Progressions

Exercise 6.1

Q. 1 A. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

The taxi fare after each km when the fare is Rs. 20 for the first km and rises by Rs. 8 for each additional km.

Answer : Given that price for 1st km is Rs. 20 so that is first term 'a' of the series.

And also we have been given that for every km after 1st km is Rs. 8

So, for 2 km it will be 20 + 8 = 28

Similarly, 3 km = 28 + 8 + 8 = 36

In the same way it adds on for another km

If we put taxi fares in series for each km. It will look like this,

20, 28, 36, 44, 52 ……

On seeing the above series we can conclude that it is in A.P.

∵ They have same common difference between two consequent numbers (d = 8).

Q. 1 B. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Answer : Let V be the initial volume of air that is present in the cylinder.

They have said that a vacuum removes $\frac{1}{4}$ of the air.

Let initial volume (v) = 1 litre

After first removal of air, the remaining will be V_1

$V_1 = 1 - \frac{1}{4} \Rightarrow \frac{3}{4}$

In second attempt it removes $\frac{1}{4}$ of the $\frac{3}{4}$ air remaining.

That means they remove $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ of air.

So, $V_2 = \frac{3}{4} - \frac{3}{16} \Rightarrow \frac{9}{16}$

After third attempt $V_3 = \frac{81}{256}$ of air will be remaining.

So, the series will be look like

$1, \frac{3}{4}, \frac{9}{16}, \frac{81}{256}, \ldots \ldots$

The above series is NOT looking like A.P.

∵ Their common difference is not same.

Q. 1 C. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

The cost of digging a well, after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

Answer : Given that cost for digging 1st meter is Rs. 150 so that is first term 'a' of the series.

And also we have been given that for every meter after 1st meter digging is Rs. 50

So, for 2 meter it will be 150 + 50 = 200

Similarly, 3 meter = 150 + 50 + 50 = 250

In the same way it adds on for every another meter digging

If we put cost of digging in series for each meter. It will look like this,

150, 200, 250, 300, ……

On seeing the above series we can conclude that it is in A.P.

∵ They have same common difference between two consequent numbers (d = 50).

Q. 1 D. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8% per annum.

Answer : Given that initially 10000 is in the account, so it is first term ‘a’.

Every year 8% compound interest will be paid for that.

That means they will pay 8% of 10000 for the first year.

$$\Rightarrow \frac{8}{100} \times 10000 = 800 \text{ will be paid.}$$

After first year 800 will be added to account.

So the amount will be become 10800.

Since it is compound interest every year principal amount will be updated.

So, they will pay 8% of 10800 in the second year.

$$\Rightarrow \frac{8}{100} \times 10800 = 864 \text{ will be paid.}$$

After first year 864 will be added to account.

So the amount will be become 11664.

So, the series will be 10000, 10800, 11664 …..

Since the common difference is not same.

The series is not in A.P

Q. 2 A. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

a = 10, d = 10

Answer : Given that first term (a) = 10

Common difference (d) = 10

The first four terms of the A.P is given by

a, a+d, a+2d, a+3d

⇒ 10, 10+10, 10+2×10, 10+3×10

⇒ 10, 20, 30, 40

Q. 2 B. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

a = -2, d = 0

Answer : Given that first term (a) = -2

Common difference (d) = 0

Series will be same as First Term. Since common difference (d) in the series is 0.

The first four terms in the series is -2,-2,-2,-2.

Q. 2 C. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

a = 4, d = -3

Answer : Given that first term (a) = 4

Common difference (d) = -3

The first four terms of the A.P is given by

a, a+d, a+2d, a+3d

⇒ 4, 4+ (-3), 4+ 2(-3), 4+ 3(-3)

⇒ 4, 1, -2, -5 .

Q. 2 D. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

a = 1, d = 1/2.

Answer :

Given that first term (a) = -1

Common difference (d) = $\frac{1}{2}$

The first four terms of the A.P is given by

a, a+d, a+2d, a+3d

$\Rightarrow -1, -1+\frac{1}{2}, -1+2\times\frac{1}{2}, -1+3\times\frac{1}{2}$

$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}$

Q. 2 E. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

a = -1.25, d = -0.25

Answer : Given that first term (a) = -1.25

Common difference (d) = -0.25

The first four terms of the A.P is given by

a, a+d, a+2d, a+3d

⇒ -1.25, -1.25+ (-0.25), -1.25+2 × (-0.25), -1.25+3 ×(-0.25)

⇒ -1.25, -1.5, -1.75, -2.

Q. 3 A. For the following APs, write the first term and the common difference.

3, 1, -1,-3

Answer : In the above series first term (a) is 3

Common difference (d) is given by 2nd term - 1st term

⇒ d = 1 -3

⇒ d = -2

Q. 3 B. For the following APs, write the first term and the common difference.

-5, -1, 3, 7

Answer : In the above series first term (a) is -5

Common difference (d) is given by 2nd term - 1st term

⇒ d = -1 – (-5)

⇒ d = 4

Q. 3 C. For the following APs, write the first term and the common difference.

$\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$

Answer : In the above series first term (a) is $\frac{1}{3}$

Common difference (d) is given by 2nd term - 1st term

$\Rightarrow d = \frac{5}{3} - \frac{1}{3}$

$\Rightarrow d = \frac{4}{3}$

Q. 3 D. For the following APs, write the first term and the common difference.

0.6, 1.7, 2.8, 3.9

Answer : In the above series first term (a) is 0.6

Common difference (d) is given by 2nd term - 1st term

⇒ d = 1.7 -0.6

⇒ d = 1.1

Q. 4 A. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

2, 4, 8, 16.

Answer : For a series to be in AP, the common difference (d) should be Equal.

D_1 = second term – first term = 4 – 2 = 2

D_2 = Third term - Second term = 8 – 4 = 4

Since common difference is not equal the above series is not in AP

Q. 4 B. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

2, 5/2, 3, 7/2

Answer : For a series to be in AP, the common difference (d) should be Equal.

d_1 = second term – first term = $\frac{5}{2}$ – 2 = 0.5

d_2 = Third term - Second term = 3 – $\frac{5}{2}$ = 0.5

Since common difference is same the above series is in AP.

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by = a + (5-1)d = a + 4d = 2 + 4(0.5) = 4

6^{th} term is a + (6-1)d = a + 5d = 2 + 5(0.5) = 4.5

7^{th} term is a + (7-1)d = a + 6d = 2 + 6(0.5) = 5.

Q. 4 C. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

-1.2, -3.2, -5.2, -7.2, . . .

Answer : For a series to be in AP, the common difference (d) should be Equal.

D_1 = second term – first term = -3.2 – (-1.2) = -2

D_2 = Third term - Second term = -5.2 – (-3.2) = -2

Since common difference is equal the above series is in AP

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by

= a + (5-1)d = a + 4d = -1.2 + 4(-2) = -9.2

6^{th} term is a + (6-1)d = a + 5d = -1.2 + 5(-2) = -11.2

7^{th} term is a + (7-1)d = a + 6d = -1.2 + 6(-2) = -13.2

Q. 4 D. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

-10, -6, -2, 2, . . .

Answer : For a series to be in AP, the common difference (d) should be

Equal.

D_1 = second term – first term = -6 – (-10) = 4

D_2 = Third term - Second term = -2 – (6) = 4

Since common difference is equal the above series is in AP

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by

= a + (5-1)d = a + 4d = -10 + 4(4) = 6

6^{th} term is a + (6-1)d = a + 5d = -10 + 5(4) = 10

7^{th} term is a + (7-1)d = a + 6d = -10 + 6(4) = 14

Q. 4 E. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

3, 3 + √ 2, 3 + 2√ 2, 3 + 3√ 2 ……

Answer : For a series to be in AP, the common difference (d) should be

Equal.

D_1 = second term – first term = 3 + √ 2 - 3 = √ 2

D_2 = Third term - Second term = 3 + 2√ 2 - 3 + √ 2 = √2

Since common difference is equal the above series is in AP

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by

= a + (5-1)d = a + 4d = 3 + 4(√2) = 3 + 4√2

6^{th} term is a + (6-1)d = a + 5d = -10 + 5(√2) = 3 + 5√2

7^{th} term is a + (7-1)d = a + 6d = -10 + 6(√2) = 3 + 6√2.

Q. 4 F. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

0.2, 0.22, 0.222, 0.2222, ...

Answer : For a series to be in AP, the common difference (d) should be

Equal.

D_1 = second term – first term = 0.22 -0.2 = 0.02

D_2 = Third term - Second term = 0.222 – 0.22 = 0.002

Since common difference is not equal the above series is not in AP

Q. 4 G. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

0, -4, -8, -12, . . .

Answer : For a series to be in AP, the common difference (d) should be

Equal.

d_1 = second term – first term = -4 – 0 = -4

d_2 = Third term - Second term = -8 – (-4) = -4

Since common difference is same the above series is in AP.

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by = a + (5-1)d = a + 4d = 0 + 4(-4) = -16

6^{th} term is a + (6-1)d = a + 5d = 0 + 5(-4) = -20

7^{th} term is a + (7-1)d = a + 6d = 0 + 6(-4) = -24.

Q. 4 H. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

$\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}$

Answer : For a series to be in AP, the common difference (d) should be

Equal.

$$d_1 = \text{second term} - \text{first term} = \frac{-1}{2} - \frac{-1}{2} = 0$$

$$d_2 = \text{Third term - Second term} = \frac{-1}{2} - \frac{-1}{2} = 0$$

Since common difference is same the above series is in AP.

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

$$5^{th} \text{ term will be given by} = a + (5-1)d = a + 4d = \frac{-1}{2} + 4(0) = \frac{-1}{2}$$

$$6^{th} \text{ term is } a + (6-1)d = a + 5d = \frac{-1}{2} + 5(0) = \frac{-1}{2}$$

$$7^{th} \text{ term is } a + (7-1)d = a + 6d = \frac{-1}{2} + 6(0) = \frac{-1}{2}.$$

Q. 4 I. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

1, 3, 9, 27, . . .

Answer : For a series to be in AP, the common difference (d) should be

Equal.

D_1 = second term – first term = 3 - 1 = 2

D_2 = Third term - Second term = 9 – 3 = 7

Since common difference is not equal the above series is not in AP

Q. 4 J. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

a, 2a, 3a, 4a..

Answer : For a series to be in AP, the common difference (d) should be

Equal.

d_1 = second term – first term = 2a – a = a

d_2 = Third term - Second term = 3a – 2a = a

Since common difference is same the above series is in AP.

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by = a + (5-1)d = a + 4d = a + 4(a) = 5a

6^{th} term is a + (6-1)d = a + 5d = a + 5(a) = 6a

7^{th} term is a + (7-1)d = a + 6d = a + 6(a) = 7a.

Q. 4 K. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

a, a^2, a^3, a^4

Answer : For a series to be in AP, the common difference (d) should be Equal.

D_1 = second term – first term = $a^2 - a = a^2 - a$

D_2 = Third term - Second term = $a^3 - a^2 = a^3 - a^2$

Since common difference is not equal the above series is not in AP

Q. 4 L. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$

Answer : He above series can be re written as

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$

$\Rightarrow \sqrt{2}, \sqrt{(2^2 \times 2)}, \sqrt{(3^2 \times 2)}, \sqrt{(4^2 \times 2)}$

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots..$

For a series to be in AP, the common difference (d) should be

Equal.

d_1 = second term – first term = $2\sqrt{2} - \sqrt{2} = \sqrt{2}$

d_2 = Third term - Second term = $3\sqrt{2} - 2\sqrt{2}$ =

Since common difference is same the above series is in AP.

The next three terms will be the 5^{th}, 6^{th}, 7^{th}.

5^{th} term will be given by

$a + (5-1)d = a + 4d = \sqrt{2} + 4(\sqrt{2}) = 5\sqrt{2} = \sqrt{50}$

6^{th} term is $a + (6-1)d = a + 5d = \sqrt{2} + 5(\sqrt{2}) = 6\sqrt{2} = \sqrt{72}$

7^{th} term is $a + (7-1)d = a + 6d = \sqrt{2} + 6(\sqrt{2}) = 7\sqrt{2} = \sqrt{98}$.

Q. 4 M. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$

Answer : For a series to be in AP, the common difference (d) should be Equal.

D_1 = second term – first term = $\sqrt{3} - \sqrt{6} = \sqrt{3}(1 - \sqrt{2})$

D_2 = Third term - Second term = $\sqrt{6} - \sqrt{9} = \sqrt{3}(\sqrt{2} - 3)$

Since common difference is not equal the above series is not in AP

Exercise 6.2

Q. 1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP:

S. No.	a	d	n	a_n
(i)	7	3	8	. . .
(ii)	-18	. . .	10	0
(iii)	. . .	-3	18	-5
(iv)	-18.9	2.5	. . .	3.6
(v)	3.5	0	105	. . .

Answer : I. Given first term (a) = 7

Common difference (d) = 3

n = 8 ,

a_n = ?

we know that $a_n = a + (n-1)d$

So, we have to find a_8

$a_8 = a + (8-1)d = 7 + 7 \times 3 = 28$

II. Given first term (a) = -18

Common difference (d) = ?

n = 10, $a_n = 0$

$a_{10} = a + (10-1)d$

$0 = -18 + 9 \times d$

d = 2

III. Given

Common difference (d) = -3

n = 18, $a_n = -5$

first term (a) = ?

$a_8 = a + (18-1)d$

$-5 = a + 17\times(-3)$

$a = -46$

IV. Given first term (a) = -18.9

Common difference (d) = 2.5

n = ?

$a_n = 3.6$

we know that $a_n = a + (n-1)d$

$3.6 = -18.9 + (n-1)2.5$

$22.5 = (n-1)2.5$

$9 = n-1$

$n = 10$

V. Given first term (a) = 3.5

Common difference (d) = 0

n = 105 ,

a_n = ?

we know that $a_n = a + (n-1)d$

$a_n = 3.5 + (105-1)0$

$a_n = 3.5$

S. No.	a	d	n	a_n
(i)	7	3	8	**28**
(ii)	-18	**2**	10	0
(iii)	**-46**	-3	18	-5
(iv)	-18.9	2.5	**10**	3.6
(v)	3.5	0	105	**3.5**

Q. 2 A. Find the

30thterm of the A.P. 10, 7, 4 ……

Answer : Given

First term (a) = 10

Common difference (d) = 2nd – 1st terms

d = 7 – 10 = -3

we need to find the 30th term

we know that $a_n = a + (n-1)d$

so n = 30

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = -77$

Q. 2 B. Find the

11thterm of the A.P. : -3, $\frac{-1}{2}$, 2, …

Answer : Given

First term (a) = -3

Common difference (d) = 2nd – 1st terms

$$d = -3 - \left(\frac{-1}{2}\right) = 2.5$$

we need to find the 11^{th} term

we know that $a_n = a + (n-1)d$

so here, n = 11

$a_{11} = -3 + (11-1)\ (2.5)$

$a_{11} = 22$

Q. 3 A. Find the respective terms for the following APs.

$a_1 = 2$, $a_3 = 26$, find a_2

Answer : Given that $a_1 = 2$ & $a_3 = 26$

That means $a_3 = a_1 + (3-1)d$

$\Rightarrow 26 = 2 + 2d$

$\Rightarrow 24 = 2d$

$\Rightarrow d = 12$

$\therefore a_2 = a_1 + (2-1)d$

$\Rightarrow a_2 = 2 + 12 = 14$

Q. 3 B. Find the respective terms for the following APs.

$a_2 = 13$, $a_4 = 3$, find a_1, a_3

Answer : Given that $a_2 = 13$ & $a_4 = 3$

$a_2 = a_1 + (2-1)d$

$13 = a_1 + d \Rightarrow \underline{1}$

$a_4 = a_1 + (4-1)d$

$3 = a_1 + 3d \Rightarrow \underline{2}$

By subtracting the both equations

$13 – 3 = (a_1 + d) - (a_1 + 3d)$

$10 = -2d$

$d = -5$

By substituting in 1 equations we get a_1

$13 = a_1 + (-5)$

$18 = a_1$

$a_3 = a_1 + (3-1)d$

$= 18 + 2 \times (-5)$

$= 8.$

Q. 3 C. Find the respective terms for the following APs.

$a_1 = 5, a_4 = 9\frac{1}{2}$, find a_2, a_3

Answer : Given that $a_1 = 5$ & $a_4 = 9.5$

$a_4 = a_1 + (4-1)d$

$9.5 = a_1 + 3d$

Substituting a_1 in above equation

$9.5 = 5 + 3d$

$4.5 = 3d$

$d = 1.5$

now, $a_2 = a_1 + (2-1)d$

$= 5 + 1.5$

$= 6.5$

$a_3 = a_1 + (3-1)d$

$= 5 + 2\times 1.5$

= 8

Q. 3 D. Find the respective terms for the following APs.

$a_1 = 4$; $a_6 = 6$. Find a_2, a_3, a_4, a_5

Answer : Given that $a_1 = 4$

And $a_6 = 6$

$a_6 = a_1 + (6-1)d$

$6 = 4 + 5d$

$D = 0.4$

To find a_2:

$a_2 = a_1 + (2-1)d$

$= 4 + 0.4$

$= 4.4$

To find a_3:

$a_3 = a_1 + (3-1)d$

$= 4 + 2\times0.4$

$= 4.8$

To find a_4:

$a_4 = a_1 + (4-1)d$

$= 4 + 3\times0.4$

$= 5.2$

To find a_5:

$a_5 = a_1 + (5-1)d$

$= 4 + 4\times0.4$

= 5.6

Q. 3 E. Find the respective terms for the following APs.

$a_2 = 38, a_6 = -22$, find a_1, a_3, a_4, a_5

Answer : Given that $a_2 = 38$ & $a_6 = -22$

$a_2 = a_1 + (2-1)d$

$38 = a_1 + d \Rightarrow \underline{1}$

$a_6 = a_1 + (6-1)d$

$-22 = a_1 + 5d \Rightarrow \underline{2}$

By subtracting the both equations

$38 – (-22) = (a_1 + d) - (a_1 + 5d)$

$60 = -4d$

$d = -15$

By substituting in $\underline{1}$ equations we get a_1

$38 = a_1 + (-15)$

$53 = a_1$

To find a_3:

$a_3 = a_1 + (3-1)d$

$= 53 + 2\times(-15)$

$= 23$

To find a_4:

$a_4 = a_1 + (4-1)d$

$= 53 + 3\times(-15)$

$= 8$

To find a_5:

$a_5 = a_1 + (5-1)d$

$= 53 + 4\times(-15)$

$= -7$

Q. 4. Which term of the AP : 3, 8, 13, 18, ….. , is 78

Answer : Given first term (a) = 3

Common difference (d) = $n_2-n_1 = 8-3 = 5$

$a_n = 78$

$\Rightarrow a_n = a + (n-1)d$

$78 = 3 +(n-1)\ 5$

$75 = (n-1)\ 5$

$(n-1) = 15$

$n = 16$

Therefore 78 is 16^{th} term of the series.

Q. 5 A. nd the number of terms in each of the following APs :

7, 13, 19, . . ., 205

Answer : Given first term (a) = 7

Common difference (d) = $n_2-n_1 = 13 - 7 = 6$

$a_n = 205$

$\Rightarrow a_n = a + (n-1)d$

$205 = 7 +(n-1)\ 6$

$198 = (n-1)\ 6$

$(n-1) = 33$

n = 34

Therefore 205 is 34^{th} term of the series.

Q. 5 B. Find the number of terms in each of the following APs :

18, $15\frac{1}{2}$, 13 …. , -47

Answer : Given first term (a) = 18

Common difference (d) = $n_2-n_1 = 15\frac{1}{2} - 18 = 2.5$

$a_n = -47$

$\Rightarrow a_n = a + (n-1)d$

$-47 = 18 +(n-1)\ 2.5$

$65 = (n-1)\ 2.5$

$(n-1) = 26$

n = 27

Therefore -47 is 27^{th} term of the series

Q. 6. Check whether, -150 is a term of the AP: 11, 8, 5, 2 ….

Answer : Given first term (a) = 18

Common difference (d) = $n_2-n_1 = 8 – 11 = -3$

Let us consider -150 is a term in above series.

Then, $a_n = -150$

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-161 = (n-1)(-3)$

$$n-1 = \frac{161}{3}$$

$$n = \frac{164}{3}$$

Since n is not an integer the given number is not in the series.

Q. 7. Find the 31st term of an AP whose 11thterm is 38 and the 16th term is 73

Answer : Given

11thterm = 38

⇒ a + (11-1)d = 38 → 1

16th term = 73

⇒ a + (16-1)d = 73 → 2

By subtracting the both equations we will get ‘d’

(a +10d) – (a+15d) = 38 -73

-5d = -35

d = 7

By substituting “d” in equation 1

a +10d = 38

a + 10×7 = 38

a = -32

a_{31} = a + (31-1) d

= -32 + 30×7

= 178

Q. 8. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Answer : Given

$a_n = 0$, n = ?

3rd term = 4

$\Rightarrow a + (3-1)d = 4 \rightarrow \underline{1}$

9th term = 73

$\Rightarrow a + (9-1)d = -8 \rightarrow \underline{2}$

By subtracting the both equations we will get 'd'

$(a +2d) – (a+8d) = 4 – (-8)$

$-6d = 12$

$d = -2$

By substituting "d" in equation $\underline{1}$

$a +2d = 4$

$a + (-2)2 = 4$

$a = 8$

$a_n = a+(n-1)d$

$0 = 8+(n-1)(-2)$

$-8 = (n-1)(-2)$

$4 = n-1$

$n = 5$

$\therefore$ 0 is the 5th term in the series.

Q. 9. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Answer : Sum of the first n term is given by $(s_n) = \frac{n}{2}$ [2a + (n-1) d]

The sum of first 7 terms $S_7 = 49 \rightarrow 7^2$

$S_7 = \frac{7}{2}$ [2×a+(7-1)d]

$49 = \frac{7}{2}$ [2a+6d]

49 = 7[a+3d]

7 = a + 3d ⇒ 1

The sum of first 17 terms $S_{17} = 289 \rightarrow 17^2$

$S_{17} = \frac{17}{2}$ [2×a+(17-1)d]

$289 = \frac{17}{2}$ [2a+16d]

289 = 17[a+8d]

17 = a + 8d ⇒ 2

By subtracting both the equations

17 – 7 = a + 8d – (a+3d)

10 = 5d

d =2

∴ common difference (d) = 2

Substituting‘d’ in equation 1

7 = a + 3d

7 = a + 3(2)

a = 1

∴ Sum of the first n term is given by $(s_n) = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{n}{2}$ [2×1 + (n-1)2]

= n[1+ (n-1)]

= n^2

Q. 10. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Answer : Given that the 2 AP series has same common difference (d)

Let x, x+d, x+2d,…. X+(n-1)d &

y, y+d, y+2d,…..y+(n-1)d are the two A.P series

given that the difference between their 100th terms is 100

that is,

x+(100-1)d – (y+(100-1)d) = 100

x-y = 100

The difference between their 1000^{th} term is

= x+(1000-1)d –(y+(1000-1)d)

= x-y

We know that value from previous equation

= 100

∴ The difference between their 1000th terms is 100

Q. 11. How many three-digit numbers are divisible by 7?

Answer : The series of three-digit numbers divisible 7 is,

105, 112, 119,……. 994

So here first term (a) is 105

Since it is divisible by 7 the common difference will be 7

Let a_n = 994

a_n = a + (n-1) d

994 = 105 + (n-1) 7

889 = (n-1)7

127 = n-1

n = 128

Q. 12. How many multiples of 4 lie between 10 and 250?

Answer : The multiples of 4 lie between 10 & 250 is

12, 16, 20,.....248

So, first term (a) = 12

Since it is multiples if 4 the common difference will be 4

a_n = 248

a_n = a + (n-1) d

248 = 12 + (n-1) 4

236 = 4(n-1)

59 = n-1

n = 60.

Q. 13. For what value of n, are the nth terms of two APs: 63, 65, 67, …. And 3, 10, 17, …. Equal?

Answer : We have to find the value of n, so that the nth term in the both series should be equal

63, 65, 67, ….

In the above series

First term (a_1) = 63,

Common difference (d_1) = 2

3, 10, 17, ….

First term (a_2) = 3,

Common difference (d_2) = 7

Since value of n^{th} term is equal we equate a_n of both the series.

$a_{n1} = a_{n2}$

$a_1 + (n-1)d_1 = a_2 + (n-1)d_2$

$63 + (n-1)2 = 3 + (n-1)7$

$61 + 2n = 7n - 4$

$5n = 65$

$n = 13$.

Q. 14. Determine the AP whose third term is 16 and the 7^{th}term exceeds the 5^{th}term by 12.

Answer : Given

$a_3 = 16$; $a_7 = a_5 + 12$

$a + (7-1)\, d = a + (5-1)\, d + 12$

$2d = 12$

$d = 12$

Common difference (d) = 12

Substituting value of d in a_3

$a_3 = 16$

$a + (3-1)\, d = 16$

$a + 2\,(6) = 16$

$a = 4$

∴ The series will be a, a+d, a+2d, a+3d,….

4, 10, 16, 22,…

Q. 15. Find the 20^{th}term from the end of the AP : 3, 8, 13, …., 253.

Answer : Given:

First term (a) = 3

Common difference (d) = $n_2 – n_1 = 8 - 3 = 5$

$a_n = a + (n-1)d$

Let us find the total number of terms in A.P. Therefore,

$253 = 3 + (n - 1) \times 5$

$253 - 3 = 5(n - 1)(n - 1) = 250/5 = 50n = 51$

Now, 20th term from end = (51 - 20 + 1)th term from beginning.

20th term from end = 32nd term from beginning.Therefore,$a_{32} = a + (32 - 1)d = 3 + 5 \times 31$
$= 3 + 153$

= 156Hence, 20th term from end is 156.

Q. 16. The sum of the 4th and 8^{th}terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

Answer : Given,

$a_4 + a_8 = 24$;

$(a + 3d) + (a + 7d) = 24$

$2a + 10d = 24 \Rightarrow a+5d = 12 \rightarrow \underline{1}$

We have been also given that

$a_6 + a_{10} = 44$

$(a + 5d) + (a + 9d) = 44$

$2a + 14d = 44 \Rightarrow a +7d = 22 \rightarrow \underline{2}$

By subtracting two equations

(a + 5d) - (a + 7d) = 12 – 22

-2d = -10

d = 5

Substitute d in any one of the equation.

a+5d = 12

a + 5×5 = 12

a = 2

The first 3 terms of the series is given by a, a+d, a+2d

⇒ 2, 2+5, 2+2×5

⇒ 2, 7, 12

Q. 17. Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his salary reach Rs. 7000?

Answer : Initial salary of Subba Rao is Rs. 5000

That is first term (a) = 5000

He receives Rs.200 every year as increment

So, common difference (d) = 200

The year when he reaches Rs. 7000

So, a_n = 7000

a_n = a +(n-1)d

7000 = 5000 + (n-1) 200

2000 = 200(n-1)

10 = n-1

n = 11

So, it takes him 11 years to reach Rs. 7000

Exercise 6.3

Q. 1 A. Find the sum of the following APs:

2, 7, 12, …., to 10 terms.

Answer : In the above series

First term (a) = 2

Common different (d) = $n_2 – n_1 = 7 – 2 = 5$

No. Of terms (n) = 10

The sum of the series is given by $S_n = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{10}{2}$ [2×2 + (10-1)5]

= 245

Q. 1 B. Find the sum of the following APs:

-37, -33, -29, ……, to 12 terms.

Answer : In the above series

First term (a) = -37

Common different (d) = $n_2 – n_1 = -33 – (-37) = 4$

No. Of terms (n) = 12

The sum of the series is given by $S_n = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{12}{2}$ [2×(-37) + (12-1)4]

= -180

Q. 1 C. Find the sum of the following APs:

0.6, 1.7,2.8, ….., to 100 terms.

Answer : In the above series

First term (a) = 0.6

Common different (d) = $n_2 – n_1$ = 1.7 - 0.6 = 1.1

No. Of terms (n) = 100

The sum of the series is given by $S_n = \frac{n}{2}$ [2a + (n-1) d]

= $\frac{100}{2}$ [2×(0.6) + (100-1)1.1]

= 5505

Q. 1 D. Find the sum of the following APs:

$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$....., to 11 terms

Answer : In the above series

First term (a) = $\frac{1}{15}$

Common different (d) = $n_2 – n_1 = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

No. Of terms (n) = 11

The sum of the series is given by $S_n = \frac{n}{2}$ [2a + (n-1) d]

= $\frac{11}{2}$ [2×($\frac{1}{15}$) + (11-1) ($\frac{1}{60}$)]

$= \frac{11}{2} [\frac{2}{15} + \frac{10}{60}]$

$= \frac{33}{20}$

Q. 2 A. Find the sums given below:

$7 + 10\frac{1}{2} + 14 + \ldots + 84$.

Answer :

We have given a formulae to find sum of the series when first and last numbers are given.

$S_n = \frac{n}{2} [2a + (n-1) d]$

Where n is the last term in the series

d is the common difference

a is the first term.

$a_n = 84$, $a = 7$

$d = n_2 – n_1 = 10\frac{1}{2} - 7 = 3.5$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)3.5$

$77 = 3.5(n-1)$

$22 = n-1$

$n = 23$

The sum of the series is $S_n = \frac{n}{2} [2a + (n-1) d]$

$= \frac{23}{2}$ [2×7 + (23-1)3.5]

$= \frac{23}{2}$ [14 + (22× 3.5)

$\Rightarrow \frac{23}{2}[91]$

$\Rightarrow \frac{2093}{2}$

$= 1046.4$

Q. 2 B. Find the sums given below:

34 + 32 + 30 +….+10

Answer : The sum of the series given by

$S_n = \frac{n}{2}$ [2a + (n-1) d]

Where n is the last term in the series

d is the common difference

a is the first term.

$a_n = 10$, $a = 34$

$d = n_2 – n_1 = 32 - 34 = 2$

$a_n = a + (n-1)d$

$10 = 34 + (n-1)(-2)$

$-24 = -2(n-1)$

$12 = n-1$

$n = 13$

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{13}{2}$ [2×34 + (13-1)(-2)]

$= \frac{13}{2}$ [44] ⇒ 286

Q. 2 C. Find the sums given below:

(-5) + (-8) + (-11) +…… + (-230)

Answer : The sum of the series given by

$S_n = \frac{n}{2}$ [2a + (n-1) d]

Where n is the last term in the series

d is the common difference

a is the first term.

a_n = -230, a = -5

d = $n_2 – n_1$ = (-8) – (-5) = -3

n^{th} term a_n = a + (n-1)d

-230 = (-5) + (n-1)(-3)

-225 = -3(n-1)

75 = n-1

n = 76

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{76}{2}$ [2×(-5) + (76-1)(-3)]

= 38 [-235]

⇒ 8930

Q. 3 A. In an AP :

given a = 5, d = 3, a_n = 50 find n and s_n

Answer : d is the common difference = 3

a is the first term = 5

$a_n = 50$

n = a + (n-1)d

50 = 5 + (n-1)(3)

45 = 3(n-1)

15 = n-1

n = 16

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

= $\frac{16}{2}$ [2×5 + (16-1)(3)]

= 8 [55]

⇒ 440

Q. 3 B. In an AP :

given a = 7, a_n = 35 find d and s_{13}

Answer : d is the common difference

a is the first term = 7

$a_{13} = 35$

a_{13} = a + (13-1)d

35 = 7 + (12)(d)

28 = 12 d

d = $\frac{7}{3}$

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

$S_{13} = \frac{13}{2}$ [2×7 + (13-1) $\frac{7}{3}$]

$= \frac{13}{2}$ [42]

= 273

Q. 3 C. In an AP :

given d = 3, a_{12} = 37 find a and s_{12}

Answer : d is the common difference = 3

a is the first term

a_{12} = 37

a_{12} = a + (12-1)d

37 = a + (11)3

a = 4

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

$S_{12} = \frac{12}{2}$ [2×4 + (12-1)×3]

= 6 [41]

= 246

Q. 3 D. In an AP :

given S_{10} = 125, a_3 = 15 find d and a_{10}

Answer : d is the common difference of the series

a is the first term of the series

a_3 = 15(Given in the Question)

We know that :-

a_n= a + (n-1)d

∴ a_3= a + (3-1)d

a_3= a + 2d=15.....(i)

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

∴S_{10} =10/2 [2×a + (10-1)×d]

⇒125 = 5[2a + 9d)

⇒ 25 = 2a + 9d..... (ii)

Solving (i) and (ii) we get:-

2a + 9d= 25 a + 2d= 15 (Multiplying the Equation by 2 and changing the sign)⇒ 2a + 9d = 25 2a + 4d = 30 - - ----------⇒ 5d = (-5)----------

∴ **d= (-1)**
After putting value of d in any of the above equation we get **a=17**
∴ **a_{10} = 17 + (10-1)(-1)**
= 8

Q. 3 E. In an AP :

given a = 2, S_n = 90, d = 8 find n and a_n

Answer : First term (a) = 2

Common difference (d) = 8

S_n = 90

The sum of the series is $S_n = \frac{n}{2}$ [2a + (n-1) d]

$90 = \frac{n}{2}$ [2×2 + (n-1)8]

$90 = 4n^2 – 2n$

$2n^2 – n - 45 = 0$

By solving the above quadratic equation we will get

$2n^2 -10n + 9n – 45 = 0$

2n(n-5) + 9(n-5) = 0

n = 5

$a_n = a + (n-1)d$

$a_5 = 2 + (5-1)8 = 34$

Q. 3 F. In an AP :

given $a_n = 4$, $d = 2$, $s_n = -14$. Find n and a

Answer : Given, series in A.P.

n^{th} term in the AP $a_n = 4$,

Common difference (d) = 2

Sum of the series is $s_n = -14$

$a_n = a + (n-1)d$

$4 = a + (n-1)2$

$6 = a + 2n$

$a = 6 - 2n \Rightarrow \underline{1}$

Sum of the series $(s_n) = \frac{n}{2}[2a + (n-1)\,d]$

$-14 = \frac{n}{2}[\,2\times(6-2n) + (n-1)2]$

$-14 = n[5-n]$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7) + 2(n-7) = 0$

$(n-7)(n+2) = 0$

$n-7 = 0 \Rightarrow n = 7$ (or) $n+2 = 0 \Rightarrow n = -2$

since n should be a positive integer

we take n = 7

substitute in equation 1

$a = 6 – 2n$

$a = 6 – 2(7)$

$a = -8$

∴ first term is -8

Q. 3 G. In an AP :

given l = 28, s_n = 144 and there are 9 terms Find and a

Answer : Given

Last term (l) = 28

Sum of the series $s_n = 144$

The number of terms in the series (n) = 9

We have another formula to find sum of the series, when common difference is not given

Sum of the series $s_n = \frac{n}{2}[a + l]$

Where a is the first term

$144 = \frac{9}{2}[a + 28]$

$32 = a + 28$

$4 = a$

∴ first term (a) = 4.

Q. 4. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer : Given first term (a) = 17, last term (a_n) = 350

Common difference (d) = 9

$a_n = a + (n-1)\, d$

350 = 17 + (n-1) 9

333 = 9(n-1)

37 = n-1

n = 38.

Sum of the series (s_n) = $\frac{n}{2}$ [2a + (n-1) d]

$= \frac{38}{2}$ (2×17 +(38-1)9)

= 19 × 367

= 6973

Q. 5. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer : Second term (a_2) = 14; Third term (a_3) = 18

a_2 = a+(2-1)d

14 = a+d ⇒ 1

a_3 = a+(3-1)d

18 = a+2d ⇒ 2

By subtracting both the equations

14 -18 = a+d –(a+2d)

-4 = -d

d = 4

By substituting d in the equation 1

a + d = 14

a + 4 = 14

a = 10

The sum of first 51 terms is given by (s_n) = $\frac{n}{2}$ [2a + (n-1) d]

$S_{51} = \frac{51}{2}$ [2×10 + (51-1) 4]

= 5610

Q. 6. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer : Given: S_7 = 49 and S_{17} = 289

Let a be the first term, and d be the common difference of the given A.P.

Therefore,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_7 = \frac{7}{2}[2a + 6d]$$

And7[a + 3d] = 49a + 3d = 7.....(1)Also,

$$S_{17} = \frac{17}{2}[2a + 16d]$$

17[a + 8d] = 289a + 8d = 17.....(2)Subtracting equation 1 from equation 2 we get,8d - 3d = 17 - 75d = 10d = 2Putting the value of d in equation 1, we get,a + 3.2 = 7a + 6 = 7a = 1

$$S_n = \frac{n}{2}[2 \times 1 + (n - 1) \times 2]$$

$$S_n = \frac{n}{2}(2n - 2 + 2)$$

$$S_n = n^2$$

Q. 7 A. Show that a_1, a_2,…..a_n form an AP where is defined as below :

a_n = 3+4n

Also find the sum of the first 15 terms in each case.

Answer : Given a_n = 3+4n

So, a_1 = 3+4(1) = 7

$a_2 = 3+4(2) = 11$

$a_3 = 3+4(3) = 15$

as of now a_1, a_2, a_3 have the same common difference (d) = 4

so given one is A.P

The sum of first 15 terms is given by $(s_n) = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{15}{2}$ [2×7 +(15-1) 4]

= 525

Q. 7 B. Show that $a_1, a_2, \ldots a_n$ form an AP where is defined as below :

$a_n = 9-5n$

Also find the sum of the first 15 terms in each case.

Answer : Given $a_n = 9-5n$

So, $a_1 = 9 – 5(1) = 4$

$a_2 = 9 -5(2) = -1$

$a_3 = 9 – 5(3) = -6$

as of now a_1, a_2, a_3 have the same common difference

so given one is A.P

The sum of first 15 terms is given by $(s_n) = \frac{n}{2}$ [2a + (n-1) d]

$= \frac{15}{2}$ [2×4 +(15-1) (-5)]

= -465

Q. 8. If the sum of first n terms of an AP is $4n-n^2$ what is the first (remember the first term is s_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd,and the 10th and the nth terms.

Answer : The sum of first n terms is given by $4n - n^2$

So the first term is

$a_1 = 4(1) - 1^2 = 3$

the sum of first two term $S_2 = 4(2) - 2^2 = 4$

The difference between S_2 & S_1 gives us 2^{nd} term

$a_2 = S_2 - S_1 = 4-3 = 1$

the sum of first 3 term $S_3 = 4(3) - 3^2 = 3$

The difference between S_2 & S_3 gives us 3^{rd} term

$a_3 = S_3 - S_2 = 3-4 = -1$

From this we can find common difference = $a_2 - a_1 = 1-3 = -2$

So the 10^{th} term is = a + (10-1)d

= 3 + 9×(-2) = -15

n^{th} term is = a + (n-1)d

= 3 + (n-1)×(-2) = 5 -2n

Q. 9. Find the sum of the first 40 positive integers divisible by 6.

Answer : The first 40 positive integers divisible by 6 are,

6, 12,…..240

Here we have n = 40 terms

First term (a) = 6;

Common difference is 6 since they are multiple of 6

The sum of 6 terms is given by $(s_n) = \frac{n}{2}$ [2a + (n-1) d]

$S_6 = \frac{40}{2}$ [2×6 + (40-1)6]

= 4920

Q. 10. A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Answer : Let p be the first prize awarded to students

Since there are 7 cash prizes, n = 7

So $a_1 = x$

Then the preceding prizes will be Rs. 20 lesser

So the series will be

p, p-20, p-40,…. p-120.

Since preceding prizes will be Rs. 20 lesser the common difference is 20

The sum of all prizes are given by (s_7) = 700

$\frac{7}{2}$ [2×(p)+(7-1)(-20)] = 700

(2p - 120) = 200

2p = 320

P = 160

So, the each prize cost will be

160, 140, 120, 100, 80, 60, 40

Q. 11. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Answer : Given that each section of each class will plant a trees.

e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections.

So the trees planted by each class will be like this

1 × 3, 2 × 3, 3 × 3,…. 12 × 3

⇒ 3, 6, 9,…..36

In the above series

First term (a) = 3

Common difference (d) = $a_2 – a_1$ = 6-3 = 3

The sum of 12 terms is given by (s_n) = $\frac{n}{2}$ [2a + (n-1) d]

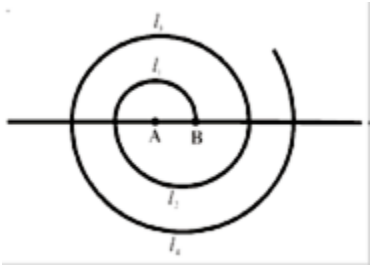
$S_{12} = \frac{12}{2}$ [2×3 + (12-1) 3]

= 6(6+33) = 234

So, totally 234 trees will be planted by students in the school.

Q. 12. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? Take $\pi = \frac{22}{7}$

[Hint : Length of successive semicircles is l_1 , l_2, l_3, l_4 ,…with centres at A, B, A, B, ….., respectively.]



Answer : Firstly, we have find circumference of each semi circle to find the total length

The circumference for semicircles is given by πr

where r is radius of each semi circle

the radii of circles is given by 0.5, 1.0,….6.5

$$l_1 = \pi r_1 = \frac{22}{7} \times 0.5 = 1.57\text{cm}$$

$$l_1 = \pi r_2 = \frac{22}{7} \times 1.0 = 3.14\text{cm}$$

Likewise we calculate all the other circumferences

1.57, 3.14,…… 20.41

The first term (a) = 1.57

Common difference (d) = $a_2 - a_1$ = 3.14 -1.57 = 1.57

The sum of 13 terms is given by (S_n) = $\frac{n}{2}$ [2a + (n-1) d]

S_{13} = $\frac{13}{2}$ [2×1.57 + (13-1) 1.57]

= 6.5×21.98

= 142.87 = 143 (app.)

So, the total length of the spiral made is 143 cm

Q. 13. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Answer : Let us consider the row number as the term in the series & the no. Of logs present in the each row as the values

20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on.

Considering bottom row as first term so (a) = 20

Second term $a_2 = 19$

Common difference (d) = $a_2 - a = 19-20 = -1$

Given that 200 logs are stacked

So, $s_n = 200$

$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Here n is the no. Of rows are used to pile the log.

$S_n = \frac{n}{2}[2\times20+ (n-1)(-1)]$

$200 = \frac{n}{2}(41-n)$

$400 = 41n - n^2$

$n^2 -41n+400 = 0$

$n^2 - 25n -16n + 400 = 0$

$n(n-25) -16(n-25) = 0$

$(n-16)(n-25) = 0$

$\therefore n = 16$ or 25

Let us take n = 16, because 25 is higher value and that is not feasible solution for the problem

$a_{16} = a+(16-1)d = 20+15\times(-1) = 5$

There will be 5 logs in top row.

Q. 14. In a bucket and ball race, a bucket is placed at the starting point, which is 5 m from the first ball, and the other balls are placed 3 m apart in a straight line. There are ten balls in the line.

A competitor starts from the bucket, picks up the nearest ball, runs back with it, drops it in the bucket, runs back to pick up the next ball, runs to the bucket to drop it in, and she continues in the same way until all the balls are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first ball and the second ball, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Answer : The distance between the first ball and the bucket is 5m

So the competitor runs 10m (forward + backward)

The second ball the placed 3m ahead of first ball

For second ball he will 16.

Likewise, the distance covered by each ball is

3^{rd} = 22, 4^{th} = 28,….. 10^{th} = 64

Common difference (d) = 3^{rd} - 2^{nd} = 22-16 = 6

Here there are 10 balls to be picked so n = 10

Last term a_{10} = 64

The sum of all the distance is given by $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{10}{2}(2 \times 10 + (10 - 1)6)$

= 5×(74) = 370

So the total distance covered by the competitor to pick all the is 370

Exercise 6.4

Q. 1 A. In which of the following situations, does the list of numbers involved in the form of a GP.?

Salary of Sharmila, when her salary is Rs. 5,00,000 for the first year and expected to receive yearly increase of 10%.

Answer : This is not likely be to in G.P

Initially the salary of sharmila is 5,00,000

They have said that she will get 10% every year

For second year her salary will be increased by 10% of 5,00,000.

So her second year salary will be 5,50,000

For third year her salary will be increased by 10% of 5,00,000.

So her second year salary will be 6,00,000

You can see the common ratios will be same r =

$$\frac{\text{second year}}{\text{firstyear}}$$

$$= \frac{550000}{500000} = \frac{11}{10}$$

$$r = \frac{\text{third year}}{\text{second year}} = \frac{600000}{550000} = \frac{12}{11}$$

Q. 1 B. In which of the following situations, does the list of numbers involved in the form of a GP.?

Number of bricks needed to make each step, if the stair case has total 30 steps. Bottom step needs 100 bricks and each successive step needs 2 brick less than the previous step.

Answer : This is not in GP

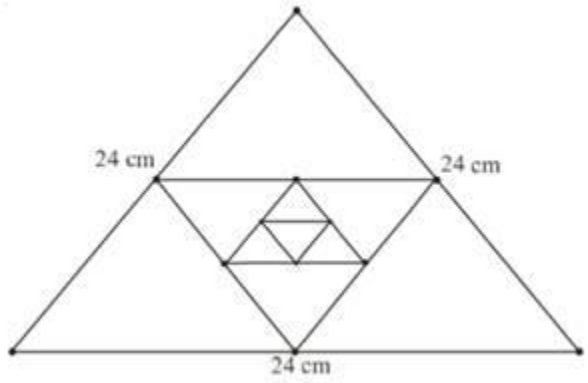
Since they have given common difference -2

They have said that successive steps need 2 bricks less than the previous.

Q. 1 C. In which of the following situations, does the list of numbers involved in the form of a GP.?

Perimeter of each triangle, when the mid points of sides of equilateral triangle whose side is 24 cm are joined to form another triangle, whose mid points in turn

are joined to form still another triangle and the process continues indefinitely.



Answer : It has been already given that the perimeter of any figure is drawn by using the midpoint of the same figure in GP.

If the perimeter of that triangle is “a” then the perimeter of the triangles formed by using midpoint is $\frac{a}{2}, \frac{a}{4}$ ….,

Perimeter of a triangle is = $s_1 + s_2 + s_3 = 24 + 24 + 24 = 72$

In the same way the perimeter of given triangle is 72

The perimeter of the triangles formed by using midpoint will be 36, 18, 9

We can see that their common ratios is same

$$R_1 = \frac{p_2}{p_1} = \frac{36}{72} = 0.5$$

$$R_2 = \frac{p_3}{p_2} = \frac{18}{36} = 0.5$$

Q. 2 A. Write three terms of the GP when the first term ‘a’ and the common ratio ‘r’ are given?

a = 4, r = 3

Answer : First term (a) = 4; common ratio (r) = 3

The three terms are given by

⇒ a, ar, ar^2.

= 4, 4×3, 4×3^2

= 4, 12, 24

Q. 2 B. Write three terms of the GP when the first term 'a' and the common ratio 'r' are given?

a = √5, r = $\frac{1}{5}$

Answer : First term (a) = 4; common ratio (r) = 3

The three terms are given by

⇒ a, ar, ar^2.

$= \sqrt{5}, \sqrt{5}\times\frac{1}{5}, \sqrt{5}\times(\frac{1}{5})^2 = \sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}$

Q. 2 C. Write three terms of the GP when the first term 'a' and the common ratio 'r' are given?

a = 81, r = $\frac{-1}{3}$

Answer :

First term (a) = 81; common ratio (r) = $\frac{-1}{3}$

The three terms are given by

⇒ a, ar, ar^2.

$= 81, 81\times\frac{-1}{3}, 81\times(\frac{-1}{3})^2 = 81, -27, 9$

Q. 2 D. Write three terms of the GP when the first term 'a' and the common ratio 'r' are given?

a = $\frac{1}{64}$, r = 2

Answer :

First term (a) = $\frac{1}{64}$; common ratio (r) = 2

The three terms are given by

$\Rightarrow a, ar, ar^2.$

$= \frac{1}{64}, \frac{1}{64} \times 2, \frac{1}{64} \times (2)^2$

$= \frac{1}{64}, \frac{1}{32}, \frac{1}{16}$

Q. 3 A. Which of the following are GP? If they are in GP. Write three more terms?

4, 8, 16,….

Answer : Here first term is (a) = 4

$r_1 = \frac{n_2}{n_1} = \frac{8}{4} = 2$

$r_2 = \frac{n_3}{n_2} = \frac{16}{2} = 2$

Common ratios are equal so the given series is in GP.

$a_4 = a\ r^3 = 4 \times 2^3 = 4 \times 8 = 32$

$a_5 = a\ r^4 = 4 \times 2^4 = 4 \times 16 = 64$

$a_6 = a\ r^5 = 4 \times 2^5 = 4 \times 32 = 128$

Q. 3 B. Which of the following are GP? If they are in GP. Write three more terms?

$\frac{1}{3}, \frac{-1}{6}, \frac{1}{12}$,……

Answer :

Here first term is (a) = $\frac{1}{3}$

$$r_1 = \frac{n_2}{n_1} = \frac{(\frac{-1}{6})}{\frac{1}{3}} = \frac{-1}{2}$$

$$r_2 = \frac{n_3}{n_2} = \frac{(\frac{1}{12})}{\frac{-1}{6}} = \frac{-1}{2}$$

Common ratios are equal so the given series is in GP.

$$a_4 = ar^3 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^3 = \frac{1}{3} \times \frac{-1}{8} = \frac{-1}{24}$$

$$a_5 = ar^4 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^4 = \frac{1}{3} \times \frac{1}{16} = \frac{1}{48}$$

$$a_6 = ar^5 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^5 = \frac{1}{3} \times \frac{-1}{32} = \frac{-1}{96}$$

Q. 3 C. Which of the following are GP? If they are in GP. Write three more terms?

5, 55, 555, ……

Answer : Here first term is (a) = 5

$$r_1 = \frac{n_2}{n_1} = \frac{55}{5} = 11$$

$$r_2 = \frac{n_3}{n_2} = \frac{555}{55} = 10.09$$

The common ratios are not equal so this is not in GP

Q. 3 D. Which of the following are GP? If they are in GP. Write three more terms?

-2, -6, -18 ……..

Answer : Here first term is (a) = -2

$$r_1 = \frac{n_2}{n_1} = \frac{-6}{-2} = 3$$

$$r_2 = \frac{n_3}{n_2} = \frac{-18}{-6} = 3$$

Common ratios are equal so the given series is in GP.

$a_4 = a\, r^3 = -2 \times 3^3 = -2 \times 27 = -54$

$a_5 = a\, r^4 = -2 \times 3^4 = -2 \times 81 = -162$

$a_6 = a\, r^6 = -2 \times 3^5 = -2 \times 243 = -486$

Q. 3 E. Which of the following are GP? If they are in GP. Write three more terms?

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ **……..**

Answer :

Here first term is (a) = $\frac{1}{2}$

$$r_1 = \frac{n_2}{n_1} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$$r_2 = \frac{n_3}{n_2} = \frac{\frac{1}{6}}{\frac{1}{4}} = 0.66$$

The common ratios are not equal so this is not in GP

Q. 3 F. Which of the following are GP? If they are in GP. Write three more terms?

3, -3^2, 3^3…….

Answer : Here first term is (a) = 3

$$r_1 = \frac{n_2}{n_1} = \frac{-9}{3} = -3$$

$$r_2 = \frac{n_3}{n_2} = \frac{27}{-9} = -3$$

Common ratios are equal so the given series is in GP.

$a_4 = a\, r^3 = 3\times(-3)^3 = 3 \times -27 = -81$

$a_5 = a\, r^4 = 3\times(-3)^4 = 3 \times 81 = 243$

$a_6 = a\, r^5 = 3\times(-3)^5 = 3 \times -243 = -729$

Q. 3 G. Which of the following are GP? If they are in GP. Write three more terms?

x, 1, $\frac{1}{x}$……

Answer : Here first term is (a) = x

$$r_1 = \frac{n_2}{n_1} = \frac{1}{x}$$

$$r_2 = \frac{n_3}{n_2} = \frac{\frac{1}{x}}{1} = \frac{1}{x}$$

Common ratios are equal so the given series is in GP.

$$a_4 = a\, r^3 = x(\frac{1}{x})^3 = \frac{1}{x^2}$$

$$a_5 = a\, r^4 = x(\frac{1}{x})^4 = \frac{1}{x^3}$$

$$a_6 = a\, r^5 = x(\frac{1}{x})^5 = \frac{1}{x^4}$$

Q. 3 H. Which of the following are GP? If they are in GP. Write three more terms?

$$\frac{1}{\sqrt{2}}, -2, \frac{8}{\sqrt{2}}$$

First term is (a) = $\frac{1}{\sqrt{2}}$

$r_1 = \frac{n_2}{n_1} = \frac{-2}{\frac{1}{\sqrt{2}}} = -2\sqrt{2}$

$r_2 = \frac{n_3}{n_2} = \frac{\frac{8}{\sqrt{2}}}{-2} = -2\sqrt{2}$

Common ratios are equal so the given series is in GP.

$a_4 = a\, r^3 = \frac{1}{\sqrt{2}} \times (-2\sqrt{2})^3 = -16$

$a_5 = a\, r^4 = \frac{1}{\sqrt{2}} \times (-2\sqrt{2})^4 = 32\sqrt{2}$

$a_6 = a\, r^5 = \frac{1}{\sqrt{2}} \times (-2\sqrt{2})^5 = -128$

Q. 3 I. Which of the following are GP? If they are in GP. Write three more terms?

0.4, 0.04, 0.004, ……

Answer : Here first term is (a) = 0.4

$r_1 = \frac{n_2}{n_1} = \frac{0.04}{0.4} = \mathbf{0.1}$

$r_2 = \frac{n_3}{n_2} = \frac{0.004}{0.04} = \mathbf{0.1}$

Common ratios are equal so the given series is in GP.

$a_4 = a\,r^3 = 0.4\times(0.1)^3 = 0.0004$

$a_5 = a\,r^4 = 0.4\times(0.1)^4 = 0.00004$

$a_6 = a\,r^5 = 0.4\times(0.1)^5 = 0.000004$

Q. 4. Find x so that x, x+2, x+6 are consecutive terms of a geometric progression.

Answer : In a GP common ratio are equal

x, x+2, x+6…..

$$r = \frac{n_2}{n_1} = \frac{n_3}{n_2}$$

$$\frac{x+2}{x} = \frac{x+6}{x+2}$$

$(x+2)^2 = x(x+6)$

$X^2 + 4 + 4x = x^2 + 6x$

$2x = 4$

$X = 2$

Q. 1 A

For each geometric progression find the common ratio 'r', and then find an

$$3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}\ldots\ldots$$

Answer : First term (a) = 3

First term (a) = 3

Common ratio (r) = $\frac{n_2}{n_1} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$

n^{th} term in the GP (a_n) = ar^{n-1}

$a_n = 3\left(\frac{1}{2}\right)^{n-1}$

Q. 1 B. For each geometric progression find the common ratio 'r', and then find a_n

2, -6, 18, -54….

Answer :

First term (a) = 2

Common ratio (r) = $\frac{n_2}{n_1} = \frac{-6}{2} = -3$

n^{th} term in the GP (a_n) = ar^{n-1}

$a_n = 2(-3)^{n-1}$

Q. 1 D. For each geometric progression find the common ratio 'r', and then find a_n

5, 2, $\frac{4}{5}, \frac{8}{5}$.....

Answer : First term (a) = 5

First term (a) = 5

Common ratio (r) = $\frac{n_2}{n_1} = \frac{2}{5}$

n^{th} term in the GP (a_n) = ar^{n-1}

$a_n = (5)\left(\frac{2}{5}\right)^{n-1} = \left(\frac{2}{5}\right)^{n-1}$

Q. 2. Find the 10th and nth term of GP. : 5, 25, 125, …….

Answer :

First term (a) = 5

$$\text{Common ratio (r)} = \frac{n_2}{n_1} = \frac{25}{5} = 5$$

n^{th} term in the GP $(a_n) = ar^{n-1}$

$a_n = 5(5)^{n-1} = 5^n$

10^{th} term in the GP $(a_{10}) = ar^{10-1}$

$a_{10} = 5 \times 5^9 = 5^{10}$

Q. 3 A. Find the indicated term of each geometric Progression

$a_1 = 9$, $r = \frac{1}{3}$ find a_7

Answer : First term (a) = 9

Common ratio (r) = $\frac{1}{3}$

7^{th} term in the GP $(a_7) = ar^{7-1} = ar^6$

$$= 9 \times \left(\frac{1}{3}\right)6 = \frac{1}{81}$$

Q. 3 B. Find the indicated term of each geometric Progression

$a_1 = -12$, $r = \frac{1}{3}$ find a_6

Answer : s First term (a) = -12

Common ratio (r) = $\frac{1}{3}$

6^{th} term in the GP (a_6) = $ar^{6-1} = ar^5$

$$= -12 \times (\frac{1}{3})5 = \frac{-4}{243}$$

Q. 4 A. Which term of the GP.

2, 8, 32, ……. is 512?

Answer : First term (a) = 2

Common ratio (r)

$$= \frac{n_2}{n_1} = \frac{8}{2} = 4$$

n^{th} term in the GP (a_n) = ar^{n-1}

$512 = 2 \times 4^{n-1}$

$256 = 4^{n-1}$

$4^4 = 4^{n-1}$

Equating powers on both sides

n-1 = 4 = 5

Q. 4 B. Which term of the GP.

√3, 3, 3√3…… is 729?

Answer : First term (a) = √3

Common ratio (r)

$$= \frac{n_2}{n_1} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

n^{th} term in the GP (a_n) = ar^{n-1}

$729 = \sqrt{3} \times (\sqrt{3})^{n-1}$

$3^6 = \sqrt{3}^n$

Equating powers on both sides

$\frac{n}{2} = 6 \Rightarrow n = 12.$

Q. 4 C. Which term of the GP.

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ **is** $\frac{1}{2187}$**?**

Answer :

$$\text{First term (a)} = \frac{1}{3}$$

$$\text{Common ratio (r)} = \frac{n_2}{n_1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$$

$$n^{th} \text{ term in the GP } (a_n) = ar^{n-1}$$

$$\frac{1}{2187} = \frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

$$\left(\frac{1}{3}\right)^7 = \left(\frac{1}{3}\right)^n$$

Equating powers on both sides

n = 7

Q. 5. Find the 12^{th} term of a GP. Whose 8^{th} term is 192 and the common ratio is 2.

Answer : Given,

common ratio (r) is 2.

8^{th} term is 192

$\Rightarrow ar^{8-1} = 192$

$a \times 2^7 = 192$

$$a = \frac{192}{128} = \frac{3}{2}$$

12^{th} term of a GP = $ar^{12-1} = ar^{11}$

$$= \frac{3}{2} \times 2^{11} = 3072.$$

Q. 6. The 4^{th} term of a geometric progression is $\frac{2}{3}$ and the seventh term is $\frac{16}{81}$ Find the geometric series.

Answer : Given,

4^{th} term of the series = $\frac{2}{3}$

$ar^3 = \frac{2}{3}$

$$a = \frac{2}{3r^3} \Rightarrow \underline{1}$$

7^{th} term of a series = $\frac{16}{81}$

$$ar^6 = \frac{16}{81} \Rightarrow \underline{2}$$

Substituting 'a' in $\underline{1}$

$$\frac{2}{3r^3} \times r^6 = \frac{16}{81}$$

$r^3 = \frac{8}{27}$

$r^3 = (\frac{2}{3})^3$

equating power and base we will get

$r = \frac{2}{3}$

The geometric series is a, ar, ar^2, ar^3

$\frac{2}{3}, \frac{2}{3} \times \frac{2}{3}, \frac{2}{3}(\frac{2}{3})^2, \ldots\ldots$

$\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots\ldots$