Binomial Distribution Ex 33.1 Q1

Ex - 33.1

Let p denote the probability of having defective item, so

$$p = 6\% = \frac{6}{100} = \frac{3}{50}$$

So, $q = 1 - p$

$$= 1 - \frac{3}{50} \quad [\text{Since } p + q = 1]$$

$$= \frac{47}{50}$$

Let X denote the number of defective items in a sample of 8 items. Then, the pro ty of getting r defective bulks is

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{3}{50}\right)^r \left(\frac{47}{50}\right)^{8-r} \quad ---(1)$$

Therefore, probability of getting not more then one defective item

$$= P(X = 0) + P(X = 1)$$

$$= {}^8C_0 \left(\frac{3}{50}\right)^0 \left(\frac{47}{50}\right)^{8-0} + {}^8C_1 \left(\frac{3}{50}\right)^1 \left(\frac{47}{50}\right)^{8-1} \quad [\text{Using equation } (1)]$$

$$= 1.1.\left(\frac{47}{50}\right)^8 + 8.\frac{3}{50}.\left(\frac{47}{50}\right)^7$$

$$= \left(\frac{47}{50}\right)^7 \left(\frac{47}{50} + \frac{24}{50}\right)$$

$$= \left(\frac{71}{50}\right)\left(\frac{47}{50}\right)^7$$

$$= (1.42) \times (0.\ 94)^7$$

The required probability is,

$$(1.42) \times (0.\ 94)^7$$

Binomial Distribution Ex 33.1 Q2

Probability of getting head on one throw of coin $= \frac{1}{2}$

So, $p = \frac{1}{2}$

$q = 1 - \frac{1}{2}$

$q = \frac{1}{2}$ $\quad$ [Since $p + q = 1$]

The coin is tossed 5 times. Let X denote the number of getting head as 5 tosses of coins.
So probability of getting r heads in n tosses of coin is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad ---(1)$$

Probability of getting at least 3 heads

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{5-3} + {}^5C_4\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5\left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \quad \text{[Using (1)]}$$

$$= {}^5C_3\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 + {}^5C_4\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5\left(\frac{1}{2}\right)^5 .1$$

$$= \frac{5.4}{2} \cdot \left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 + 1.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q3

Let p be the probability getting tail on a toss of a fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number tail obtained on the toss of coin 5 times. So probability of getting r tails in n tosses of coin is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting tail an odd number of times

$$= P\left(X = 1\right) + P\left(X = 3\right) + P\left(X = 5\right)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \qquad \left[\text{Using } (1)\right]$$

$$= 5.\left(\frac{1}{2}\right)^5 + \frac{5.4}{2}.\left(\frac{1}{2}\right)^5 + 1.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 \left[5 + 10 + 1\right]$$

$$= 16\left(\frac{1}{2}\right)^5$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q4

Let p be the probability of getting a sum of 9 and it considered as success.
Sum of a 9 on a pair of dice

$$= \{(3,6),(4,5),(5,4),(6,3)\}$$

So, $p = \frac{4}{36}$

$$p = \frac{1}{9}$$

$$q = 1 - \frac{1}{9}$$

$$q = \frac{8}{9} \qquad [\text{Since } p + q = 1]$$

Let X denote the number of success in throw of a pair of dice 6 times. So probability of getting r success out of n is given by

$$P(X = r) = {}^nC_r p^r q^{n-r} \qquad ---(1)$$

Probability of getting at least 5 success

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^{6-5} + {}^6C_6 \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^{6-6} \qquad [\text{Using } (1)]$$

$$= 6\left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^1 + 1.\left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^0$$

$$= \left(\frac{1}{9}\right)^5 \left[\frac{48}{9} + \frac{1}{9}\right]$$

$$= \frac{49}{9} \times \left(\frac{1}{9}\right)^5$$

$$= \frac{49}{9^6}.$$

So,

Required probability $= \frac{49}{9^6}.$

Binomial Distribution Ex 33.1 Q5

Let p be the probability of getting head in a throw of coin. So,

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number of heads on tossing the coin 6 times. Probability of getting r in tossing the coin n times is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r} \qquad ---(1)$$

Probability of getting at least three heads

$$= P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right) + P\left(X = 6\right)$$

$$= 1 - \left[P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)\right]$$

$$= 1 - \left[{}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{6-0} + {}^6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{6-1} + {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2}\right] \qquad \left[\text{Using } (1)\right]$$

$$= 1 - \left[1.\left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^6 + \frac{6.5}{2}.\left(\frac{1}{2}\right)^6\right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^6\left(1 + 6 + 15\right)\right]$$

$$= 1 - \left[\frac{22}{64}\right]$$

$$= \frac{64 - 22}{64}$$

$$= \frac{42}{64}$$

$$= \frac{21}{32}$$

Required probability $= \dfrac{21}{32}$

Binomial Distribution Ex 33.1 Q6

Let p denote the 4 turning up in a toss of a fair die, so

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6}$$

$$q = \frac{5}{6} \qquad [\text{Since } p + q = 1]$$

Let X denote the variable showing the number of turning 4 up in 2 tosses of die.
Probability of getting 4, r times in n tosses of a die is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{2}C_{r} \left(\frac{1}{6}\right)^{2} \left(\frac{5}{6}\right)^{2-r} \qquad ---(1)$$

Probability of getting 4 at least once in tow tosses of a fair die

$$= P(X = 1) + P(X = 2)$$

$$= 1 - P(X = 0)$$

$$= 1 - \left[{}^{2}C_{0} \left(\frac{1}{6}\right)^{0} \left(\frac{5}{6}\right)^{2-0}\right] \qquad [\text{Using } (1)]$$

$$= 1 - \left[1.1.\left(\frac{5}{6}\right)^{2}\right]$$

$$= 1 - \left[\frac{25}{36}\right]$$

$$= \frac{36 - 25}{36}$$

$$= \frac{11}{36}$$

So,

Required probability $= \frac{11}{36}$

Binomial Distribution Ex 33.1 Q7

Let p denote the probability of getting head in a toss of fair coin. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the variable representing number of heads on 5 tosses of a fair coin.
Probability of getting r an n tosses of a fair coin, so

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P\left(X = r\right) = {}^{5}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting head on an even number of tosses of coin

$$= P\left(X = 0\right) + P\left(X = 2\right) + P\left(X = 4\right)$$

$$= {}^{5}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{5-0} + {}^{5}C_{2} \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{5-2} + {}^{5}C_{4} \left(\frac{1}{2}\right)^{4} \left(\frac{1}{2}\right)^{5-4} \qquad \left[\text{Using } (1)\right]$$

$$= 1.1.\left(\frac{1}{2}\right)^{5} + \frac{5.4}{2}.\left(\frac{1}{2}\right)^{5} + 5.\left(\frac{1}{2}\right)^{5}$$

$$= \left(\frac{1}{2}\right)^{5} \left[1 + 10 + 5\right]$$

$$= 16 \times \frac{1}{32}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q8

Let p be the probability of hitting the target, so

$$p = \frac{1}{4}$$

$$q = 1 - p \qquad [\text{Since } p + q = 1]$$

$$= 1 - \frac{1}{4}$$

$$q = \frac{3}{4}$$

Let X denote the variable representing the number of times hittintg the target out of 7 fires. Probability of hitting the target r times out of n fires is given by,

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{7}C_{r} \left(\frac{1}{4}\right)^{r} \left(\frac{3}{4}\right)^{7-r} \qquad ---(1)$$

Probability of hitting the target at least twice

$$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[{}^{7}C_{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{7-0} + {}^{7}C_{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{7-1}\right] \qquad [\text{Using } (1)]$$

$$= 1 - \left[1.1.\left(\frac{3}{4}\right)^{7} + 7.\frac{1}{4}.\left(\frac{3}{4}\right)^{6}\right]$$

$$= 1 - \left(\frac{3}{4}\right)^{6}\left(\frac{3}{4} + \frac{7}{4}\right)$$

$$= 1 - \left(\frac{3}{4}\right)^{6}\left(\frac{10}{4}\right)$$

$$= 1 - \frac{7290}{16384}$$

$$= \frac{9194}{16384}$$

$$= \frac{4547}{8192}$$

Binomial Distribution Ex 33.1 Q9

Let the probability of one telephone number out of 15 is busy between 2 PM and 3 PM be 'p'. then
P = 1/15 ; probability that number is not busy, q = 1-p

Q = 14/16. Binomial distribution is $\left(\frac{14}{15}+\frac{1}{15}\right)^6$

Since 6 numbers are called we find the probability for none of the numbers are busy is P(0)
One number is busy P(1); Two numbers are busy is P(2)
Three numbers are busy is P(3); Four numbers are busy is P(4) ; Five numbers are busy is P(5); Six numbers are busy is P(6).

$P(0) = {}^6C_0 \left(\frac{14}{15}\right)^6$

$P(1) = {}^6C_1 \left(\frac{14}{15}\right)^5 \left(\frac{1}{15}\right)^1$

$P(2) = {}^6C_2 \left(\frac{14}{15}\right)^4 \left(\frac{1}{15}\right)^2$

$P(3) = {}^6C_3 \left(\frac{14}{15}\right)^3 \left(\frac{1}{15}\right)^3$

$P(4) = {}^6C_4 \left(\frac{14}{15}\right)^2 \left(\frac{1}{15}\right)^4$

$P(5) = {}^6C_5 \left(\frac{14}{15}\right)^1 \left(\frac{1}{15}\right)^5$

$P(6) = {}^6C_6 \left(\frac{14}{15}\right)^0 \left(\frac{1}{15}\right)^6$

Probability that at least 3 of the numbers will be busy

P(3) + P(4) + P(5) + P(6) = 0.05

Binomial Distribution Ex 33.1 Q10

p denote the probability of success

p = Probability of getting 5 or 6 in a throw of die.

$$= \frac{2}{6}$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in six throws of a dic. Probability of getting r success in six throws of an unbiased dic is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$= {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r} \qquad ---(1)$$

$$P\left(X \geq 4\right)$$

$$= P\left(X = 4\right) + P\left(X = 5\right) + P\left(X = 6\right)$$

$$= {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{6-4} + {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} + {}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{6.5}{2}\left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + 6\left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right) + 1.\left(\frac{1}{3}\right)^6 .1$$

$$= 15.\frac{1}{81}.\frac{4}{9} + 6.\frac{1}{243}.\frac{2}{3} + \frac{1}{729}$$

$$= \frac{60}{729} + \frac{12}{729} + \frac{1}{729}$$

$$= \frac{73}{729}$$

Required probability $= \dfrac{73}{729}$

Binomial Distribution Ex 33.1 Q11

Let p denote the probability of getting head on a throw of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{2}$$

Let X denote the variable representing the number of getting heads on throw of 8 coins.
Probability of getting r heads in a throw of n coins is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{8}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{8-r} \qquad ---(1)$$

Probability of getting at least six heads

$$= P\left(X = 6\right) + P\left(X = 7\right) + P\left(X = 8\right)$$

$$= {}^{8}C_{6}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{8-6} + {}^{8}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{8-7} + {}^{8}C_{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{8-8} \qquad \left[\text{Using } (1)\right]$$

$$= \frac{8.7}{2}\left(\frac{1}{2}\right)^{8} + 8\left(\frac{1}{2}\right)^{8} + 1.\left(\frac{1}{2}\right)^{8}.1$$

$$= \left(\frac{1}{2}\right)^{8}\left[28 + 8 + 1\right]$$

$$= \frac{1}{256}\left(37\right)$$

$$= \frac{37}{256}$$

Required probability $= \dfrac{37}{256}$

Binomial Distribution Ex 33.1 Q12

Let p denote the probability of getting one spade out of a deck of 52 cards, so

$$p = \frac{13}{52}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{}{4}$$

Let X denote the radom variable of number of spades out of 5 cards. Probability of getting r spades out of n cards is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{5}C_{r} \left(\frac{1}{4}\right)^{r} \left(\frac{3}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability of getting all five spades

$$= P(X = 5)$$

$$= {}^{5}C_{5} \left(\frac{1}{4}\right)^{5} \left(\frac{3}{4}\right)^{5-5}$$

$$= \frac{1}{1024}$$

Probability of getting 5 spades $= \dfrac{1}{1024}$

(ii)

Probability of getting only 3 spades

$$= P(X = 3)$$

$$= {}^{5}C_{3} \left(\frac{1}{4}\right)^{3} \left(\frac{3}{4}\right)^{5-3}$$

$$= \frac{5.4}{2} \left(\frac{1}{64}\right) \left(\frac{9}{16}\right)$$

$$= \frac{45}{512}$$

Probability of getting 3 spades $= \dfrac{45}{512}$

(iii)

Probability that none is spade

$$= P(X = 0)$$

$$= {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$$

$$= \frac{243}{1024}$$

Probability of getting non spade $= \frac{243}{1024}$

Binomial Distribution Ex 33.1 Q13

Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. So

$$p = \frac{5}{20}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let X denote the random variable of number of selecting white ball with replacement out of 4 balls. Probability of getting r white balls out of n balls is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^4C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r} \qquad ---(1)$$

(i)

Probability of getting none white ball

$$= P(X = 0)$$

$$= {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{4-0} \qquad \left[\text{Using } (1)\right]$$

$$= \left(\frac{3}{4}\right)^4$$

$$= \frac{81}{256}$$

Probability of getting none white ball $= \frac{81}{256}$

(ii)

Probability of getting all white balls

$$= P(X = 4)$$

$$= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-0}$$

$$= \left(\frac{1}{4}\right)^4$$

$$= \frac{1}{256}$$

Probability of getting all white balls $= \frac{1}{256}$

(iii)

Probability of getting any two are white

$$= P(X = 2)$$

$$= {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{4-2}$$

$$= \frac{4.3}{2} \cdot \frac{1}{16} \cdot \frac{9}{16}$$

$$= \frac{27}{128}$$

Probability of getting any two are white balls $= \frac{27}{128}$

Binomial Distribution Ex 33.1 Q14

Let p denote the probability of getting a ticket bearing number divisible by 10, So

$$p = \frac{10}{100} \qquad \left[\begin{array}{l}\text{Since there are } 10, 20, 30, 40, 50, 60, 70, 80, \\ 90, 100 \text{ which are divisible by } 10\end{array}\right]$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the variable representing the number of tickets bearing a number divisible by 10 out of 5 tickets. Probability of getting r tickets bearing a number divisible by 10 out of n tickets is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r} \qquad ---(1)$$

Probability of getting all the tickets bearing a number divisible by 10

$$= {}^5C_5 \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^{5-5} \qquad [\text{Using } (1)]$$

$$= 1.\left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^0$$

$$= \left(\frac{1}{10}\right)^5$$

Required probability $= \left(\frac{1}{10}\right)^5$

Binomial Distribution Ex 33.1 Q15

Let p denote the probability of getting a ball marked with 0. So

$$p = \frac{1}{10} \qquad [\text{Since balls are marked with } 0,1,2,3,4,5,6,7,8,9]$$

$$q = 1 - \frac{1}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the variable presenting the number of balls marked with 0 out of four balls drawn. Probability of drawing r balls out of n balls that are marked 0 is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^4C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{4-r} \qquad ---(1)$$

Probability of getting none balls marked with 0

$$= P(X = 0)$$

$$= {}^4C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{4-0}$$

$$= 1.1. \left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^4$$

Probability of getting none balls marked with 0 $= \left(\frac{9}{10}\right)^4$

Binomial Distribution Ex 33.1 Q16

Let p denote the probability of getting one defective item out of hundred. So

$$p = 5\% \qquad \text{[Since 5\% are defective items]}$$

$$= \frac{5}{100}$$

$$p = \frac{1}{20}$$

$$q = 1 - \frac{1}{20} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{19}{20}$$

Let X denote the random variable representing the number of defective items out of 10 items. Probability of getting r defective items out of n items selected is given by,

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{10}C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{10-r} \qquad ---(1)$$

Probability of getting not more than one defective items

$$= P(X = 0) + P(X = 1)$$

$$= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{10-1}$$

$$= 1.1.\left(\frac{19}{20}\right)^{10} + 10.\frac{1}{20}\left(\frac{19}{20}\right)^9$$

$$= \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \frac{29}{20}\left(\frac{19}{20}\right)^9$$

The required probability $= \frac{29}{20}\left(\frac{19}{20}\right)^9$

Binomial Distribution Ex 33.1 Q17

Let p denote the probability that one bulb produced will fuse after 150 days, so

$$p = 0.05$$

$$= \frac{5}{100} \qquad \text{[It is given]}$$

20

$q = 1 - \frac{1}{20}$ $\quad\quad\quad\quad$ [Since $p + q = 1$]

$q = \frac{19}{20}$

Let X denote the number of fuse bulb out of 5 bulbs. Probability that r bulbs out of n will fuse in 150 days is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{5-r} \quad\quad --- (1)$$

(i)

Probability that none is fuse $= P(X = 0)$

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{5-0}$$

$$= \left(\frac{19}{20}\right)^5$$

Probability that none will fuse $= \left(\frac{19}{20}\right)^5$

(ii)

Probability that not more than 1 will fuse

$$= P(X = 0) + P(X = 1)$$

$$= \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{5-1}$$

$$= \left(\frac{19}{20}\right)^4 \left[\frac{19}{20} + \frac{5}{20}\right]$$

$$= \left(\frac{24}{20}\right)\left(\frac{19}{20}\right)^4$$

Probability not more than one will fuse $= \left(\frac{6}{5}\right)\left(\frac{19}{20}\right)^4$

(iii)

Probability that more than one will fuse

$$= P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$= 1-\left[P(X=0)+P(X=1)\right]$$

$$= 1-\left[\frac{6}{5}\left(\frac{19}{20}\right)^4\right]$$

Probability that more than one will fuse $= 1-\left[\frac{6}{5}\left(\frac{19}{20}\right)^4\right]$

(iv)

Probability that that at least one will fuse

$$= P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$= 1-P(X=0)$$

$$= 1-\left[{}^5C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{5-0}\right]$$

$$= 1-\left[\left(\frac{19}{20}\right)^5\right]$$

Probability that that at least one will fuse $= 1-\left(\frac{19}{20}\right)^5$

Binomial Distribution Ex 33.1 Q18

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = \mathrm{P}(\text{right-handed}) = \frac{9}{10}$$
$$q = \mathrm{P}(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

= 1 − P (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

Binomial Distribution Ex 33.1 Q19

Let p denote the probability of getting 1 red ball out of 7 green, 4 white and 5 red balls, so

$$p = \frac{5}{16}$$

$$q = 1 - \frac{5}{16} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{11}{16}$$

Let X denote the number of red balls drawn out of four balls. Probability of getting r red balls out of n drawn balls is given by

$$P\left(X = r\right) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{4}C_r \left(\frac{5}{16}\right)^r \left(\frac{11}{16}\right)^{4-r} \qquad ---\left(1\right)$$

Probability of getting one red ball

$$= P\left(X = 1\right)$$

$$= {}^{4}C_1 \left(\frac{5}{16}\right)^1 \left(\frac{11}{16}\right)^{4-1}$$

$$= 4.\left(\frac{5}{16}\right)\left(\frac{11}{16}\right)^3$$

$$= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$$

Required probability $= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$

Binomial Distribution Ex 33.1 Q20

X	$P(X)$
0	$\frac{7}{9}\times\frac{6}{8}=\frac{21}{36}$
1	$\frac{7}{9}\times\frac{2}{8}\times 2=\frac{14}{36}$
2	$\frac{2}{9}\times\frac{1}{8}=\frac{1}{36}$

Binomial Distribution Ex 33.1 Q21

X	$P(X)$
0	${}^3C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^{3-0}=\left(\frac{4}{7}\right)^3=\frac{64}{343}$
1	${}^3C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^{3-1}=3.\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)^2=\frac{144}{343}$
2	${}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^{3-2}=3.\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)=\frac{108}{343}$
3	${}^3C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^{3-0}=\left(\frac{3}{7}\right)^3=\frac{27}{343}$

Binomial Distribution Ex 33.1 Q22

Let p be the probability of getting doublet is a throw of a pair of dice, so

$$p=\frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \text{ are doublets}\right]$$

$$p=\frac{1}{6}$$

$$q=1-\frac{1}{6} \qquad \left[\text{Since } p+q=1\right]$$

$$=\frac{5}{6}$$

Let X denote the number of getting doublets out of 4 times. So probability distribution is given by

Binomial Distribution Ex 33.1 Q23

X	$P(X)$
0	${}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{3-0} = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$
1	${}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{3-1} = 3\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)^2 = \frac{25}{72}$
2	${}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{3-2} = 3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) = \frac{5}{72}$
3	${}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{3-3} = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$

Binomial Distribution Ex 33.1 Q24

We know that, probability of getting head in a toss of coin $p = \frac{1}{2}$

Probability of not getting head $q = 1 - \frac{1}{2}$

$$q = \frac{1}{2}$$

The coin is tossed 5 times. Let X denote the number of times head occur is 5 tosses.

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{5-r}$$

Probability distribution is given by

Binomial Distribution Ex 33.1 Q25

Let p be the probability of a getting a number greater than 4 in a toss of die, so

$$p = \frac{2}{6} \qquad \left[\text{Since, numbers greater than 4 coin a die } = 5,6\right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in 2 throws of a die. Probability of getting r success in n thrown of a die is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$= {}^2C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{2-r} \qquad ---(1)$$

Probability distribution of number of success is given by

X	$P(X)$
0	${}^2C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{2-0} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
1	${}^2C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{2-1} = 2.\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$
2	${}^2C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{2-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

Binomial Distribution Ex 33.1 Q26

Let n denote the number of throws required to get a head and X denote the amount won/lost.

He may get head on first toss or lose first and 2nd toss or lose first and won second toss probability distribution for X

Number of throws (n):	1	2	2
Amount won/lost (X):	1	0	-2
Probability $P(X)$:	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

So probability distribution is given by

X	$P(X)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Binomial Distribution Ex 33.1 Q27

Let p denote the probability of getting 3,4 or 5 in a throw of die. So

$$p = \text{probability of success}$$
$$= \frac{3}{6}$$

$$p = \frac{1}{2}$$
$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$
$$q = \frac{1}{2}$$

Let X denote the number of success in throw of 5 dice simultaneously. Probability of getting r success out of n throws of die is given by

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability getting at least 3 success

$$= P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right)$$
$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$$
$$= \frac{5.4}{2}\left(\frac{1}{2}\right)^5 + 5.\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$$
$$= \left(\frac{1}{2}\right)^5 \left[10 + 5 + 1\right]$$
$$= \frac{16}{32}$$
$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q28

Let p denote the probability of getting defective items out of 100 items, so

$$p = 10\%$$
$$= \frac{10}{100}$$

$$p = \frac{1}{10}$$
$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$
$$q = \frac{9}{10}$$

Let X denote the number of defective items drawn out of 8 items. Probability of getting r defective items out of a sample of 8 items is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$
$$= {}^{8}C_{r} \left(\frac{1}{10}\right)^{r} \left(\frac{9}{10}\right)^{8-r} \qquad ---\left(1\right)$$

Probability of getting 2 defective items

$$= P\left(X = 2\right)$$
$$= {}^{8}C_{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{8-2}$$
$$= \frac{8 \times 7}{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{6}$$
$$= \frac{28 \times 9^{6}}{10^{8}}$$

Required probability $= \dfrac{28 \times 9^{6}}{10^{8}}$

Binomial Distribution Ex 33.1 Q29

Let p denote the probability of drawing a heart from a deck of 52 cards, so

$$p = \frac{13}{52} \qquad \left[\because \text{ There are 13 hearts in deck}\right]$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let the card is drawn n times. So Binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

where X denote the number of spades drawn and $r = 0, 1, 2, 3, \ldots n$

(i)

We have to find the smallest value of n for which $P(X = 0)$ is less than $\frac{1}{4}$

$$P(X = 0) < \frac{1}{4}$$

$${}^{n}C_{0} \left(\frac{1}{1}\right)^{0} \left(\frac{3}{4}\right)^{n-0} < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^{n} < \frac{1}{4}$$

Put $n = 1$, $\left(\frac{3}{4}\right) \not< \frac{1}{4}$

$n = 2$, $\left(\frac{3}{4}\right)^{2} \not< \frac{1}{4}$

$n = 3$, $\left(\frac{3}{4}\right)^{3} \not< \frac{1}{4}$

So, smallest value of $n = 3$

$\therefore$ We must draw cards at least 3 times

(ii)

Given, the probability of drawing a heart $> \frac{3}{4}$

$$1 - P(X = 0) > \frac{3}{4}$$

$$1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0} > \frac{3}{4}$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{3}{4}$$

$$1 - \frac{3}{4} > \left(\frac{3}{4}\right)^n$$

$$\frac{1}{4} > \left(\frac{3}{4}\right)^n$$

For $n = 1$, $\left(\frac{3}{4}\right)^1 \not< \frac{1}{4}$

$n = 2$, $\left(\frac{3}{4}\right)^2 \not< \frac{1}{4}$

$n = 3$, $\left(\frac{3}{4}\right)^3 \not< \frac{1}{4}$

$n = 4$, $\left(\frac{3}{4}\right)^4 \not< \frac{1}{4}$

$n = 5$, $\left(\frac{3}{4}\right)^5 \not< \frac{1}{4}$

So, card must be drawn 5 times.

Binomial Distribution Ex 33.1 Q30

Here $\quad x = 8, p = \frac{1}{2}, q = \frac{1}{2}$

Let there be k desks and X be the number of students studying in office.

Then we want that

$$P\left(X \le k\right) > .90$$

$$\Rightarrow \quad P\left(X > k\right) < .10$$

$$\Rightarrow \quad P\left(X = k+1, k+2, \ldots 8\right) < .10$$

Clearly $P\left(X > 6\right) = P\left(X = 7 \text{ or } X = 8\right)$

$$= {}^{8}C_{7}\left(\frac{1}{2}\right)^{8} + {}^{8}C_{8}\left(\frac{1}{2}\right)^{8}$$

$$= .04$$

and $\quad P\left(X > 5\right) = P\left(X = 6,\ X = 7 \text{ or } X = 8\right)$

$$= .15$$

$$\therefore \quad P\left(X > 6\right) < 0.10$$

$\Rightarrow$ If there are 6 desks then there is at least 90% chance for every graduate assistant to get a desk.

Binomial Distribution Ex 33.1 Q31

Binomial Distribution formula is given by

$P(x) = {}^nC_x\, p^x\, q^{n-x}$, where x = 0, 1, 2, ...n

Let x = No. of heads in a toss

We need probability of 6 or more heads

X = 6, 7, 8

Here p = ½ and q = ½

$P(6)$ = Prob of getting 6 heads, 2 tails $= {}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2$

$P(7)$ = Prob of getting 7 heads, 1tails $= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1$

$P(8)$ = Prob of getting 8 heads, 0 tails $= {}^8C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$

The probability of getting at least 6 heads (not more than 2 tails) is then

$${}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2 + {}^8C_1 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_2 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{256} + 8\frac{1}{256} + 28\frac{1}{256} = \frac{37}{256}$$

Binomial Distribution Ex 33.1 Q32

Let p represents the probability of getting head in a toss of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{2}$$

Let X denote the random variable representing the number heads in 6 tosses of coin. Probability of getting r sixes in n tosses of a fair coin is given by,

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{6}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{6-r} \qquad ---(1)$$

(i)

Probability of getting 3 heads

$$= P\left(X = 3\right)$$

$$= {}^{6}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{6-3}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{3}$$

$$= \frac{20}{64}$$

Probability of getting 3 heads $= \frac{20}{64} = \frac{5}{16}$

(ii)

Probability of getting no heads

$$= P(X = 0)$$

$$= {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0}$$

$$= \left(\frac{1}{2}\right)^6$$

$$= \frac{1}{64}$$

Probability of getting no heads $= \frac{1}{64}$

(iii)

Probability of getting at least one head

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

Probability of getting at least one head $= \frac{63}{64}$

Binomial Distribution Ex 33.1 Q33

Let p be the probability that a tube function for more than 500 hours. So

$$p = 0.2$$

$$p = \frac{1}{5}$$

$$q = 1 - \frac{1}{5} \qquad \left[\text{Since } p + q = 1\right]$$

$$= \frac{4}{5}$$

Let X denote the random variable representing the number of tube that functions for more than 500 hours out of 4 tubes. Probability of functioning r tubes out n tubes selected for more than 500 hours is given by,

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{4}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{4-r} \qquad ---(1)$$

Probability that exactly 3 tube will function for more than 500 hours

$$= {}^{4}C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{4-3}$$

$$= 4.\left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)$$

$$= \frac{16}{625}$$

Required probability $= \dfrac{16}{625}$

Binomial Distribution Ex 33.1 Q34

Let p be the probability that component survive the shock test. So

$$p = \frac{3}{4}$$

$$q = 1 - \frac{3}{4} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{1}{4}$$

Let X denote the random variable representing the number of components that survive shock test out of 5 components. Probability of that r components that survive shock test out of n components is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that exactly 2 will survive the shock test

$$= P(X = 2)$$

$$= {}^5C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{5-2}$$

$$= \frac{5.4}{2}\left(\frac{9}{16}\right)\left(\frac{1}{64}\right)$$

$$= \frac{45}{512} = 0.0879$$

Probability that exactly 2 survive $= 0.0879$

(ii)

Probability that at most 3 will survive

$$= P(X=0)+P(X=1)+P(X=3)+P(X=4)$$

$$= 1-[P(X=4)+P(X=5)]$$

$$= 1-\left[{}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^{5-4} + {}^5C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^{5-5}\right]$$

$$= 1-\left[5.\frac{81}{1024}+\frac{243}{1024}\right]$$

$$= 1-\left[\frac{405+243}{1024}\right]$$

$$= \frac{1024-648}{1024}$$

$$= \frac{376}{1024} = 0.3672$$

Binomial Distribution Ex 33.1 Q35

Probability that bomb strikes a target p = 0.2

Probability that a bomb misses the target = 0.8

n = 6

let x = number of bombs that strike the target

P(x=2) = exactly 2 bombs strike the target

$$= {}^6C_2\left(\frac{2}{10}\right)^2 \times \left(\frac{8}{10}\right)^4 = 15\times\frac{16384}{10^6} = 0.24576$$

$P(x\geq2)$ = at least 2 bombs strike the target

$= 1- P(x<2)$

$= 1 - [P(x=0)+P(x=1)]$

$$= 1 - \left[{}^6C_0\left(\frac{2}{10}\right)^0 \times \left(\frac{8}{10}\right)^6 + {}^6C_1\left(\frac{2}{10}\right)^1 \times \left(\frac{8}{10}\right)^5\right]$$

= 1 − [0.0.262144 + 0.393216] = 1 − 0.65536

= 0.34464

Binomial Distribution Ex 33.1 Q36

Let p be the probability that a mouse get contract the desease. So

$$p = 40\%$$
$$= \frac{40}{100}$$
$$= \frac{2}{5}$$
$$q = 1 - \frac{2}{5} \qquad [\text{Since } p + q = 1]$$
$$q = \frac{3}{5}$$

Let X denote the variable representing number of mice contract the disease out of 5 mice.

Probability the r mice get contract the disease out of n mice inoculated is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{2}{5}\right)^r \left(\frac{3}{5}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that none contract the disease $= P(X = 0)$

$$= {}^5C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{5-0}$$
$$= \left(\frac{3}{5}\right)^5$$

Probability that none contract the disease $= \left(\frac{3}{5}\right)^5$

(ii)

Probability that more than 3 contract disease

$$= P(X = 4) + P(X = 5)$$
$$= {}^5C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^{5-4} + {}^5C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^{5-5}$$
$$= 5.\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^5$$

$$= \left(\frac{\quad}{5}\right) \left[3 + \frac{\quad}{5}\right]$$

$$= \frac{17}{5}\left(\frac{2}{5}\right)^4$$

Binomial Distribution Ex 33.1 Q37

Let p be the probability of success is experiments, q be the probability of failure,

Given, $p = 2q$

but $\quad p + q = 1$

$$2q + q = 1$$
$$3q = 1$$
$$q = \frac{1}{3}$$
$$p = \frac{2}{3}$$

Let X denote the random variable representing the number of success out of 6 experiments.
Probability of getting r success out of n experiments is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r} \qquad ---(1)$$

Probability of getting at least 4 success

$$= P(X = 4) + P(X = 5) + P(X = 6)$$
$$= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{6-4} + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{6-5} + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^{6-6}$$
$$= \frac{6 \times 5}{2} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + 6 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + \left(\frac{2}{3}\right)^6$$
$$= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9} + \frac{4}{3} + \frac{4}{9}\right]$$
$$= \left(\frac{2}{3}\right)^4 \left[\frac{15 + 12 + 4}{9}\right]$$
$$= \left(\frac{31}{9}\right)\left(\frac{2}{3}\right)^4$$
$$= \frac{496}{729}$$

Required probability $= \dfrac{496}{729}$

Binomial Distribution Ex 33.1 Q38

Let x = number of out of service machines
p = probability that machine will be out of service on the same day
= 2/100
q = probability that machine will be in service on the same day
= 8/100
P(x=3) = probability exactly 3 machines will be out of service on the same day

$$P(x=3) = {}^{20}C_3 \times \left(\frac{2}{100}\right)^3 \left(\frac{8}{100}\right)^0 = 1140 \times 0.000008$$

= 0.00912
For low probability events Poisson' distribution is used instead of Binomial distribution. Then,
$\lambda = np = 20 \times 0.02 = 0.4$

$$P(x=r) = \frac{e^{-\lambda} \times \lambda^{3}}{r!}$$

$$P(x=3) = \frac{e^{-0.4} \times 0.4^{3}}{3!} = 0.6703 \times 0.064/6 = 0.0071$$

Binomial Distribution Ex 33.1 Q39

Let p be the probability that a student entering a university will graduate, so

$p = 0.4$

$q = 1 - 0.4$ [Since $p + q = 1$]

$= 0.6$

Let X denote the random variable representing the number of students entering a university will graduate out of 3 students of university. Probability that r students will graduate out of n entering the university is given by

$$P(X = r) = {}^{n}C_r p^r (q)^{n-r}$$

$$= {}^{3}C_r (0.4)^r (0.6)^{3-r} \qquad ---(1)$$

(i)

Probability that none will graduate

$= P(X = 0)$

$= {}^{3}C_0 (0.4)^0 (0.6)^{3-0}$

$= 0.216$

Probability that none will graduate $= 0.216$

(ii)
Probability that one will graduate

$= P(X = 1)$

$= {}^3C_1 (0.4)^1 (0.6)^{3-1}$

$= 3 \times (0.4)(0.36)$

$= 0.432$

Probability that only one will graduate $= 0.432$

(iii)
Probability that all will graduate

$= P(X = 3)$

$= {}^3C_3 (0.4)^3 (0.6)^{3-3}$

$= (0.4)^3$

$= 0.064$

Probability that all will graduate $= 0.064$

Binomial Distribution Ex 33.1 Q40

Let X denote the number of defective eggs in the 10 eggs drawn.
Since the drawing is done with replacement, the trials are Bernoulli trials.

Clearly, X has the binomial distribution with n=10 and $p = \frac{10}{100} = \frac{1}{10}$

Therefore, $q = 1 - \frac{1}{10} = \frac{9}{10}$

Now, P(at least one defective egg) $= P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

Binomial Distribution Ex 33.1 Q41

Let p be the probability of answering a true. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$= \frac{1}{2}$$

Thus the probability that he answers at least 12 questions correctly among 20 questions is

$$P(X \geq 12) = P(X = 12) + P(X = 13) + P(X = 14) + P(X = 15) + P(X = 16) +$$

$$P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$$

$$= \left(\frac{1}{2}\right)^{20} \left\{ {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20} \right\}$$

$$= \frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Therefore, the required answer is

$$\frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Binomial Distribution Ex 33.1 Q42

X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Then, } P(X = x) = {}^nC_x q^{n-x} p^x$$

$$= {}^6C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x$$

$$= {}^6C_x \left(\frac{1}{2}\right)^6$$

It can be seen that $P(X = x)$ will be maximum, if ${}^6C_x$ will be maximum.

$$\text{Then, } {}^6C_0 = {}^6C_6 = \frac{6!}{0! \cdot 6!} = 1$$

$${}^6C_1 = {}^6C_5 = \frac{6!}{1! \cdot 5!} = 6$$

$${}^6C_2 = {}^6C_4 = \frac{6!}{2! \cdot 4!} = 15$$

$${}^6C_3 = \frac{6!}{3! \cdot 3!} = 20$$

The value of ${}^6C_3$ is maximum. Therefore, for $x = 3$, $P(X = x)$ is maximum.

Thus, $X = 3$ is the most likely outcome.

Binomial Distribution Ex 33.1 Q43

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p=\frac{1}{3}$

$$\therefore q=1-p=1-\frac{1}{3}=\frac{2}{3}$$

Clearly, X has a binomial distribution with n = 5 and $p=\frac{1}{3}$

$$\therefore \mathrm{P}(\mathrm{X}=x)={}^{n}\mathrm{C}_{x}q^{n-x}p^{x}$$

$$={}^{5}\mathrm{C}_{x}\left(\frac{2}{3}\right)^{5-x}\cdot\left(\frac{1}{3}\right)^{x}$$

P (guessing more than 4 correct answers) = P(X ≥ 4)

$$=\mathrm{P}(\mathrm{X}=4)+\mathrm{P}(\mathrm{X}=5)$$

$$={}^{5}\mathrm{C}_{4}\left(\frac{2}{3}\right)\cdot\left(\frac{1}{3}\right)^{4}+{}^{5}\mathrm{C}_{5}\left(\frac{1}{3}\right)^{5}$$

$$=5\cdot\frac{2}{3}\cdot\frac{1}{81}+1\cdot\frac{1}{243}$$

$$=\frac{10}{243}+\frac{1}{243}$$

$$=\frac{11}{243}$$

Binomial Distribution Ex 33.1 Q44

(b) P (winning exactly once) = P(X = 1)

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) = P(X ≥ 2)

$$= 1 - P(X < 2)$$

$$= 1 - P(X \le 1)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= [1 - P(X = 0)] - P(X = 1)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$$

$$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

Binomial Distribution Ex 33.1 Q45

Let the shooter fire n times.
n fires are Bernoulli trials.

In each trial, p= probability of hitting the target= $\frac{3}{4}$

And q = probability of not hitting the target= $1-\frac{3}{4}=\frac{1}{4}$

Then, $P(X=x)={}^{n}C_{x}\,q^{n-x}p^{x}={}^{n}C_{x}\left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^{x}={}^{n}C_{x}\frac{3^{x}}{4^{n}}$

Now, given that

P(hitting the target atleast once) > 0.99

i.e. $P(x\ge 1)>0.99$

$\Rightarrow\quad 1-P(x=0)>0.99$

$\Rightarrow\quad 1-{}^{n}C_{0}\frac{1}{4^{n}}>0.99$

$\Rightarrow\quad {}^{n}C_{0}\frac{1}{4^{n}}<0.01$

$\Rightarrow\quad \frac{1}{4^{n}}<0.01$

$\Rightarrow\quad 4^{n}>\frac{1}{0.01}=100$

The minimum value of n to satisfy this inequality is 4
Thus, the shooter must fire 4 times.

Binomial Distribution Ex 33.1 Q46

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

$$\Rightarrow p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$$

$$\therefore P(X = x) = {}^nC_x p^{n-x} q^x = {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that,

P (getting at least one head) > $\frac{90}{100}$

$P(x \geq 1) > 0.9$

$1 - P(x = 0) > 0.9$

$$1 - {}^nC_0 \cdot \frac{1}{2^n} > 0.9$$

$$ {}^nC_0 \cdot \frac{1}{2^n} < 0.1$$

$$\frac{1}{2^n} < 0.1$$

$$2^n > \frac{1}{0.1}$$

$$2^n > 10 \qquad \ldots(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Binomial Distribution Ex 33.1 Q47

Let the man toss the coin n times.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

So,

$$p=\frac{1}{2}$$

$$q=1-\frac{1}{2} \qquad [\text{Since } p+q=1]$$

$$=\frac{1}{2}$$

$$\therefore P(X=x)={}^{n}C_{x}p^{n-x}q^{x}$$

$$={}^{n}C_{x}\left(\frac{1}{2}\right)^{n-x}\left(\frac{1}{2}\right)^{x}$$

$$={}^{n}C_{x}\left(\frac{1}{2}\right)^{n}$$

It is given that

$$p(\text{getting atleast one head})>\frac{80}{100}$$

$$P(x\geq 1)>0.8$$

$$1-P(x=0)>0.8$$

$$1-{}^{n}C_{0}\cdot\frac{1}{2^{n}}>0.8$$

$${}^{n}C_{0}\cdot\frac{1}{2^{n}}<0.2$$

$$\frac{1}{2^{n}}<0.2$$

$$2^{n}>\frac{1}{0.2}$$

$$2^{n}>5$$

The minimum value of n that satisfies the given inequality is 3.
Thus, the man should toss the coin 3 or more than 3 times.

Binomial Distribution Ex 33.1 Q48

Let p be the probability of getting a doublet in a throw of a pair of dice, so

$$p = \frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\right]$$

$$= \frac{1}{6}$$

$$q = 1 - \frac{1}{6} \qquad [\text{Since } p + q = 1]$$

$$= \frac{5}{6}$$

Let X denote the number of getting doublets i.e. success out of 4 times. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{4-0} = \left(\frac{5}{6}\right)^4$
1	${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{4-1} = 4\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = \frac{2}{3}\left(\frac{5}{6}\right)^3$
2	${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{4-2} = \frac{4\cdot 3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216}$
3	${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{4-3} = \frac{4\cdot 3}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) = \frac{5}{324}$
4	${}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{4-4} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

Binomial Distribution Ex 33.1 Q49

Let p be the probability of defective bulbs, so

$$p = \frac{6}{30}$$
$$= \frac{1}{5}$$
$$q = 1 - \frac{1}{5} \qquad [\text{Since } p + q = 1]$$
$$= \frac{4}{5}$$

Here, 4 bulbs is drawn at random with replacement. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{4-0} = \frac{256}{625}$
1	${}^4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{4-1} = \frac{4}{5} \times \frac{4^3}{5^3} = \frac{256}{625}$
2	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{4-2} = \frac{6}{5^2} \times \frac{4^2}{5^2} = \frac{96}{625}$
3	${}^4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^{4-3} = \frac{4}{5^3} \times \frac{4}{5} = \frac{16}{625}$
4	${}^4C_4\left(\frac{1}{5}\right)^4\left(\frac{6}{5}\right)^{4-4} = 1 \cdot \frac{1}{625} = \frac{1}{625}$

Binomial Distribution Ex 33.1 Q50

Here success is a score which is multiple of 3 i.e. 3 or 6.

$$\therefore p(3 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$$

Now P(at least 8 successes) = P(8) + P(9) + P(10)

$$= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^0$$

$$= \frac{1}{3^{10}}\left[45 \times 4 + 10 \times 2 + 1\right]$$

$$= \frac{201}{3^{10}}$$

Binomial Distribution Ex 33.1 Q51

Here success is an odd number i.e. 1,3 or 5.

$\therefore p(1,3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$

The probability of r successes in 5 throws is given by

$$P(r) = {}^{5}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{5-r}$$

Now $P(\text{exactly 3 times}) = P(3)$

$$= {}^{5}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{2}$$

$$= \frac{10}{2^{5}}$$

$$= \frac{5}{16}$$

Binomial Distribution Ex 33.1 Q52

Probablity of a man hitting a target is 0.25.

$$\therefore p = 0.25 = \frac{1}{4}, \quad q = 1 - p = \frac{3}{4}$$

The probability of r successes in 7 shoots is given by

$$P(r) = {}^7C_r (0.25)^r (0.75)^{7-r}$$

Now P(at least twice) = 1 – P(less than 2)

$$= 1 - {}^7C_0 (0.25)^0 (0.75)^7 + {}^7C_1 (0.25)^1 (0.75)^6$$

$$= 1 - \frac{3^7}{4^7} + 7 \times \frac{3^6}{4^7}$$

$$= \frac{4547}{8192}$$

Binomial Distribution Ex 33.1 Q53

Pr obablity of a bulb to be defective is $\frac{1}{50}$.

$\therefore p = \frac{1}{50}, \ q = 1 - p = \frac{49}{50}$

The probability of r defective bulbs in 10 bulbs is given by

$P(r) = {}^{10}C_r \left(\frac{1}{50}\right)^r \left(\frac{49}{50}\right)^{10-r}$

(i) P(none of the bulb is defective) = P(0)

$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10}$

$= \left(\frac{49}{50}\right)^{10}$

(ii) P(exactly two bulbs are defective) = P(2)

$= {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$

$= 45 \times \frac{(49)^8}{(50)^{10}}$

(iii) P(more than 8 bulbs work properly)

= P(at most two bulbs are defective)

$$= {}^{10}C_0\left(\frac{1}{50}\right)^0\left(\frac{49}{50}\right)^{10} + {}^{10}C_1\left(\frac{1}{50}\right)^1\left(\frac{49}{50}\right)^9 + {}^{10}C_2\left(\frac{1}{50}\right)^2\left(\frac{49}{50}\right)^8$$

$$= \left(\frac{49}{50}\right)^{10} + 10 \times \frac{(49)^9}{(50)^{10}} + 45 \times \frac{(49)^8}{(50)^{10}}$$

$$= \frac{(49)^8}{(50)^{10}}\left[(49)^2 + 490 + 45\right]$$

$$= \frac{(49)^8 \times 2936}{(50)^{10}}$$

Ex 33.2

Binomial Distribution Ex 33.2 Q1

Let X be a binomial variate with parameters n and p.

$$\text{Mean} = np$$
$$\text{Varience} = npq$$

$$\begin{aligned}\text{Mean} - \text{Variance} &= np - npq \\ &= np(1-q) \\ &= np.p \\ &= np^2\end{aligned}$$

$$\text{Mean} - \text{Variance} > 0$$
$$\text{Mean} > \text{Variance}$$

So, mean can never be less than varience.

Binomial Distribution Ex 33.2 Q2

Let X denote the variance with parameters n and p

$$p + q = 1$$
$$q = 1 - p$$

Goven, $\text{Mean} = np = 9 \qquad ---(i)$

$$\text{Variance} = npq = \frac{9}{4} \qquad ---(ii)$$

$$\frac{npq}{np} = \frac{\frac{9}{4}}{9} \qquad [\text{By dividing } (i) \text{ by } (ii)]$$

$$q = \frac{1}{4}$$

So, $\quad p = 1 - q$

$$= 1 - \frac{1}{4}$$
$$p = \frac{3}{4}$$

Put p in equation (i),

$$n\left(\frac{3}{4}\right) = 9$$

$\Rightarrow \quad n = \frac{36}{3}$

So, $\quad n = 12$

The distribution is given by

$$= {}^nC_r p^r (q)^{n-r}$$

$$P(X = r) = {}^{12}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r}$$

for $r = 0, 1, 2, ..12$

Binomial Distribution Ex 33.2 Q3

Let n and p be parameters of binomial distribution. Here

$$\text{Mean } = np = 9 \quad ---(\text{i})$$
$$\text{Variance } = npq = 6 \quad ---(\text{ii})$$

$$\frac{npq}{np} = \frac{6}{9}$$
$$q = \frac{2}{3}$$

So, $p = 1 - \frac{2}{3}$ $\quad [\text{Since } p + q = 1]$

$$p = \frac{1}{3}$$

Using equation (i), $np = 9$

$$n\left(\frac{1}{3}\right) = 9$$
$$n = 27$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$$
$$r = 0, 1, 2, \ldots 27$$

Binomial Distribution Ex 33.2 Q4

Given that,

$$n = 5$$

Also, $\text{Mean} + \text{Varience} = 4.8$

$$np + npq = 4.8$$
$$np(1 + q) = 4.8$$
$$5p(1 + q) = 4.8$$
$$5(1 - q)(1 + q) = 4.8 \quad [\text{Since } p + q = 1]$$
$$5(1 - q^2) = 4.8$$
$$1 - q^2 = \frac{4.8}{5}$$
$$q^2 = 1 - \frac{4.8}{5}$$
$$= \frac{0.2}{5}$$
$$q^2 = \frac{1}{25}$$
$$q = \frac{1}{5}$$

$\Rightarrow$ $p = 1 - q$

$$= 1 - \frac{1}{5}$$
$$p = \frac{4}{5}$$

So, $n = 5,\ p = \frac{4}{5},\ q = \frac{1}{5}$

Here binomial distribution is

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = 5C_r \left(\frac{4}{5}\right)^r \left(\frac{1}{5}\right)^{5-r}$$
$$r = 0, 1, 2, 3, \ldots 5$$

Binomial Distribution Ex 33.2 Q5

Given that,

Mean $= np = 20$ --- (i)

Variance $= npq = 16$ --- (ii)

Let n and p be the parameters of distribution dividing equation (ii) by (i)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put p in equation (i),

$$np = 20$$

$$n\left(\frac{1}{5}\right) = 20$$

$$n = 20 \times 5$$

$$n = 100$$

So, binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{100}C_{r} \left(\frac{1}{5}\right)^{r} \left(\frac{4}{5}\right)^{100-r}$$

$$r = 0, 1, 2, 3, \ldots 100$$

Binomial Distribution Ex 33.2 Q6

Let n and p be the parameters of distribution binomial distribution. So

$$q = 1 - p \qquad \text{as } p + q = 1$$

$$\text{Mean} + \text{Variance} = \frac{25}{3}$$

$$np + npq = \frac{25}{3}$$

$$np(1+q) = \frac{25}{3}$$

$$np = \frac{25}{3(1+q)} \qquad ---(1)$$

$$\text{Mean} \times \text{Variance} = \frac{50}{3}$$

$$np \times npq = \frac{50}{3}$$

$$n^2p^2q = \frac{50}{3}$$

$$\left[\frac{25}{3(1+q)}\right]^2 .q = \frac{50}{3} \qquad [\text{Using } (1)]$$

$$625q = \frac{50}{3}\left[9(1+q)^2\right]$$

$$625q = 150(1+q)^2$$

$$25q = 6(1+q)^2$$

$$6 + 6q^2 + 12q - 25q = 0$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$3q(2q-3) - 2(2q-3) = 0$$

$$(2q-3)(3q-2) = 0$$

$$\Rightarrow \quad 2q - 3 = 0 \quad \text{or} \quad 3q - 2 = 0$$

$$\Rightarrow \quad q = \frac{3}{2} \quad \text{or} \quad q = \frac{2}{3}$$

Since $q \le 1$, so

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Binomial Distribution Ex 33.2 Q7

Let n and p be the parameters of binomial distribution.

Given that,

Mean $= np - 20$ $\quad ---(i)$

Standard deviation $= \sqrt{npq} = 4$

Squaring both the sides,

$npq = 16$ $\quad ---(ii)$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ $\quad$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put value of p in equation (i),

$$np = 20$$

$$\frac{n}{5} = 20$$

$$n = 100$$

$$p = \frac{1}{5}$$

Binomial Distribution Ex 33.2 Q8

Let p denotes the probability of selecting a defective bolt, so

$$p = 0.1$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{9}{10}$$

Given, $n = 400$

(i)

Mean $= np$

$$= 400 \times \frac{1}{10}$$

Mean $= 40$

(ii)

Standard deviation $= \sqrt{npq}$

$$= \sqrt{400 \times \frac{1}{10} \times \frac{9}{10}}$$

$$= \sqrt{36}$$

Standard deviation $= 6$

Binomial Distribution Ex 33.2 Q9

Let n and p be the parameters of binomial distribution.

Given, Mean $= np = 5 \quad ---(i)$

$$\text{Variance } = npq = \frac{10}{3} \quad ---(ii)$$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{\frac{10}{3}}{5}$$

$$q = \frac{2}{3}$$

So, $p = 1 - q \quad$ [Since $p + q = 1$]

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = 5$$
$$n = 5 \times 3$$
$$n = 15$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{15}C_{r} \left(\frac{1}{3}\right)^{r} \left(\frac{2}{3}\right)^{15-r}$$

$$r = 0, 1, 2, \ldots 15$$

Binomial Distribution Ex 33.2 Q10

Let p be the probability of a ship returning safely to parts, so

$$p = \frac{9}{10}$$

$$q = 1 - \frac{9}{10} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{1}{10}$$

Given, $n = 500$

$$\text{Mean } = np$$
$$= 500 \times \frac{9}{10}$$
$$\text{Mean } = 450$$

$$\text{Standard deviation } = \sqrt{npq}$$
$$= \sqrt{500 \times \frac{9}{10} \times \frac{1}{10}}$$
$$= \sqrt{45}$$
$$= 6.71$$

Mean $= 450$

Standard deviation $= 6.71$

Binomial Distribution Ex 33.2 Q11

Given that, parameters for binomial distribution are n and p.

Also, Mean $= np = 16$ $\quad ---(i)$

Variance $= npq = 8$ $\quad ---(ii)$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{8}{16}$$

$$q = \frac{1}{2}$$

So, $p = 1 - \frac{1}{2}$ $\quad [\text{as } p + q = 1]$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{32}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{32-r} \quad ---(iii)$$

$$P(X = 0)$$

$$= {}^{32}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0} \quad [\text{Using (iii)}]$$

$$= \left(\frac{1}{2}\right)^{32}$$

$$P(X = 1)$$

$$= {}^{32}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1}$$

$$= 32 . \frac{1}{2}\left(\frac{1}{2}\right)^{31}$$

$$= \left(\frac{1}{2}\right)^{27}$$

$$P(X \ge 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{32} + \left(\frac{1}{2}\right)^{27}\right]$$

$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{1}{32} + 1\right)$$

$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{33}{32}\right)$$

$$= 1 - \frac{33}{2^{32}}$$

Hence

$$P(X = 0) = \left(\frac{1}{2}\right)^{32}, \; P(X = 1) = \left(\frac{1}{2}\right)^{27}, \; P(X \ge 2) = 1 - \frac{33}{2^{32}}$$

Binomial Distribution Ex 33.2 Q12

Let p be the probability of success in a single throw of die

$$p = \frac{2}{6} \quad \left[\text{Since success is occurance of 5 or 6}\right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \quad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Given, $n = 8$

$$\text{Mean} = np = \frac{8}{3} = 2.66$$

$$\text{Standard deviation} = \sqrt{npq} = \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} = \frac{4}{3} = 1.33$$

Mean $= 2.66$, Standard deviation $= 1.33$

Binomial Distribution Ex 33.2 Q13

Let n and p be the parameters of binomial distribution.
Let p = probability of having a boy in the family

Given, $p = q$

Since, $p + q = 1$

$$p + p = 1$$
$$2p = 1$$
$$p = \frac{1}{2}$$
$$q = \frac{1}{2}$$
$$n = 8$$

$$\text{The expected number of boys} = np = 8 \times \frac{1}{2} = 4$$

The expected number of boys $= 4$

Binomial Distribution Ex 33.2 Q14

Let p denot the probability of a defective item produced in the factory, so

$$p = 0.02$$
$$= \frac{2}{100}$$
$$p = \frac{1}{50}$$
$$q = 1 - \frac{1}{50} \qquad [\text{Since } p + q = 1]$$
$$= \frac{49}{50}$$

Given $n = 10{,}000$

Expected number of defective item $= np$

$$= 10000 \times \frac{1}{50}$$
$$= 200$$

Standard deviation $= \sqrt{npq}$

$$= \sqrt{10000 \times \frac{1}{50} \times \frac{49}{50}}$$
$$= 14$$

Expected No. of defective items $= 200$

Standard deviation $= 14$

Binomial Distribution Ex 33.2 Q15

Let p be the probability of success, so

$$p = \frac{2}{6} \qquad [\text{Since success in occurance of 1 or 6 on the die}]$$
$$p = \frac{1}{3}$$

Given, $n = 3$

$$q = 1 - p \qquad [\text{Since } p + q = 1]$$
$$= 1 - \frac{1}{3}$$
$$q = \frac{2}{3}$$

Mean $= np$

$$= 3\left(\frac{1}{3}\right)$$
$$= 1$$

Variance $= npq$

$$= 3 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$
$$= \frac{2}{3}$$

Mean $= 1$

Variance $= \frac{2}{3}$

Binomial Distribution Ex 33.2 Q16

Let n and p be the parameters of binomial distribution
Given,

$$\text{Mean } = np = 3 \qquad ---(\text{i})$$

$$\text{Variance } = npq = \frac{3}{2} \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{3}{2}}{3}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \qquad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i)

$$np = 3$$

$$n\left(\frac{1}{2}\right) = 3$$

$$n = 6$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{6}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} \qquad ---(\text{iii})$$

$$P(X \le 5)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^{6}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6}, \qquad [\text{Using (iii)}]$$

$$= 1 - \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$P(X \le 5) = \frac{63}{64}$$

Binomial Distribution Ex 33.2 Q17

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean } = np = 4 \quad \text{---(i)}$$

$$\text{Variance } = npq = 2 \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{2}\right) = 4$$

$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \quad \text{---(iii)}$$

$$P(X \geq 5)$$

$$= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$= {}^8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8$$

[Using equation (iii)]

$$= \frac{8 \times 7 \times 6}{3 \times 2}\left(\frac{1}{2}\right)^8 + \frac{8 \times 7}{2}\left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 [56 + 28 + 8 + 1]$$

$$= \frac{93}{256}$$

$$P(X \geq 5) = \frac{93}{256}$$

Binomial Distribution Ex 33.2 Q18

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{16}{81}$$

$$= \frac{65}{81}$$

$$P(X \geq 1) = \frac{65}{81}$$

Binomial Distribution Ex 33.2 Q19

Let n and p be the parameters of binomial distribution,

Given, $n = 6$

$$\text{Mean} + \text{Variance} = \frac{10}{3}$$

$$np + npq = \frac{10}{3}$$

$$6p + 6pq = \frac{10}{3}$$

$$6p(1+q) = \frac{10}{3}$$

$$6(1-q)(1+q) = \frac{10}{3} \qquad [\text{Since } p + q = 1]$$

$$6(1-q^2) = \frac{10}{3}$$

$$1 - q^2 = \frac{10}{18}$$

$$-q^2 = \frac{5}{9} - 1$$

$$-q^2 = -\frac{4}{9}$$

$$q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Hence, the binomial distribution is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

as $r = 0, 1, 2, \ldots 6$

Binomial Distribution Ex 33.2 Q20

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $= 36$

Let p be the probability of success therefore

$p = 6/36 = 1/6$

Let q be the probability of failure therefore $q = 1 - p = 1 - 1/6 = 5/6$

Since the dice is thrown 4 times, $n = 4$

Let X be the random variable for getting doublet, therefore X can take at max 4 values.

$$P(X=0) = {}^4C_0 p^0 q^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 p^1 q^3 = 4 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2 p^2 q^2 = \frac{4 \cdot 3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3 p^3 q^1 = 4 \cdot \left(\frac{1}{6}\right)^3 \cdot \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4 p^4 q^0 = 1 \cdot \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

Mean

$$\mu = \sum_{i=1}^{4} X_i P(X_i) = 0 \cdot \frac{625}{1296} + 1 \cdot \frac{500}{1296} + 2 \cdot \frac{150}{1296} + 3 \cdot \frac{20}{1296} + 4 \cdot \frac{1}{1296}$$

$$= \frac{500 + 300 + 60 + 4}{1296} = \frac{54}{81} = \frac{2}{3}$$

Hence the mean is $= \frac{2}{3}$

Binomial Distribution Ex 33.2 Q21

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $= 36$

Let p be the probability of success therefore

$$p = 6/36 = 1/6$$

Let q be the probability of failure therefore $q = 1 - p = 1 - 1/6 = 5/6$

Since there is three rows of dice so $n = 3$

Let X be the random variable for getting doublet, therefore X can take at max 3 values.

$$P(X=0) = {}^3C_0 p^0 q^3 = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X=1) = {}^3C_1 p^1 q^2 = 3 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X=2) = {}^3C_2 p^2 q^1 = 3 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) = \frac{15}{216}$$

$$P(X=3) = {}^3C_3 p^3 q^0 = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

Mean

$$\mu = \sum_{i=1}^{3} X_i P(X_i) = 0 \cdot \frac{125}{216} + 1 \cdot \frac{75}{216} + 2 \cdot \frac{15}{216} + 3 \cdot \frac{1}{216}$$

$$= \frac{75+30+3}{216} = \frac{108}{216} = \frac{1}{2}$$

Hence the mean is $= \frac{1}{2}$

Binomial Distribution Ex 33.2 Q22

Out of 15 bulbs 5 are defective.

Hence, the probability that the drawn bulb is defective is

$$P(\text{Defective}) = \frac{5}{15} = \frac{1}{3}$$

$$P(\text{Not defective}) = \frac{10}{15} = \frac{2}{3}$$

Let X denote the number of defective bulbs out of 4.

Then, X follows binomial distribution with

$n = 4$, $p = \frac{1}{3}$ and $q = \frac{2}{3}$ such that

$$P(X = r) = {}^4C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{4-r}; r = 0,1,2,3,4$$

$$\text{Mean} = \sum_{r=0}^{4} rP(r) = 1 \times {}^4C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + 2 \times {}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$$

$$+3 \times {}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right) + 4 \times {}^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0$$

$$= \frac{32}{81} + \frac{48}{81} + \frac{24}{81} + \frac{4}{81} = \frac{108}{81} = \frac{4}{3}$$

Binomial Distribution Ex 33.2 Q23

Let p be the probablity of getting 2 when a dice is thrown.

Then $p = \frac{1}{6}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{6}$.

$\therefore$ Expectation $= E(X) = np = 3 \times \frac{1}{6} = \frac{1}{2}$

Binomial Distribution Ex 33.2 Q24

Let p be the probablity of getting an even number on the toss when a dice is thrown.

Let q be the probablity of not getting an even number on the toss when a dice is thrown.

Then $p = \frac{3}{6} = \frac{1}{2}$ and $q = \frac{1}{2}$

Clearly, X follows binomial distribution with $n = 2$, $p = \frac{1}{2}$.

$\therefore$ Variance $= npq = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

Binomial Distribution Ex 33.2 Q25

Let p be the probablity of getting a spade card.
Let q be the probablity of getting a spade card.

Then $p = \frac{13}{52} = \frac{1}{4}$ and $q = \frac{3}{4}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$.

Probablity distribution is given by,

$$P(X = r) = {}^3C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{3-r};\ r = 0, 1, 2$$

$$\therefore \text{Mean} = np = 3 \times \frac{1}{4} = \frac{3}{4}$$