8

Construction of Triangles

8.1 Introduction

Every triangle has six components—three sides and three angles. We have already studied about the essential condition for congruency of triangles. To construct a triangle three components are required i.e. :

(i) Three sides, or

(ii) Two sides and angle between them, or

(iii) Two angles and one side, or

(iv) In right angled triangle hypotenuse and one side

Remark :

(i) If all three angles are given, then construction of triangle is not possible,

(ii) If two sides and their one opposite acute angle be given, then construction of triangle is in ambiguous situation.

In this chapter, we will learn to construct triangles in various conditions.

Construction 8.1 : Construction of triangle whose three sides are given

Construct a triangle ABC whose sides a = 5 cm, b = 4 cm and c – 3.5 cm

Here, 'a' stands for side opposite to $\angle A$; 'b' stands for side opposite to $\angle B$; 'c' stands for side opposite to $\angle C$.

Before the construction of the triangle, we will draw a rough sketch of the triangle of given measurements. On the basis of that we will draw the required triangle.

Construction :

(i) Draw a line segment $BC - a = 5$ cm

(ii) Taking B as centre and 3.5 cm as radius draw an arc.

(iii) Taking C as centre and 4 cm as radius draw another arc which cuts the previous arc at A.

(iv) Join A to B and C

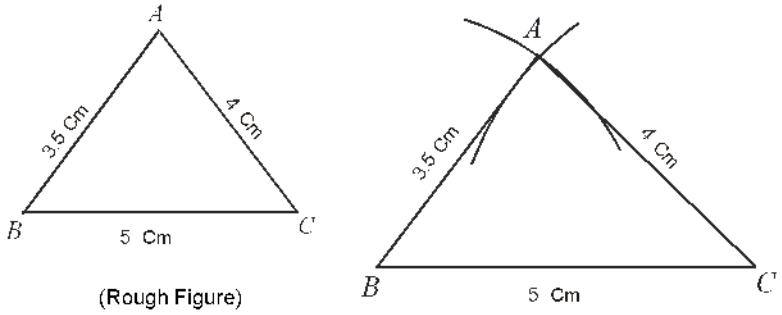


Fig. 8.01

Thus, ABC is the required triangle.

Exercise 8.1

1. Construct a triangle ABC in which $AB = 4$ cm, $BC - 5$ cm and $CA - 6$ cm.
2. Two points A and B are at a distance 6.5 cm to each other. Find the position of point C which is at a distance of 7 cm and 6 cm from points A and B respectively.
3. Construct a triangle ABC in which sides $a = 6.5$ cm, $b = 7.2$ cm and $c = 8$ cm. Draw the bisector of $\angle$ B, which meets AC at point M.
4. Construct a Δ ABC such that $a - 7$ cm, $b = 5$ cm and $c - 4$ cm. Draw a perpendicular from A on BC.
5. Construct an equilateral triangle ABC whose each side is 5.5 cm.
6. Construct an isosceles triangle whose base is 3 cm and equal sides are 5 cm.

Construction 8.2

Construction of a triangle whose two sides and angle between them is given

Construct a triangle ABC whose sides a = 4.5 cm, $c - 3.5$ cm and $\angle$ B $-$ 45°.

Construction :

(i) Draw a line segment $BC - a - 4.5$ cm

(ii) At point B with the help of ruler and compass draw $\angle$ DBC = 45° with BC

(iii) Taking B as a centre and 3.5 cm as radius, draw an arc cutting BA from BD.

(iv) Join A to C.

Thus, Δ ABC is the required triangle.

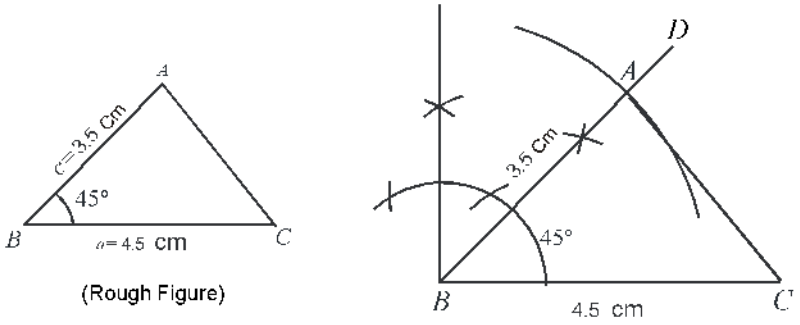


Fig. 8.02

Exercise 8.2

1. Construct a triangle ABC in which $a = 4$ cm, $b = 5$ cm and $\angle C = 60°$.
2. Construct a triangle LMN in which $\angle L = 120°$, $LM - 4$ cm, $LN - 5$ cm.
3. Construct a triangle ABC in which side $AB = AC = 8$ cm and $\angle A - 75°$. Also draw the bisector of $\angle B$ which meets the opposite side.
4. Construct an isosceles triangle whose vertex angle is 120° and each equal sides is of length 5.5 cm.

Construction 8.3

Construction of a triangle whose one side and two angles are given :

Construct a ΔABC in which $\angle B - 60°$, $\angle C = 75°$ and side $BC - 4.6$ cm.

Construction :

(i) Draw a line segment $BC = 4.6$ cm

(ii) With the help of ruler and compass, at point B, draw $\angle EBC - 60°$ and at point C, draw $\angle FCB = 75°$ with BC.

(iii) BE and CF intersect at point A. Thus, Δ ABC is the required triangle.

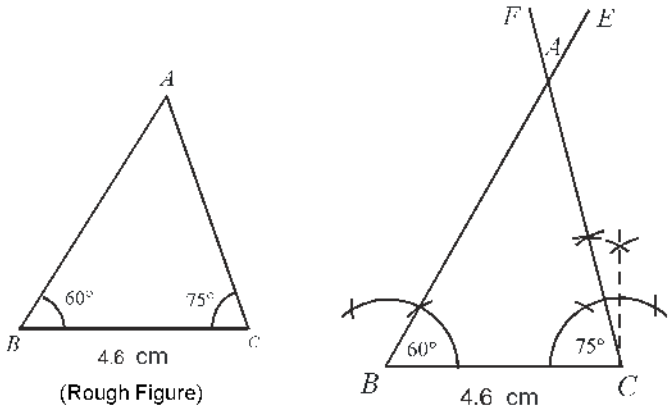


Fig. 8.03

Remark :

If one angle is given at the ends B and C on side BC and a vertex angle is given, then to find the base angle, we can subtract the sum of base and vertex angle from 180° and can get the other base angle, then construct the triangle by above geometric construction.

Exercise 8.3

1. Construct a triangle PQR in which $QR - 8$ cm, $\angle Q = 120°$ and $\angle R = 30°$.
2. Construct a triangle ABC in which $b - 7$ cm, $\angle A = 90°$ and $\angle C = 60°$.
3. Construct an isosceles triangle whose base is 4 cm and vertex angle is 30°. Draw perpendicular from the vertex on base.

Construction 8.4

Construction of a right angled triangle in which hypotenuse and one other side be given.

Construct a right-angled triangle whose hypotenuse $AC = 6.5$ cm and side $AB - 3.5$ cm.

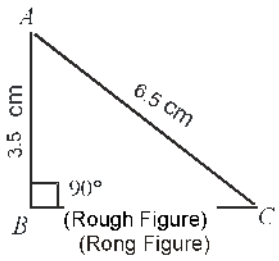
Construction :

(i) Draw a line segment BD of suitable length and at B draw $\angle DBE - 90°$.

(ii) Taking B as centre and 3.5 cm as radius, draw an arc which cuts BE at A.

(iii) Taking A as centre and 6.5 cm as radius, draw an arc which cuts BD at C.

(iv) Join A and C.



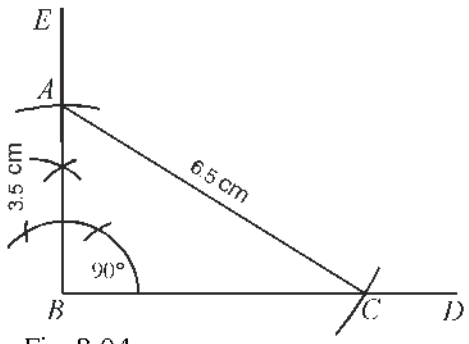


Fig. 8.04

Thus, ABC is the required triangle.

Exercise 8.4

1. Construct a right angled triangle whose hypotenuse is 5 cm and other side is 3 crn.
2. Construct a triangle ABC in which $\angle A - 90°$, side $AC = 5.4$ cm and hypotenuse $BC - 10cm$.
3. Construct a right angle triangle ABC in which $\angle A - 90°$ and side a = 10 cm and side b = 6 cm. Draw a perpendicular on hypotenuse from vertex A.

Construction 8.5

Construct of a triangle whose two sides and an angle opposite to one of the given side, is given.

Construct a triangle Δ ABC in which $AB = 5$ cm, $AC = 3$ cm, and $\angle B = 30°$.

Construction :

1. Draw a line segment BF. Draw an angle $\angle EBF = 30°$ at point B
2. With B as centre and 5 cm as radius, draw an arc which will intersect BE at point A.
3. With A as centre and 3 cm as radius, draw an arc which intersect BF at C and C'. Now join C and C' to A

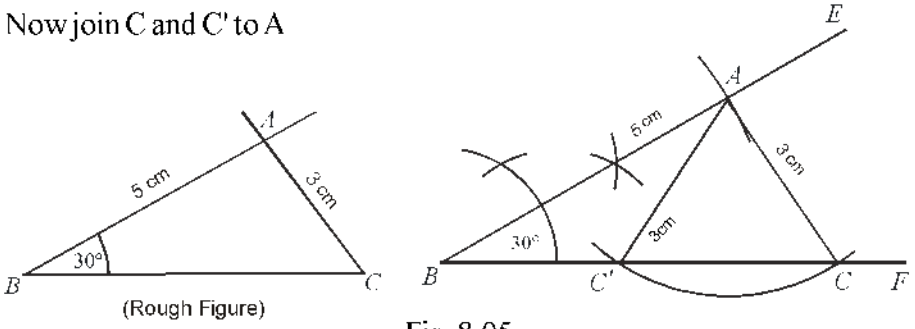


Fig. 8.05

4. Hence, Δ ABC and Δ ABC' are the required triangles.

Various conditions related to above construction

Condition 1 : When length of side b is less than the length of perpendicular p drawn from A to BF. Then taking A as centre and radius as b, drawn arc does not cut BF. In this condition construction of triangle is not possible (Fig 8.06)

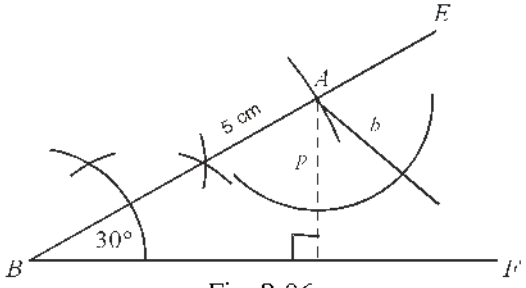


Fig. 8.06

Condition 2 : When length of side b is equal to the length of perpendicular p, then taking A as centre and b as radius, draw an arc will touch BF. In this condition only one triangle can be constructed which will be right-angled triangle (Fig. 8.07).

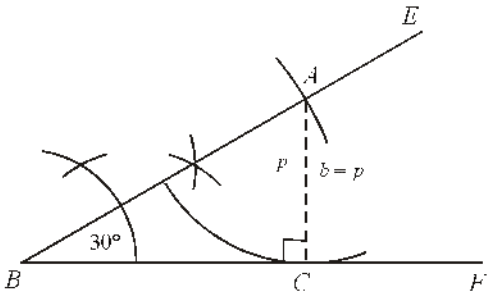


Fig. 8.07

Condition 3 : If side b is greater than perpendicular p and equal to c, then an isosceles triangle can be made in which side AB and AC are equal (Fig. 8.08).

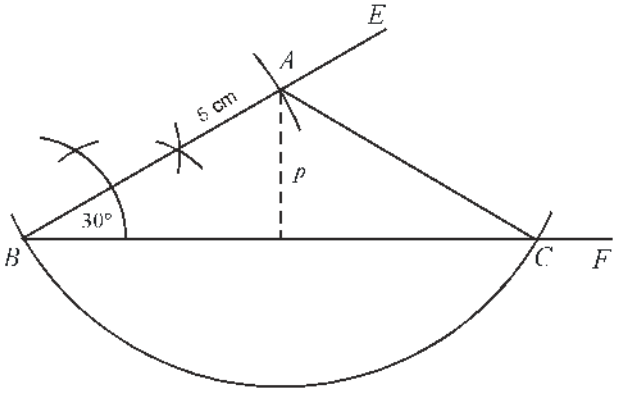


Fig. 8.08

Condition 4 : If side b, is greater than both the perpendicular p and side c, then arc will intersects line BF towards the right side of point B, at point C. In this condition, only one triangle ABC can be constructed. In this condition when arc CB *is* extended then it will intersect at C'. But Δ ABC' is not a triangle because in which $\angle ABC' \neq 30°$ (Fig. 8.09).

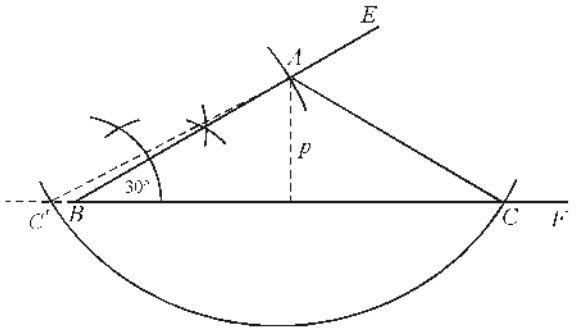


Fig. 8.09

Condition 5 : If side b is greater than perpendicular p and smaller than side c, then arc will intersect BF at different points, and so ΔABC and Δ ABC' are required triangles shown as in (Fig. 8.05)

Remark:

From above conditions, it is clear that, if $\angle$B is not acute angle, then there is only one triangle is possible because in this condition side b is greater than side c. If $\angle$B is an acute angle, then above any condition may be possible.

Exercise 8.5

1. Construct a $\angle$XYZ in which $\angle$XYZ = 60°, XY = 5 cm and XZ = 4.5 cm. How many triangles of such type can be drawn?
2. Construct a triangle PQR in which $\angle$PQR = 45°, side PQ = 6 cm and PR = 5 cm.

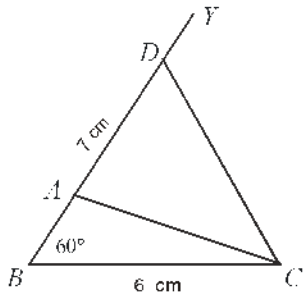
3. Construct a triangle ABC in which a = 5.4 cm, b = 6.8 and $\angle A = 45^\circ$. Also find the measurement of side AB.

8.02 Difficult Construction of Triangles

Illustrative Example

Example 1. Construct a triangle ABC, when base BC = 6 cm, $\angle ABC = 60^\circ$ and AB + AC = 7 cm.

Solution : Draw a rough sketch of given measurements. Extend BA to point D such that AC = AD so BD = BA + AC. Join D to C Draw perpendicular bisector of DC. Then find the Δ ABC.



(Rough Figure)

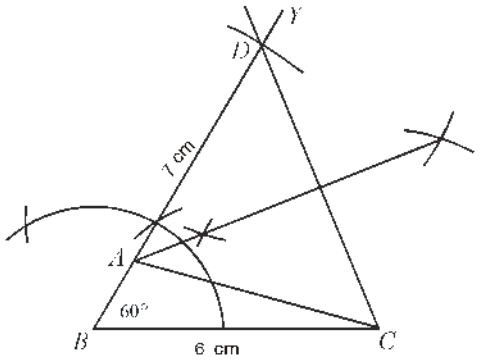


Fig. 8.10

Construction :

(i) Draw the BC = 6 cm and at point B, construct $\angle YBC = 60^\circ$.

(ii) Cut a line segment BD = 7 cm equal to (BA + AC) from BY.

(iii) Join D to C

(iv) Draw perpendicular bisector of DC which intersects BD at point A.

(v) Join A and C.

Thus, ABC is required triangle in which BA + AC = 7 cm

Note : A lies on the perpendicular bisector of CD, so AD = DC.

Example 2. Construct a triangle in which AB + BC + CA = 8 cm, $\angle B = 60^\circ$ and $\angle C = 80^\circ$

Solution : Draw the rough sketch with the given measurements

Extend BC in both side such that BP = AB and CQ = AC. Join AP and AQ, obtain Δ ABP and Δ ACQ.

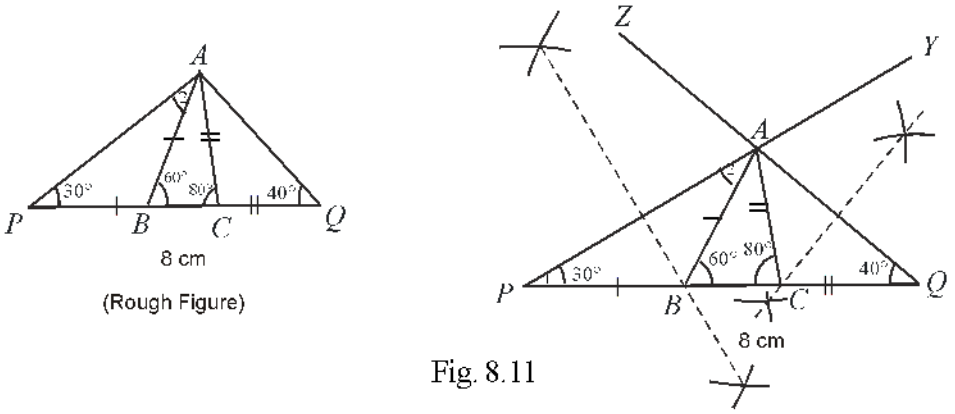


Fig. 8.11

Construction :

(i) Draw a line segment PQ = 8 cm.

(ii) At point P and Q, construct $\angle YPQ = 30^\circ$ and $\angle ZQP = 40^\circ$ respectively.

(iii) PY and QZ intersects at point A

(iv) Draw perpendicular bisectors of AP and AQ which intersects PQ at point B and C respectively.

(v) Join AB and AC (Fig. 8.11)

Thus, ABC is the required triangle.

Example 3. Construct a triangle AB in which BC = 7 cm, b – c = 3.5 cm and $\angle C = 30^\circ$

Solution : Draw a rough sketch figure with the given dimensions.

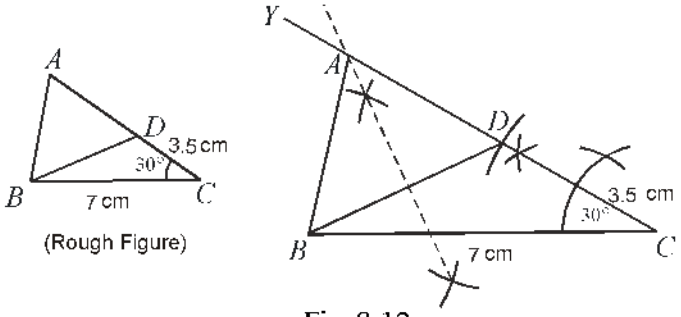


Fig. 8.12

In the figure, AC is bigger then AB. To get required triangle cut line segment AB from AC.

Steps of construction :

(i) Draw a line segment BC = 7 cm and at point C, Construct $\angle YCB = 30^\circ$.

(ii) Cut the line segment CD equal to b – c = 3.5 cm from CY.

(iii) Join BD and draw the perpendicular bisector of BD.

(iv) Let it intersect CY at point A. Join AB.

Thus, ABC is the required triangle.

Example 4. Construct a triangle ABC in which BC = 6 cm, and medians AD and CF are 9 cm and 7.5 cm respectively.

Solution : Draw the rough sketch with the given measurements.

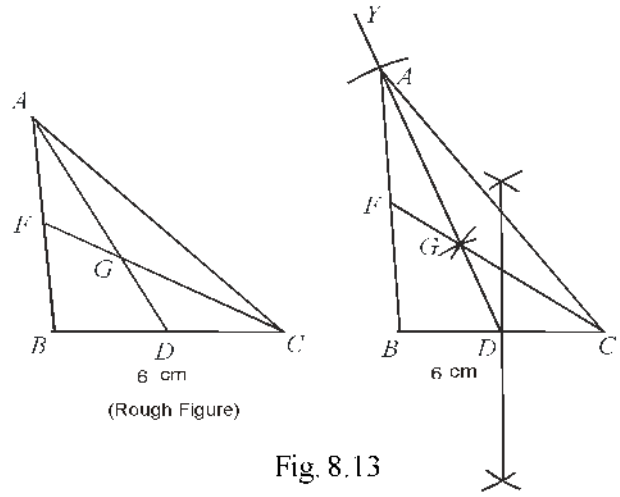


Fig. 8.13

$DC = \frac{1}{2}BC = \frac{1}{2} \times 6 = 3$ cm

$GC = \frac{2}{3}CF = \frac{2}{3} \times 7 \cdot 5 = 5$ cm

$GD = \frac{1}{3} \times 9 = 3$ cm

Construction :

Draw $BC - 6$ cm. Bisect BC at D. Taking D as a centre and radius $\left[GD = \frac{1}{3}AD\right]$ $= 3$ cm, draw an arc. Similarly, taking C as centre and radius $\left[GC = \frac{2}{3}CF\right]$ $= 5$ cm, draw another arc, which will intersect the first arc at point G.

Extend DG such that $DA - 9$ cm. Join AB and AC.

Thus, ABC is the required triangle.

Example 5. Construct a triangle ABC whose medians are 3.6 cm, 2.7 cm and 4.2 cm respectively.

Solution : Draw a rough sketch with given measurement

From figure, $OB = \frac{2}{3} \times BE = \frac{2}{3} \times 2 \cdot 7 = 1 \cdot 8$ cm

$OC = \frac{2}{3} \times CF = \frac{2}{3} \times 4 \cdot 2 = 2 \cdot 8$ cm

$OA = \frac{2}{3} \times AD = \frac{2}{3} \times 3 \cdot 6 = 2 \cdot 4$ cm

Complete the figure by extending AD in such a way that $KC - OB$. Find the mid-point of OK say D and complete the triangle ABC.

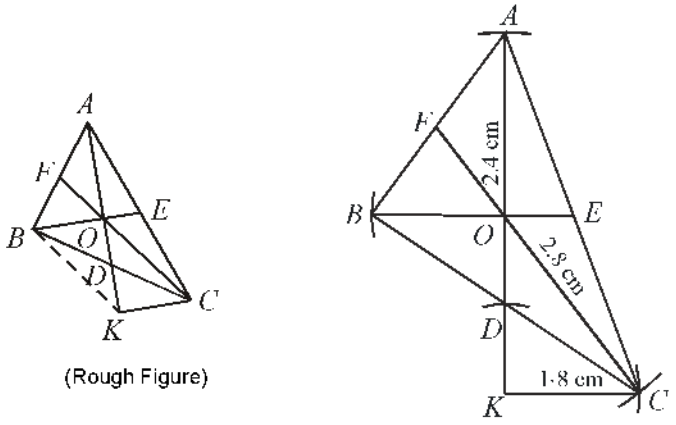


Fig. 8.14

Construction :

Construct triangle OKC in such a way that $OK - AO = 2.4$ cm, $OC = 2.8$ cm and $KC - OB - 1.8$ cm. Find the mid-point of OK say D. Extend KD forward such that $AD - 3.6$ cm. Extend CD backward such that $CD - BD$. Join AB and AC.

Thus, ΔABC is required triangle whose median $AD = 3.6$ cm, $BE - 2.7$ cm and $CF - 4.2$ cm.

Exercise 8.6

1. Construct a triangle ABC in which $BC - 7$ cm, $\angle C = 50°$ and $AC \quad AB - 8$ cm.
2. Construct a triangle PQR in which $PQ - 6$ cm, $\angle Q = 60°$ and $PQ - PR = 8$ cm.
3. Construct a triangle PQR in which $QR = 5$ cm, $\angle R - 40°$ and $PR - PQ - 1$ cm.
4. Construct a triangle ABC whose perimeter is 12 cm and base angles are 50° and 70°.
5. Construct a triangle ABC in which $BC = 6$cm, medians AD and CFare 6 cm and 7.5 cm respectively.
6. Construct a triangle ABCwhose, three medians are 3 cm, 2.7 cm and 6 cm respectively.

Important Points

To construct a triangle, following components are necessarilly known:

(i) All three sides (side-side-side).

or

(ii) Two angles and one side (angle-side-angle).

or

(iii) Two sides and angle between them (side-angle-side).

or

(iv) In right angled triangle, hypotenuse and one side. (hypotenuse- side)

2. In the following conditions, construction of triangle is not possible,

(i) If all three angles are given.

(ii) If two sides and one of the angles opposite to them be acute angle given (In this condition, two triangles can be made but the final triangle cannot be decided) therefore, condition is not clear.

Miscellaneous Exercise-8

1. Construct a triangle whose perimeter is 12 cm and ratio of sides is 1 : 2 : 3.
2. Construct a triangle ABC in which $\angle B - 90°$, $\angle C - 60°$ and $c - 5$ cm.
3. Construct a right angled triangle ABC in which hypotenuse BC is 8.2 cm and one side is 4.2 cm
4. Construct a triangle ABC *in* which $\angle B - 45°$, $\angle C - 60°$ and perpendicular from A on BC is AD and its length is 4 cm.
5. Construct a triangle ABC in which $a - 5.6$ cm, b $c - 10.2$ cm and $\angle B$ $\angle C - 30°$

(Hint: Vertex angle B is $90° + \frac{1}{2}(\angle B - \angle C) = 105°$)

6. Construct a triangle whose all three medians are 4.2 cm, 4.8 cm and 5.4 cm.
7. Construct an isosceles triangle whose height is 6 cm and equal sides are 7 cm each. Find the measurement of base.

Answers

Exercise 8.5

3. 2.8 cm, 6.8 cm

Miscellaneous Exercise-8

7. 7 cm