CHAPTER 18

Set Theory

Set

A well defined collection of objects is called a set.

The objects in a set are called its members or elements.

If x is an element of a set A, we write $x \in A$, which means that 'x belongs to A' or that x is an element of A.

If x does not belong to A, we write, $x \notin A$.

e.g.,

(i) The collection of vowels in English alphabet is a set containing five elements namely a, e, i, o, u.
Clearly, $A = \{a, e, i, o, u\}$ and $a \in A, b \notin A, e \in A$ etc.

(ii) The collection of first four prime numbers is a set containing the elements 2, 3, 5, 7.

(iii) The collection of all honest persons of India is not a set. Similarly, collection of all beautiful girls of India is not a set, since the term beautiful is vague and it is not well defined.

How to Specify a Set?

1. **Roster Method** (Tabular form) Under this method, we only make a list of the objects of the set and put them in the braces *i.e.*, { } separated by commas,

e.g.,

(i) If A is a set of first eight prime numbers, then
$$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

(ii) If B is a set of squares of first five natural numbers, then $B = \{1, 4, 9, 16, 25\}$

(iii) If A is a set of vowels of English alphabets, then
$$A = \{a, e, i, o, u\}$$

> The order in which the elements are written in a set makes no difference and also the repetition of an element has no effect.

2. **Set Builder Form** (Property form) In this method, a set is described by a characterising property $P(x)$ of its elements x. In such a case the set is described by $\{x : P(x) \text{ holds}\}$ or $\{x \mid P(x) \text{ holds}\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol '|' or ':' is read as "such that".

e.g.,

(i) The set B of all even natural numbers can be written as
$B = \{x : x \text{ is a natural number and } x = 2n \text{ for } n \in N\}$
or $B = \{x \mid x \in N, x = 2n, n \in N\}$

(ii) The set $A = \{3, 5, 7, 9, 11\}$, then it is represented as
$A = \{x : x = 2n + 1 \text{ where } n \in N, n < 6\}$

(iii) The set $A = \{0, 1, 4, 9, 16, \ldots\}$ can be written as
$A = \{x^2 \mid x \in Z\}$ where Z is the set of integers.

Types of Sets

1. **Empty Set** A set is said to be empty or null or void set, if it has no element.
- It is denoted by ϕ.
- In Roster method, ϕ is denoted by { }.
- The empty set is a subset of every set.

e.g., (i) $\{x \in R \mid x^2 = -2\} = \phi$ (ii) $\{x \in N \mid 5 < x < 6\} = \phi$

2. **Singleton Set** A set consisting of a single element is called a singleton set.
e.g., (i) The set {5} is a singleton set.
(ii) $\{x : x + 6 = 6\} = \{0\}$, which is a singleton set.

3. **Finite Set** A set is called a finite set, if it is either void set or its elements can be counted by natural numbers. Suppose, the total elements are 'n', then 'n' is called as cardinal number of a finite set and is denoted as $n(A)$, if A is a finite set.

e.g.,
(i) Set of even natural numbers less than 50.
(ii) Set of officers in Army.
(iii) Set of population of India.
(iv) Set of even prime natural numbers.

4. **Infinite Set** A set is called an infinite set, if it is not finite.
e.g., (i) Set of all points in a plane.
(ii) $\{x \in R \mid 0 < x < 1\}$ there R is a set of real number.
(iii) Set of all points on a line.

5. **Equal Sets** Two set A and B are said to be equal, if every element of A is a member of B and every element of

B is a member of A. If A and B are two sets, then $A = B$ and $A \neq B$, if A and B are not equal. If $A = \{7, 4, 6, 5\}$ and $B = \{4, 5, 6, 7\}$, then $A = B$ as each element of A is an element of B and *vice-versa*.

- The repetition of elements in a set is meaningless. So, $\{1, 2, 3, 3, 2, 1, 1\} = \{1, 2, 3\}$.
- $\phi \neq \{0\} \neq 0$, since ϕ is a set having no element at all, $\{0\}$ is a singleton set and 0 is not a set.

Cardinal Number of a Set

The number of distinct elements contained in a set A is called the cardinal number of A and is denoted as $n(A)$.

If $A = \{a, e, i, o, u\}$, then $n(A) = 5$.

6. **Equivalent Sets** Two sets A and B are equivalent, if their cardinal numbers are same *i.e.*, $n(A) = n(B)$
7. **Subsets** Let A and B be two sets and if every element of A is an element of B, then A is called a subset of B. So, if A is subset of B, then we write $A \subseteq B$ and read as "A is subset of B" or "A is contained in B".
   So, if $a \in A \Rightarrow a \in B$
   If A is not subset of B, then we write $A \not\subseteq B$.
8. **Super Set** If A is subset of B, then we say that B is superset of A and we write $B \supseteq A$.
   e.g., (i) If $A = \{1, 2, 3, 4\}$
   $B = \{1, 2, 3, 4, 5, 6\}$,
   then there $A \subseteq B$ and $B \supseteq A$.
   (ii) $N \subseteq W \subseteq I \subseteq Q \subseteq R$
9. **Comparable Sets** Two sets A and B are said to be comparable, if either $A \subseteq B$ or $B \subseteq A$. *i.e.*, either A is subset of B or B is subset of A.
10. **Non-comparable Sets** Two sets A and B are said to be non-comparable sets, if A is not a subset of B and B is not a superset of A.
11. **Disjoint Sets** Two sets A and B are said to be disjoint sets, if no element of A is in B and no element of B is in A. *i.e.*, $A \cap B = \phi$.
12. **Proper Subset** If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.
13. **Universal Set** A set that contains all sets in a given context is called the universal set and it is denoted by U.
   e.g.,
   (i) If $A = \{1, 2, 3\}, B = \{2, 3, 4, 5, 7\}$ and $C = \{2, 4, 6, 8\}$, then U is the universal set and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.
   (ii) When we study two dimensional coordinate geometry, then the set of all points in the xy-plane is the universal set.
14. **Power Set** Let A be a set, then the collection of all the possible subsets of A is called the power set of A and it is denoted by P (A). *i.e.*, $P(A) = \{S \mid S \subset A\}$
   - The power set of a given set is always non-empty as it contain the set itself and the empty set.

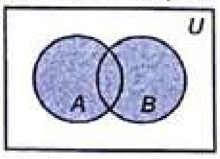
Venn Diagram

'Venn diagram' is the geometrical representation of various types of sets and operations of sets.

The universal set is denoted by a rectangular region and other subsets of the universal set are denoted by closed bounded figures inscribed in it. Each inscribed circle represents a set (subset of universal set).

Operations on Sets

1. **Union of Sets** Let A and B be any two sets, then union of A and B is the set of all those elements which belongs either to A or to B to both A and B. The notation for union of A and B is $A \cup B$ (read as "A union B").



Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

So, $x \in A \cup B \Leftrightarrow x \in A \text{ or } x \in B$

and $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$

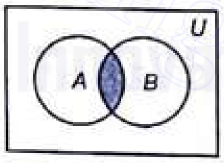
So, we can see that $A \subseteq A \cup B, B \subseteq A \cup B$.

e.g., if $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 5, 7\}$, then

$A \cup B = \{1, 2, 3, 4, 5, 7\}$

If $A_1, A_2, A_3, \ldots, A_n$ is a finite family of sets, then their union is denoted by $\bigcup_{i=1}^{n} A_i$ or $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$

2. **Intersection of Sets** Let A and B be any two sets. The intersection of A and B is the set of all those elements that belongs to both A and B. The intersection of A and B is denoted by $A \cap B$.



(read as "A intersection B").

Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

The shaded region represents $A \cap B$ in the Venn diagram. Here, we can see that $A \cap B \subseteq A, A \cap B \subseteq B$.

e.g., if $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 7, 9\}$, then $A \cap B = \{1, 3\}$.

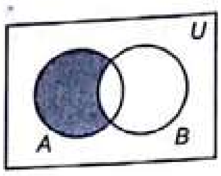
If $A_1, A_2, A_3, \ldots, A_n$ is a finite family of sets, then their intersection is represented as

$$\bigcap_{i=1}^{n} A_i \text{ or } A_1 \cap A_2 \cap \ldots \cap A_n$$

3. **Disjoint Sets** Two sets A and B are said to be disjoint, if $A \cap B = \phi$. If $A \cap B \neq \phi$, then A and B are said to be intersecting or overlapping sets.

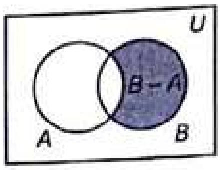
e.g., If $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$ and $C = \{7, 8, 9\}$, then here A and B are intersecting sets while A and C are disjoint sets.

4. **Difference of Sets** Let A and B be two sets. Then, the difference of A and B written as $A-B$, is the set of all those elements of A which do not belongs to B. Thus,



$$A-B=\{x: x \in A \text{ and } x \notin B\}$$
$$A-B=\{x \in A: x \notin B\}$$

The shaded part in the diagram represents $A-B$.



Similarly, the difference $B-A$ is the set of all those elements of B that do not belong to A. *i.e.*,

$$B-A=\{x: x \in B, x \notin A\}$$

The shaded part in the Venn diagram represents $B-A$.

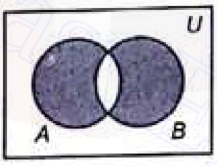
e.g., (i) If $A=\{2,4,6,8,10\}$ and $B=\{2,4,6,12,14\}$, then $A-B=\{8,10\}$ and $B-A=\{12,14\}$.

(ii) If $A=\{1,2,3\}$ and $B=\{1,2,3,4\}$, then here $A-B=\phi$.

	$A \subseteq B \Rightarrow A-B=\phi$ and
Also,	$A-B=B-A \Leftrightarrow A=B$

Symmetric Difference of Two Sets

Let A and B be two sets. The symmetric difference of sets A and B is the set $(A-B) \cup (B-A)$ and is denoted by $A \Delta B$. *i.e.*, it is a set consisting of all those members of A which are not in B or those which are in B but not in A.



Thus, $A \Delta B=\{x: x \in A \text{ but } x \notin B\} \cup \{x: x \in B \text{ but } x \notin A\}$

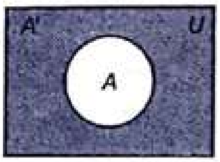
$A \Delta B=(A-B) \cup (B-A)$

The shaded part in figure represents $A \Delta B$.

e.g., if $A=\{1,2,3,4,5,6,7,8\}$ and $B=\{1,3,5,6,7,8,9\}$, then $A-B=\{2,4\}$, $B-A=\{9\}$, then $A \Delta B=\{2,4,9\}$.

Complement of a Set

Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^C or $U-A$ and is defined as the set of all those elements of U which are not in A.



Thus, $A'=\{x \in U: x \notin A\}$

So, $x \in A' \Leftrightarrow x \in A$

The shaded portion in the figure shows the complement of set A. *e.g.*,

(i) Let set of natural numbers be the universal set and let $A=\{1,3,5,7,\ldots\}$. Then, the complement of A is
$$A'=\{2,4,6,8,\ldots\}$$

(ii) If $U=\{a,b,c,d,e,f\}$ and $A=\{a,e\}$, then $A^c=\{b,c,d,f\}$.

(iii) If U is the set of all letters in English alphabet and A is the set of all vowels, then A^c is the set of all consonants.

Some Results to be Remember

On Subsets

- Every set is subset of itself.
- The empty set is subset of every set.
- The total number of subsets of a finite set containing n elements is 2^n.
- The number of all proper subsets of a set containing n elements is (2^n-1).
- the total number of non-empty proper subsets of a set having n elements is (2^n-2).
- The set of all subsets of a given set A is called as the power set of A, denoted by $P(A)$.

On Complementation

- $U'=\{x \in U: x \notin U\}=\phi$
- $\phi'=\{x \in U: x \notin \phi\}=U;$
- $(A')'=\{x \in U: x \notin A'\}=\{x \in U; x \in A\}=A$
- $A \cup A'=\{x \in U: x \in A\} \cup \{x \in U: x \notin A\}=U;$
- $A \cap A'=\{x \in U: x \in A\} \cap \{x \in U: x \notin A\}=\phi$

Properties of Algebra of Sets

1. **Idempotent Laws** For any set A, we have
(i) $A \cup A=A$ (ii) $A \cap A=A$

2. **Identity Laws** For any set A,
(i) $A \cup \phi=A$ (ii) $A \cap U=A$
i.e., ϕ and U are identity elements for union and intersection respectively.

3. **Commutative Laws** For any two sets A and B, we have
(i) $A \cup B=B \cup A$ (ii) $A \cap B=B \cap A$

4. **Associative Laws** If A, B and C are any three sets, then
(i) $(A \cup B) \cup C=A \cup (B \cup C)$
(ii) $A \cap (B \cap C)=(A \cap B) \cap C$
i.e., union and intersection are associative.

5. **Distributive Laws** If A, B and C are any three sets, then
(i) $A \cup (B \cap C)=(A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C)=(A \cap B) \cup (A \cap C)$
i.e., union and intersection are distributive over intersection and union respectively.

6. **De-Morgan's Law** If A and B are any two sets, then
(i) $(A \cup B)'=A' \cap B'$ (ii) $(A \cap B)'=A' \cup B'$

More Results on Operations on Sets

Theorem 1 If A and B are any two sets, then
(i) $A-B=A \cap B'$
(ii) $B-A=B \cap A'$

(iii) $A-B=A \Leftrightarrow A\cap B=\phi$
(iv) $(A-B)\cup B=A\cup B$
(v) $(A-B)\cap B=\phi$
(vi) $A\subseteq B \Leftrightarrow B'\subseteq A'$
(vii) $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$

Theorem 2 If A, B and C are any three sets, then

(i) $A-(B\cap C)=(A-B)\cup(A-C)$
(ii) $A-(B\cup C)=(A-B)\cap(A-C)$
(iii) $A\cap(B-C)=(A\cap B)-(A\cap C)$
(iv) $A\cap(B\,\Delta\, C)=(A\cap B)\,\Delta\,(A\cap C)$

Theorem 3 If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
(ii) $n(A\cup B)=n(A)+n(B) \Leftrightarrow$ A, B are disjoint sets.
(iii) $n(A-B)=n(A)-n(A\cap B)$
i.e., $n(A-B)+n(A\cap B)=n(A)$
(iv) Number of elements which belong to exactly one of A or B, then

$$n(A\,\Delta\, B)=n(A)+n(B)-2n(A\cap B)$$

(v) $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)$
$-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$
(vi) Number of elements in exactly two of the sets A, B and C
$=n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$
(vii) Number of elements in exactly one of the sets A, B and C
$=n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)$
$-2n(A\cap C)+3n(A\cap B\cap C)$
(viii) $n(A'\cup B')=n((A\cap B)')=n(U)-n(A\cap B)$
(ix) $n(A'\cap B')=n((A\cup B)')=n(U)-n(A\cup B)$

Ordered Pairs

If A is a set and $a,b\in A$, then the ordered pair of elements a and b in A, is denoted by (a,b).

The natural numbers and their squares can be represented by ordered pair in the following way

$$(1, 1), (2, 4), (3, 9), (4, 16), \ldots$$

Two ordered pairs (a,b) and (c,d) will be equal, if and only if $a=c$ and $b=d$.

Cartesian Product of Two Sets

If A and B be two sets and both are non-empty sets, then the cartesian product $A\times B$ is defined as

$$A\times B=\{(a,b): a\in A \text{ and } b\in B\}$$

- If $A=\phi$ or $B=\phi$, then $A\times B=\phi$
- In general $A\times B\neq B\times A$
- $n(A\times B)=n(A)\times n(B)$, where $n(A)$ and $n(B)$ denotes the number of elements of A and B, respectively.
- If A or B any one is infinite set, then $A\times B$ is an infinite set.

e.g., (i) If $A=\{0,3\}, B=\{2,3,4\}$, then
$A\times B=\{(0,2), (0,3), (0,4), (3,2), (3,3), (3,4)\}$
$B\times A=\{(2,0)\ (2,3), (3,0), (3,3), (4,0)\ (4,3)\}$

(ii) If $A=\{1,2,3\}, B=\{3,4\}$ and $C=\{4,5,6\}$, then
$A\times(B\cap C)$. Here, $B\cap C=\{4\}$
$A\times(B\cap C)=\{(1,4),(2,4),(3,4)\}$ while
$(A\cap B)\times C=\{(3,4),(3,5),(3,6)\}$

Partition of a Set

Let A be a non-empty set and let $A_1, A_2, A_3, \ldots$ be non-empty subsets of A, then the set P which is the set of subsets of A such that

$P=\{A_1, A_2, \ldots\}$ is called the partition of a set A.

1. $A_1\cup A_2\cup A_3\cup\ldots=A$
2. If A_1 and A_2 are subsets of A, then
$A_1\cap A_2=\phi,\ \forall A_1, A_2\in P$

Exercise

1. If $A=\{5,6,7\}$ and $B=\{7,8,9\}$, then $A\cup B$ is equal to
(a) $\{5, 6, 7, 8, 9\}$ (b) $\{7, 8, 9\}$
(c) $\{5, 6, 7\}$ (d) ϕ

2. Given that $A=\{2,6,8,9\}$, $B=\{7,8,9,12\}$, then $B-A$ is equal to
(a) $\{7, 8, 9, 12\}$ (b) $\{7, 12\}$
(c) $\{2, 6, 7, 8, 9, 12\}$ (d) $\{2, 6, 8, 9, 12\}$

3. If U is the universal set and $A=\{1,2,3,4,5\}$, then compute $A\cap U$.
(a) $\{1,2,3,4\}$ (b) ϕ
(c) $\{1,2,3,4,5\}$ (d) U

4. Which one of the following is correct? **(CDS 2007 I)**
(a) $\{\phi\}\subset\{\{\phi\},\{\{\phi\}\}\}$
(b) $\{\phi\}\in\{\{\phi\},\{\{\phi\}\}\}$
(c) $\phi\in\{\{\phi\},\{\{\phi\}\}\}$
(d) $\phi=\{\{\phi\},\{\{\phi\}\}\}$

5. The set $\{2,4,16,256,\ldots\}$ can be represented as which one of the following?
(a) $\{x\in N \mid x=2^{2^n}, n\in N\}$
(b) $\{x\in N \mid x=2^{2^n}, n=0,1,2\ldots\}$
(c) $\{x\in N \mid x=2^{4^n}, n=0,1,2\ldots\}$
(d) $\{x\in N \mid x=2^{2n}, n=0,1,2\ldots\}$

6. Match each of the sets on the left described in the Roster form with the same set on the right described in set builder form

1.	{1, 2, 3, 6}	(A)	{$x : x$ is a prime and a divisor of 6}
2.	{2, 3}	(B)	{$x : x$ is an odd natural number less than 10}
3.	{H, A, Y, R, N}	(C)	{$x : x$ is a natural number and divisor of 6}
4.	{1, 3, 5, 7, 9}	(D)	{$x : x$ is a letter of the word 'HARYANA'}

	A	B	C	D		A	B	C	D
(a)	2	4	1	3	(b)	4	3	2	1
(c)	1	2	3	4	(d)	3	2	1	4

7. If P and Q are any two sets and $P \subset Q$, then
(a) $P \cap Q = \phi$ (b) $P' \cap Q = P$
(c) $P \cap Q = P$ (d) $P \cap Q = Q$

8. Which one of the following is a true statement?
(a) $(A \cup B)' = A' \cup B'$ (b) $(A \cap B)' = A \cap B$
(c) $(A \cap B)' = A' \cup B'$ (d) $(A \cap B)' = A' \cap B'$

9. Which one of the following is a true statement?
(a) $(A - B) \cap (B - A) = \phi$ (b) $(A - B) \cap (B - A) = A$
(c) $(A - B) \cap (B - A) = U$ (d) $(A - B) \cap (B - A) = B$

10. If P and Q are any two sets, then $P \cup Q = P \cap Q$, if
(a) P is the empty set
(b) Q is the empty set
(c) Both P and Q are empty sets
(d) P and Q are non-empty sets

11. If $n(P) = 4$ and $n(Q) = 3$, numbers of elements in $P \times Q$ is
(a) 12 (b) 7 (c) 1 (d) 81

12. Let two sets A and B have $2n$ and $4n$ elements respectively, where n is a natural number. What can be the minimum number of elements in $A \cup B$? **(CDS 2007 II)**
(a) $2n$ (b) $3n$ (c) $4n$ (d) $6n$

13. If A and B are two sets, then $A \cap (A \cup B)$ equals to
(a) A (b) B
(c) ϕ (d) None of these

14. The smallest set B such that $B \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(a) $\{3, 5, 9\}$ (b) $\{3, 5, 8\}$
(c) $\{1, 2, 3\}$ (d) None of these

15. State which of the sets given below are infinite set?
I. Set of all concentric circles.
II. $\{x : x$ is a multiple of $2, x$ is an integer.$\}$
III. The set of lines which are parallel to x-axis.
IV. The set of positive integers greater than 100.
(a) I and II (b) II and III
(c) I only (d) All of the sets

16. Given that the set $A = \{0, 1, 2, 3\}$, which of the following statements about A are true?
I. A is a finite set.
II. A is a subset of the set of integers.
III. $\{1, 2\}$ is a proper subset of A.
IV. A is the null set.
(a) I, II, III are true (b) I and IV are true
(c) I and III are true (d) All are true

17. Let $A = \left\{3, \pi\ \sqrt{2}, \frac{2}{7}, -5, 3 + \sqrt{7}\right\}$. The subset of A containing all the elements from it which are irrational numbers is
(a) $\left\{3, \frac{2}{7}, -5\right\}$ (b) $\left\{3, \pi, \frac{2}{7}, -5, 3 + \sqrt{7}\right\}$
(c) $\{\pi, \sqrt{2}, 3 + \sqrt{7}\}$ (d) $\{3, -5\}$

18. The set $\{x : (x - 2)(x - 3) > 0\}$ is equal to
(a) $\{x : x > 3\} \cup \{x : x < 2\}$
(b) $\{x : x < 3\} \cup \{x : x < 2\}$
(c) $\{x : 2 < x < 3\}$
(d) None of these

19. The set $S = \{x \in N. x + 3 = 3\}$ is a **(CDS 2011 I)**
(a) null set (b) singleton set
(c) infinite set (d) None of these

20. If $P = \{x : x^2 - 3x + 2 = 0\}, Q = \{x : x^2 + 4x - 12 = 0\}$, then $P - Q$ is
(a) $\{1, 2\}$ (b) $\{2\}$ (c) $\{1\}$ (d) $\{4, 3\}$

21. If $U = \{x : x \in N\}, A = \{x : x$ is an odd number$\}$, $B = \{x : x$ is an even number$\}$, then A' is equal to
(a) $\{x : x$ is an even number$\}$
(b) $\{x : x$ is an odd number$\}$
(c) $\{x : x$ is a natural number$\}$
(d) $\{x : x$ is an integer$\}$

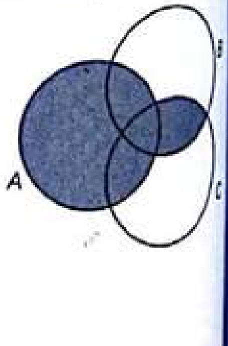
22. If two sets are disjoint, then their intersection is
(a) null set (b) singleton
(c) a infinite set (d) None of these

23. If $|4x + 3| > 7$ for $x \in R$, then the solution set is given by
(a) $\left\{x \in R : 1 < x < -\frac{5}{2}\right\}$
(b) $\left\{x \in R : -1 < x < \frac{5}{2}\right\}$
(c) $\{x \in R : x > 1\} \cup \left\{x \in R : x < \frac{-5}{2}\right\}$
(d) $\{x \in R : x < 1\} \cup \left\{x \in R : x > \frac{-5}{2}\right\}$

24. If A, B and C are any three non-empty sets, then $A - (B \cup C)$ equals to
(a) $(A - B) \cup (A - C)$ (b) $(A - C) \cup B$
(c) $(A - B) \cup C$ (d) $(A - B) \cap (A - C)$

25. Which one of the following is an infinite set? **(CDS 2010 I)**
(a) $\{x : x$ is a whole number less than or equal to 1000$\}$
(b) $\{x : x$ is a natural number less than 1000$\}$
(c) $\{x : x$ is a positive integer less than or equal to 1000$\}$
(d) $\{x : x$ is an integer and less than 1000$\}$

26. The shaded portion in the following Venn diagram represents



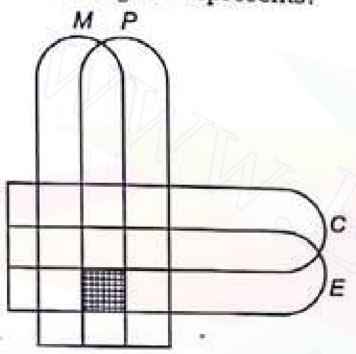
(a) $A \cup (B \cap C)$
(b) $A \cup (B \cup C)$
(c) $(A \cap B) \cap C$
(d) $(A \cap B) \cup C$

27. If $X = \{1, 2, 3\}$ and $Y = \{3, 4, 5\}$, then $X \times Y$ is equal to
(a) $(\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\} \{2, 5\}, \{3, 3\}, \{3, 4\}, \{3, 5\})$
(b) $(\{1, 3\}, \{3, 4\}, \{3, 5\})$
(c) $(\{1, 3\}, \{1, 4\}, \{1, 5\})$
(d) $(\{1, 5\} \{2, 4\}, \{3, 3\}, \{3, 6\})$

28. If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $P = \{0, 1, 2, 3\}$, $Q = \{2, 3, 4, 5\}$, $R = \{4, 5, 6\}$, then $Q' \cap (P \cup R)$ is equal to
(a) $\{0, 1, 6\}$ (b) $\{0, 1, 2, 3, 4, 5\}$
(c) $\{6, 7, 8, 9\}$ (d) $\{2, 3\}$

29. If P is a non-empty set, then $(P')'$ is equal to
(a) ϕ (b) U (c) $U - P$ (d) P

30. The Venn diagram given above represents four sets of students who have opted for Mathematics (M), Physics (P), Chemistry (C) and Electronics (E). What does the shaded region represents? **(CDS 2007 II)**



(a) Students who opted for Physics, Chemistry and Electronics
(b) Students who opted for Mathematics, Physics and Chemistry
(c) Students who opted for Mathematics, Physics and Electronics
(d) Students who opted for Mathematics, Chemistry and Electronics

31. Which of the following statements are true. If X and Y are two subsets of $\cup$, then
I. $(X \cup Y)' = X \cup Y'$ II. $(X \cap Y)' = X' \cap Y'$
III. $(X \cup Y)' = X' \cap Y'$ IV. $(X \cap Y)' = X' \cup Y'$
(a) III and IV are true (b) II, III and IV are true
(c) I, II and III are true (d) All are true

32. Which of the following sets are equivalent?
I. $A = \{1, 2, 3, 4, 5\}$, $B = \{7, 8, 9, 10, 11\}$
II. $A = \{x, y, z\}$ $B = \{p, q\}$
III. $P = \{2, 4, 6, 8\}$, $R = \{a, b, c, d\}$
IV. $A = \{36, 39, 42, 45\}$, $B = \{42, 39, 45, 36\}$
(a) I, II, IV are equivalent sets
(b) II, III, IV are equivalent sets
(c) I, III and IV are equivalent sets
(d) None of the above

33. Which of the following are examples of empty set?
I. $A = \{x : x + 3 = 3, x \in I\}$
II. $B = \{x : x$ is a positive even integer and prime$\}$
III. $C = \{x : x^2 = 16, x$ is odd integer$\}$
IV. $D = \{x : x$ is a multiple of 4, less than 8$\}$
V. $E = \{x : x + 7 = 4$ and $x \in N\}$
(a) I, II, V only (b) III, IV and V only
(c) I, III and V only (d) III and V only

34. State which of the following statements about sets are true?
I. Every subset of a finite set is finite.
II. ϕ is a subset of $\{0\}$.
III. The number of subsets of $\{0\}$ is 2.
IV. $\{0\}$ is subset of every set.
V. Every subset of an infinite set in finite.
(a) IV and V are true (b) II, IV are true
(c) I, II, III are true (d) Only IV is true

35. Given that, $R = \{$All right triangles$\}$
$E = \{$All equilateral triangles$\}$, then $n(E \cap R)$ is
(a) 3 (b) 2 (c) 1 (d) 0

36. ϕ is the same as
(a) 0 (b) $\{0\}$ (c) $\{\phi\}$ (d) $\{\}$

37. If $A = \{(2^{2n} - 3n - 1) \mid n \in N\}$ and $B = \{9(n-1) \mid n \in N$, then which one of the following is correct? **(CDS 2007 I)**
(a) $A \subset B$
(b) $A \subset A$
(c) $A = B$
(d) Neither A is a subset of B nor B is a subset of A

38. If A, B and C are any three sets, then
(a) $A - (B \cup C) = (A - B) \cup (A - C)$
(b) $A - (B \cup C) = (A - B) \cup (A - C)$
(c) $A - (B \cup C) = (A - B) \cap (A - C)$
(d) $A - (B \cup C) = (A \cup B) - (A \cup C)$

39. Consider the following statements,
For any two set P and Q
I. $(P - Q) \cup Q = P$ II. $(P - Q) \cup P = P$
III. $(P - Q) \cap Q = \phi$ IV. $P \subseteq Q \Leftrightarrow P \cup Q = Q$
Of these statements
(a) I, II, III are correct (b) II, III, IV are correct
(c) I, II, IV are correct (d) All are correct

40. Consider the following statements
I. $A' \cup B = (A \cap B)'$ II. $(\phi')' = X$
III. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
IV. $\phi' = X$
Of these statements
(a) II and III are correct (b) I and III are correct
(c) III and IV are correct (d) II and IV are correct

41. Consider the following statements
If A' denotes the complement of the set A and B' denotes the complement of the set B, then
I. $(A \cap B)' = A' \cap B'$ II. $(A \cup B)' = A' \cup B'$
III. $(A - B)' = A' - B'$
Of these statements
(a) All are true (b) I is true
(c) II is true (d) None is true

42. If P and Q are two sets such that $n(P) = m$, $n(Q) = n$ and $n(P \cap Q) = p$, then $n(A \cup B)$ is equal to
(a) $m + n$ (b) $m + n + p$ (c) $m + n - p$ (d) $m - n - p$

43. If $A = \{2, 3, 5\}$, $B = \{4, 5, 6\}$, then $(A \cap B) \times A$ is
(a) $\{(2, 5), (3, 5)\}$ (b) $\{(5, 2), (5, 3)\}$
(c) $\{(5, 2), (5, 3), (5, 5)\}$ (d) $\{(5, 2), (2, 5), (3, 5)\}$

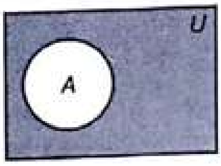
44. If $P = \{-1, 0, 1\}$, $Q = \{-1, 0\}$ and $R = \{0, 1\}$, then
I. $(P \times Q) \cap (P \times R) = \{(-1, 0) (0, 0) (1, 0)\}$

II. $(P \times Q) \cup (P \times R) = \{(-1, -1), (-1, 0), (-1, 1), (0, -1)$
$(0, 0), (0, 1)\ (1, -1), (1, 0)\ (1, 1)\}$

III. $(P \cap R) \times Q = \{(0, -1), (0, 0), (1, -1)\}$

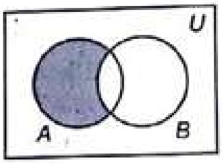
(a) I and III are correct (b) III alone is correct
(c) I and II are correct (d) II and III are correct

45. The shaded region in the adjoining diagram is



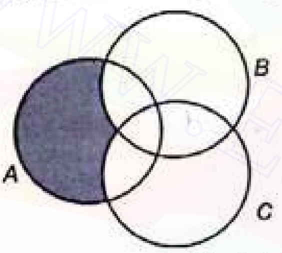
(a) $A \cup A'$
(b) U
(c) A'
(d) $A \cap A'$

46. The shaded region in the adjoining diagram represents



(a) $(A \cap B)'$
(b) $A \cup B$
(c) $A - B$
(d) $A \cap B$

47. The shaded region in the adjoining diagram is



(a) $A \cap (B - C)$ (b) $A - (B \cup C)$
(c) $A \cap (B \cup C)$ (d) $A \cup (B \cap C)$

48. A and B are two sets such that $n(A) = 17$, $n(B) = 23$, $n(A \cup B) = 38$. Then, $n(A \cap B)$ is
(a) 40 (b) 78
(c) 2 (d) None of these

49. In a group of 70 people, 45 speak Hindi language and 33 speak English language and 10 speak neither Hindi nor English. How many can speak both English is well as Hindi language?
(a) 20 (b) 18 (c) 19 (d) 17

50. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English. How many can speak Hindi only?
(a) 600 (b) 150 (c) 300 (d) 500

51. If A and B are subsets of the universal set U, then $(A - B) \cap (B - A)$ is equal to
(a) $A \cup B$ (b) B (c) A (d) ϕ

52. Let $A = \{x : x^2 - 6x + 8 = 0\}$ and $B = \{x : 2x^2 + 3x - 2 = 0\}$ Then, which one of the following is correct? **(CDS 2007 II)**
(a) $A \subseteq B$
(b) $B \subseteq A$
(c) Neither $A \subseteq B$ nor $B \subseteq A$
(d) $A = B$

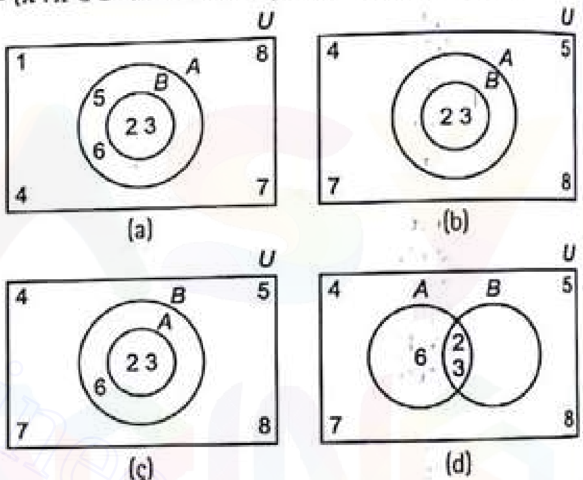
53. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak atleast one of these two languages?
(a) 60 (b) 50 (c) 30 (d) 70

54. If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5\}$, $C = \{1, 3, 4, 5, 6, 7\}$, then $A \cap (B \cup C)$ is equal to
(a) $\{1, 2, 3, 4, 5, 6, 7\}$ (b) $\{1, 2, 3, 4\}$
(c) $\{1, 2, 3, 4, 5\}$ (d) ϕ

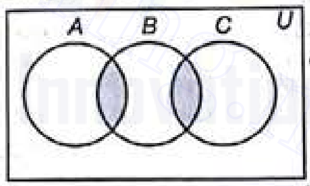
55. Let the set A and B be given by $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8, 10\}$ and the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $(A \cup B)'$ is
(a) $\{2, 4\}$ (b) U
(c) $\{1, 3, 5, 6, 7, 8, 9, 10\}$ (d) $\{5, 7, 9\}$

56. What is $\{[(A \cup B)' \cap A]\} - (A - B)$ equal to? **(CDS 2007 I)**
(a) ϕ (b) A (c) B (d) B'

57. The Venn diagram for the following sets is
$U = \{x : x$ is a natural number and $2 \le x \le 8\}$
$A = \{x : x \in U$ and x divides $18\}$
$B = \{x : x \in U$ and x is a prime divisor of $18\}$



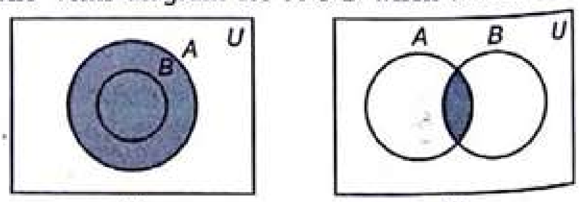
58. In the Venn diagram below, shaded portion represents



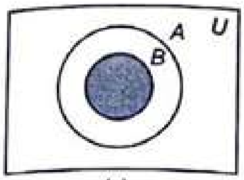
(a) $A \cup B \cup C$ (b) $A \cap B \cap C$
(c) $(A \cap B) \cup (B \cap C)$ (d) $A \cup B \cap C$

59. Which one of the following is not correct is respect of the sets A and B? **(CDS 2009 II)**
(a) If $A \subseteq B$, then $B \cup A = B$
(b) If $A \subseteq B$, then $A \cap (A - B) = \phi$
(c) If $A \subseteq B$, then $B \cap A = A$
(d) If $A \cap B = \phi$, then either $A = \phi$ or $B = \phi$

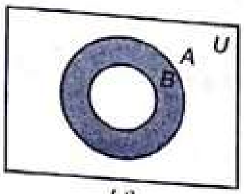
60. The Venn diagram for $A \cup B$ when $B \subset A$ is



(a) (b)



(c)



(d)

61. Which one of the following is a correct statement? **(CDS 2008 II)**

(a) $\phi \in \phi$ (b) $\phi \notin P(\phi)$
(c) $\phi = P(\phi)$ (d) $\phi \in P(\phi)$

62. Let $A = \{2, 3, 4\}$, $X = \{0, 1, 2, 3, 4\}$, which of the following statements are correct?

I. $\phi \in A'$ w.r.t. X II. $0 \subset A'$ w.r.t. X
III. $0 \in A'$ w.r.t. X IV. $\{0\} \in A'$ w.r.t. X

(a) I, II and III (b) III and IV only
(c) II, III and IV (d) All of these

63. If A is set of all regular polygons and B is a set of all quadrilateral, then $A \cap B$ is a set of all

(a) squares (b) rectangles
(c) rhombus (d) parallelograms

64. If $P \subset Q$, then which of the following is true?

(a) $P \cup Q = P$ (b) $P \cup Q = \phi$ (c) $P \cup Q = Q$ (d) $P \cap Q = Q$

65. Every student in a class of 42 students, studies atleast one of the subjects, Mathematics, English and Commerce, 14 students study Mathematics, 20 Commerce and 24 English. 3 students study Mathematics and Commerce, 2 English and Commerce and there is no student who studies all the three subjects. The number of students who study Mathematics but not Commerce is

(a) 4 (b) 3 (c) 12 (d) 11

66. In a class 22 students offered Mathematics, 18 students offered Chemistry and 24 students offered Physics. All of them have to offer atleast one of the three subjects of those, 11 are in both Mathematics and Chemistry, 13 in Chemistry and Physics and 7 have offered all the three subjects. The number of students in the class is

(a) 33 (b) 35
(c) 36 (d) 39

67. The function $g : N \to N : g(x) = 2x$, where N is the set of natural numbers is

(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto

68. Which one of the following is a correct statement? **(CDS 2010 I)**

(a) $\{a\} \in \{\{a\}, \{b\}, c\}$ (b) $\{a\} \subseteq \{\{a\}, b, c\}$
(c) $\{a, b\} \subseteq \{\{a\}, b, c\}$ (d) $a \subseteq \{\{a\}, b, c\}$

69. The range of the function $f(x) = a\cos(bx + c) + d, a > 0$ is

(a) $[-a, a]$ (b) $[-a - d, a - d]$
(c) $[-d - a, d + a]$ (d) $[d - a, d + a]$

70. If Y is the set of all integers and f is defined on Y by $f(x) = x^2$, then the image of the set $\{-2, -1, 0, 1, 2\}$ is

(a) $\{-2, -1, 0, 1, 2\}$ (b) $\{-2, 1, 0\}$
(c) $\{0, 1, 4\}$ (d) $\{0, 1, 2\}$

71. If $A = \{1, 2, 3, 4\}$, what is the number of subsets of A with atleast three elements? **(CDS 2008 I)**

(a) 3 (b) 4 (c) 5 (d) 10

72. Consider the following statements

I. Set of points of a given line is a finite set.
II. Intelligent students in a class is a set.
III. Good books in a school library is a set.

Which of the above statements is/are not correct? **(CDS 2007 II)**

(a) I only (b) II and III only
(c) I and II only (d) I, II and III

73. If $g(x) = x^2$ and $h(x) = x^3$, x being real, then

(a) g is one-one but h is not one-one
(b) g is not one-one and h is one-one
(c) Neither g nor h is one-one
(d) Both g and h are one-one

74. Let P = Set of all integral multiples of 3
Q = Set of all integral multiples of 4
R = Set of all integral multiples of 6

Consider the following relations

I. $P \cup Q = R$ II. $P \subset R$
III. $R \subset (P \cup Q)$

Which of the relations given above is/are correct? **(CDS 2007 II)**

(a) Only I (b) Only II (c) Only III (d) II and III

75. In an examination, 52% candidates failed in English and 42% failed in Mathematics. If 17% candidates failed in both English and Mathematics, what percentage of candidates passed in both the subjects? **(CDS 2011 I)**

(a) 18% (b) 21% (c) 23% (d) 25%

Answers

1. (a)	2. (b)	3. (c)	4. (b)	5. (b)	6. (a)	7. (c)	8. (c)	9. (a)	10. (c)
11. (a)	12. (c)	13. (a)	14. (a)	15. (d)	16. (a)	17. (c)	18. (a)	19. (a)	20. (c)
21. (a)	22. (a)	23. (c)	24. (d)	25. (d)	26. (a)	27. (a)	28. (a)	29. (d)	30. (c)
31. (a)	32. (c)	33. (d)	34. (c)	35. (d)	36. (d)	37. (a)	38. (c)	39. (b)	40. (c)
41. (d)	42. (c)	43. (c)	44. (c)	45. (c)	46. (c)	47. (b)	48. (c)	49. (b)	50. (a)
51. (d)	52. (c)	53. (a)	54. (b)	55. (d)	56. (a)	57. (b)	58. (c)	59. (d)	60. (a)
61. (d)	62. (b)	63. (a)	64. (a)	65. (b)	66. (a)	67. (b)	68. (a)	69. (d)	70. (c)
71. (c)	72. (d)	73. (b)	74. (c)	75. (c)					

Hints and Solutions

1. $A \cup B = \{5, 6, 7, 8, 9\}$

2. $B - A = \{7, 8, 9, 12\} - \{2, 6, 8, 9\} = \{7, 12\}$

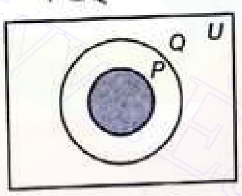
3. Here, $U = \{1, 2, 3, 4, 5, \ldots\}$ and $A = \{1, 2, 3, 4, 5\}$
$A \cap U = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, \ldots\} = \{1, 2, 3, 4, 5\}$

4. Here, $\{\phi\} \in \{\{\phi\}, \{\{\phi\}\}\}$ is correct.

5. Let $A = \{2, 4, 16, 256, \ldots\}$
This set can be rewritten as $\{x \in N \mid x = 2^{2^n}, n = 0, 1, 2, \ldots\}$

6. Here, $\{1, 2, 3, 6\} = \{x : x$ is a natural number and divisor of 6$\}$
$\{2, 3\} = \{x : x$ is a prime and a divisor of 6$\}$
$\{H, A, Y, R, N\} = \{x : x$ is a letter of the word 'HARYANA'$\}$
$\{1, 3, 5, 7, 9\} = \{x : x$ is an odd natural number less than 10$\}$

7. $\because$ $P \subset Q$



So, $P \cap Q = P$

8. $(A \cap B)' = A' \cup B'$ (by De-Morgan's law)

9. Let $x \in A - B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \notin B - A$
Also, $x \in B - A \Rightarrow x \in B$ and $x \notin A$
$\Rightarrow$ $x \notin A - B$
So, $(A - B) \cap (B - A) = \phi$

10. Only if P and Q both are empty set or P and Q have same elements.

11. $n(P \times Q) = n(P) \times n(Q) = 4 \times 3 = 12$

12. $\because n(A \cap B) = 2n$
$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 2n + 4n - 2n = 4n$
Hence, minimum number of elements of $A \cup B$ is $4n$.

13. Clearly, $A \cap (A \cup B) = A$

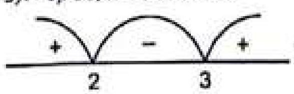
14. $B \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$
$\Rightarrow$ $B = \{3, 5, 9\}$ is the smallest set B.

15. All sets are infinite set.

16. I. A is a finite set.
II. As all elements of A are integers, so A is subest of integers.
III. $\{1, 2\}$ is a proper subset of A.
IV. $A \neq \phi$. So, I, II and III are correct.

17. As $\pi, \sqrt{2}, 3 + \sqrt{7}$ are irrational numbers.
So, $\{\pi, \sqrt{2}, 3 + \sqrt{7}\}$ is a set.

18. $\{x : (x - 2)(x - 3) > 0\}$. So, $x > 3$ or $x < 2$



$\therefore$ $\{x : x > 3\} \cup \{x : x < 2\}$ [see quadratic in equation]

19. Given, $S = x \in N : \{x + 3 = 3\}$
$S = \{\}$ $(\because 0 \notin N)$
So, S is a null set.

20. Here, $P = \{x : (x - 1)(x - 2) = 0\} \Rightarrow P = \{x : x = 1, x = 2\}$
So, $P = \{1, 2\}$ and
$Q = \{x : x^2 + 4x - 12 = 0\}$
$Q = \{x : (x + 6)(x - 2) = 0\}$
So, $Q = \{-6, 2\}$
So, $P - Q = \{1, 2\} - \{-6, 2\} = \{1\}$

21. Here, $U = \{1, 2, 3, 4, 5, \ldots\}$
and $A = \{1, 3, 5, 7, 9, 11, \ldots\}$
So, $A' = U - A = \{2, 4, 6, 8, 10, \ldots\}$
So, $A = \{x : x$ is an even integer$\}$

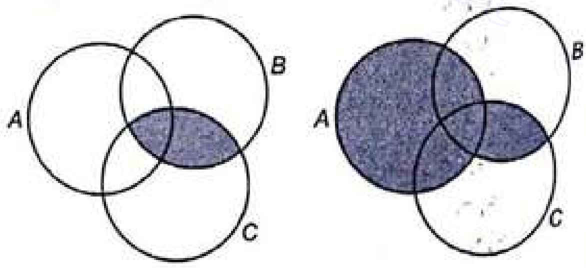
22. Disjoint sets have no common element.
e.g., $A = \{1, 2, 3\}, B = \{4, 5, 6\}$
$A \cap B = \phi$

23. $|4x + 3| > 7$
$\therefore$ $4x + 3 > 7$ or $-(4x + 3) > 7$
or $4x > 4$ or $-4x > 10$
or $x > 1$ or $4x < -10$
or $x > 1$ or $x < \frac{-5}{2}$
Hence, solution set is
$\{x \in R : x > 1\} \cup \left\{x : x < \frac{-5}{2}\right\}$

24. Here, $A - (B \cup C) = (A - B) \cap (A - C)$
Hence, the answer is (d).

25. In a given option only
$\{x : x$ is an integer and less than 1000$\}$
i.e., $x \in (-\infty, 1000)$ is an infinite set.

26. Here, $(B \cap C)$ is $A \cup (B \cap C)$
Now,



27. $X \times Y = (\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{3, 3\}\ \{3, 4\}, \{3, 5\}, \{2, 4\}, \{2, 5\}$

28. $(P \cup R) = \{0, 1, 2, 3, 4, 5, 6\}$
$Q' = \{0, 1, 6, 7, 8, 9\}$
So, $Q' \cap (P \cup R) = \{0, 1, 6\}$

29. Clearly, $(P')' = P$

30. It is clear from the given Venn diagram that shaded portion represent the students who opted for Mathematics, Physics and Electronics.

31. Clearly, III and IV are De-Morgan's law.

32. Equivalent sets have same cardinal numbers. Here, cardinal numbers of I, III, IV sets is 4.

33. I. $A=\{0\}$
II. $B=\{2\}$
III. $C=\{\}$; 4 is not an odd integer
IV. $D=\{4\}$;
V. $E=\{\}$; not a possible case
Here, III and V are examples of empty set.

34. Clearly, I, II and III are true for IV : Let $A=\{1,2\}$, so $\{0\}$ is not a subset of A.

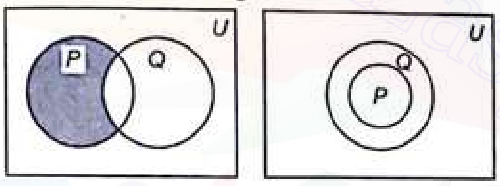
35. No equilateral triangle is a right triangle.
So, $n(E \cap R)=0$
as $(E \cap R)=\{\}$

36. ϕ is the same as $\{\}$.

37. Given, $A=\{(2^{2n}-3n-1)|n \in N\}=\{0,9,54,243,\ldots\}$
and $B=\{9(n-1)|n \in N\}=\{0,9,18,27,\ldots\}$
From the above, it is clear that $A \subset B$.

38. Clearly, by De-Morgan's law
$A-(B \cup C)=(A-B) \cap (A-C)$

39. With the help of Venn diagram, we have



I. $(P-Q) \cup Q \neq P$
II. $(P-Q) \cup P=P$
III. $(P-Q) \cap Q=\phi$
IV. $P \subseteq Q \Rightarrow P \cup Q=Q$
So, only, II, III and IV are correct.

40. By distributive law in sets
III. $A \cap (B \cup C)=(A \cap B) \cup (A \cap C)$
and IV. $\phi'=X$
So, only III and IV are correct.

41. By using De-Morgan's law
I. $(A \cap B)' \neq A' \cap B'$ II. $(A \cup B)' \neq A' \cup B'$ III. $(A-B)' \neq A'-B'$

42. $n(P \cup Q)=n(P)+n(Q)-n(P \cap Q)$
$n(P \cup Q)=m+n-p$

43. $A \cap B=\{5\}$ and $A=\{2,3,5\}$
$\therefore (A \cap B) \times A=\{(5,2)(5,3)(5,5)\}$ [ordered pair]

44. $P \times Q=\{(-1,-1),(-1,0),(0,-1),(0,0),(1,-1),(1,0)\}$
$P \times R=\{(-1,0),(-1,1),(0,0),(0,1),(1,0),(1,1)\}$
$\therefore (P \times Q) \cap (P \times R)=\{(-1,0),(0,0),(1,0)\}$
Here, I is correct.
Again, $(P \times Q) \cup (P \times R)=\{(-1,-1)(-1,0)(-1,1)$
$(0,-1)(0,0)(0,1)(1,-1)(1,0)(1,1)\}$
Here, II is correct.
But $P \cap R=\{0,1\}$ and $Q=\{-1,0\}$
$\therefore (P \cap R) \times Q=\{(0,-1),(0,0),(1,-1),(1,0)\}$
Here, III is not correct.

45. The shaded region is $U-A=A'$.

46. The shaded region represents the elements in A and not in B, so it represents $A-B$.

47. The shaded region represents the elements in A and not in B or C or $A-(B \cup C)$.

48. $n(A \cup B)=n(A)+n(B)-n(A \cap B)$
$n(A \cap B)=n(A)+n(B)-n(A \cup B)=17+23-38=2$

49. Here, $n(H)=45, n(E)=33$
$n(H \cup E)'=10, n(U)=70$
$\therefore n(H \cup E)=n(U)-n(H \cup E)'=70-10=60$
$n(H \cap E)=n(E)+n(H)-n(H \cup E)=45+33-60=18$
18 people can speak both English and Hindi.

50. $n(H \cup E)=1000, n(H)=750, n(E)=400$
$n(H \cup E)=n(H)+n(E)-n(H \cap E)$
$1000=750+400-n(H \cap E)$
$\Rightarrow n(H \cap E)=1150-1000=150$
Number of people who can speak Hindi only
$=n(H \cap E')=n(H)-n(H \cap E)=750-150=600$

51. Any $x \in A-B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \notin B-A$
Also, $x \in B-A \Rightarrow x \in B$ and $x \notin A \Rightarrow x \notin A-B$
$\Rightarrow (A-B) \cap (B-A)=\phi$

52. Given, $A \equiv x^2-6x+8=0 \Rightarrow (x-4)(x-2)=0 \Rightarrow x=4,2$
and $B=2x^2+3x-2=0$
$\Rightarrow (2x-1)(x+2)=0 \Rightarrow x=\frac{1}{2},-2$
Hence, neither $A \subseteq B$ nor $B \subseteq A$.

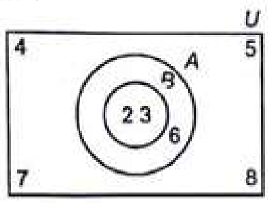
53. Let S be the set of people who speak Spanish and F be the set of people who speak French
$\therefore n(S)=20, n(F)=50, n(S \cap F)=10$
$\therefore n(S \cup F)=n(S)+n(F)-n(S \cap F)=20+50-10=60$

54. $(B \cup C)=\{1,2,3,4,5,6,7\}$
$\therefore A \cap (B \cup C)=\{1,2,3,4\} \cap \{1,2,3,4,5,6,7\}=\{1,2,3,4\}$

55. $(A \cup B)=\{1,2,3,4,6,8,10\}$
$(A \cup B)'=U-(A \cup B)$
$=\{1,2,3,4,5,6,7,8,9,10\}-\{1,2,3,4,6,8,10\}$
$=\{5,7,9\}$

56. $\{(A \cup B)' \cap A\}-(A-B)=\{(U-(A \cup B)) \cap A\}-(A-B)$
$=\{(U \cap A)-\{(A \cup B) \cap A\}\}-(A-B)$
$=\{A-A\}-(A-B)=\phi-(A-B)=\phi$

57. Here, $U=\{2,3,4,5,6,7,8\}$
$A=\{2,3,6\}$
$B=\{2,3\}$
$\therefore$ Venn diagram is



58. Clearly, $(A \cap B) \cup (B \cap C)$

59. If $A \cap B = \phi$, then it is not necessary that either

$$A = \phi \text{ or } B = \phi$$

60. Clearly, (a) is true.

61. In the given options, the correct statement is $\phi \in P(\phi)$

62. I. $\phi \in A'$ in X is false because ϕ is not an element of A' in X.

II. $0 \subset A'$ w.r.t. X is false because 0 is not a set.

III. $0 \in A'$ in X is correct.

IV. $\{0\} \in A'$ in X is correct.

63. As $A \cap B$ is a set of all squares.

64. Clearly, $P \cup Q = Q$

65. Here,

M = Set of students of Math

E = Set of students of English

C = Set of students of Commerce

$\therefore n(M \cup E \cup C) = 42$

$n(M) = 14, n(E) = 24, n(C) = 20, n(M \cap E) = 3,$

$n(E \cap C) = 2, n(M \cap E \cap C) = 0$

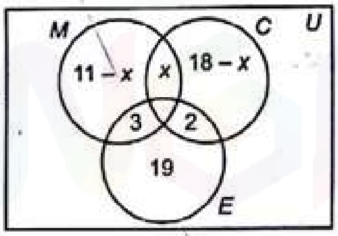
Let x be the number of students who studies Mathematics and Commerce, *i.e.*, $n(M \cap C) = x$

$\therefore$ From Venn diagram $11 - x + x + 18 - x + 2 + 3 + 19 = 42$

$\Rightarrow \quad 53 - x = 42$

$\Rightarrow \quad x = 52 - 42 = 11$

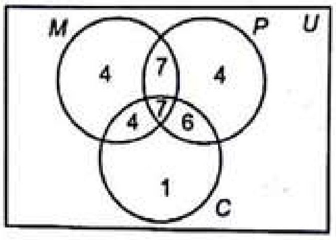
$\therefore$ Required number of students who study Mathematics but not Commerce $= 14 - x = 14 - 11 = 3$



66. Here, $n(M) = 22, n(C) = 18, n(P) = 24,$

$n(M \cap C) = 11, n(C \cap P) = 13,$

$n(M \cap P) = 14, n(M \cap P \cap C) = 7$



$$\therefore n(M \cup P \cup C) = [n(M) + n(P) + n(C)] - [n(M \cap P) + n(M \cap C) + n(P \cap C)] + n(M \cap P \cap C)$$

$$= (22 + 24 + 18) - (14 + 11 + 13) + 7$$

$$= 64 - 38 + 7 = 33$$

There are 33 students in the class.

67. Here, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$\Rightarrow \quad 2x_1 \neq 2x_2 \Rightarrow x_1 \neq x_2$

or if $\quad f(x_1) = f(x_2)$

$\Leftrightarrow \quad 2x_1 = 2x_2$

$\Leftrightarrow \quad x_1 = x_2$

So, f is one-one.

Let $y = 2x$, then $x = \frac{1}{2}y$, then if $y = 1$

$$x = \frac{1}{2} \notin N$$

So, f one-one but not onto.

68. It is true that, $\{a\} \in \{\{a\}, \{b\}, c\}$

69. $-1 \leq \cos(bx + c) \leq 1$

$\Rightarrow \quad -a \leq a\cos(bx + c) \leq a$

$\Rightarrow \quad -a + d \leq a\cos(bx + c) + d \leq a + d$

$\therefore \quad$ Range $f = [d - a, d + a]$

70. $f(-2) = (-2)^2 = 4,$

$f(-1) = (-1)^2 = 1,$

$f(0) = 0^2 = 0,$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

So, image of given set $= \{0, 1, 4\}$

71. Given, $A = \{1, 2, 3, 4,\}$

So, required subsets are $\{1,2,3\}$, $\{1,2,4\}, \{1,3,4\}, \{2,3,4\}$ and $\{1,2,3,4\}$.

Hence, the number of subsets is 5.

72. I. The set of points of a given line is not a finite set.

II. Here, we cannot decide, whose student is intelligent.

III. Here, we cannot decide, which good books are in a school library.

73. g is not one-one, since $1 \neq -1$

But $\quad g(1) = g(-1) = 1$

also $\quad h(x_1) = h(x_2) \Rightarrow x_1^3 = x_2^3$

$\Rightarrow \quad x_1 = x_2$, so h is one-one.

74. Here, $P = \{\ldots -6, -3, 0, 3, 6, \ldots\}$

$Q = \{\ldots -8, -4, 0, 4, 8, \ldots\}$

and $\quad R = \{\ldots -36, -6, 0, 6, 36, \ldots\}$

I. $P \cup Q = \{\ldots -8, -6, -4, -3, 0, 3, 4, 6, 8, \ldots\} \neq R$

II. Here, $P \not\subset R$

III. Here, $R \subset (P \cup Q)$ is true.

75. Failed candidates in English, $n(E) = 52\%$

Failed candidates in Maths, $n(M) = 42\%$

Failed candidates in both English and Maths, $n(E \cap M) = 17\%$

$\because$ Total failed candidates

$$n(E \cup M) = n(E) + n(M) - n(E \cap M)$$

$$= 52 + 42 - 17 = 77\%$$

$\therefore$ Total passed candidates $= (100 - 77) = 23\%$