Chapter 5. Complex Numbers and Quadratic Equations

Question-1

If $z_1, z_2 \in C$, show that $(z_1 + z_2)^2 = z_1^2 + 2z_1z_2 + z_2^2$

Solution:

Let $z_1 = x_1 + iy_1$

$z^2 = x_2 + iy_2$

$(z_1 + z_2)^2 = [(x_1 + iy_1) + (x_2 + iy_2)]^2$

$= [(x_1 + x_2) + i(y_1 + y_2)]^2$

$= (x_1 + x_2)^2 + 2i(x_1 + x_2)(y_1 + y_2) - (y_1 + y_2)^2$

$= x_1^2 + 2x_1x_2 + x_2^2 + 2ix_1y_1 + 2ix_1y_2 + 2ix_2y_1 + 2ix_2y_2 - y_1^2 - 2y_1y_2 - y_2^2$

$= x_1^2 + 2ix_1y_1 - y_1^2 + x_2^2 + 2ix_2y_2 - y_2^2 + 2(x_1 + iy_1)(x_2 + iy_2)$

$= x_1^2 + 2ix_1y_1 + (iy_1)^2 + x_2^2 + 2ix_2y_2 + (iy_2)^2 + 2(x_1 + iy_1)(x_2 + iy_2)$

$= (x_1 + iy_1)^2 + (x_2 + iy_2)^2 + 2z_1z_2$

$= z_1^2 + 2z_1z_2 + z_2^2$

Question-2

Write the following as complex numbers

i. $\sqrt{-16}$

ii. $1 + \sqrt{-1}$

iii. $-1 - \sqrt{-5}$

iv $\frac{\sqrt{3}}{2} - \frac{\sqrt{-2}}{\sqrt{7}}$

v. $\sqrt{x}$, $(x>0)$

vi. $-b + \sqrt{-4ac}$, $(a, c > 0)$

Solution:

i. $\sqrt{-16} = \sqrt{-1 \times 16} = \sqrt{-1}\sqrt{16} = i\sqrt{16}$

ii. $1 + \sqrt{-1} = 1 + i$

iii. $-1 - \sqrt{-5} = -1 - \sqrt{-1}\sqrt{5} = -1 - i\sqrt{5}$

iv. $\frac{\sqrt{3}}{2} - \frac{\sqrt{-2}}{\sqrt{7}} = \frac{\sqrt{3}}{2} - \frac{\sqrt{-1}\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{3}}{2} - \frac{i\sqrt{2}}{\sqrt{7}}$

v. $\sqrt{x} = \sqrt{x} + i0$

vi. $-b + \sqrt{-4ac} = -b + \sqrt{-1}\sqrt{4ac} = -b + 2i\sqrt{ac}$

Question-3

Obtain a quadratic equation whose root are 2 and 3.

Solution:

Let α, β be the roots of the equation.

Sum of the roots $\alpha + \beta = 2+3 = 5$

Product of the roots $\alpha \times \beta = 2 \times 3 = 6$

∴ The equation is given by

x^2 - (sum of roots)x + product of roots = 0

∴ Equation is $x^2 - 5x + 6 = 0$.

Question-4

Solve the following equation: $25x^2 - 30x + 9 = 0$.

Solution:

$25x^2 - 30x + 9 = 0$

$D = b^2 - 4ac = 900 - 4 \times 25 \times 9 = 900 - 900 = 0$

Hence the two real equal roots of the equation are : $\frac{30}{50}, \frac{30}{50}$

i.e $\frac{3}{5}, \frac{3}{5}$

Question-5

Write the real and imaginary parts of the following complex numbers below:

- **i.** $\frac{\sqrt{17}}{2}+\frac{i2}{\sqrt{70}}$
- **ii.** $-\frac{1}{5}+\frac{i}{5}$
- **iii.** $\sqrt{37}+\sqrt{-19}$
- **iv.** $\sqrt{3}+i\frac{\sqrt{2}}{76}$
- **v.** 7
- **vi.** 3i

Solution:

i. Let z = $\frac{\sqrt{17}}{2}+\frac{i2}{\sqrt{70}}$

Re z = $\frac{\sqrt{17}}{2}$, Im z = $\frac{2}{\sqrt{70}}$

ii. Let z = $-\frac{1}{5}+\frac{i}{5}$

Re z = $-\frac{1}{5}$, Im z = $\frac{1}{5}$

iii. Let z = $\sqrt{37}+\sqrt{-19}=\sqrt{37}+i\sqrt{19}$

Re z = $\sqrt{37}$, Im z = $\sqrt{19}$

iv. $\sqrt{3}+i\frac{\sqrt{2}}{76}$

Re z = $\sqrt{3}$, Im z = $\frac{\sqrt{2}}{76}$

v. 7

Re z = 7, Im z = 0

vi. 3i

Re z = 0, Im z = 3

Question-6

Without computing the roots of $3x^2 + 2x+6 = 0$, find (i) $\frac{1}{\alpha} + \frac{1}{\beta}$ (ii) α2 +β2 (iii) $\alpha^3+\beta^3$

Solution:

If α and β an the root of the equation $3x^2 +2x+6 = 0$
Sum of the roots $\alpha +\beta = \frac{-b}{a} = \frac{-2}{3}$
Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{3} = 2$

(i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-2}{3} \times \frac{1}{2} = \frac{-1}{3}$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-2}{3}\right)^2 - 2\times 2 = \frac{4}{9} - 4 = \frac{4-36}{9} = -\frac{32}{9}$

(iii $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(\frac{-2}{3}\right)^3 - 3(2)\left(\frac{-2}{3}\right) = \frac{-8}{27} + 4 = 3\frac{19}{27}$.

Question-7

Show that $(1-i)^2 = -2i$.

Solution:

$(1-i)^2 = 1^2 - 2(i)(1) + (i)^2 = 1-2(-i) + (i)^2 = 1-2i-1 = -2i$

Question-8

Solve the following equation: $2x^2-2\sqrt{3}x + 1 = 0$.

Solution:

$2x^2-2\sqrt{3}x + 1 = 0$

$D = b^2-4ac = 12 - 4\times 2\times 1 = 12 - 8 = 4 > 0$
$\sqrt{D} = 2$
Hence the two real and unequal roots are : $\frac{2\sqrt{3}+2}{4}, \frac{2\sqrt{3}-2}{4}$
i.e $\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}-1}{2}$

Question-9

Solve the equation $\sqrt{x}$ = (x - 2) in C.

Solution:

Squaring both sides

$(\sqrt{x})^2=(x-2)^2$

$x = x^2 - 2(x)(2)+4$

$0 = x^2 - 4x+ 4 - x$

$= x^2 - 5x + 4$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$x(x - 4) - (x - 4) = 0$

$x = 4$ or $x = 1$

x=1 doesn't satisfy the equation

$\therefore x = 4$.

Question-10

Find the conjugate of the following complex numbers

i. 3 + i
ii. 3 – i
iii. $-\sqrt{5} - i\sqrt{7}$
iv. $-i\sqrt{5}$

v. 4/5

vi. 49 – i/7

Solution:

i. Conjugate of 3 + i is 3 – i.
ii. Conjugate of 3 – i is 3 + i.
iii. Conjugate of $-\sqrt{5} - i\sqrt{7}$ is $-\sqrt{5} + i\sqrt{7}$
iv. Conjugate of $-i\sqrt{5}$ is $i\sqrt{5}$.

v. Conjugate of 4/5 is 4/5.

vi. Conjugate of 49 – i/7 is 49 + i/7.

Question-11

Solve the following equation: $\sqrt{3x+1}-\sqrt{x-1}=2$.

Solution:

$\sqrt{3x+1}-\sqrt{x-1}=2$

Squaring,

$(3x+1)+(x-1)-2\sqrt{3x+1}\times\sqrt{x-1}=4$

$4x-2\sqrt{3x+1}\times\sqrt{x-1}=4$

$\sqrt{3x+1}\times\sqrt{x-1}=2x-2$

Squaring,

$3x^2-2x-1=(2x-2)^2$

$3x^2-2x-1=4x^2-8x+4$

$x^2-6x+5=0$

$D=b^2-4ac=36-4\times1\times5=16>0$

$\sqrt{D}=4$

Hence the two real and unequal roots are : $\frac{6+4}{2},\frac{6-4}{2}$

i.e 5,1

Question-12

Find the conjugate of $\frac{1-i}{1+i}$.

Solution:

$\frac{1-i}{1+i}=\frac{(1-i)(1-i)}{(1+i)(1-i)}=\frac{(1-i)^2}{1^2-(i)^2}=\frac{1-2i+i^2}{1+1}=\frac{1-2i-1}{2}=\frac{-2i}{2}=-i$.

$\therefore$Conjugate of $\frac{1-i}{1+i}=i$

Question-13

Show that if a,b,c,d,∈ R, $\overline{(a+ib)(c+id)}=(a-ib)(c-id)$ **= (a-ib) (c-id).**

Solution:

$(a+ib)(c+id)=ac+iad+ibc+i^2bd=ac+i(ad+bc)-bd=(ac-bd)+i(ad+bc)$

$\therefore\overline{ac-bd+i(ad+bc)}$

$=(ac-bd)-i(ad+bc)$ ---- (1)

$(a-ib)(c-id)=(ac-bd)-i(bc+ad)$ ------(2)

From (1) and (2) $\overline{(a+ib)(c+id)}=(a-ib)(c-id)$

Question-14

Find the value of x and y, if 4x + i(3x - y) = 3 – i6.

Solution:

$4x + i(3x - y) = 3 - i6$

Equating the real and imaginary, we have

$4x = 3$

$x = 3/4$

$3x - y = -6$

$3(3/4) - y = -6$

$9/4 - y = -6$

$-y = -6 - 9/4$

$y = 33/4$

Question-15

Solve the following equation: $2x^2+1=0$.

Solution:

$2x^2+1=0 \Rightarrow x^2=-1/2$

Hence the complex roots of the equation are $+i\frac{\sqrt{2}}{2}, -i\frac{\sqrt{2}}{2}$.

Question-16

Does the equation $2x^2 - 4x + 3 = 0$ have equal roots? Find the roots.

Solution:

The given equation is $2x^2 - 4x + 3 = 0$.

Comparing with $ax^2 + bx + c = 0$

$a = 2, b = -4, c = 3$

$b^2 - 4ac = (-4)^2 - 4\times2\times3 = 16 - 24 = -8 < 0$

The roots are not equal.

Hence the roots of the given equation is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{-8}}{2\times2} = \frac{4 \pm 2\sqrt{2}i}{4} = 1 \pm \frac{1}{\sqrt{2}}i$

Question-17

Find the value of x and y, if (3y - 2) + i(7 − 2x) = 0.

Solution:

$(3y - 2) + i(7 - 2x) = 0$

Equating the real and imaginary, we have

$3y - 2 = 0$

$y = 2/3$

$7 - 2x = 0$

$2x = 7$

$x = 7/2$

The value of $x = 7/2$ and $y = 2/3$.

Question-18

If two complex numbers z_1z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$?

Solution:

$|z_1| = |x_1 + iy| = \sqrt{x_1^2 + y_1^2}$

$|z_2| = |x_2 + iy_2| = \sqrt{x_2^2 + y_2^2}$

$\therefore \quad |z_1| = |z_2|$

$\sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2}$

$x_1^2 + y_1^2 = x2^2 + y_2^2$

$x_1^2 = x_2^2$ and $y_1^2 = y_2^2$

$x_1 = \pm x_2 \; y_1 = \pm y$

$\therefore z_1$ need not be z_2

Question-19

Solve the following equation: $x^2 - 4x + 7 = 0$.

Solution:

$x^2 - 4x + 7 = 0$

$D = b^2 - 4ac = 16 - 4 \times 1 \times 7 = 16 - 28 = -12 < 0$

$\sqrt{D} = 2\sqrt{3}\,i$

Hence the two complex roots are : $\frac{4+2\sqrt{3}i}{2}, \frac{4-2\sqrt{3}i}{2}$

i.e $2+\sqrt{3}\,i$, $2+\sqrt{3}\,i$

Question-20

For what values of a is one of the roots of the equation $x^2 + (2a+1)x + a^2 + 2 = 0$ twice the value of the other.

Solution:

Let the roots be α, 2α.

$\alpha + 2\alpha = \frac{-(2a+1)}{1}$

$\Rightarrow 3\alpha = -2\alpha - 1$

$\Rightarrow \alpha = \frac{-2a-1}{3}$

$\alpha \cdot 2\alpha = \frac{a^2+2}{1}$

$2\alpha^2 = \frac{a^2+2}{1}$

$2\left(\frac{-(2\alpha+1)}{3}\right)^2 = \alpha^2 + 2$

$2(4\alpha^2 + 4\alpha + 1) = 9(\alpha^2 + 2)$

$8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 18$

$-\alpha^2 + 8\alpha - 16 = 0$

$\alpha^2 - 8\alpha + 16 = 0$

$\alpha^2 - 4\alpha - 4\alpha + 16 = 0$

$\alpha(\alpha - 4) - 4(\alpha - 4) = 0$

$\therefore \alpha = 4$ or $\alpha = 4$

Question-21

If the difference of the root of $x^2 - bx + c = 0$ is the same as that of the roots of $x^2 - cx + b = 0$ then b+c+4 = 0 unless b - c = 0.

Solution:

Let α,β be the root of the equation x^2-bx+c = 0; γ, δand be the roots of the equation $x^2 - cx + b = 0$.

Then a + b = b, αβ=c , γ+ δ=c and γ δ = b

Given that a - b = g - δ

$(\alpha - \beta)^2 = (\gamma - \delta)^2$

$(\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$b^2 - 4c = c^2 - 4b$

$b^2 - c^2 + 4b - 4c = 0$

(b-c)(b+c)+4(b-c) = 0

(b-c) (b+c+4) = 0

Hence b-c = 0 or b+c+4 = 0

(ie) b+c+4 = 0 or b = c

Question-22

Find the value of x and y, if $\left(\frac{3}{\sqrt{5}}x - 5\right) + i2\sqrt{5}\,y = \sqrt{2}$.

Solution:

$\left(\frac{3}{\sqrt{5}}x - 5\right) + i2\sqrt{5}\,y = \sqrt{2}$

Equating the real and imaginary, we have

$\left(\frac{3}{\sqrt{5}}x - 5\right) = \sqrt{2}$

$\frac{3}{\sqrt{5}}x = \sqrt{2} + 5$

$x = \sqrt{5}\,(\sqrt{2} + 5)/3$

$2\sqrt{5}\,y = 0$

y = 0

The value of $x = \sqrt{5}\,(\sqrt{2} + 5)/3$ and y = 0.

Question-23

If z_1, z_2, z_3 are 3 complex numbers such that there exists a z with $|z_1-z|=|z_2-z|=|z_3-z|$ show that z_1, z_2, z_3 lie on a circle in the plane diagram.

Solution:

Let z_1, z_2, z_3 be $x_1 + iy_1$, $x_2 + iy_2$ and $x_3 + iy_3$ respectively.

Representing points P, Q, R

Let the z be point O given by x + iy.

$|z_1 - z| = \sqrt{(x_1 - x)^2 + (y_1 - y)^2} = OP$

Similarly $|z_2 - z| = OQ$

and $|z_3 - z| = OR$

$|z_1 - z| = |z_2 - z| = |z_3 - z|$

$OP = OQ = OR = r$

This means P, Q, R are points on a circle with centre O and radius r.

Or z_1, z_2, z_3 lie on a circle.

Question-24

Solve the following equation: $x^2+x+1=0$.

Solution:

$x^2+x+1=0$

$D = b^2-4ac = 1 - 4 = -3 < 0$

$\sqrt{D} = \sqrt{3}\,i$

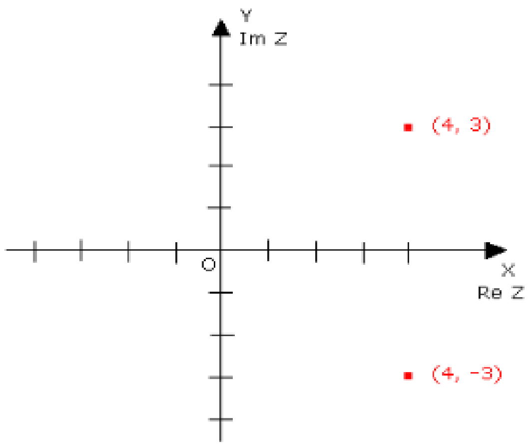
Hence the two complex roots are : $\frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

Question-25

Plot the following number and their complex conjugates on a complex number plane and find their absolute values: 4 – i3.

Solution:

Conjugate of 4 – 3i is 4 + 3i.



The absolute value of 4 – 3i

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

= 5

Question-26

A group of students decided to buy a tape-records from 170 to 195 rupee. But at the last moment two student backed out of the decision so that the remaining student had to pay 1rupee more than they had planned. What was the price of the tape recorder if the student paid equal shares ?

Solution:

Let the price of the tape recorder be Rs. x

Let no. of student be n.

At the last moment

No. of students = (n-2)

Increased contribution = $\frac{x}{n-2}$

Original contribution = $\frac{x}{n}$

According to the question

$\frac{x}{n-2} = \frac{x}{n} + 1$

$\frac{x}{n-2} = \frac{x+n}{n}$

$nx = (n-2)(x+n) = nx + n^2 - 2x - 2n$

$n^2 - 2n = 2x$

$x = \frac{n^2-2n}{2}$, Also $170 < x < 195$

$170 < \frac{n^2-2n}{2} < 195$

$\Rightarrow 340 \le n^2 - 2n \le 390$

Either $340 \le n^2 - 2n$

$n^2 - 2n - 340 \ge 0$

Roots are given by

$n = \frac{2 \pm \sqrt{4+1360}}{2} = \frac{1 \pm \sqrt{341}}{2}$

$n \ge 1 + \sqrt{341}$ or $n \le 1 - \sqrt{341}$

$n = 20$ or $n^2 - 2n - 390 < 0$ ----- (1)

$n = \frac{2 \pm \sqrt{1564}}{2}$ $n = \frac{1 \pm \sqrt{391}}{2}$

$\therefore 1 - \sqrt{391} \le n < 1 + \sqrt{391}$

Since n is a natural no. n = 1, 2, 3 ....20 ---- (2)

From (1) and (2),

n = 20

Cost of tape - recorder $x = \frac{n^2-2n}{2} = \frac{20^2 - 2(20)}{2}$ = = Rs. 180.

Question-27

Solve the following equation: $x^2 + 2x + 2 = 0$.

Solution:

$x^2 + 2x + 2 = 0$

$D = b^2 - 4ac = 4 - 4 \times 2 = -4 < 0$

$\sqrt{D} = 2i$

Hence the two complex roots are : $\frac{-2+2i}{2}, \frac{-2-2i}{2}$

i.e -1-i , -1+i

Question-28

Plot the following number and their complex conjugates on a complex number plane and find their absolute values: $-3 + i5$

Solution:

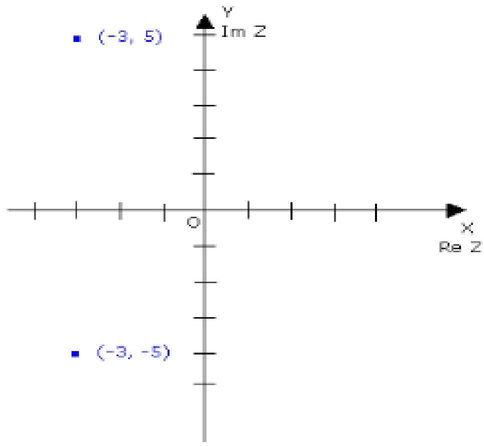
Conjugate of $-3 + i5$ is $-3 - i5$.

The absolute value of $-3 + i5$

$= \sqrt{(-3)^2 + 5^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$



Question-29

Solve $(x^2-5x+7)^2 - (x-2)(x-3) = 1$.

Solution:

$(x^2 - 5x + 7)^2 - (x - 2)(x - 3) = 1$

$(x^2 - 5x + 7)^2 - [x^2 - (2 + 3)x + 2 \times 3] = 1$

$(x^2-5x+7)^2 - [x^2 -5x+6] -1 = 0$

Let $x^2 - 5x = y$ -------------(1)

$(y+7)^2 - (y+6) -1 = 0$

$y^2 + 14y + 49 - y - 6 - 1 = 0$

$y^2 + 13y + 42 = 0$

$y = \frac{-13 \pm \sqrt{13^2 - 4(1)(42)}}{2 \times 1}$

$= \frac{-13 \pm \sqrt{169-168}}{2}$

$= \frac{-13 \pm \sqrt{1}}{2}$

$= \frac{-13+1}{2}$ or $\frac{-13-1}{2}$

$= \frac{-12}{2}$ or $\frac{-14}{2}$

= -6 or -7

$\therefore$ y = -6 or -7

Substituting y = -6 in (1)

$x^2 - 5x = -6$

$x^2 - 5x + 6=0$

x = 3 or x = 2

Substituting y = -7 in (1)

$x^2 - 5x = -7$

$x^2 - 5x + 7=0$

$x = \frac{5 \pm \sqrt{3}i}{2}$.

Question-30

Solve the following equation: $25x^2 - 30x + 11=0$.

Solution:

$25x^2 - 30x + 11=0$

$D = b^2 - 4ac = 900 - 4 \times 25 \times 11 = -200 < 0$

$\sqrt{D} = 10\sqrt{2}\,i$

Hence the two complex roots are : $\frac{30+10\sqrt{2}i}{50}, \frac{30-10\sqrt{2}i}{50}$

i.e $\frac{3+\sqrt{2}i}{5}, \frac{3-\sqrt{2}i}{5}$

Question-31

Prove that $x^4+4 = (x+1+i)(x+1-i)(x-1+i)(x-1-i)$.

Solution:

$(x+1+i)\,(x+1-i)\,(x-1+i)\,(x-1-i) = [(x+1)^2-i^2]\,[(x-1)^2 - i^2]$

$= (x^2+2x+1+1)\,(x^2-2x+1+1)$

$= [(x^2+2)+2x]\,[(x+2)-2x]$

$= (x^2+2)^2 - 4x^2$

$= x^4+4x^2+4-4x^2$

$= x^4+4$

Question-32

Solve the following equation: $5x^2 - 6x + 2 = 0$.

Solution:

$5x^2 - 6x + 2 = 0$

$D = b^2 - 4ac = 36 - 4\times 5\times 2 = -4 < 0$

$\sqrt{D} = 2i$

Hence the two complex roots are : $\frac{6+2i}{10}, \frac{6-2i}{10}$

i.e $\frac{3+i}{5}, \frac{3-i}{5}$

Question-33

Plot the following number and their complex conjugates on a complex number plane and find their absolute values: 5

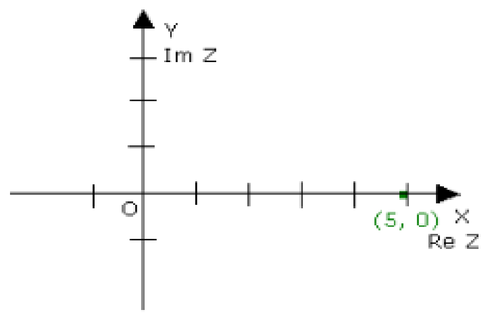
Solution:

Conjugate of 5 is 5.

The absolute value of 5 = $\sqrt{5^2+0^2}$

= $\sqrt{25}$

= 5



Question-34

If $(1+x)^n = p_0+p_1x+p_2x^2+ \ldots\ldots\ldots\ldots p_nx^n$, prove that : $p_0+p_3+p_6+\ldots\ldots\ldots\ldots = \frac{1}{3}(2^n + 2\cos\frac{n\pi}{3})$.

Solution:

$(1+x)^n = p_o+p_1x + p_2x^2+ \ldots\ldots\ldots\ldots p_nx^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

Put x = 1,w,w^2 in (1) and add

[$1+w = -w^2$ and $1+w^2 = -w$]

$3(p_o+p_3+p_6 \ldots\ldots\ldots\ldots) = 2^n+(-w^2)^n + (-w)^n \ldots\ldots\ldots\ldots\ldots (2)$

Now $w = \frac{-1+i\sqrt{3}}{2}$

$\therefore -w = \frac{1}{2} - i\frac{\sqrt{3}}{2} = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$, ($\because r = 1, \theta = \frac{\pi}{3}$)

$-w^2 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\therefore (-w)^n + (-w^2)^n = 2\cos\frac{n\pi}{3}$ (Demoivre' s Theorem)

Substituting in (2), $3(p_o+p_3+p_6\ldots\ldots\ldots\ldots) = 2^n + 2\cos\frac{n\pi}{3}$

or $p_o+p_3+p_6\ldots\ldots\ldots\ldots = \frac{1}{3}(2^n + 2\cos\frac{n\pi}{3})$.

Question-35

From an equation whose roots are the squares of the sum and difference of the roots of
$2x^2 + 2(m + n)x + m^2 + n^2 = 0$.

Solution:

Let α , β be the roots of the equation $2x^2 + 2(m + n)x + m^2 + n^2 = 0$.

Then $\alpha + \beta = -2(m + n)/2 = -(m + n)$

$\alpha\beta = (m^2 + n^2)/2$

The roots of the required equation are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

Sum of the roots = $(\alpha + \beta)^2 + (\alpha - \beta)^2 = (\alpha + \beta)^2 + [(\alpha + \beta)^2 - 4\alpha\beta]$

$= (m + n)^2 + [(m + n)^2 - \frac{4(m^2 + n^2)}{2}]$

$= 4mn$

Product of the roots = $(\alpha + \beta)^2 (\alpha - \beta)^2 = (\alpha + \beta)^2 [(\alpha + \beta)^2 - 4\alpha\beta]$

$= (m + n)^2 [(m + n)^2 - \frac{4(m^2 + n^2)}{2}]$

$= (m + n)^2 [2mn - m^2 - n^2]$

The required equation is $x^2 - 4mnx + (m + n)^2[2mn - m^2 - n^2] = 0$

or $x^2 - 4mnx + (m + n)^2[-(m - n)^2] = 0$

or $x^2 - 4mnx - (m^2 - n^2)^2 = 0$

Question-36

Find the values of the root $\sqrt{1-i}$.

Solution:

$1-i = \sqrt{2}(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4})$

$= \sqrt{2}\left[\cos(2n\pi + \frac{\pi}{4})\right] - i\sin(2n\pi + \frac{\pi}{4})$

$= \sqrt{2}(\cos(8n + 1)\frac{\pi}{4} - \sin(8n + 1)\frac{\pi}{4})$

$\sqrt{1-i} = 2^{1/4}[\cos(8n+1)\frac{\pi}{8} - i\sin(8n + 1)\frac{\pi}{8}]$

$= 2^{1/4}(\cos\frac{\pi}{8} - i\sin\frac{\pi}{8})$ for n = 0

$= 2^{1/4}(\cos(\pi + \frac{\pi}{8}) - i\sin(\pi + \frac{\pi}{8})$ for n = 1

$= -2^{1/4}(\cos\frac{\pi}{8} - i\sin\frac{\pi}{8})$ where

$\cos\frac{\pi}{8} = \sqrt{\left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right)}$, $\sin\frac{\pi}{8} = \sqrt{\left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)}$

Question-37

Solve the following equation: $3x^2 - 7x + 5 = 0$.

Solution:

$3x^2 - 7x + 5 = 0$

$D = b^2 - 4ac = 49 - 4\times 3\times 5 = -11 < 0$

$\sqrt{D} = \sqrt{11}\, i$

Hence the two complex roots are : $\frac{7+\sqrt{11}i}{6}, \frac{7-\sqrt{11}i}{6}$

Question-38

Solve the equation $25x^2 - 30x + 9 = 0$.

Solution:

$X = \frac{+30 \pm \sqrt{30^2 - 4(25)(9)}}{2\times 25} = \frac{30 \pm \sqrt{900-900}}{50} = \frac{30}{50}$

$X = \frac{3}{5}, \frac{3}{5}$

Question-39

Plot the following number and their complex conjugates on a complex number plane and find their absolute values: 2i

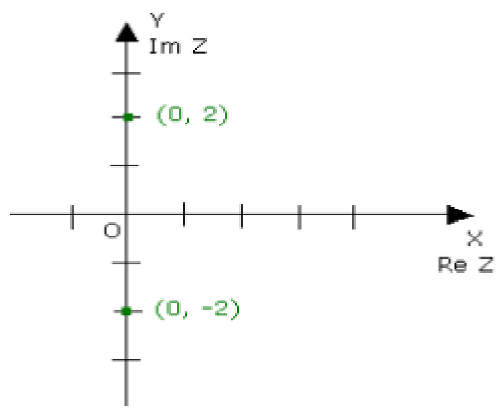
Solution:

Conjugate of 2i is -2i.

The absolute value of 2i = $\sqrt{0^2 + 2^2}$

$= \sqrt{4}$

$= 2$



Question-40

Plot the following number and their complex conjugates on a complex number plane and find their absolute values: -1/2 - 3i

Solution:

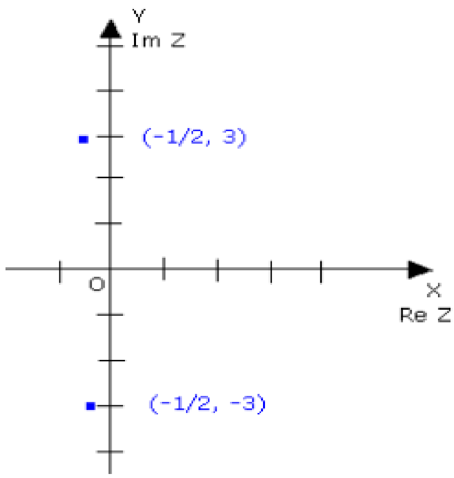
Conjugate of $-\frac{1}{2}-i3$ is $-\frac{1}{2}+i3$

The absolute value of $-\frac{1}{2}+i3$

$$= \sqrt{\left(-\frac{1}{2}\right)^2+3^2}$$

$$= \sqrt{\frac{1}{4}+9}$$

$$= \sqrt{\frac{37}{4}}$$



Question-41

If the roots of $x^2 - lx + m = 0$ differ by 1, then prove that $l^2 = 4m + 1$.

Solution:

Let a , b be the roots of the equation $x^2 - lx + m = 0$.

$\alpha + \beta = l$

$\alpha\beta = m$

$\alpha - \beta = 1$

$(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$l^2 = 1 + 4m$

Question-42

Solve the following equation: $13x^2 - 7x + 1 = 0$.

Solution:

$13x^2 - 7x + 1 = 0$

$D = b^2 - 4ac = 49 - 4\times 13\times 1 = -3 < 0$

$\sqrt{D} = \sqrt{3}\,i$

Hence the two complex roots are : $\frac{7+\sqrt{3}i}{26}, \frac{7-\sqrt{3}i}{26}$

Question-43

If z = x+iy and $z^{1/3}$= a-ib then show that $\frac{x}{a}-\frac{y}{b}=4(a^2-b^2)$.

Solution:

z = x+iy and $2^{1/3}$ = a-ib

$(x+iy)^{1/3}$ = a-ib

Cubing both sides,

$x+iy = (a-ib)^3$

$= a^3+b^3i-3abi(a-ib)$

$= a^3+b^3i-3a^2bi-3ab^2$

Equating the real and imaginary,

$x = a^3-3ab^2$

$y = b^3-3a^2b$

$\frac{x}{y}-\frac{y}{b}=a^2-3b^2-b^2+3a^2$

$= 4(a^2-b^2)$

Question-44

$(1-w+w^2)(1-w^2+w^4)(1-w^4+w^8)$......... to 2n factors = 2^{2n}.

Solution:

$(1-w+w^2)(1-w^2+w^4)(1-w^4+w^8)$ .... 2n factors.

$= (1-w+w^2)(1-w^2+w)(1-w+w^2)$ .... 2n factors.

(since $w^4=w$, $w^8 = w^2$ ....)

$= (-2w)(-2w^2)(-2w)(-2w^2)$ .... 2n factors.

$= (2^2w^3)(2^2w^3)$ .... n factors.

$= (2^2)^n = 2^{2n}$

Question-45

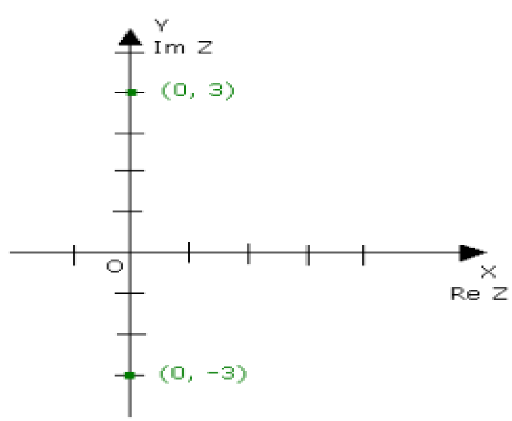
Plot the following number and their complex conjugates on a complex number plane and find their absolute values: √(-3)

Solution:

$\sqrt{-3}$ = 3i

Conjugate of 3i is −3i.

The absolute value of 3i = $\sqrt{3^2}$ = $\sqrt{9}$ = 3



Question-46

Solve the following equation: $9x^2+10x+3=0$.

Solution:

$9x^2+10x+3=0$

$D = b^2-4ac = 100-4\times 9\times 3 = -8 < 0$

$\sqrt{D} = 2\sqrt{2}i$

Hence the two complex roots are : $\frac{-10+2\sqrt{2}i}{18}, \frac{-10-2\sqrt{2}i}{18}$

i.e $\frac{-5+\sqrt{2}i}{9}, \frac{-5-\sqrt{2}i}{9}$