CBSE Board
Class XI Mathematics
Sample Paper – 7

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Find $\lim_{x\to 1}\frac{x^n - 1}{x - 1}$.

2. Write the statement in the form "if p, then ": You can access the website only if you pay a subscription fee.

3. Write complex conjugate of -4i – 8.

OR

Find argument of 4 + 4i.

4. If standard deviation of a distribution is 4 then find variance of the distribution.

SECTION – B

5. If X = {a, b, c, d} and Y = {f, b, d, g} find X – Y and Y – X.

6. Find the domain of the function $f(x) = \log_{3+x}(x^2 - 1)$

OR

If $f(x) = 2x\sqrt{1-x^2}$ then show that f(sin x/2) = sinx

7. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

OR

Find in degrees and radians the angle between the hour hand and the minute-hand of a clock at half past three.

8. If R is the set of all real numbers, what do the Cartesian products R × R and R × R × R represent?

9. Prove that: $\frac{1+\cos 4x}{\cot x-\tan x}=\frac{1}{2}\sin 4x$

OR

Prove that $8\cos^3\frac{\pi}{9}-6\cos\frac{\pi}{9}=1$.

10. Find the component statement and check whether it is true or not?
All integers are positive or negative.

11. Find the range of the function f(x) = $\frac{4-x}{x-4}$.

12. Find the distance between the directrices the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$

SECTION – C

13. Given that sin A = $\frac{3}{5}$ and that A is an acute angle, find without using tables, the values of sin2A, cos2A and tan2A. Hence find the value of sin4A.

14. Let A be the set of two positive integers. Let f : A → Z^+ (set of positive integers) be defined by f(n) = p where p is the highest prime factor of n. If range of f = {3}. Find set A. Is A uniquely determined?

15. Sum to n terms the series : 0.7 + 0.77 + 0.777 + ...

16. Show that a real value of x will satisfy the equation $\frac{1-ix}{1+ix}=a-ib$ if $a^2+b^2=1$ where a and b are real.

17. Tickets are numbered from 1 to 100. They are well shuffled and a ticket is drawn at random. What is the probability that the drawn ticket has

1. An even number
2. A number 5 or multiple of 5
3. A number which is greater than 75

4. A number which is a square

18. The side of a given square is equal to a. The mid-points of its sides are joined to form a new square. Again, the mid-points of the sides of this new square are joined to form another square. This process is continued indefinitely. Find the sum of the areas of the square and the sum of the perimeters of the squares.

19. A committee of 4 is to be selected from amongst 5 boys and 6 girls. In how many ways can this be done so as to include

i. exactly one girl

ii. At least one girl.

OR

If the letters of the word "AGAIN" be arranged in a dictionary, what is the 50^{th} word?

20. Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

OR

A circle has radius 3 units and its centre lies on the line y = x – 1. Find the equation of the circle, if it passes through (7, 3).

21. Differentiate xe^x from first principles.

OR

If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$ prove that $2xy\frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$

22. Find the equation of the ellipse with foci at $(\pm 5, 0)$ and x = 36/5 as one of the directrices.

SECTION – D

23. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$ show that $\cos(\alpha+\beta) = \frac{b^2 - a^2}{b^2 + a^2}$ and

$\sin(\alpha+\beta) = \frac{2ab}{b^2 + a^2}$

SECTION – D

24. Given below is the frequency distribution of weekly study hours of a group of class 11 students. Find the mean, variance and standard deviation of the distribution using the short cut method.

Classes	Frequency
0 - 10	5
10 - 20	8
20 - 30	15
30 - 40	16
40 - 50	6

25. If $x \in Q_3$ and $\cos x = -\frac{1}{3}$, then show that $\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$.

OR

If $\tan(\alpha+\theta) = n\tan(\alpha-\theta)$ show that $(n + 1)\sin 2\theta = (n - 1)\sin 2\alpha$

26. A man wants to cut three lengths form a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

OR

Solve the following system of inequalities graphically: $5x + 4y \leq 20$, $x \geq 1$, $y \geq 2$

27. Find the term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$

28. The sum of three numbers in G. P. is 42. If the first two numbers are increased by 2 and third is decreased by 4, the resulting numbers form A.P. Find the numbers of G.P.

OR

Suppose x and y are two real numbers such that the r^{th} mean between x and 2y is equal to the r^{th} mean between 2x and y when n arithmetic means are inserted between them in both the cases. Show that $\frac{n+1}{r} - \frac{y}{x} = 1$

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Sample Paper – 7 Solution

SECTION – A

1.

$$\lim_{x\to 1}\frac{x^n-1}{x-1}$$
$$=\lim_{x\to 1}\frac{x^n-1^n}{x-1}$$
$$=n(1)^{n-1} \qquad \because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$
$$=n$$

2. If you access the website, then you pay a subscription fee.

3. The complex conjugate of a + bi is a – bi. Hence, complex conjugate of -4i – 8 i.e. -8 – 4i is -8 + 4i.

OR

z = 4 + 4i comparing with z = a + bi we get a = 4 = b
tan Θ = b/a = 1
∴ Θ = 45°

4. Standard deviation = 4
Standard deviation = √Variance
4 = √Variance
Variance = 4^2 = 16

SECTION – B

5. X = {a, b, c, d} and Y = {f, b, d, g}
X – Y = {a, c} and Y – X = {f, g}

6. f(x) is defined when $x^2 - 1 > 0$ ∴ $x^2 > 1$ and 3 + x > 0
x < -1 or x > 1 and x > -3 and x ≠ -2, since base 3 + x ≠ 1
Domain is (-3, -2) ∪ (-2, -1) ∪ (1, ∞)

OR

$f(x) = 2x\sqrt{1-x^2}$

$$f\left(\sin\frac{x}{2}\right) = 2\sin\frac{x}{2}\sqrt{1-\sin^2\frac{x}{2}}$$

$$f\left(\sin\frac{x}{2}\right) = 2\sin\frac{x}{2}\sqrt{\cos^2\frac{x}{2}} \qquad \because 1-\sin^2 x = \cos^2 x$$

$$f\left(\sin\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$f\left(\sin\frac{x}{2}\right) = \sin x \qquad \because \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

7. Let s be the length of the arc subtending an angle Θ^c at the centre of a circle of radius r.
Then, s = rΘ

$s = 5\times 15\times\frac{\pi}{180} = \frac{5\pi}{12}$

OR

The angle traced by the hour hand in 12 hours = 360°

The angle traced by the hour hand in 3 hrs 30 min i. e. 7/2 hrs = $\left(\frac{360}{12}\times\frac{7}{2}\right) = 105°$

The angle traced by the minute hand in 60 min = 360°

The angle traced by minute hand in 30 min = $\left(\frac{360}{60}\times 30\right) = 180°$

Hence, the required angle between two hands = 180° - 105° = 75° = $75\times\frac{\pi}{180} = \frac{5\pi}{12}$

8. The Cartesian product of the set R of all real numbers with itself i. e. R × R is the set of all ordered pairs (x, y) where x, y $\in$ R. In other words R × R = {(x, y) : x, y $\in$ R }
R × R is the set of all points in XY-plane. The set of R × R is also denoted by R^2.
R × R × R = {(x, y, z) : x, y, z $\in$ R}
It represents set of all points in space denoted by R^3.

9. $$\frac{1+\cos 4x}{\cot x - \tan x} = \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}$$

$$= \frac{2\cos^2 2x \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{2\cos^2 2x \sin x \cos x}{\cos 2x}$$

$= \cos 2x \sin 2x$

$= \frac{1}{2}(2\cos 2x \sin 2x)$

$= \frac{1}{2}\sin 4x$

$$\frac{1+\cos 4x}{\cot x - \tan x} = \frac{1}{2}\sin 4x$$

OR

$$8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9} = 2\left(4\cos^3\frac{\pi}{9} - 3\cos\frac{\pi}{9}\right)$$

$$= 2\cos\left(3\times\frac{\pi}{9}\right) \text{ reason}$$

$$= 2\cos\frac{\pi}{3} = 1$$

10. The component statements are

p : All integers are positive.

q : All integers are negative.

p and q both are false. The connecting word is 'or'.

11. For any $x \in$ domain f we have,

$$f(x) = \frac{4-x}{x-4} = \frac{-(x-4)}{x-4} = -1$$

Hence, the range of the function = {-1}

12. $\frac{x^2}{36} + \frac{y^2}{20} = 1$ comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = 36$, $b^2 = 20$

$b^2 = a^2(1 - e^2)$

$20 = 36(1 - e^2)$

$e^2 = 16/36$

$e = 2/3$

Distance between the directrices = $2a/e = 18$.

SECTION – C

13. sin A = $\frac{3}{5}$

cos A = $\sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

tan A = $\frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

$$\sin 2A = 2\sin A\cos A = 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$$

$$\cos 2A = 1-2\sin^2 A = 1-2\times\frac{9}{25} = \frac{7}{25}$$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$$

$$\sin 4A = 2\sin 2A\cos 2A = 2\times\frac{24}{25}\times\frac{7}{25} = \frac{336}{625}$$

14. It is given that the set A consists of two positive integers. So, let A = {n, m}. Since, range of f = {3}.

f(n) = 3 and f(m) = 3

Highest prime factors of n ad m both are equal to 3.

n = 3 and m = 6 or n = 3 and m = 9 or n = 3 and m = 12 or n = 6 and m = 12 etc

A = {3, 6} or A = {3, 9} or A = {3, 12} or A = {6, 12} etc.

Clearly, A is not uniquely determine.

15. S_n = 0.7 + 0.77 + 0.777 + ...+ to n terms

= 7 (0.1 + 0.11 + 0.111 + ... to n terms)

= $\frac{7}{9}(0.9+0.99+0.999+\text{....to n terms})$

= $\frac{7}{9}\left[(1-0.1)+(1-0.01)+(1-0.001)+\text{...to n terms}\right]$

= $\frac{7}{9}\left[n-\frac{0.1\left[1-0.1^n\right]}{1-0.1}\right]$

= $\frac{7}{9}\left[n-\frac{\frac{1}{10}\left[1-\frac{1}{10^n}\right]}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

16.

$$\frac{1-ix}{1+ix}=a-ib$$

$$\frac{(1-ix)+(1+ix)}{(1-ix)-(1+ix)}=\frac{a-ib+1}{a-ib-1}$$

$$\frac{2}{-2ix}=\frac{1+a-ib}{-(1-a+ib)}$$

$$ix=\frac{1-a+ib}{1+a-ib}$$

$$ix=\frac{1-a+ib}{1+a-ib}\times\frac{1+a+ib}{1+a+ib}$$

$$ix=\frac{1-a^2-b^2+2ib}{(1+a)^2-i^2b^2}$$

$$ix=\frac{1-a^2-b^2+2ib}{(1+a)^2+b^2}$$

$$ix=\frac{2ib}{(1+a)^2+b^2} \qquad \because a^2+b^2=1$$

$$x=\frac{2b}{(1+a)^2+b^2}$$

Hence, x is real as denominator is always positive.

17. Total number of tickets = 100

∴ Total number of exhaustive, mutually exclusive and equally likely cases n(S) = 100

1. A : getting an even number. Since there are 50 even numbered tickets, n(A) = 50
   P(A) = 50/100 = ½
2. B : getting 5 or multiple of 5
   The favourable outcomes are 5, 10, 15, 20,….95, 100
   n(B) = 20 ∴ P(B) = 20/100 = 1/5
3. C : getting a number greater than 75
   The favourable outcomes are 76, 77,….100
   n(C) = 25 ∴ P(C) = 25/100 = ¼
4. D : getting a square number
   The favourable outcomes are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
   n(D) = 10 ∴ P(D) = 10/100 = 1/10

18. Since the diagonal of a square is √2 times the side of a square, we get the following series as infinite G. P. of the areas of the squares :

$$a^2+\left(\sqrt{2}\times\frac{a}{2}\right)^2+\left(\sqrt{2}\times\sqrt{2}\times\frac{a}{4}\right)^2+\left(\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\frac{a}{8}\right)^2+\text{...to }\infty$$

$$=a^2+\frac{a^2}{2}+\frac{a^2}{4}+\frac{a^2}{8}+\text{...to }\infty$$

$$S_\infty=\frac{\text{First term}}{1-\text{common ratio}}=\frac{a^2}{1-\frac{1}{2}}=2a^2\text{ sq. units}$$

Similarly, since the diagonal of a square is √2 times the side of a square, we get the following series as an infinite G. P. of the sum of the perimeters of the squares:

$$4\left[a+\left(\sqrt{2}\times\frac{a}{2}\right)+\left(\sqrt{2}\times\sqrt{2}\times\frac{a}{4}\right)+\text{...to }\infty\right]$$

$$=4\left[a+\frac{a}{\sqrt{2}}+\frac{a}{2}+\text{...to }\infty\right]$$

$$=4\times\frac{a}{1-\frac{1}{\sqrt{2}}}=\frac{4\sqrt{2}a}{\sqrt{2}-1}=\frac{4\sqrt{2}\left(\sqrt{2}+1\right)a}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}=4\sqrt{2}\left(\sqrt{2}+1\right)a\text{ units}$$

19. i.

In this case we have to select one girl out of 6 and 3 boys out of 5.

The number of ways selecting 3 boys = $^5C_3=10$

The number of ways selecting one girl = $^6C_1=6$

The required committee can be formed in 6 × 10 = 60 ways.

ii.

The committee can be formed with

a. one boy and three girls

b. 2 boys and 2 girls

or c. 3 boys and one girl or d. 4 girls alone

The required number of ways of forming a committee

= $^5C_1\times{}^6C_3+{}^5C_2\times{}^6C_2+{}^5C_3\times{}^6C_1+{}^6C_4$

= 100 + 150 + 60 + 15 = 325 ways

OR

In a dictionary the words are arranged in an alphabetical order.

i. Starting with A, the remaining 4 letters G, A, I, N can be arranged in 4! = 24 ways. These are the first 24 words.
ii. Then starting with G, the remaining letters A, A, I, N can be arranged in 4!/2! = 12 ways. Thus, there are 12 words starting with G.
iii. Now the words will start with I. Starting with I, the remaining letters A, G, A, N can be arranged in 4!/2! = 12 ways. So, there are 12 words which start with I.
iv. Thus, so far we have constructed 24 + 12 + 12 i. e. 48 words. The 49^{th} word will start with N and is NAAGI. Hence, the 50^{th} word is NAAIG.

20. Let 2a and 2b be the transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes. Then, the equation of the hyperbola is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ....(i)

2b = 5 and 2ae = 13

$b^2 = a^2(e^2 - 1)$

$b^2 = a^2e^2 - a^2$

$\frac{25}{4}=\frac{169}{4}-a^2$

$a^2=\frac{144}{4}$

a = 6

$\frac{x^2}{36}-\frac{y^2}{\frac{25}{4}}=1$ from (i)

$25x^2 - 144y^2 = 900$

OR

The coordinates of any point on the line y = x – 1 can be taken as (t, t – 1).

So, let C(t, t – 1) be the centre of the required circle. Its radius is 3. Therefore, equation of the required circle is

$(x - t)^2 + [y - (t - 1)]^2 = 3^2$ ....(i)

It passes through (7, 3)

$\therefore (7 - t)^2 + [3 - (t - 1)]^2 = 3^2$

$\therefore (7 - t)^2 + (4 - t)^2 = 9$

$t^2 - 11t + 28 = 0$

$(t - 4)(t - 7) = 0$

t= 4, 7

Substituting the values of t in (i) we get the equations of the required circles are

$(x - 4)^2 + (y - 3)^2 = 3^2$ and $(x - 7)^2 + (y - 6)^2 = 3^2$

21. f(x) = xe^x then f(x + h) = $(x + h)e^{x + h}$

$$\frac{d}{dx}f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(x+h)e^{x+h}-xe^x}{h}$$

$$= \lim_{h\to 0}\frac{xe^{x+h}-xe^x+he^{x+h}}{h}$$

$$= \lim_{h\to 0}xe^x\left(\frac{e^h-1}{h}\right)+e^{x+h}$$

$$= xe^x\lim_{h\to 0}\frac{e^h-1}{h}+\lim_{h\to 0}e^{x+h}$$

$= xe^x + e^x$

$= (x + 1)e^x$

OR

$$y = \sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a}}\times\frac{1}{2\sqrt{x}}-\frac{1}{2}\frac{\sqrt{a}}{x\sqrt{x}}$$

$$2xy\frac{dy}{dx} = 2x\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}\right)\left(\frac{1}{\sqrt{a}}\times\frac{1}{2\sqrt{x}}-\frac{1}{2}\frac{\sqrt{a}}{x\sqrt{x}}\right)$$

$$2xy\frac{dy}{dx} = 2x\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}\right)\left(\frac{x-a}{2x\sqrt{x}\sqrt{a}}\right)$$

$$2xy\frac{dy}{dx} = \left(\frac{x+a}{\sqrt{xa}}\right)\left(\frac{x-a}{\sqrt{xa}}\right)$$

$$2xy\frac{dy}{dx} = \frac{x^2-a^2}{ax}$$

$$2xy\frac{dy}{dx} = \frac{x}{a}-\frac{a}{x}$$

22. The equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and let e be its eccentricity. The coordinates of its foci and the equations of the directrices are $(\pm ae, 0)$ and x = $\pm$ ae respectively.

But, it is given that the coordinates of foci are $(\pm 5, 0)$ and the equations of one of the directrices is x = 36/5.

ae = 5 and a/e = 36/5

$$ae\times\frac{a}{e} = 5\times\frac{36}{5}$$

$a^2 = 36$

a = 6

$b^2 = a^2 (1 - e^2)$
$b^2 = a^2 - (ae)^2$
$b^2 = 36 - 25$
$b^2 = 11$
$b = \sqrt{11}$
Substitute a = 6 and $b = \sqrt{11}$ in the equation of ellipse we get

$$\frac{x^2}{36} + \frac{y^2}{11} = 1$$

SECTION – D

23. $b^2 + a^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$b^2 + a^2 = (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$b^2 + a^2 = 1 + 1 + 2\cos(\alpha - \beta)$$

$$b^2 + a^2 = 2 + 2\cos(\alpha - \beta)$$

And

$$b^2 - a^2 = (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2$$

$$b^2 - a^2 = \cos^2\alpha + \cos^2\beta - \sin^2\alpha - \sin^2\beta + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$$b^2 - a^2 = \cos^2\alpha - \sin^2\beta + \cos^2\beta - \sin^2\alpha + 2\cos(\alpha + \beta)$$

$$b^2 - a^2 = \cos(\alpha + \beta)\cos(\alpha - \beta) + \cos(\alpha + \beta)\cos(\beta - \alpha) + 2\cos(\alpha + \beta)$$

$$b^2 - a^2 = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta)$$

$$b^2 - a^2 = \cos(\alpha + \beta)\left[2\cos(\alpha - \beta) + 2\right]$$

$$b^2 - a^2 = \cos(\alpha + \beta)(b^2 + a^2)$$

$$\cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2}$$

$$= \sqrt{\frac{4a^2b^2}{(a^2 + b^2)^2}} = \frac{2ab}{a^2 + b^2}$$

24. Let assumed mean be a = 25

Classes	f_i	x_i	$y_i = (x - a)/10$	y_i^2	$f_i y_i$	$f_i y_i^2$
0 - 10	5	5	-2	4	-10	20
10 - 20	8	15	-1	1	-8	8
20 - 30	15	25	0	0	0	0
30 - 40	16	35	1	1	16	16
40 - 50	6	45	2	4	12	24
	50				10	68

$$\sum_{i=1}^{n} f_i y_i = 10, \quad \sum_{i=1}^{n} f_i y_i^2 = 68, \quad \sum_{i=1}^{n} f_i = 50, \quad h = 10$$

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} \times h$$

We get, $\bar{x} = 25 + \frac{10 \times 10}{50} = 27$

$$\sigma_x = \frac{h}{N}\sqrt{N \sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}$$

$$\sigma_X = \frac{10}{50}\left[\sqrt{50 \times 68 - (10)^2}\right]$$

$$\sigma_X = \frac{1}{5} \times 10\sqrt{33} = 11.49$$

$$\sigma_X^2 = 132.02$$

So for the given data Mean = 27, Standard Deviation = 11.49 and Variance = 132.02

25. $x \in Q_3$ III quadrant and $\cos x = -\frac{1}{3}$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\Rightarrow \cos x = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow -\frac{1}{3} + 1 = 2\cos^2\frac{x}{2} \Rightarrow \frac{2}{3\times 2} = \cos^2\frac{x}{2}$$

$$\Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1}{3}}$$

Now, $x \in Q_3$

$$\Rightarrow 2n\pi + \pi < x < 2n\pi + \frac{3\pi}{2}$$

$$\Rightarrow \frac{2n\pi + \pi}{2} < \frac{x}{2} < \frac{2n\pi + \frac{3\pi}{2}}{2}$$

$$\Rightarrow n\pi + \frac{\pi}{2} < \frac{x}{2} < n\pi + \frac{3\pi}{4}$$

Case I: When n is even $= 2k$ (say)

$$\Rightarrow (2k)\pi + \frac{\pi}{2} < \frac{x}{2} < (2k)\pi + \frac{3\pi}{4}$$

$$\Rightarrow \frac{x}{2} \in Q_2$$

Case I: When n is odd $= 2k+1$ (say)

$$\Rightarrow (2k+1)\pi + \frac{\pi}{2} < \frac{x}{2} < (2k+1)\pi + \frac{3\pi}{4}$$

$$\Rightarrow (2k)\pi + \pi + \frac{\pi}{2} < \frac{x}{2} < (2k)\pi + \pi + \frac{3\pi}{4}$$

$$\Rightarrow (2k)\pi + \frac{3\pi}{2} < \frac{x}{2} < (2k)\pi + \frac{7\pi}{4}$$

$$\Rightarrow \frac{x}{2} \in Q_4$$

$$\sin\frac{x}{2} = \pm\sqrt{1 - \left(\cos\frac{x}{2}\right)^2} = \pm\sqrt{1 - \left(\pm\sqrt{\frac{1}{3}}\right)^2} = \pm\sqrt{1 - \frac{1}{3}} = \pm\sqrt{\frac{2}{3}}$$

In Q_2 $\sin\frac{x}{2} = \sqrt{\frac{2}{3}}$

In Q_4 $\sin\frac{x}{2} = -\sqrt{\frac{2}{3}}$

So $\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$

OR

$$\tan(\alpha + \theta) = n\tan(\alpha - \theta)$$

$$\frac{\tan(\alpha + \theta)}{\tan(\alpha - \theta)} = \frac{n}{1}$$

$$\frac{\tan(\alpha + \theta) + \tan(\alpha - \theta)}{\tan(\alpha + \theta) - \tan(\alpha - \theta)} = \frac{n+1}{n-1}$$

$$\frac{\sin(\alpha+\theta)\cos(\alpha-\theta)+\cos(\alpha+\theta)\sin(\alpha-\theta)}{\sin(\alpha+\theta)\cos(\alpha-\theta)-\cos(\alpha+\theta)\sin(\alpha-\theta)}=\frac{n+1}{n-1}$$

$$\frac{\sin(\alpha+\theta+\alpha-\theta)}{\sin(\alpha+\theta-\alpha+\theta)}=\frac{n+1}{n-1}$$

$$\frac{\sin 2\alpha}{\sin 2\theta}=\frac{n+1}{n-1}$$

$(n + 1)\sin 2\Theta = (n - 1)\sin 2\alpha$

26. Let the length of the shortest piece be x cm. Then, length of the second piece and the third piece are x + 3 cm and 2x cm respectively.

Since the three lengths are to be cut from a single piece of board of length 91 cm,

$x + x + 3 + 2x \le 91$

$4x + 3 \le 91$

$4x \le 91 - 3$

$4x \le 88$

$x \le 22$…….(i)

Also, the third piece is at least 5 cm longer than the second piece.

$2x \ge x + 3 + 5$

$2x \ge x + 8$

$x \ge 8$……….(ii)

From (i) and (ii)

$8 \le x \le 22$

OR

$5x + 4y \le 20$….(i)

$x \ge 1$……(ii)

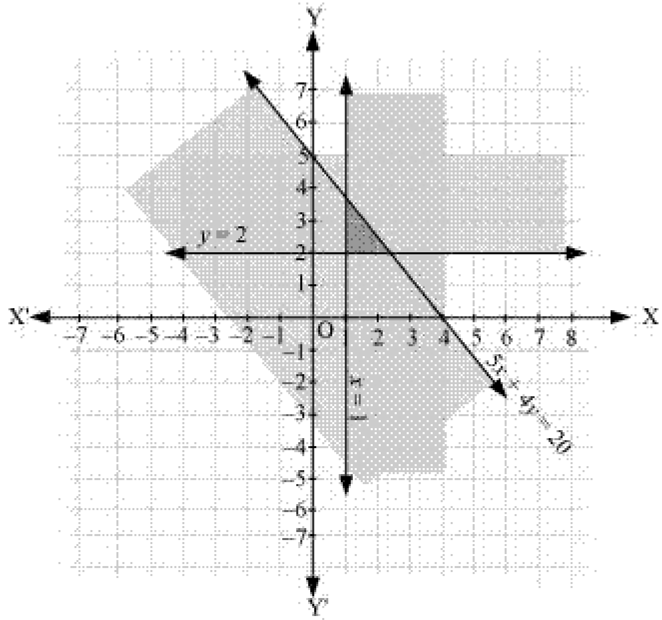
$y \ge 2$…..(iii)

Inequality (i) represents the region below the line, 5x + 4y = 20 (including the line 5x + 5y = 20)

Inequality (ii) represents the region on the right hand side of the line, x = 1 (including the line x = 1)

Inequality (iii) represents the region above the line, y = 2 (including the line y = 2)

Hence, the solution of the given system of linear equalities is represented by the common shaded region including the points on the respective lines as follows



27. $\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}$

$=\frac{\left(x^{\frac{1}{3}}\right)^3+1^3}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{\left(x^{\frac{1}{2}}\right)^2-1^2}{x-x^{\frac{1}{2}}}$

$=\frac{\left(x^{\frac{1}{3}}+1\right)\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}+1\right)}{x^{\frac{1}{2}}\left(x^{\frac{1}{2}}-1\right)}$

$=\left(x^{\frac{1}{3}}+1\right)-\frac{\left(x^{\frac{1}{2}}+1\right)}{x^{\frac{1}{2}}}$

$=x^{\frac{1}{3}}+1-1-x^{\frac{-1}{2}}$

$=x^{\frac{1}{3}}-x^{\frac{-1}{2}}$

$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}=\left(x^{\frac{1}{3}}-x^{\frac{-1}{2}}\right)^{10}$

$$t_{r+1} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r} \left(-x^{\frac{-1}{2}}\right)^r$$

t_{r+1} is the term independent of x if $\frac{10-r}{3} - \frac{r}{2} = 0$

if 20 – 2r – 3r = 0 if 5r = 20

if r = 4

Hence, required independent term is ${}^{10}C_4 (-1)^4 = {}^{10}C_4 = 210$

28. Let a, ar, ar^2 be three numbers.

According to the question,

$a + ar + ar^2 = 42$.....(i)

Also, a + 2, ar + 2, $ar^2 - 4$ are in A. P.

$2(ar + 2) = (a + 2) + (ar^2 - 4)$

$2ar + 4 = a + ar^2 - 2$

$a(1 - 2r + r^2) = 6$…….(ii)

Dividing (ii) by (i)

$$\frac{1-2r-r^2}{1+r+r^2} = \frac{1}{7}$$

$7 - 14r + 7r^2 = 1 + r + r^2$

$6r^2 - 15r + 6 = 0$

$3r^2 - 5r + 2 = 0$

$(r - 1)(3r - 2) = 0$

r = 1, 2/3

As r = 1 does not give the required result.

Hence, r = 2/3

Put it in (ii)

$$a\left(1 - \frac{4}{3} + \frac{4}{9}\right) = 6$$

$$a\left(\frac{9-12+4}{9}\right) = 6$$

$$a\left(\frac{1}{9}\right) = 6$$

a = 54

Hence, the numbers are 54, 36, 24.

OR

Let $A_1, A_2, \ldots, A_n$ be n A. M.'s between x and 2y.

x, $A_1, A_2, \ldots, A_n$, 2y are in A. P. having common difference d_1.

$d_1 = \frac{2y - x}{n + 1}$ ……….(i)

r^{th} mean, $A_r = x + rd_1 = x + r\left(\frac{2y - x}{n + 1}\right)$ ……..(ii)

Let $A'_1, A'_2, \ldots, A'_n$ be n A. M.'s between 2x and y.

2x, $A'_1, A'_2, \ldots, A'_n$, y are in A. P. having common difference d_2.

$d_2 = \frac{y - 2x}{n + 1}$ ……….(iii)

r^{th} mean $A'_r = 2x + rd_2 = 2x + r\left(\frac{y - 2x}{n + 1}\right)$ ……..(iv)

According to the question,

$A_r = A'_r$

$x + r\left(\frac{2y - x}{n + 1}\right) = 2x + r\left(\frac{y - 2x}{n + 1}\right)$

$(n + 1)x + r(2y - x) = (n + 1)2x + r(y - 2x)$

$(n + 1)x - ry = rx$

Hence, $\frac{n + 1}{r} - \frac{y}{x} = 1$