CONTROL SYSTEMS TEST 3

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following are properties of signal flow graph?

(P) Signal flow graph is applicable to linear network only.

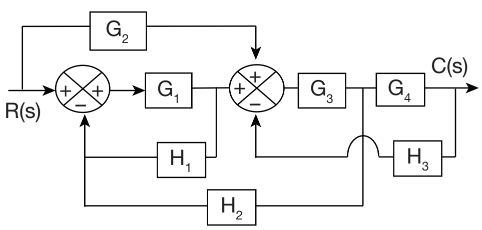
(Q) Signals travel along branches only in the marked direction and it gets multiplied by the gain of the branch.

(R) The algebraic equations must be in the form of cause and effect.

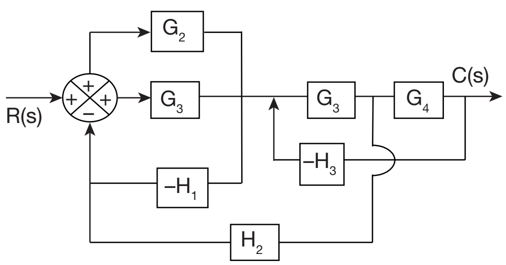
(A) *P* and *Q* (B) *P* and *R*

(C) Q and *R* (D) *P, Q* and *R*

2. Which of the block diagram is equivalent to the given block diagram?



(A)



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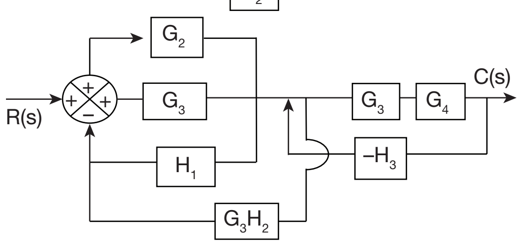

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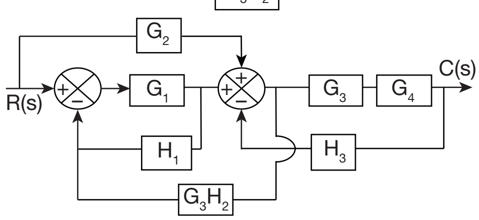
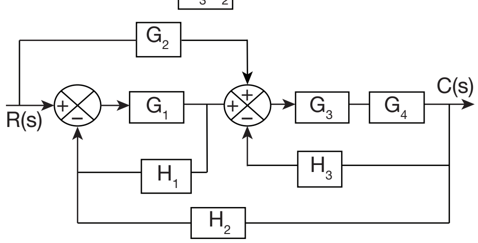


(D)



3. The transmittance is

(A) The node point in the signal flow graph

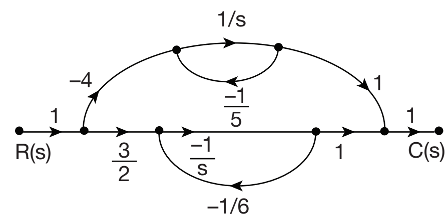
(B) The gain acquired by the signal when it travel from one node to another node in signal flow graph.

(C) The source input signal applied to the node in signal flow graph.

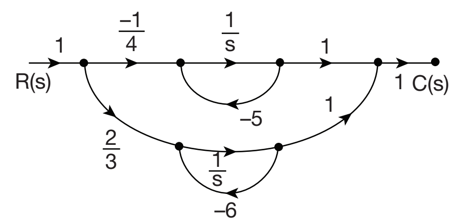
(D) None of the above

4. The signal flow graph for the transfer function $\dfrac{\frac{-1}{4}}{s+5}+\dfrac{\frac{2}{3}}{s+6}$ is

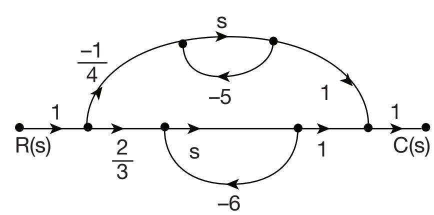
(A)



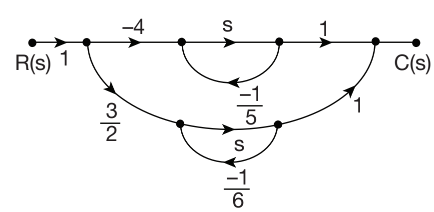
(B)



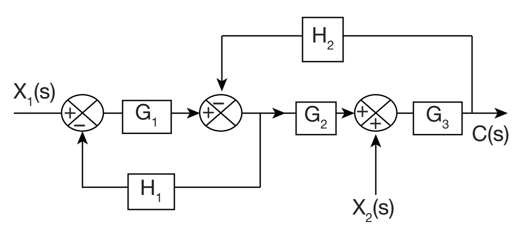
(C)



(D)



5. Evaluate the closed loop transfer function of $\dfrac{C(s)}{X_2(s)}$



(A) $\dfrac{G_1\, G_2\, G_3}{1+G_1\, H_1+G_2\, G_3\, H_2}$

(B) $\dfrac{G_3}{1+G_3 H_2+G_1 H_1+G_3 G_1 H_2}$

(C) $\dfrac{G_3}{1+G_3 G_2 H_2}$

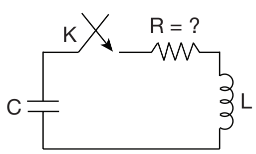
(D) $\dfrac{G_3}{1+G_3 G_2 H_2+G_1 H_1}$

6. The overall transfer function of a control system is given by $\dfrac{C(s)}{R(s)}=\dfrac{9}{s^2+1.8s+9}$

Calculate the new damping ratio when derivate feedback with a constant of 0.4 is used.

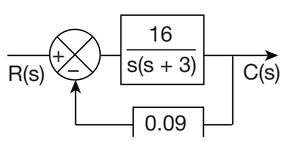
(A) 0.9 (B) 0.12
(C) 0.6 (D) 0.75

7. What is the value of R for the following circuit? (Assume the initial voltage across capacitor is 2V) $I(S)=\dfrac{1}{s^2+4s+2}$



(A) 2 Ω (B) 4 Ω
(C) 8 Ω (D) 16 Ω

8. Determine the sensitivity of the closed loop transfer function with respect to forward path of the following block diagram at ω = 2 rad/sec



(A) 0.51 (B) 1.10
(C) 0.90 (D) 1.54

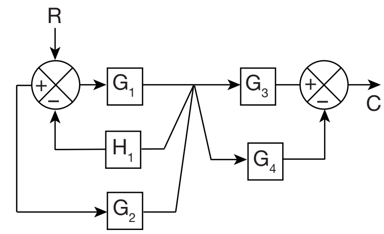
9. A unity feedback control system has open loop transfer function of $G(s)=\dfrac{4s+2}{2s^2}$. Then the expression for the time response when the system is subjected to unit step input function is

(A) $0.5 + 0.5e^{-t} - 0.5t\,e^{-t}$
(B) $-0.5 + 0.5e^{-t} + 0.5te^{-t}$
(C) $0.5 - 0.5e^{-t} + 0.5te^{-t}$
(D) $0.5 + 0.5e^{-t} + 0.5te^{-t}$

10. A closed loop transfer function of unity feedback system is $\dfrac{5s+10}{s^2+6s+10}$. Then the velocity error constant is

(A) 10 (B) 0
(C) infinite (D) 0.1

11. The C/R of the following block diagram.



(A) $\dfrac{G_1 G_2 (G_3-G_4)}{1+G_1 H_1}$

(B) $\dfrac{G_1+G_2+G_3-G_4}{1+G_1 H_1}$

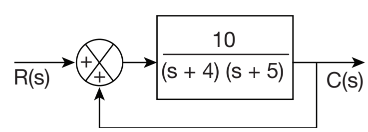
(C) $\left(\dfrac{G_1+G_2}{1+G_1 H_1}\right)(G_3+G_4)$

(D) $\left(\dfrac{G_1+G_2}{1+G_1H_1+G_2H_1}\right)(G_3-G_4)$

12. The unity feedback system is characterized by an open loop transfer function $G(s)=\dfrac{400}{s(s+20)}$. Determine the peak overshoot for a unit step input.

(A) 16.3% (B) 5.08%
(C) 32.6% (D) 8%

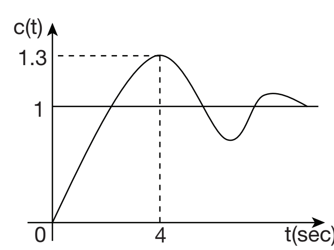
13. The block diagram of a unity feedback control system is shown below.



At what time the first undershoot occurs

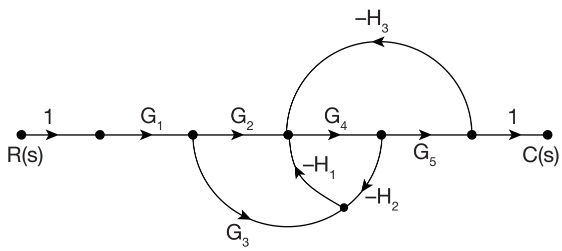
(A) 2 sec
(B) 1 sec
(C) 3 sec
(D) Under shoot does not occur

14. Calculate the natural frequency of oscillation of the following response.



(A) 0.783 rad/sec (B) 0.838 rad/sec
(C) 1.676 rad/sec (D) 1.57 rad/sec

15. The transfer function for the following signal flow graph



(A) $\dfrac{G_1 G_2 G_4 G_5}{1 - G_4 H_1 H_2 + G_4 G_5 H_3}$

(B) $\dfrac{G_1 G_2 G_4 G_5 + G_1 G_3 G_5}{1 + G_4 H_2 H_1 + G_4 G_5 H_3}$

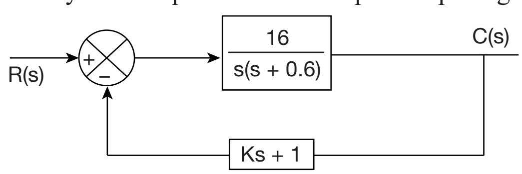
(C) $\dfrac{G_1 G_2 G_4 G_5 - G_1 G_3 H_1 G_4 G_5}{1 - G_4 H_1 H_2 + G_4 G_5 H_3}$

(D) $\dfrac{G_1 G_2 G_4 G_5 + G_1 G_3 G_4 G_5}{1 - G_1 H_1 H_2 + G_4 G_5 H_3}$

16. The open loop transfer function of a unity feedback system is $G(s) = \dfrac{100}{s^2(s+5)(s^2+5s+10)}$. Then the steady state error when the input is $R(s) = 6/s^3$

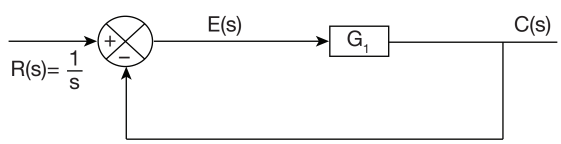
(A) 0 (B) α
(C) 3 (D) 2

17. The damping ratio of the following system is 0.8 then the steady state response for unit impulse input signal



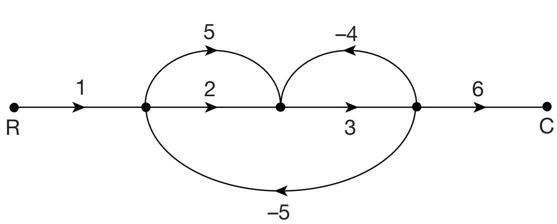
(A) 6.67 cos 2.4t (B) 6.67 sin 2.4t
(C) 6.67 cos 5.76t (D) 6.67 sin 5.76t

18. Calculate the steady state error for the following $E(S) = \dfrac{s^2+5s+4}{s(s^2+3s+4)}$ for unit step input signal.



(A) 0 (B) 1
(C) α (D) Insufficient data

19. The value of transfer function



(A) 1.18 (B) 2.93
(C) 0.95 (D) 1.06

20. The peak overshoot of the following transfer function $T(S) = \dfrac{36}{s^2+12s+36}$

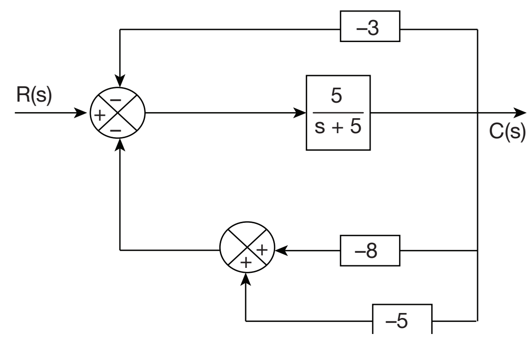
(A) 100% (B) 10.28%
(C) 5.14% (D) zero

21. The system response for a unit impulse input is $e^{-2t} - e^{-3t}$, if unit step input is applied to the system then the corresponding output is

(A) $\dfrac{1}{6} - \dfrac{1}{2}e^{-2t} + \dfrac{1}{3}e^{-3t}$ (B) $\dfrac{1}{6} - \dfrac{1}{3}e^{-3t} + \dfrac{1}{2}e^{-2t}$

(C) $6 - 2e^{-2t} + 3e^{-3t}$ (D) $6 - 3e^{-3t} + 2e^{-2t}$

22. Calculate the steady state error for the following system when a step signal of magnitude 3 is applied to the input.



(A) 1 (B) −0.2
(C) ∞ (D) 0

23. The output equation $y(t)$ of the following system when unit step input is applied $\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 6y = 8u(t)$

(A) $\dfrac{4}{3} + 4e^{-2t} + \dfrac{8}{3}e^{-3t}$ (B) $\dfrac{4}{3} - 4e^{-2t} - \dfrac{8}{3}e^{-3t}$

(C) $\dfrac{4}{3} - 4e^{-2t} + \dfrac{8}{3}e^{-3t}$ (D) $\dfrac{4}{3} + 4e^{-2t} + \dfrac{8}{3}e^{-3t}$

24. Calculate the settling time for 5% error of the system when the system has open loop gain of $\dfrac{4}{s(s+4)}$ and negative feedback gain of 4.

(A) 1.5 sec (B) 2 sec
(C) 0.5 sec (D) 2.5 sec

25. A unity feedback second order system has 3% of peak overshoot and 2sec of settling time for 2% tolerance error in the system. Then the corresponding open loop transfer function of

(A) $\dfrac{7.29}{s(s+4)}$ (B) $\dfrac{2.7}{s(s+4)}$

(C) $\dfrac{2.7}{s(s+1.48)}$ (D) $\dfrac{5.4}{s(s+2.7)}$

Answer Keys

1. D	**2.** C	**3.** B	**4.** B	**5.** D	**6.** A	**7.** C	**8.** B	**9.** C	**10.** A
11. D	**12.** A	**13.** D	**14.** B	**15.** C	**16.** C	**17.** B	**18.** B	**19.** D	**20.** D
21. A	**22.** B	**23.** C	**24.** A	**25.** A					

Hints and Explanations

1. Choice (D)

2. Choice (C)

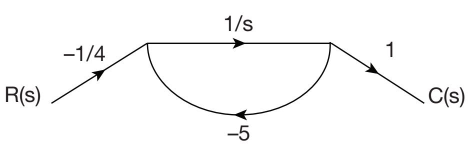
3. Choice (B)

4. Given transfer function $T(S) = \dfrac{\frac{-1}{4}}{s+5} + \dfrac{\frac{2}{3}}{s+6}$

The transfer function has two parallel paths

I^st Path

$$T_1 = \frac{-\frac{1}{4}}{s+5}$$



By using Mason's gain formula.

Forward gain $-\,{}^{1}\!/_{4s}$

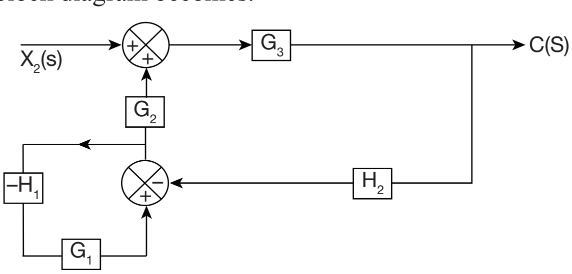
Feedback gain $-\dfrac{5}{s}$

$$\therefore \quad T_1 = \frac{-\,{}^{1}\!/_{4s}}{1+{}^{5}\!/_{s}} = \frac{-\frac{1}{4}}{s+5}$$

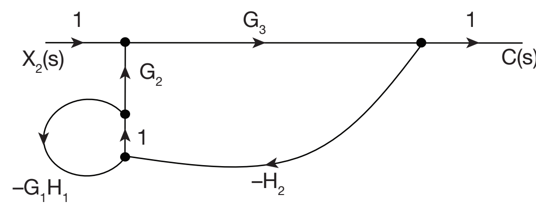
Similarly for second parallel path $T_2 = \dfrac{{}^{2}\!/_{3}}{s+6}$

Choice (B)

5. For input $X_2(S)$, the input $X_1(S)$ is inactive. Then the block diagram becomes.



Signal flow graph



$$\frac{C(s)}{X_2(s)} = \frac{G_3}{1+G_3\,G_2\,H_2+G_1\,H_1}$$ Choice (D)

6. The overall transfer function of the system using derivative feedback control is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+\left(2\xi\omega_n+\omega_n^2\,K\right)s+\omega_n^2}$$

K – derivative feedback constant = 0.4

From the given system $\omega_n = 3$

$2\,\xi\,\omega_n = 1.8$

$\xi = 0.3$

New damping ratio = ξ^1

$\therefore \quad 2\,\xi^1\,\omega_n = 2\,\xi\,\omega_n + \omega_n^2\,k$

$2 \times \xi^1 \times 3 = 5.4$

$\xi^1 = 0.9$ Choice (A)

7. The loop equation

$$sLI(s) + R(s)\,I(s) + \frac{1}{sC}\,I(s) - \frac{V_o}{s} = 0$$

$$I(S) = \frac{V_o}{s\left(sL+R+\dfrac{1}{sC}\right)} = \frac{V_o}{s^2L+sR+\dfrac{1}{C}}$$

$$= \frac{V_0/L}{\left(s^2+s.\dfrac{R}{L}+\dfrac{1}{LC}\right)}$$

Compare with given equation $I(S) = \dfrac{1}{s^2+4s+2}$

$$\Rightarrow \frac{V_o}{L} = 1 \Rightarrow L = 2\text{H}$$

$$\Rightarrow \frac{R_o}{L} = 4 \Rightarrow R = 8\Omega$$ Choice (C)

8. $S_G^M = \dfrac{1}{1+G(s)H(s)} = \dfrac{1}{1+\dfrac{16}{s(s+3)}\times 0.09}$

$= \dfrac{s^2+3s}{s^2+3s+1.44}$

$S = j\omega$

$S_G^M = \dfrac{-\omega^2+3j\omega}{-\omega^2+3j\omega+1.44}$

$= \dfrac{-4+6j}{-4+6j+1.44} = \dfrac{-4+6j}{-2.56+6j} = 1.10$ Choice (B)

9. The closed loop transfer function $T(S) = \dfrac{G(s)}{1+G(s)H(s)}$

$= \dfrac{4s+2}{2s^2+4s+2} = \dfrac{s+0.5}{s^2+2s+1}$

$T(S) = \dfrac{s+0.5}{s^2+2s+1} = \dfrac{s+0.5}{(s+1)^2}$

$\dfrac{C(s)}{R(s)} = \dfrac{s+0.5}{(s+1)^2}$

$C(s) = \dfrac{s+0.5}{s(s+1)^2} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(s+1)^2}$

$A = 0.5 = C$

$B = -0.5$

$C(s) = \dfrac{0.5}{s} - \dfrac{0.5}{s+1} + \dfrac{0.5}{(s+1)^2}$

$= 0.5 - 0.5e^{-t} + 0.5te^{-t}$ Choice (C)

10. Closed loop transfer function $= \dfrac{5s+10}{s^2+6s+10}$

Open loop transfer function $G(s) = \dfrac{5(s+2)}{s(s+1)}$

Velocity error constant $K_a = \underset{S\to 0}{Lt}\, sG(s)$

$= \underset{S\to 0}{Lt}\, s\times\dfrac{5(s+2)}{s(s+1)} = 10$ Choice (A)

11. From the given block diagram G_1 and G_2 are two parallel forward paths and H_1 is the feedback path for $G_1 + G_2$. Similarly G_3 and G_4 are two parallel forward paths

$\therefore \dfrac{C}{R} = \left(\dfrac{G_1+G_2}{1+(G_1+G_2)H_1}\right)(G_3 - G_4)$ Choice (D)

12. Open loop transfer function $G(s) = \dfrac{400}{s(s+20)}$

Closed loop transfer function $= \dfrac{400}{s^2+20s+400}$

Compare the above equation with standard equation

$\omega_n = 20$

$2\xi\omega_n = 20 \Rightarrow \xi = 0.5$

Percentage peak overshoot $\% M_P = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$= 16.3\%$ Choice (A)

13. The overall transfer function $\dfrac{C(s)}{R(s)} = \dfrac{10}{(s+4)(s+5)-10}$

$= \dfrac{10}{s^2+9s+10}$

The characteristic equation $s^2 + 9s + 10 = 0$

$\therefore\ \omega_n = \sqrt{10}\quad 2\xi\omega_n = 9$

$\Rightarrow\ \xi = \dfrac{9}{2\times\sqrt{10}} = 1.42$

The given system is over damped system therefore there is no undershoot in the waveform.

Choice (D)

14. From the given waveform $m_P = 0.3$

$0.3 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$\ln(0.3) = -\dfrac{\pi\xi}{\sqrt{1-\xi^2}}$

$\Rightarrow\ -1.2 = \dfrac{-\pi\xi}{\sqrt{1-\xi^2}}$

$\Rightarrow\ (1-\xi^2)1.44 = 9.86\,\xi^2$

$\Rightarrow\ \xi^2 = \dfrac{1.44}{11.3}$

$\Rightarrow\ \xi = 0.356$

$t_P = 4 = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\Rightarrow\ \omega_n = \dfrac{\pi}{4\sqrt{1-0.35^2}} = 0.838$ rad/sec Choice (B)

15. The signal flow graph has two forward paths and two loops and the corresponding gains are

1st forward path gain $= +G_1 G_2 G_4 G_5$

2nd forward path gain $= -G_1 G_3 G_4 G_5 H_1$

Ist loop gain $= +G_4 H_1 H_2$

2nd loop gain $= -G_4 G_5 H_3$

By using mason's gain formulae the transfer function is

$\dfrac{G_1 G_2 G_4 G_5 - G_1 G_3 G_4 G_5 H_1}{1 - G_4 H_1 H_2 + G_4 G_5 H_3}$ Choice (C)

16. Acceleration error constant $k_a = \underset{S\to 0}{Lt}\, S^2G(s)$

$K_a = \underset{S\to 0}{Lt}\, S^2 \times \dfrac{100}{s^2(s+5)(s^2+5s+10)} = \dfrac{100}{50}$

$K_a = 2$

$\therefore$ steady state error $e_{ss} = \frac{6}{2} = 3$

Choice (C)

17. The transfer function of given system is

$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s(s+0.6)}}{1+\frac{16\,(Ks+1)}{s(s+0.6)}}$$

$$= \frac{16}{s^2+(0.6+k16)s+16}$$

Compare the characteristic equation with standard equation $2\,\xi\,\omega_n = 0.6 + 16k$

$$\Rightarrow K = \frac{2\times0.8\times\sqrt{16}-0.6}{16} = 0.3625$$

$$\frac{C(s)}{R(s)} = \frac{16}{S^2+6.4S+16}$$

$R(s) = 1$

$$\Rightarrow C(s) = \frac{16}{(s+3.2)^2+(2.4)^2}$$

$$= \frac{16}{2.4}\frac{2.4}{(s+3.2)^2+(2.4)^2} = 6.67\sin 2.4t$$

Choice (B)

18. Given signal is error signal $E(S) = \frac{s^2+5s+4}{s(s^2+3s+4)}$

Steady state error $e_{ss} = \underset{S\to0}{Lt}\, s.E(s)$

$$= \underset{S\to0}{Lt}\, S.\left[\frac{s^2+5s+4}{s(s^2+3s+4)}\right] = 1$$

Choice (B)

19. In the given signal flow graph two forward paths and three loops.

Ist forward path gain = 36

2nd forward path gain = 90

1st loop gain = –30

2nd loop gain = –75

3rd loop gain = –12

The transfer function $= \frac{36+90}{1+30+75+12}$

= 1.06

Choice (D)

20. The transfer function $T(s) = \frac{36}{s^2+12s+36}$

The characteristic equation $s^2 + 12s + 36 = 0$

Compare with the standared equation then

$\omega_n = 6$ $2\,\xi\,\omega_n = 12$

$\xi = 1$

$\therefore$ $\xi = 1$ the system is critically damped then peak overshoot is zero.

Choice (D)

21. $C(t) = e^{-2t} - e^{-3t}$

$$C(s) = \frac{1}{s+2} - \frac{1}{s+3}$$

$$C(S) = \frac{1}{(s+2)(s+2)}$$

$R(s) = 1$

The transfer function of the system is

$$T(S) = \frac{C(s)}{R(s)} = \frac{1}{(s+2)(s+3)}$$

If $R(S) = \frac{1}{s}$ then $C(S) = \frac{1}{s(s+2)(s+3)}$

$$= \frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}$$

$$C(t) = \frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}$$

Choice (A)

22. The closed loop transfer function $= \frac{5}{s+5-16\times5}$

$$= \frac{5}{s-75}$$

$$G(s)\,H(s) = \left(\frac{5}{s+5}\right)(-16)$$

Then $k_p = \underset{S\to0}{Lt}\, G(s)\,H(s) = -16$

$$R(s) = \frac{3}{s}$$

Steady state error $e_{ss} = \underset{S\to0}{Lt}\, \frac{S.R(s)}{1+G(s)H(s)}$

$$e_{ss} = \underset{S\to0}{Lt}\, \frac{s.\frac{3}{s}}{1+G(s)H(s)}$$

$$= \frac{3}{1+\underset{S\to0}{Lt}\, G(s)H(s)} = \frac{3}{1-16} = -\frac{1}{5}$$

= –0.2

Choice (B)

23. Given $\frac{d^2y}{dt^2} + 5.\frac{dy}{dt} + 6y = 8u(t)$

Given $u(t) = U(s) = \frac{1}{s}$

$\Rightarrow$ $s^2\,y(s) + 5sy(s) + 6y(s) = 8U(s)$

$$\Rightarrow\ y(s) = \frac{8}{s(s^2+5s+6)}$$

$$Y(s) = \frac{8}{s(s+3)(s+2)}$$

Apply Inverse Laplace $y(t) = \frac{4}{3} - 4e^{-2t} + \frac{8}{3}e^{-3t}$

Choice (C)

24. Given $G(s) = \frac{4}{s(s+4)}$

$H(s) = 4$

Closed loop transfer function $T(s) = \frac{G(s)}{1+G(s)H(s)}$

$= \frac{4}{s^2+4s+16}$

The characteristic equation $s^2 + 4s + 16 = 0$

$\omega_n = 4$

$2\xi\omega_n = 4$

$\xi = 0.5$

Settling time T_S (5% error) $= \frac{3}{\xi\omega_n}$

$= \frac{3}{0.5\times 4} \quad = \frac{3}{2} = 1.5$

Choice (A)

25. Given settling time for 2% tolerance

$T_S = \frac{4}{\xi\omega_n} = 2$

%peak overshoot $= e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$0.03 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$\ln(0.03) = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$

$-3.5 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$

$\Rightarrow \quad \xi = 0.74$

$\Rightarrow \quad T_S = \frac{4}{\xi\omega_n} = 2$

$\Rightarrow \quad \omega_n = \frac{4}{0.74\times 2} = 2.70$

For second order system the standard closed loop transfer function $T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$T(s) = \frac{2.7^2}{s^2+2\times 0.74\times 2.7s+2.7^2}$

$= \frac{7.29}{s^2+4s+7.29}$

Open loop transfer function $= \frac{7.29}{s(s+4)}$

Choice (A)