta - \frac{x^2\tan\theta}{\frac{u^2\sin 2\theta}{g}}$

$y = x\tan\theta - \frac{x^2\tan\theta}{R}$

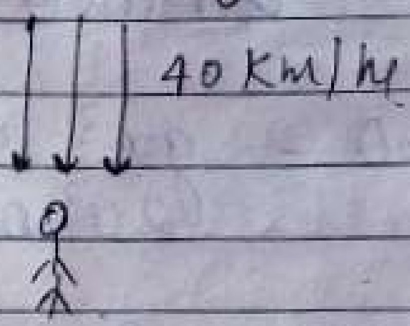
$\left\{ y = x\tan\theta\left(1 - \frac{x}{R}\right) \right\}$.

This is also equation of trajectory.

Relative Velocity 2-D, Rain Man Problem, Umbrella Woman Problem.

$\boxed{v_{AB} = v_{AG} - v_{BG}}$ 2-D

Ques- The speed of rain is 40 km/hr. A man is standing on ground. What is the speed of Rain as seen by man



Ans-
$$v_{RM} = v_{RG} - v_{MG}$$

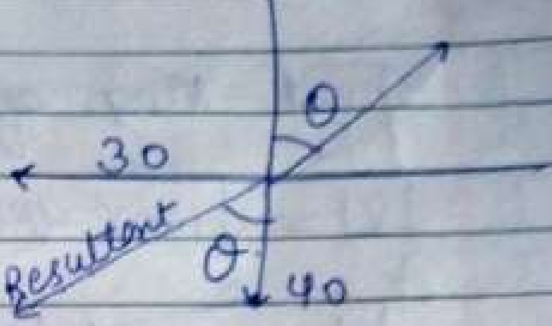
$$v_{RM} = -40\hat{j} - 0$$

$$v_{RM} = -40\hat{j}$$ (The man will open its umbrella opp. to the direction of rain).

⇒ Now the man is in a cycle with speed 30 km/hr at what angle should he bend his umbrella from vertical,

$v_{RM} = v_{RG} - v_{MG}$

$v_{RM} = -40\hat{j} - 30\hat{i}$

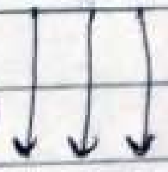


$\tan\theta = \frac{30}{40} = \frac{3}{4}$

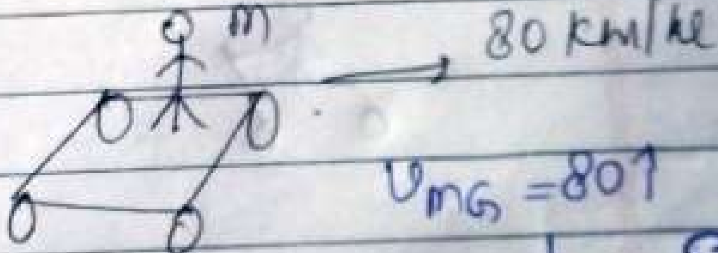
$\theta = 37^\circ$.

The man should hold the umberella 30° from vertical position

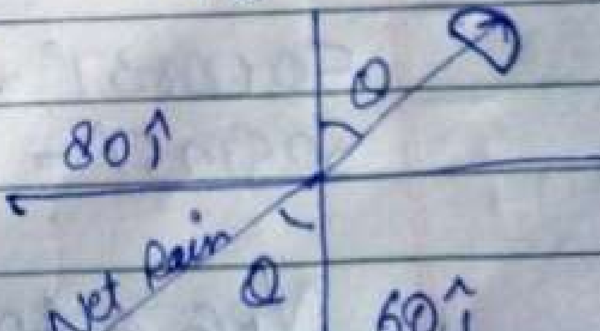
Ques 8- If the speed of rain is 60km/hr vertically. A man is in trolley of speed 80km/hr, At what angle will the man open's its umberella.



$v_{RG} = -60\hat{j}$



$v_{MG} = 80\hat{i}$



$v_{RM} = v_{RG} - v_{MG}$

$v_{RM} = -60\hat{j} - 80\hat{i}$

$\tan\theta = \frac{80}{60} = \frac{4}{3}$

$\theta = 53^\circ$

⇒ What is the speed of rain w.r.t man.

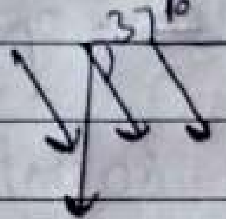
speed = Mag. of velocity

$|v| = \sqrt{(60)^2 + (80)^2}$

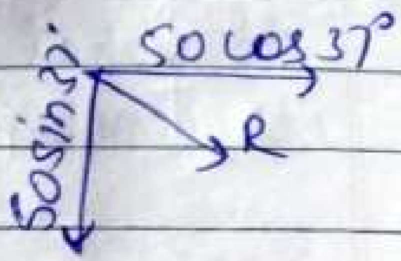
$|v| = 100 \cancel{km/hr}$

$|v| = 100 \text{ km/hr}$

Ques6 If rain is at 37° angle & $v_{RG} = 50$ km/hr a man in cycle at a speed of 40 km/hr v_{mG} (+x direction). find the position of umberalla.



$v_{RG} = 50$ km/hr



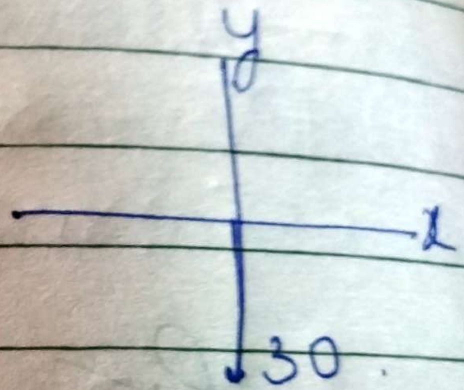
$50\cos 37° = 40$
$50\sin 37° = 30$

$v_{RG} = 40\hat{i} - 30\hat{j}$

$V_{Rm} = V_{RG} - V_{mG}$

$= 40\hat{i} - 30\hat{j} - 40\hat{i}$

$V_{Rm} = -30\hat{j}$



Umberalla should be verticallly upward.