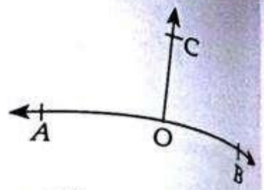
04

Lines and Angles

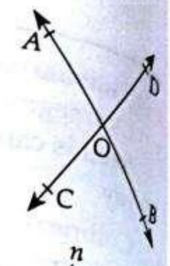
Fundamental terms and Definitions :

1. Line segment and ray : The part of a straight line whose both ends are fixed is called a line segment. If one point of a line is fixed, it is called a ray.
2. Collinear points and Non-collinear points : If three or more points lie on a straight line, they are called collinear point. If three or more points donot lie on a straight line, they are called non-collinear points.
3. Types of angles : According to measurement, angles are of following types.
   - 3.1. Acute angle : If an angle lies between 0° and 90°, it is called acute angle.
   - 3.2. Right angle : An angle whose measurement is 90° is called a right angle.
   - 3.3. Obtuse angle : If an angle lies between 90° and 180°, it is called obtuse angle.
   - 3.4. Straight angle : An angle whose measurement is 180° is called a straight angle.
   - 3.5. Reflex angle : If an angle lies between 180° and 360°, it is called Reflex angle.
4. Complementary angles and Supplementary angles : If sum of two angles

6. **Linear pair of angles** : In the adjacent figure ∠AOC and ∠COB are adjacent angles and AOB is a straight line i.e., uncommon sides of adjacent sides form a straight line. Such angles are called linear pair of angles.

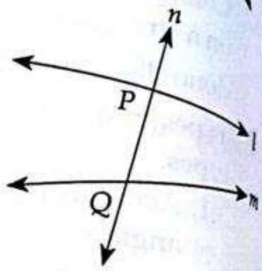


7. **Vertically opposite angles** : If two straight lines AB and CD intersect each other at point O, then angles facing each other is called vertically opposite angles. In the adjacent figure, ∠AOD and ∠BOC are one pair of vertically opposite angles, while ∠AOC and ∠BOD are another pair of vertically opposite angles.

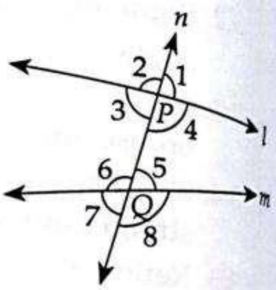


8. **Transversal line** : A straight line intersecting two or more lines at different points is called a transversal line.

In the given figure straight line n intersects two different lines l and m respectively at point P and Q, so line n is a transversal line.



9. **Exterior angles and Interior angles** : In the figure given below, a transversal line n intersects two straight lines l and m respectively at P and Q. Around each point P and Q, four angles are formed, among these angles ∠1, ∠2, ∠7, ∠8 are called exterior angles while ∠3, ∠4, ∠5, ∠6 are called interior angles.



10. **Corresponding angles and Alternate angles** : In the figure drawn above

10.1. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7 and ∠4 and ∠8 are called pair of corresponding angles.

10.2. "∠4 and ∠6" and "∠3 and ∠5" are called pair of alternate interior angles.

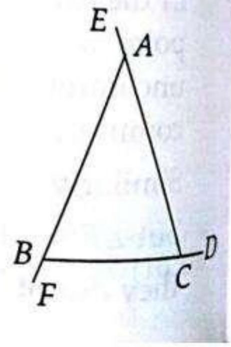
10.3. "∠1 and ∠7" and "∠2 and ∠8" are called alternate exterior angles.

10.4. "∠4 and ∠5" and "∠3 and ∠6" are called consecutive interior angles or Alternate interior/exterior allied angles or co-interior angle.

All type of alternate angles are commonly known as alternate angles.

11. **Exterior angle and Interior opposite angle of a triangle** : In the adjacent figure sides BC, CA and AB of triangle ABC are respectively produced to points D, E and F. ∠ACD, ∠BAE and ∠CBF thus formed are called exterior angles of the triangle.

Interior angles ∠A and ∠B are called interior opposite



angles to the exterior angle $\angle ACD$. Similarly $\angle B$ and $\angle C$ are interior opposite angles to the exterior angle BAE etc.

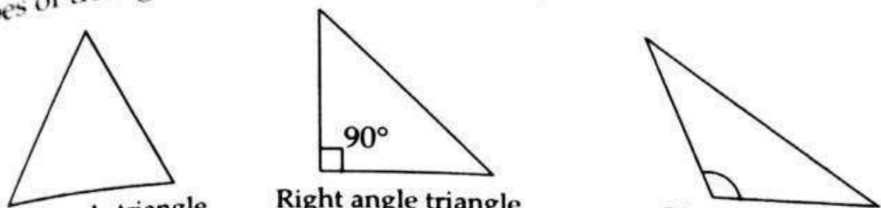
12. Types of triangles according to sides :

12.1. Equilateral triangle : When all the sides of a triangle are equal, it is called an equilateral triangle.

12.2. Isosceles triangle : If any two sides of a triangle are equal, it is called an isosceles triangle.

12.3. Scalene triangle : If sides of a triangle are unequal, it is called a scalene triangle.

13. Types of triangles according to their angles :



Acute angle triangle Right angle triangle Obtuse angle triangle

13.1. Acute angle triangle : If all the three angles of a triangle are acute, then the triangle is called an acute angle triangle.

13.2. Right angle triangle : If one of the angle of a triangle is right angled (= 90°) then it is called a right angle triangle.

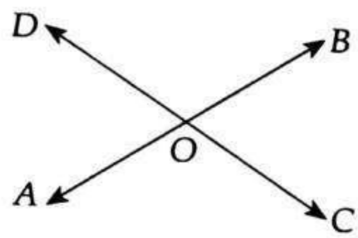
A triangle has at most one right angle.

13.3 Obtuse angle triangle : If one of the angle of a triangle is obtuse (lies between 90° and 180°) then it is called an obtuse angle triangle.

A triangle has at most one obtuse angle.

Some Theorems (Results) based on angles and straight line

1. If two straight lines intersect each other then vertically opposite angles are equal. In the given figure,



$\angle BOC = \angle AOD$ (Vertically opposite angles)

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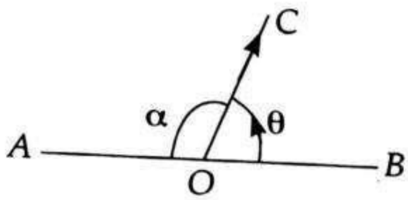
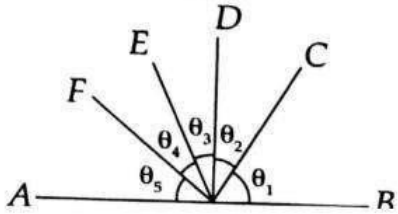
2. If a ray is inclined on a line then the sum of linear pair of angles thus formed is equal to 180° and its converse is also true.



In the given figure ray OC is standing (inclined) on the line AB,

Therefore $\theta + \alpha = 180°$

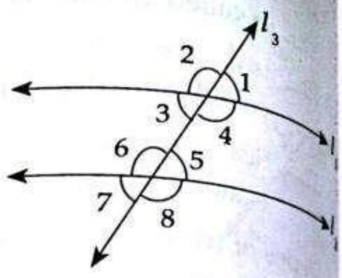
Conversely, if $\theta + \alpha = 180°$ then AOB will be a straight line.



In general, in the adjacent figure many rays are coming out from a point O on the straight line AB,

$\therefore \quad \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 180°$

Converse of this statement is also true.



3. If a transversal line intersects two parallel lines then the pair of corresponding angles thus formed are equal and its converse is also true.

In the adjacent figure, two parallel lines l_1 and l_2 are intersected by a transversal line l_3 and thus the following pair of corresponding angles are equal—

$\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 3 = \angle 7$ and $\angle 4 = \angle 8$

Conversely, it at least one of the pair of corresponding angles are equal (say $\angle 1 = \angle 5$) the lines l_1 and l_2 are parallel. If $\angle 2 = \angle 6$ then the two lines are parallel etc.

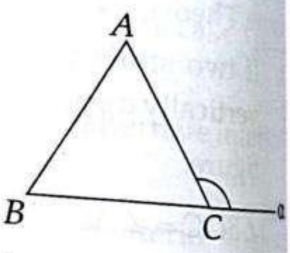
4. If a transverse line intersects two parallel lines then pair of alternate angles are equal and its converse is also true. In above figure,

$\angle 3 = \angle 5$ and $\angle 4 = \angle 6$ (Alternate interior angles)

$\angle 1 = \angle 7$ and $\angle 2 = \angle 8$ (Alternate exterior angles)

5. When a transversal line intersects two parallel lines, sum of consecutive interior angles (or allied angles or co-interior angles is equal to 180° (i.e., consecutive interior angles are supplementary), its converse is also true. In the above figure,

$\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$

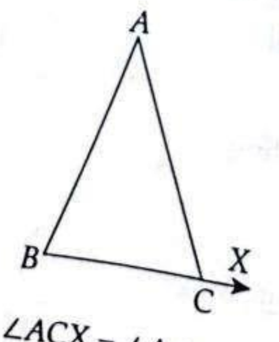


6. The sum of all the three angles of a triangle is equal to 180° (i.e., two right angle)

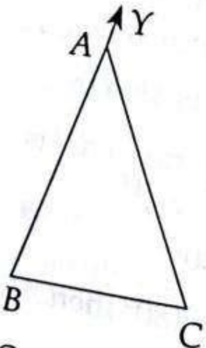
$\angle A + \angle B + C = 180°$

or, $\angle BAC + \angle ABC + \angle BCA = 180°$

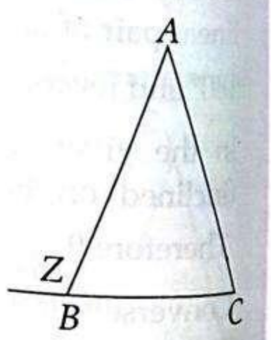
7. If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of two interior opposite angles.



In fig, $\angle ACX = \angle A + \angle B$



$\angle CAY = \angle B + \angle C$



$\angle ABZ = \angle A + \angle C$

angles to the exterior angle $\angle ACD$. Similarly $\angle B$ and $\angle C$ are interior opposite angles to the exterior angle BAE etc.

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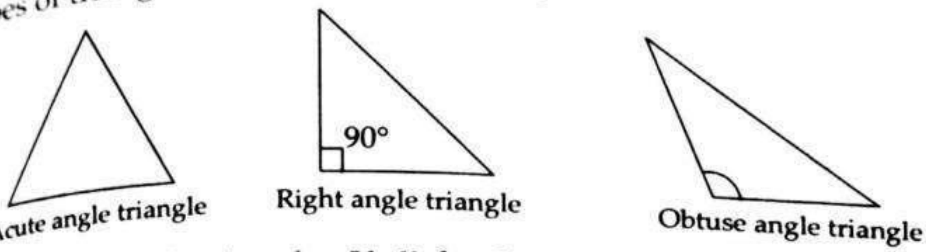

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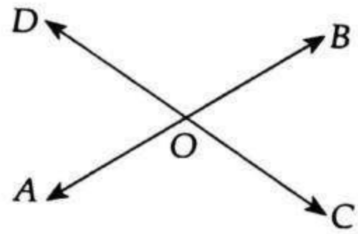

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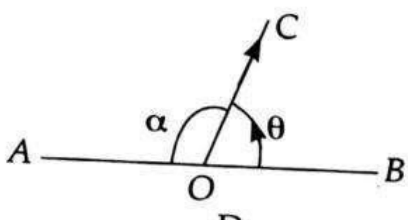

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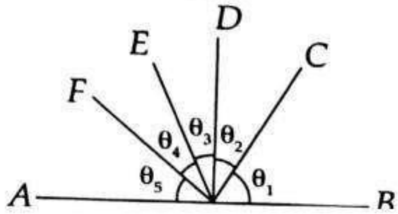

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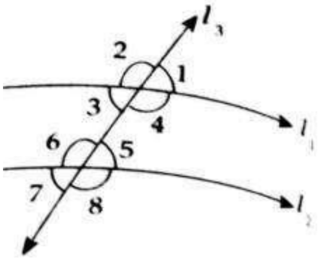

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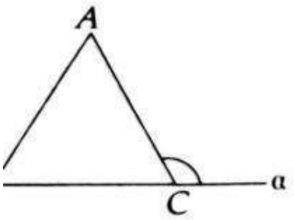

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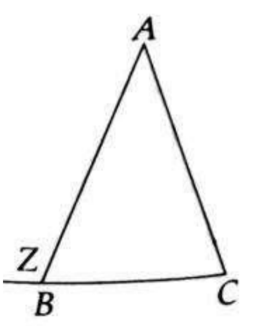
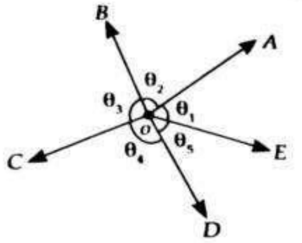


$\angle ABZ = \angle A + \angle C$

Some Important Points to Solve Objective Questions.

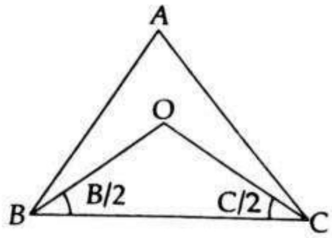
1. Sum of all angles around a point is 360°

   In the figure given below $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360°$



2. If OB and OC are angle bisector of base angles $\angle B$ and $\angle C$ of ΔABC, then $\angle BOC = 90° + \frac{\angle A}{2}$

   Proof : In the adjacent figure,



$$\angle OBC = \frac{\angle B}{2} \text{ and } \angle OCB = \frac{\angle C}{2}$$

$$\therefore \angle BOC = 180° - \frac{\angle B}{2} - \frac{\angle C}{2}$$

$$= 180° - \left(\frac{B + C}{2}\right)$$

$$= 180° - \left(\frac{180° - A}{2}\right)$$

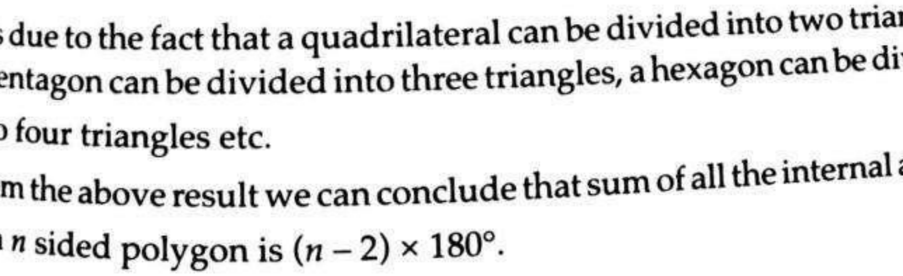
$$(\because \angle A + \angle B + \angle C = 180° \Rightarrow \angle B + \angle C = 180° - \angle A)$$

$$= 90° + \frac{\angle A}{2}$$

3. (i) Sum of all the internal angles of a Quadrilateral is 360°

   (ii) Sum of all the internal angles of a pentagon (five sides) is 540°.

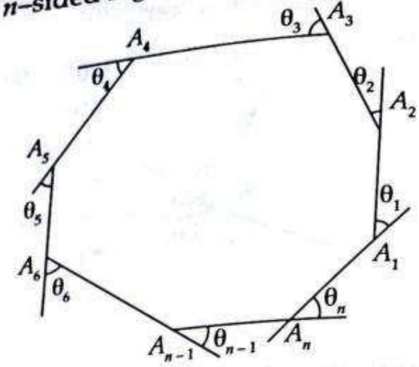
   (iii) Sum of all the internal angles of a hexagon (six sides) is 720° etc.



   It is due to the fact that a quadrilateral can be divided into two triangles, a pentagon can be divided into three triangles, a hexagon can be divided into four triangles etc.

4. From the above result we can conclude that sum of all the internal angles of a n sided polygon is $(n - 2) \times 180°$.

5. Each angle of a n–sided regular polygon is $\frac{(n-2)180^\circ}{n}$.



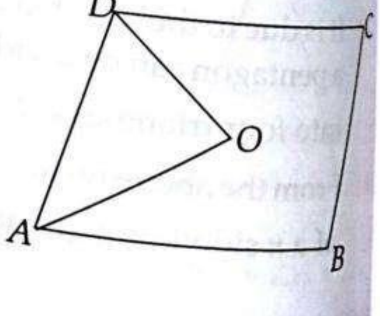
6. In the above figure, side of a polygon are produced in the same order. Angle $\theta_1 + \theta_2 + \theta_3 + \ldots + \theta_n$ thus formed are called exterior angles.
The sum of all the exterior angles of a polygon is 360°.
i.e., $\theta_1 + \theta_2 + \theta_3 + \ldots + \theta_n = 360^\circ$
If polygon is a regular polygon then each angle $= \frac{360^\circ}{n}$.
From the above discussion we can conclude that if side of a triangle or a quadrilateral or a pentagon or a hexagon etc. are produced in the same order, sum of exterior angles in all cases = 360°

7. n sided polygon has $\frac{n(n-3)}{2}$ diagonals.

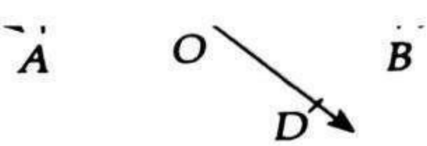
[n] sided polygon has $\frac{n(n-3)}{2}$ diagonals.
[a]t $n = 4$, Quadrilateral has $\frac{4(4-3)}{2} = 2$ diagonals.
at $n = 5$, Pentagon has $\frac{5(5-3)}{2} = 5$ diagonals.
at $n = 6$, Hexagon has $\frac{6(6-3)}{2} = 9$ diagonals etc.

8. The angle between angle bisector of two adjacent angles of a quadrilateral is equal to half the sum of remaining angles.
In the given figure OD and OA are internal bisector of $\angle D$ and $\angle A$ respectively.

$$\angle AOD = 180^\circ - \frac{\angle A}{2} - \frac{\angle D}{2}$$
$$= 180^\circ - \frac{1}{2}(\angle A + \angle D)$$
$$= 180^\circ - \frac{1}{2}(360^\circ - \angle B - \angle C)$$
$$= \frac{1}{2}(\angle B + \angle C)$$

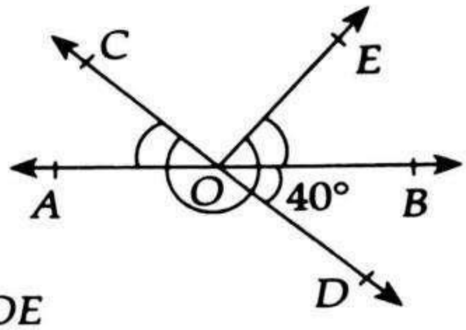


1 In the ... intersects at O. If ... $\angle BOD = 40^\circ$ then find $\angle BOE$ and reflexive $\angle COE$.



Solution : $\angle AOC = \angle BOD$ (Vertically opposite angle)

$\therefore \quad \angle AOC = 40^\circ$



According to question, $\angle AOC + \angle BOE = 70^\circ$

or, $40^\circ + \angle BOE = 70^\circ$

$\therefore \quad \angle BOE = 30^\circ$

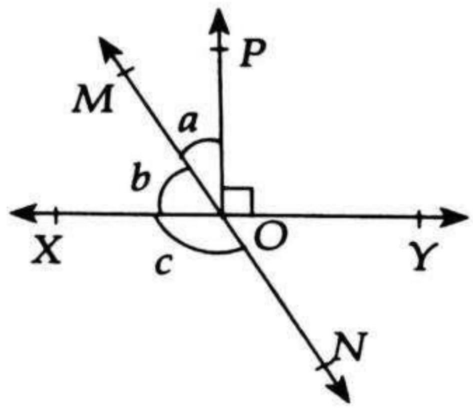
Now, $\angle COD + \angle DOB + \angle BOE =$ reflexive $\angle COE$

$\therefore \quad 180^\circ + 40^\circ + 30^\circ =$ reflexive $\angle COE$

$\therefore \quad$ reflexive $\angle COE = 250^\circ$

2. In the figure given below lines XY and MN intersect at O.

If $\angle POY = 90^\circ$ and $a : b = 2 : 3$ then find the measure of c.



Solution : Given, $\frac{a}{b} = \frac{2}{3}$

Let $a = 2k$ and $b = 3k$

$\because \quad \angle POY = 90^\circ \quad \therefore \quad \angle POX = 90^\circ$

or, $\angle a + \angle b = 90^\circ$ or, $2k + 3k = 90^\circ$

or, $5k = 90^\circ$ or, $k = 18^\circ$

$\therefore \quad \angle b = 3k = 3 \times 18^\circ = 54^\circ$

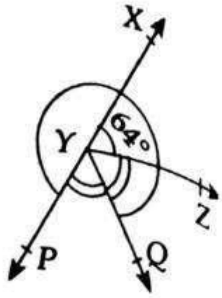
Now, $\angle XOM = \angle YON$ (Vertically opposite angle)

$\therefore \quad \angle YON = 54^\circ$

Again, $\angle XON + \angle YON = 180^\circ$ (Linear pair of angles axiom)

or, $c + 54^\circ = 180^\circ \quad \therefore \quad c = 126^\circ$

3. Given that $\angle XYZ = 64^\circ$ and line XY is produced to point P. Draw a diagram from the given inf...



or, $2\angle ZYQ = 116^\circ$

$\therefore \quad \angle ZYQ = 58^\circ$

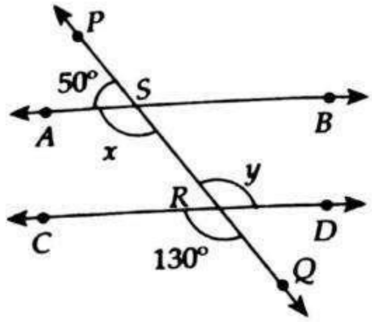
$\because \quad \angle XYQ = \angle XYZ + \angle ZYQ$

$\therefore \quad \angle XYQ = 64^\circ + 58^\circ = 122^\circ$

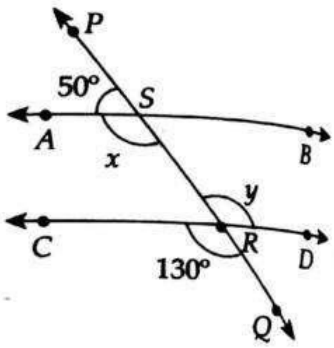
Now reflex, $\angle QYP = \angle PYX + \angle XYQ$

$= 180^\circ + 122^\circ = 302^\circ$

4. In the figure given below find x and y and hence prove that $AB \parallel CD$.



Solution : $\angle DRS = \angle CRQ$ *(Vertically opposite angle)*



$\therefore \quad y = 130^\circ$

Also, $\angle ASP + \angle ASR = 180^\circ$

(Linear pair of angles)

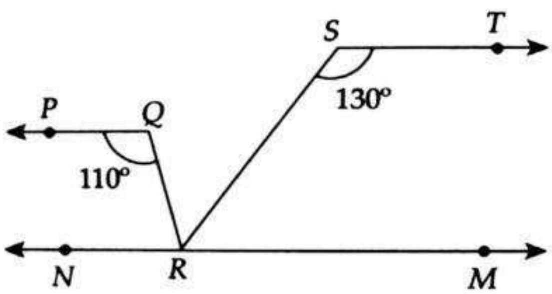
or, $50^\circ + x = 180^\circ$

or, $x = 130^\circ$

$\because \quad x = y = 130^\circ$

$\therefore \quad AB \parallel CD$ *(Alternate angle)*

5. In the given figure if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, Find $\angle QRS$.



Solution : From point R draw a line RM parallel ST.

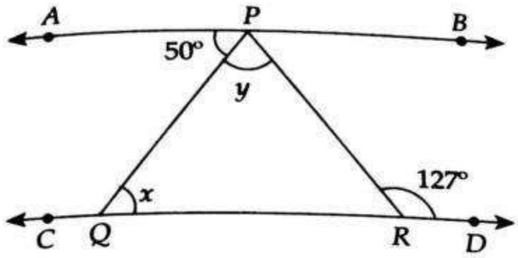
$\angle RST + \angle SRM = 180^\circ$ *(Consecutive interior angle)*

or, $130^\circ + \angle SRM = 180^\circ \quad \therefore \quad \angle SRM = 50^\circ$ ... (i)

Now, $\angle QRM = \angle PQR$ *(Alternate angle)* or, $\angle QRM = 110^\circ$ or, $\angle QRS + \angle SRM = 110^\circ$

or, $\angle QRS + 50^\circ = 110^\circ$ (from (i)) $\therefore\ \angle QRS = 60^\circ$

6. In the given figure if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Solution : $\because AB \parallel CD \quad \therefore \quad \angle APR = \angle PRD$ *(Alternate angle)*

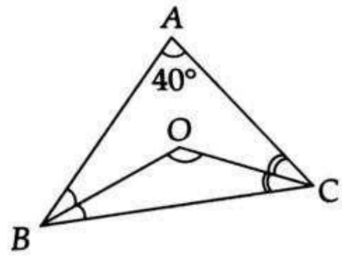
or, $\angle APQ + \angle QPR = \angle PRD$

or, $50^\circ + y = 127^\circ$ or, $y = 77^\circ$

and $\angle APQ = \angle PQR$ *(Alternate angle)*

or, $50^\circ = x$, $\therefore\ x = 50^\circ$

7. In ΔABC, $\angle A = 40^\circ$. If bisector of $\angle B$ and $\angle C$ meets at O then prove that $\angle BOC = 110^\circ$



Solution : In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

or, $40^\circ + \angle B + \angle C = 180^\circ$

or, $\angle B + \angle C = 140^\circ$

or, $\frac{\angle B}{2} + \frac{\angle C}{2} = 70^\circ$ ... (i)

Now, In ΔBOC, $\angle OBC + \angle OCB + \angle BOC = 180^\circ$

or, $\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BOC = 180^\circ$ or, $70^\circ + \angle BOC = 180^\circ$ (from (i))

$\therefore\ \angle BOC = 110^\circ$ **Proved.**

[Shortcut : $\angle BOC = 90^\circ + \frac{A}{2} = 90^\circ + 20^\circ = 110^\circ$ **Proved.**]

8. If angles of a triangle are is the ratio 2 : 3 : 4, then find the least and greatest angle.

Solution : Let angle be $2x^\circ$, $3x^\circ$ and $4x^\circ$.

$\therefore\ 2x^\circ + 3x^\circ + 4x^\circ = 180^\circ$

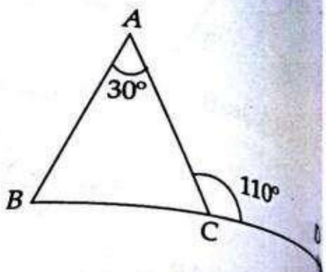
or, $9x^\circ = 180^\circ$ or, $x^\circ = \frac{180^\circ}{9} = 20^\circ$

$\therefore$ least angle $= 2x^\circ = 2 \times 20^\circ = 40^\circ$

greatest angle $= 4x^\circ = 4 \times 20^\circ = 80^\circ$

9. **The exterior angle of a triangle is 110° and one of its interior opposite angle is 30°, find other angles.**

Solution : Consider the triangle ABC in which exterior $\angle ACD = 110°$ and $\angle A = 30°$, we have to find $\angle B$ and $\angle C$.



$\because \quad \angle ABC + \angle BAC = \angle ACD$

or, $\angle ABC + 30° = 110°$

or, $\angle ABC = 80°$

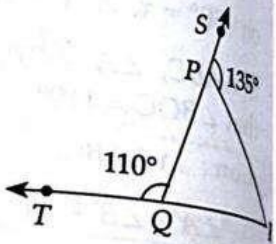
or, $\angle ACB + \angle ACD = 180°$

or, $\angle ACB + 110° = 180°$

$\therefore \quad \angle ACB = 70°$

10. **In triangle PQR, sides QP and RQ respectively produced to point S and T. If $\angle SPR = 135°$ and $\angle PQT = 110°$ find $\angle PRQ$.**

Solution : $\angle SPR + \angle QPR = 180°$ (linear pair of angles)



or, $135° + \angle QPR = 180°$

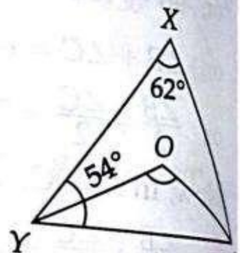
or, $\angle QPR = 45°$ ... (i)

Now, $\angle QPR + \angle PRQ = \angle PQT$

or, $45° + \angle PRQ = 110°$ (from (i))

$\therefore \quad \angle PRQ = 65°$

11. **In the adjacent figure $\angle X = 62°$ and $\angle XYZ = 54°$. If YO and ZO respectively bisects $\angle XYZ$ and $\angle XZY$ then find $\angle OZY$ and $\angle YOZ$.**



Solution : In ΔXYZ, $\angle YXZ + \angle XYZ + \angle XZY = 180°$

or, $62° + 54° + \angle XZY = 180°$

$\therefore \quad \angle XZY = 64°$

From question,

$$\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2} \times 54° = 27°$$

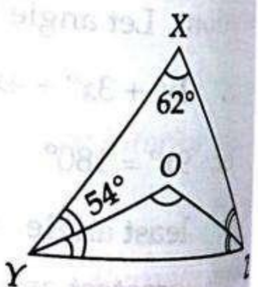
and $\angle OZY = \frac{1}{2}\angle XZY = \frac{1}{2} \times 64° = 32°$

Now, In ΔOYZ,

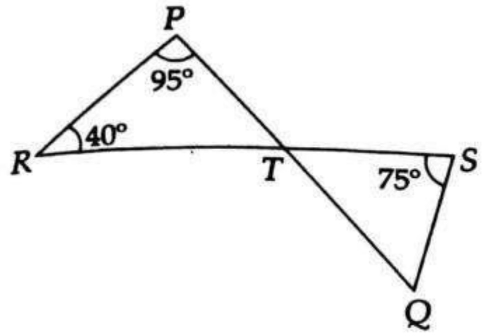
$$\angle OYZ + \angle OZY + \angle YOZ = 180°$$

or, $27° + 32° + \angle YOZ = 180°$

$\therefore \quad \angle YOZ = 121°$



12. In the given figure lines PQ and RS intersect at point T such that $\angle PRT = 40°$; $\angle RPT = 95°$ and $\angle TSQ = 75°$, Find $\angle SQT$.



Solution : In ΔPRT, $\angle RPT + \angle PRT = \angle PTS$

or, $95° + 40° = \angle PTS$

$\therefore$ $\angle PTS = 135°$ ... (i)

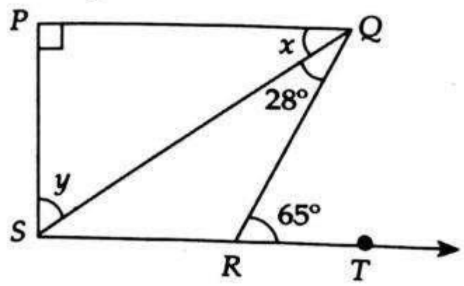
Again in,

ΔTSQ, $\angle TSQ + \angle SQT = \angle PTS$

or, $75° + \angle SQT = 135°$ $(\because \angle TSQ = 75°$ and $\because \angle PTS = 135°)$

$\therefore$ $\angle SQT = 135° - 75° = 60°$.

13. In the figure given below if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$ find x and y.



Solution : $\because$ $PQ \parallel SR$

$\therefore$ $\angle PQR = \angle QRT$ (Alternate angle)

or, $x + 28° = 65°$

or, $x = 65° - 28° = 37°$ ... (i)

Now, In

ΔPQS, $\angle SPQ + \angle PQS + \angle QSP = 180°$

or, $90° + x + y = 180°$

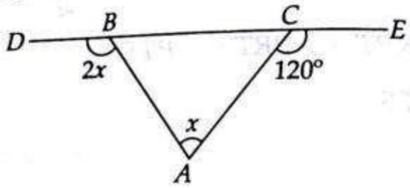
or, $90° + 37° + y = 180°$ (from (i))

or, $y = 180° - 127° = 53°$

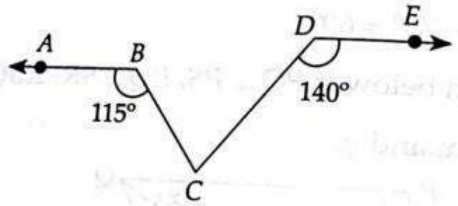
Hence, $x = 37°$ and $y = 53°$.

Exercise–4A

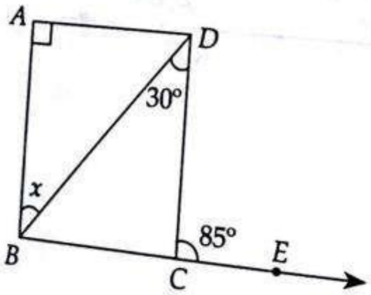
1. The angle between two legs of a compass is 60° and length of each leg is 10 cm. The distance between end points of the leg is
   (a) 5 cm (b) 10 cm (c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm
2. Angle at vertex of an isosceles triangle is 15° more than one of the angles at base. Angle at vertex is
   (a) 35° (b) 55° (c) 65° (d) 70°
3. In the given figure, value of x is



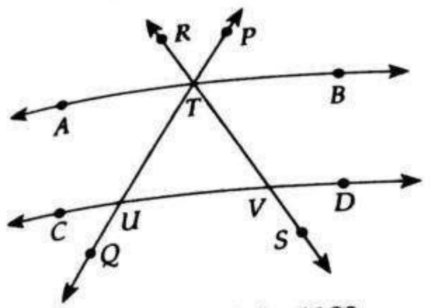
   (a) 30° (b) 40° (c) 45° (d) 60°
4. Given that $AB \parallel DE$, $\angle ABC = 115°$, $\angle CDE = 140°$, What is the value of $\angle BCD$?



   45° (b) 55° (c) 65° (d) 75°
5. that $AD \parallel BE$, $AB \perp rAD$ $\angle DEC = 85°$, $\angle BDC = 30°$, What is the of x?

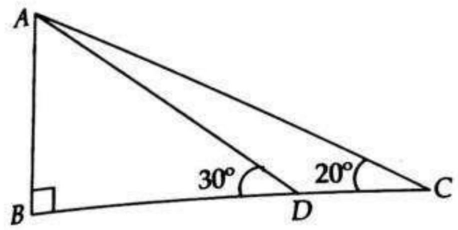


   (a) 30° (b) 35° (c) 45° (d) 55°
6. O is a point on the line LM. A line ON is drawn which is neither coincident with OL nor with OM. If $\angle MON$ is one third of $\angle LON$, then $\angle MON$ is equal to
   (a) 45° (b) 60° (c) 75° (d) 80°
7. In the given figure $AB \parallel CD$, $\angle PTB = 55°$ and $\angle DVS = 45°$. Sum of $\angle CUQ$ and $\angle RTP$ is



(a) 180° (b) 135° (c) 110° (d) 100°

In the given figure, $\angle ABD = 90°$, $\angle BDA = 30°$ and $\angle BCA = 20°$. What is the value of $\angle CAD$?



(a) 10° (b) 20° (c) 30° (d) 15°

Length of line segment AB is 2 unit. It is divided by point C such that $AC^2 = AB \times CB$, the length of CB is

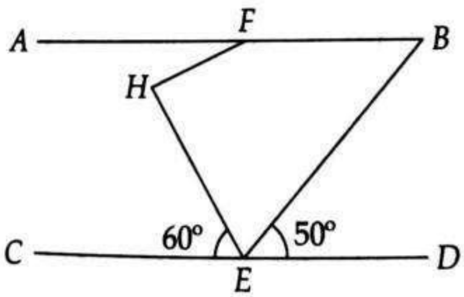
(a) $3 + \sqrt{5}$ unit (b) $3 - \sqrt{5}$ unit

(c) $2 - \sqrt{5}$ unit (d) $\sqrt{3}$ unit

Sides AB and AC of a triangle ABC are equal. Side BC is produced to point D. From a point E on AC, line EF is drawn parallel to AB. Consider the quadrilateral $ECDF$ thus formed. If $\angle ABC = 65°$ and $\angle EFD = 80°$, then what is the value of $\angle FDC$?

(a) 43° (b) 41° (c) 37° (d) 35°

In the given figure AB is parallel to CD and BE is parallel to FH. Measure of $\angle FHE$ is

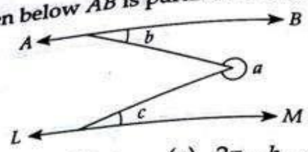


(a) 110° (b) 120° (c) 125° (d) 130°

Which of the following cannot be number of diagonals of a polygon ?

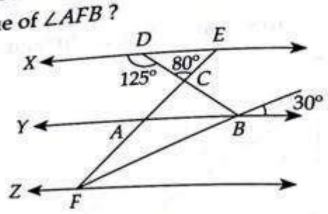
(a) 14 (b) 20 (c) 28 (d) 35

13. In the figure given below AB is parallel to LM. Angle a is equal to



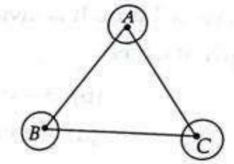
(a) $\pi + b + c$ (b) $2\pi - b + c$ (c) $2\pi - b - c$ (d) $2\pi + b - c$

14. Three lines X, Y and Z are parallel and angles are as shown in the figure. What is the value of $\angle AFB$?



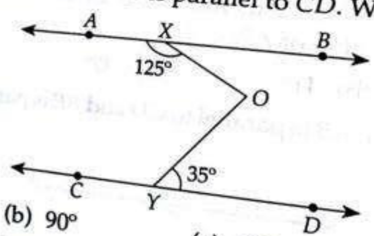
(a) 20° (b) 15° (c) 30° (d) 10°

15. In the figure given below sum of angles around the point A, B, C ex-cluded angles of the triangle is



60° (b) 720° (c) 900° (d) 1000°

figure given below AB is parallel to CD. What is $\angle XOY$?



(a) 80° (b) 90° (c) 95° (d) 100°

17. Side BC of the triangle ABC is produced to point D. If $\angle ACD = 120°$ $\angle ABC = \frac{2}{3}\angle CAB$, then what is the measure of $\angle BAC$?

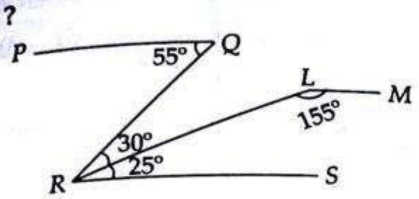
(a) 60° (b) 45° (c) 30° (d) 72°

18. Line segments AB and CD intersects at O. OF is the internal bisector of obtuse angle BOC and OE is the internal bisector of acute angle AOC. If $\angle BOC = 130°$ then measure of $\angle FOE$ is

(a) 90° (b) 110° (c) 115° (d) 120°

19.
20.
21.
22.
23.

19. In the figure given below RS is parallel to PQ. What is the angle between lines PQ and LM ?

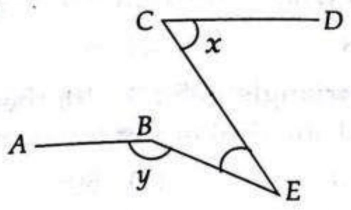


(a) 175° (b) 177° (c) 179° (d) 180°

20. Which angle is two third of its complementary angle ?

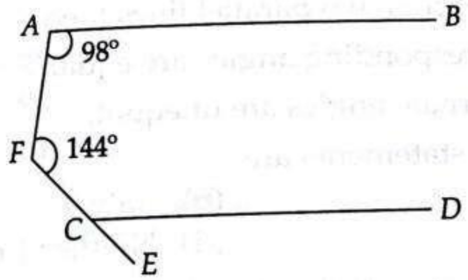
(a) 36° (b) 45° (c) 48° (d) 60°

21. In the figure given below AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then $\angle CEB$ is equal to



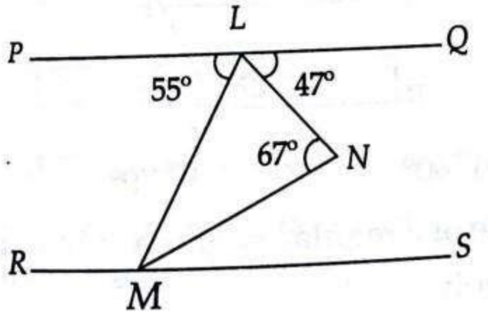
(a) $y - x$ (b) $\frac{(x+y)}{2}$ (c) $x + y - \left(\frac{\pi}{2}\right)$ (d) $x + y - \pi$

22. In the figure given below AB and CD are parallel. If $\angle BAF = 98°$ and $\angle AFC = 144°$, then $\angle ECD$ equals



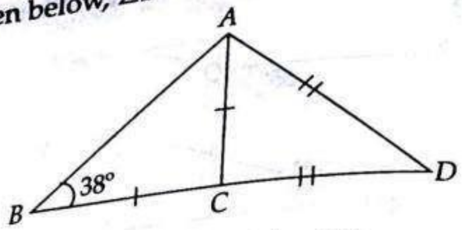
(a) 62° (b) 64° (c) 82° (d) 84°

23. In the figure given below PQ is parallel to RS. What is the measure of $\angle NMS$?



(a) 20° (b) 23° (c) 27° (d) 47°

24. In the figure given below, $\angle B = 38°$, AC ... CD. $\angle D$ is equal



(a) 26°   (b) 28°   (c) 38°   (d) 52°

25. If two angles are complementary to each other then each angle is
(a) Obtuse angle   (b) Right angle
(c) Acute angle   (d) Supplementary angle

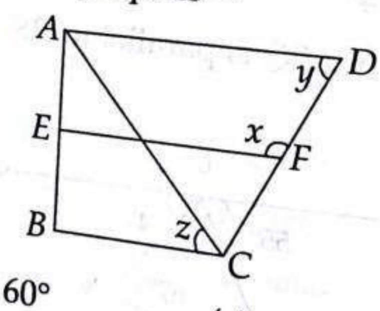
26. If each interior angle of a regular polygon is $\frac{7}{6}$ times each angle of regular hexagon, then what is the number of sides in the polygon?
(a) 7   (b) 8   (c) 9   (d) 10

27. If ratio of angles of a triangle is 5 : 3 : 10, then what is the difference between its largest and smallest angle ?
(a) 20°   (b) 30°   (c) 50°   (d) 70°

28. What is the measure of the angle which is one fifth of its supplementary part ?
(a) 15°   (b) 30°   (c) 36°   (d) 75°

29. Consider the following statements :
If a transversal line cuts two parallel lines then
1. Each pair of corresponding angles are equal.
2. Each pair of alternate angles are unequal.
Among these, true statements are—
(a) only 1   (b) only 2
(c) both 1 and 2   (d) Neither 1 nor 2

30. *ABCD* is a trapezium such that $AD \parallel BC$. If *EF* is parallel to *BC*, $\angle x = 120°$ and $\angle z = 50°$, then $\angle y$ equals—



(a) 50°   (b) 60°   (c) 70°   (d) 80°

31. If each interior angle of a regular polygon is 144°, then what is the number of sides in the polygon ?
(a) 10   (b) 20   (c) 24   (d) 36

... sides in the polygon is

(d) 30

33. ... of a regular polygon is 1080°, then number of sides in the polygon is [SSC Tier-I 2014]

(a) 6 (b) 8 (c) 10 (d) 12

34. The ratio of sides of two regular polygon is 1 : 2 and ratio of their internal angle is 2 : 3. What is the number of sides of polygon having more sides?

(a) 4 (b) 8 (c) 6 (d) 12

35. In the two regular polygon number of sides are in the ratio 5 : 4. If difference between their internal angles is 6°, then number of sides in the polygon is

(a) 15, 12 (b) 5, 4 (c) 10,8 (d) 20, 16

36. If each of interior angle of a polygon is double its each exterior angle, then number of sides in the polygon is

(a) 8 (b) 6 (c) 5 (d) 7

37. Which the following cannot be measure of an interior angle of a regular polygon ?

(a) 150° (b) 105° (c) 108° (d) 144°

38. Number of diagonals in a polygon having 10 sides is

(a) 20 (b) 40 (c) 35 (d) 32

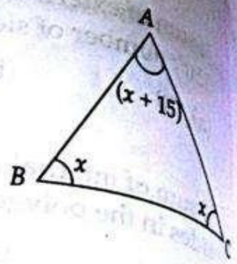
39. If one internal angle of a regular polygon is 135°, then number of diagonals in the polygon is

(a) 16 (b) 18 (c) 24 (d) 20

Answers –4A

1. (b)	2. (b)	3. (d)	4. (d)	5. (b)	6. (a)	7. (b)	8. (a)
9. (b)	10. (d)	11. (a)	12. (c)	13. (c)	14. (b)	15. (c)	16. (b)
17. (d)	18. (a)	19. (d)	20. (a)	21. (d)	22. (a)	23. (a)	24. (b)
25. (c)	26. (c)	27. (d)	28. (b)	29. (a)	30. (b)	31. (a)	32. (c)
33. (b)	34. (b)	35. (a)	36. (b)	37. (b)	38. (c)	39. (d)	

2. **(b)** Let each base angle of isosceles triangle = x



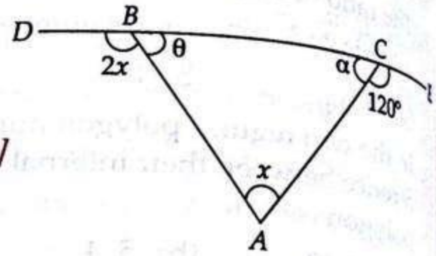
$\therefore$ Angle at vertex = $(x + 15°)$

We know that

$\therefore$ $x + 15° + x + x = 180°$

$\Rightarrow$ $x = \frac{165}{3} = 55°$

3. **(d)** Let $\angle ABC = \theta$ and $\angle ACB = \alpha$



then, $\alpha + 120° = 180°$

$\Rightarrow$ $\alpha = 60°$

$\Rightarrow$ $\alpha + x = 2x$

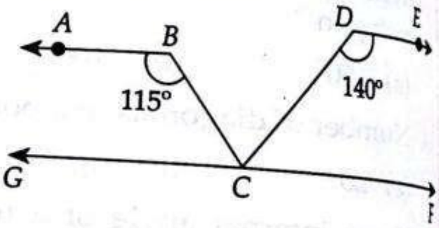
[Sum of interior opposite angle]

$\Rightarrow$ $60° + x = 2x$

$\Rightarrow$ $2x - x = 60°$

$\therefore$ $x = 60°$

4. **(d)** Draw a line through point C, which is parallel to each of AB and DE



$\because$ $AB \parallel GF \parallel DE$

$\therefore$ $\angle BCG = 180° - \angle ABC$

$= 180° - 115° = 65°$

and $\angle DCF = 180° - \angle CDE$

$= 180° - 140° = 40°$

Now $\angle BCG + \angle BCD + \angle DCF = 180°$ *(Linear pair of angles)*

$\Rightarrow$ $65° + \angle BCD + 40° = 180°$

$\therefore$ $\angle BCD = 180° - 105° = 75°$

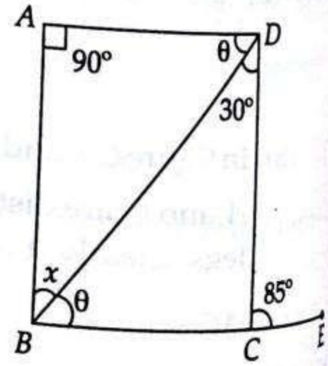
5. **(b)** Let $\angle ADB = \theta$

$\because$ $AD \parallel BE$

$\therefore$ $\angle CBD = \theta$ *(alternate angle)*

$\therefore$ $\theta + 30° = 85°$

$\Rightarrow$ $\theta = 55°$ *[sum of interior opposite angle]*



In ΔABD,

$90° + \theta + x = 180°$

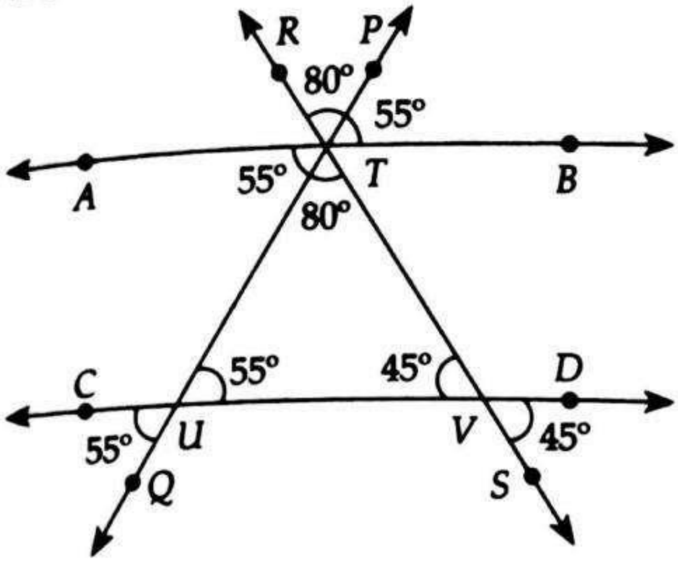
or, $90° + 55° + x = 180°$

$\therefore$ $x = 35°$

$3x + x = 18$[redacted]

$\Rightarrow x = 45°$

(c) Since $\angle PTB = 55°$



then $\angle TUV = 55°$ *(Corresponding angle)*

and $\angle CUQ = \angle TUV = 55°$ *(Vertically opposite angle)*

Given, $\angle DVS = 45°$

$\therefore \angle UVT = 45°$ *(Alternate angle)*

In ΔUTV,

$\angle T = 180° - (55° + 45°) = 80°$

$\Rightarrow \angle T = \angle PTR = 80°$ *(Vertically opposite angles)*

$\therefore \angle CUQ + \angle RTP = 55° + 80° = 135°$

a) $\angle CAD = \angle CAB - \angle DAB$

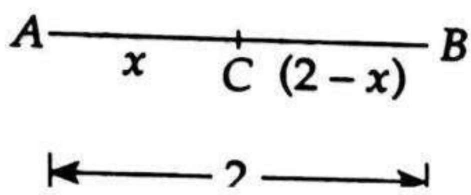
$= (90° - 20°) - (90° - 30°) = 10°$

b) Given, $AC^2 = AB \times CB$

$\Rightarrow x^2 = 2 \times (2 - x)$

$\Rightarrow x^2 = 4 - 2x$

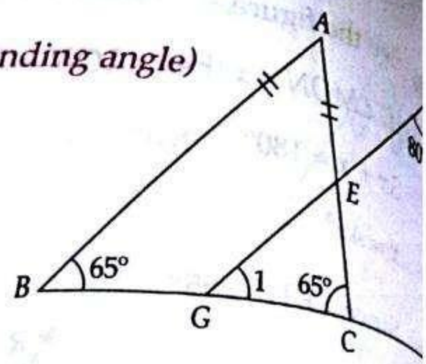
$\Rightarrow x^2 + 2x - 4 = 0$



$\Rightarrow x = \frac{-2 \pm \sqrt{4 + 16}}{}$

10. **(d)** Here, $\angle B = \angle C = 65^\circ$

$\angle 1 = \angle B = 65^\circ$ *(Corresponding angle)*



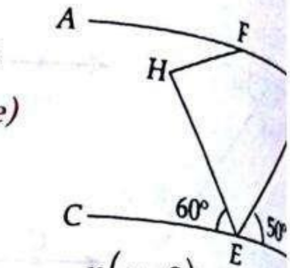
In ΔFGD,

$\angle 1 + \angle F + \angle D = 180^\circ$

$\Rightarrow$ $65^\circ + 80^\circ + \angle D = 180^\circ$

$\Rightarrow$ $\angle D = 35^\circ$

11. **(a)** $\angle HEB = 180^\circ - 60^\circ - 50^\circ = 70^\circ$



Since $HF \parallel BE$ and HE is a transversal line

$\therefore$ $\angle FHE + \angle HEB = 180^\circ$ *(co-interior angle)*

$\Rightarrow$ $\angle FHE + 70^\circ = 180^\circ$

$\Rightarrow$ $\angle FHE = 110^\circ$

12. **(c)** Number of diagonals is a n sided polygon $= \frac{n(n-3)}{2}$

No. of Sides (n)	4	5	6	7	8	9	1
No. of diagonals	2	5	9	14	20	27	3

Clearly 28 doesnot occur is the list.

For, $n = 4$, $\frac{n(n-3)}{2} = 2$

$= 5$, $\frac{n(n-3)}{2} = 5$

$= 6$, $\frac{n(n-3)}{2} = \frac{6 \times 3}{2} = 9$

$= 7$, $\frac{n(n-3)}{2} = \frac{7 \times 4}{2} = 14$

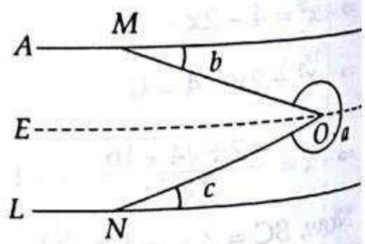
$= 8$, $\frac{n(n-3)}{2} = \frac{8 \times 5}{2} = 20$

$= 9$, $\frac{n(n-3)}{2} = \frac{9 \times 6}{2} = 27$

$= 10$, $\frac{n(n-3)}{2} = \frac{10 \times 7}{2} = 35$

So, number of diagonals cannot be 28.

13. **(c)** Let EF is drawn parallel to AB



$\therefore$ $\angle EOM = \angle OMB$ *(Alternate angle)*

$\Rightarrow$ $\angle EOM = b$

and $\angle EON = \angle ONM$ *(Alternate angle)*

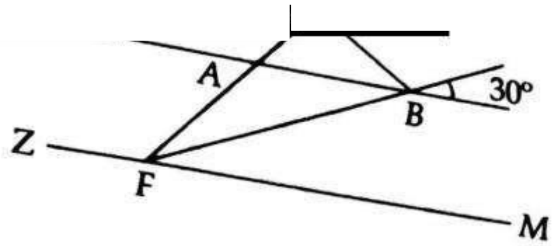
$\Rightarrow$ $\angle EON = c$ $\qquad \therefore$ $\angle MON = b + c$

$\because$ $\angle MON + a = 2\pi$ $\therefore$ $a = 2\pi - (b + c)$

In ΔDC...

and $\angle B = \angle E$

(Corresponding angle)



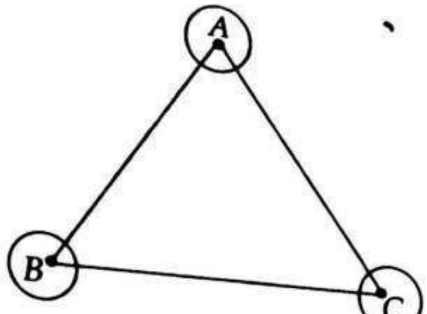
From figure,

$\angle DEF = \angle EFM$ (Alternate angle)

$\Rightarrow \angle EFM = 45°$ $\quad \Rightarrow \angle EFB + \angle BFM = 45°$

$\Rightarrow \angle EFB = 45° - 30° \Rightarrow \angle AFB = 15°$

[illegible] (c) $\because$ $\angle A = 360° -$ External $\angle A$



Similarly, $\angle B = 360° -$ External $\angle B$

and $\angle C = 360° -$ External $\angle C$

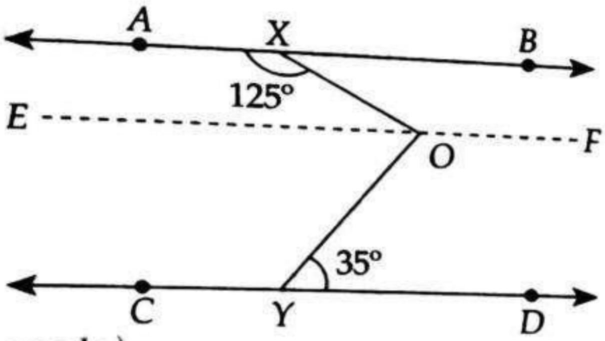
We know that, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $360° -$ External $\angle A + 360° -$ External $\angle B + 360° -$ External $\angle C = 180°$

$\Rightarrow$ External $\angle A +$ External $\angle B +$ External $\angle C = 1080° - 180° = 900°$

[illegible] (b) Draw a line EF through O such that

$EF \parallel AB \parallel CD$



Now, $AB \parallel EF$

$\therefore \angle AXO + \angle XOE = 180°$

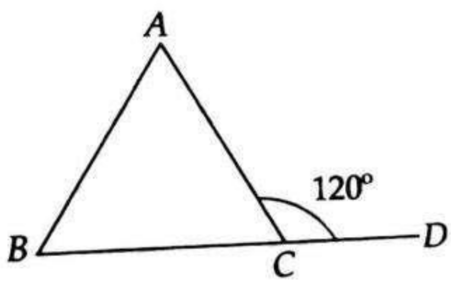
$\Rightarrow \angle XOE = 180° - 125° = 55°$

But $EF \parallel CD$

$\Rightarrow \angle EOY = \angle OYD = 35°$ (Alternate angle)

Hence, $\angle XOY = \angle XOE + \angle EOY = 55° + 35° = 90°$

(d) $\therefore$ $\angle ACD = 120°$



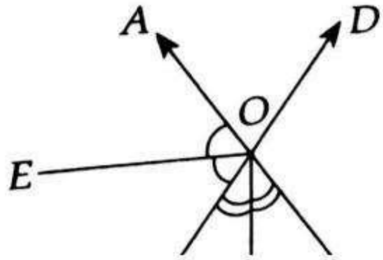
$\Rightarrow \angle CAB + \angle ABC = 120°$

(Since exterior angle of triangle is equal to sum of co-interior angles)

$\Rightarrow \angle CAB + \frac{2}{3} \angle CAB = 120°$

$\Rightarrow \angle CAB = \frac{120° \times 3}{5} = 72°$

(a) $\because$ $\angle BOC = 130°$



$\therefore$ $\angle BOC + \angle AOC = 180°$

Now, $\angle BOC = 130^\circ$

$\Rightarrow \angle BOF + \angle FOC = 130^\circ$

$\Rightarrow \angle FOC + FOC = 130^\circ$ $\quad$ ($\because$ OF is bisector of $\angle BOC$)

$\Rightarrow \angle FOC = 65^\circ$

and $\angle AOC = 50^\circ$

$\Rightarrow \angle AOE + \angle EOC = 50^\circ$

$\Rightarrow \angle EOC + \angle EOC = 50^\circ$ $\quad$ ($\because$ OE is bisector of $\angle AOC$)

$\Rightarrow \angle EOC = 25^\circ$

$\Rightarrow \angle EOF = \angle EOC + \angle FOC$

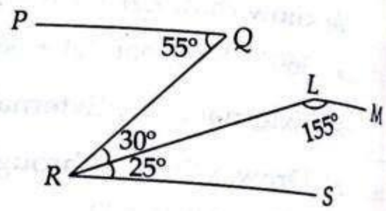
$= 65^\circ + 25^\circ = 90^\circ$

19. (d) $\because$ $\angle PQR = \angle QRS$ $\quad$ (Alternate angle)

$\therefore$ $PQ \parallel RS$ $\quad$ ... (i)

and $\angle SRL + \angle RLM = 180^\circ$ $\quad$ ... (ii)

$\Rightarrow RS \parallel LM$



From (i) and (ii),

$PQ \parallel LM$

$\therefore$ Angle between PQ and LM is 180°.

20. (a) We know that if α and β. are complementary then

$\because$ $\alpha + \beta = 90^\circ$ $\quad$ $\therefore$ $\alpha = 90 - \beta$ $\quad$ ... (i)

According to question, β tis two third of its complementary angle α.

$\therefore$ $\beta = \frac{2}{3}\alpha$

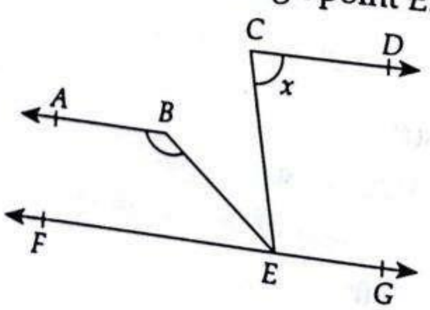
$\Rightarrow \beta = \frac{2}{3}(90^\circ - \beta)$ $\quad$ [from (i)]

$\Rightarrow \beta = 60^\circ - \frac{2}{3}\beta$

$\Rightarrow \frac{5\beta}{3} = 60^\circ \Rightarrow \beta = 36^\circ$

Thus, Required angle $= 36^\circ$

21. (d) $FG \parallel AB \parallel CD$ is drawn through point E.



$\angle BEF = \pi - y$

[co-interior angles]

$\angle CEG = (\pi - x)$ [co-interior angles]

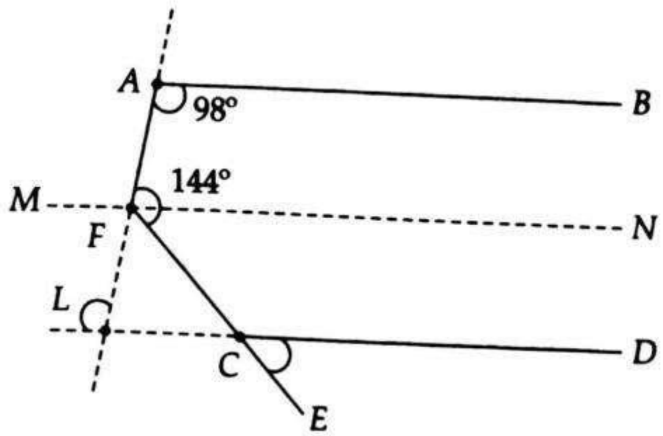
$\angle BEF + \angle BEC + \angle CEG = \pi$

$\pi - y + \angle BEC + \pi - x = \pi$

$2\pi - x - y + \angle BEC = \pi$

$\therefore \angle BEC = x + y - \pi$

22. (a) In figure, $\angle A = \angle L = 98^\circ$



In ΔFLC,

$\angle FLC = 180^\circ - 98^\circ = 82^\circ$

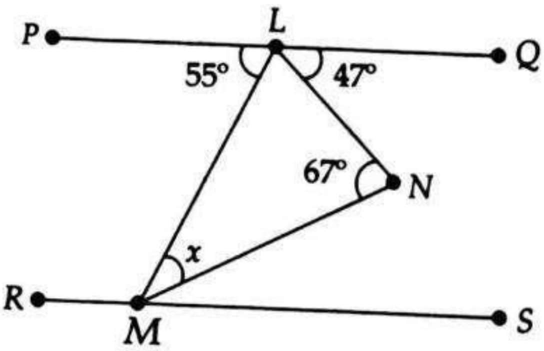
and $\angle F = 180^\circ - 144^\circ = 36^\circ$

$\Rightarrow \angle FCL = 180^\circ - (36^\circ + 82^\circ)$

$= 180^\circ - 118^\circ = 62^\circ$

Hence, $\angle ECD = \angle FCL = 62^\circ$

23. (a) In figure,



$\angle PLM = \angle LMS = 55^\circ$

$\angle LMS = \angle LMN + \angle NMS = 55^\circ$

$\Rightarrow x + \angle NMS = 55^\circ$

$\Rightarrow \angle NMS = 55^\circ - x$

$\angle MLN = 180^\circ - (\angle PLM + \angle QLN)$

$= 180^\circ - (55^\circ + 47^\circ)$

$= 180^\circ - 102^\circ = 78^\circ$

In ΔMLN, $\angle LMN + \angle MNL + \angle MLN = 180^\circ$

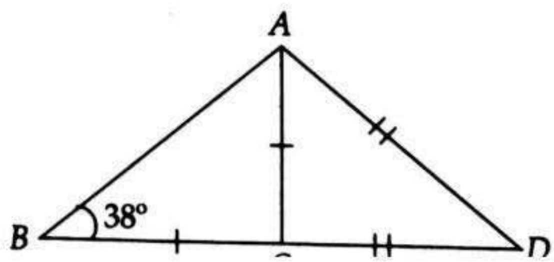
$\Rightarrow x + 67^\circ + 78^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 145^\circ = 35^\circ$

$\therefore$ From equation (i),

$\Rightarrow \angle NMS = 55^\circ - 35^\circ = 20^\circ$

24. (b) Given that $AC = BC$



$\therefore$ In ΔABC,

$\angle A = \angle B = 38^\circ$

$\therefore \angle ACB = 180^\circ - (\angle A + \angle B)$

25. **(c)** If two angles are complementary (sum = 90°) then each of them are acute angle.

26. **(c)** Since each interior angle fo regular hexagon = 120°

$\therefore$ Each interior angle of polygon $= \frac{7}{6} \times 120° = 140°$

Let number of sides in polygon be n.

$$\frac{(n-2)\times 180°}{n} = 140°$$

$\Rightarrow$ $18n - 36 = 14n$ $\Rightarrow$ $4n = 36$ $\Rightarrow$ $n = 9$

27. **(d)** Required difference $= \frac{10-3}{5+3+10} \times 180° = \frac{7}{18} \times 180° = 70°$

28. **(b)** Let required angle be x then its supplementary angle is $(180° - x)$

According to question,

$x = \frac{1}{5}(180° - x)$

$\Rightarrow$ $5x = 180° - x$

$\therefore$ $x = \frac{180°}{6} = 30°$

29. **(a)** Statement (1) is true. Statement (2) is wrong.

30. **(b)** $\because$ $ABCD$ is a trapezium

$\therefore$ $AD \parallel BC$

$EF \parallel BC$ (Given)

Hence $EF \parallel AD$

$\therefore$ $\angle x + \angle y = 180°$ (Linear pair of angle)

$\therefore$ $\angle y = 180° - 120° = 60°$

. **(a)** $\because$ Let number of sides be n

According to question, $\frac{(n-2)180}{n} = 144$

$\Rightarrow$ $(n-2)5 = 4n$

$\Rightarrow$ $5n - 10 = 4n$ $\therefore$ $n = 10$

32. **(c)** If number of sides in regular polygon be n then

$$\frac{(2n-4)}{n} \times 90° - \frac{360°}{n} = 150°$$

$\Rightarrow$ $\frac{(2n-4)\times 3}{n} - \frac{12}{n} = 5$

$\Rightarrow$ $\frac{6n-12-12}{n} = 5$

$\Rightarrow$ $6n - 24 = 5n$ $\therefore$ $n = 24$

33. **(b)** Sum of interior angles of a regular polygon of n sides $= (2n-4) \times 9$

$\therefore \quad (2n-4)\times 90^\circ = 1080^\circ$

$\Rightarrow \quad 2n-4 = 1080 \div 90 = 12$

$\Rightarrow \quad 2n = 12 + 4 = 16$

$\therefore \quad n = \frac{16}{2} = 8$

34. (b) Let number of sides in two regular polygon are respectively n and $2n$, Then their each internal angle are respectively $\frac{n\pi - 2\pi}{n}$ and $\frac{2n\pi - 2\pi}{2n}$

According to question, $\dfrac{\left(\frac{n\pi - 2\pi}{n}\right)}{\left(\frac{2n\pi - 2\pi}{2n}\right)} = \frac{2}{3}$

or, $\frac{(n-2)\pi}{(n-1)2\pi} \times 2 = \frac{2}{3}$

or, $\frac{n-2}{n-1} = \frac{2}{3}$

or, $3n-6 = 2n-2$

$\Rightarrow n = 4$

$\therefore 2n = 8$

35. (a) Let number of sides be respectively $5x$ and $4x$.

$\therefore \quad \frac{(2\times 5x-4)90^\circ}{5x} - \frac{(2\times 4x-4)\times 90^\circ}{4x} = 6^\circ$

$\left[\text{each interior angle} = \left(\frac{2n-4}{n}\right)\times 90^\circ\right]$

$\Rightarrow (10x-4)\times 360^\circ - (8x-4)\times 450^\circ = 20x \times 6^\circ$

$\Rightarrow (10x-4)\times 12 - (8x-4)15 = 4x$

$\Rightarrow 120x - 48 - 120x + 60 = 4x$

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

$\therefore$ Number of sides are respectively 15 and 12.

36. (b) Each internal angle of polygon $= \left[\frac{(n-2)180}{n}\right]^\circ$

Each exterior angle of polygon $= \left[\frac{360}{n}\right]^\circ$

According to question, $\frac{(n-2)180}{n} = 2\times\frac{360}{n}$

$\Rightarrow n-2 = 4 \qquad \therefore n = 6$

37. **(b)** Each interior angle of polygon = $\frac{n-2}{n} \times 180°$.

$= 60°$, when $n = 3$ $\quad$ $90°$, when $n = 4$ $\quad$ $108°$, when $n = 5$

$120°$, when $n = 6$ $\quad$ $135°$, when $n = 8$ $\quad$ $140°$, when $n = 9$

$144°$ when $n = 10$ $\quad$ $150°$, when $n = 12$

38. **(c)** Since number of diagonals in n sided polygon = $\frac{n(n-3)}{2}$

For, $n = 3$, Number of diagonals = $\frac{10 \times 7}{2} = 35$

39. **(d)** Each interior angle of a regular polygon = $\frac{n-2}{n} \times 180°$

Given $\frac{n-2}{n} \times 180° = 135°$ $\Rightarrow 4(n-2) = 3n \Rightarrow n = 8$

$\therefore$ Number of diagonals = $\frac{n(n-3)}{2} = \frac{8(8-3)}{2} = 20$

Exercise–4B

1. A, O, B are three points on a line segment and C is a point not lying on AOB. If $\angle AOC = 40°$ and OX, OY, are the internal and external bisectors of $\angle AOC$ respectively, then $\angle BOY$ is

(a) 72° (b) 68° (c) 70° (d) 80°

[SSC Tier-I 2012]

If each interior angle is double of each exterior angle of a regular polygon with n sides, then the value of n is

(a) 5 (b) 6 (c) 8 (d) 10

[SSC Tier-I 2012]

3. Side BC of ΔABC produces to D. If $\angle ACD = 108°$ and $\angle B = \frac{1}{2} \angle A$ then $\angle A$ is

(a) 108° (b) 59° (c) 36° (d) 72°

[SSC Tier-I 2012]

4. The external bisectors of $\angle B$ and $\angle C$ of ΔABC meet at point P. If $\angle BAC = 80°$, then $\angle BPC$ is

(a) 50° (b) 40° (c) 80° (d) 100°

[SSC Tier-I 2012]

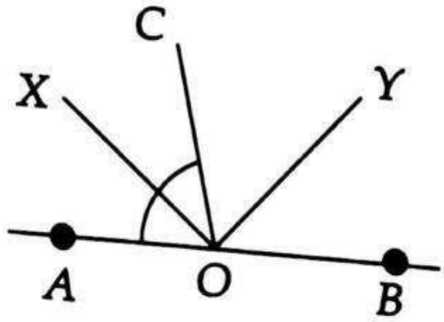
5. By decreasing 15° of each angle of a triangle, the ratios of their angles are 2 : 3 : 5. The radian measure of greatest angle is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{24}$ (c) $\frac{5\pi}{24}$ (d) $\frac{11\pi}{24}$

[SSC Tier-I 2012]

Explanation

1. (c)



$\because \angle BOC = 180° - 40° = 140°$

$\therefore \angle BOY = \frac{140°}{2} = 70°$

2. (b) Each internal angle of polygon $= \frac{(n-2)\pi}{n}$

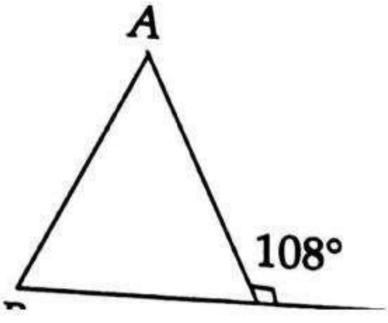
Each exterior angle of polygon $= \frac{2\pi}{n}$

According to question $= \frac{(n-2)\pi}{n} = 2 \cdot \frac{2\pi}{n}$

$\Rightarrow n-2=4 \Rightarrow n=6$

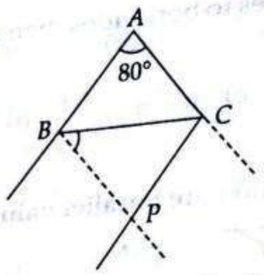
3. (d) Clearly, $\angle C = 180° - 108° = 72°$

or, $\angle A + \angle B = 180° - 72° = 108°$



$\therefore \angle A + \frac{1}{2}\angle A = 108°$

$\Rightarrow \frac{3\angle A}{2} = 108°$



$\therefore \quad \angle CBP + \angle BCP = 180° - \frac{B+C}{2}$

$= 180° - 50° = 130° \qquad (\because B + C = 180° - 80° = 100°)$

Hence, $\angle BPC = 180° - 130° = 50°$

5. **(d)** $2k + 3k + 5k = 180° - 3 \times 15°$

or, $10k = 135°$

$\Rightarrow \quad k = \frac{135°}{10}$

$\therefore$ Greatest angles $= 5k + 15°$

$= 5 \times \frac{135°}{10} + 15°$

$= \left(\frac{135°}{2} + 15°\right)$

$= \frac{165°}{2} = \left(\frac{165}{2} \times \frac{\pi}{180}\right)$ rad

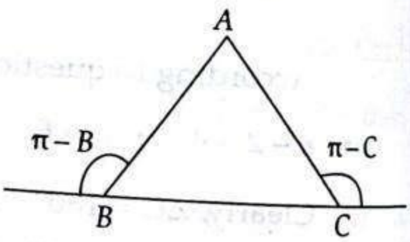
$= \left(\frac{11}{2} \times \frac{\pi}{12}\right) = \frac{11\pi}{24}$ rad

) Required sums $= (\pi - B) + (\pi - C)$

$= 2\pi - (B + C)$

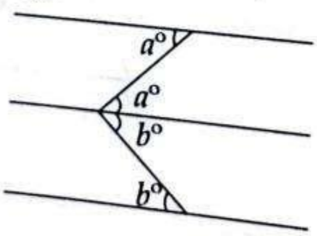
$= 2\pi - (\pi - A)$

$= \pi + A$



7. **(c)** From alternate angle,

$a° + b° = 45°$



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