Chapter - 2
Polynomial

Exercise 2.4

Q. 1 Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Answer:

(i) $P(x) = 2x^3 + x^2 - 5x + 2$

Now for zeroes, putting the given values in x.

$P(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$

$= (1/4) + (1/4) - (5/2) + 2 = (1 + 1 - 10 + 8)/2 = 0/2 = 0$

$P(1) = 2 \times 1 + 1 - 5 \times 1 + 2 = 2 + 1 - 5 + 2 = 0$

$P(-2) = 2\times (-2)^3 + (-2)^2 - 5(-2) + 2 = (2\times -8) + 4 + 10 + 2 = -16 + 16 = 0$

Thus, 1/2, 1 and -2 are zeroes of given polynomial.

Comparing given polynomial with $ax^3 + bx^2 + cx + d$ and Taking zeroes as α, β, and γ, we have

$a = 2$, $b = 1$, $c = -5$, $d = 2$ and $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

Now, We know the relation between zeroes and the coefficient of a standard cubic polynomial as

$\alpha + \beta + \gamma = -\frac{b}{a}$

Substituting value, we have

$\frac{1}{2} + 1 - 2 = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

Since, LHS = RHS (Relation Verified)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\left(\frac{1}{2} \times 1\right) + (1 \times -2) + \left(-2 \times \frac{1}{2}\right) = -\frac{5}{2}$

$\frac{1}{2} - 2 - 1 = -\frac{5}{2}$

$-\frac{5}{2} = -\frac{5}{2}$

Since LHS = RHS, Relation verified.

$\alpha\beta\gamma = -\frac{d}{a}$

$\left(\frac{1}{2} \times 1 \times -2\right) = -\frac{2}{2}$

$-\frac{2}{2} = -\frac{2}{2}$

Since LHS = RHS, Relation verified.

Thus, all three relationships between zeroes and the coefficient is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Now for zeroes , put the given value in x.

$P(2) = 2^3 - 4(2)^2 + 5 \times 2 - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$

$P(1) = 1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

$P(1) = 1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

Thus, 2, 1, 1 are the zeroes of the given polynomial.

Now,

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

a = 1, b =- 4. c = 5, d = -2 and $\alpha = 2, \beta = 1, \gamma = 1$

Now,

$2 + 1 + 1 = \frac{4}{1}$

4 = 4

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$(2 \times 1) + (1 \times 1) + (1 \times 2) = \frac{5}{1}$

2 + 1 + 2 = 5

5 = 5

$\alpha\beta\gamma = -\frac{d}{a}$

$2 \times 1 \times 1 = 2$

2 = 2

Thus, all three relationships between zeroes and the coefficient is verified.

Q. 2 Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Answer:

For a cubic polynomial equation, ax3 + bx^2 + cx + d, and zeroes α, β and γ

we know that

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Let the polynomial be $ax^3 + bx^2 + cx + d$, and zeroes α, β and γ.

A cubic polynomial with respect to its zeroes is given by,x^3 - (sum of zeroes) x^2 + (Sum of the product of roots taken two at a time) x - Product of Roots = 0

$x^3 - (2)\ x^2 + (-\ 7)\ x - (-\ 14) = 0$

$x^3 - (2)\ x^2 + (-\ 7)\ x + 14 = 0$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q. 3 If the zeroes of the polynomial $x^3 – 3x^2 + x + 1$ are (a - b), a and (a + b). Find a and b.

Answer:

Given

$P(x) = x^3 - 3x^2 + x + 1$

Zeroes are = a – b , a+ b, a

Comparing the given polynomial with $mx^3 + nx^2 + px + q$, we get,

= m = 1, n = - 3, p = 1, q = 1

Sum of zeroes = a – b + a+ a + b = - $\frac{n}{m}$

$3a = -\frac{-3}{1} = 3$

$a = \frac{3}{3} = 3$

The zeroes are = (1 - b), 1 and (1 + b)

Product of zeroes = (1 - b)(1 + b)

(1 - b)(1 + b) = -q/m

$1 - b^2 = -\frac{1}{1} = -1$

$b^2 = 2$

b $= \pm\sqrt{2}$

So,

We get, a = 1 and b $= \pm\sqrt{2}$

Q. 4 If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$find other zeroes.

Answer:

Given:

2+√3 and 2-√3 are zeroes of given equation,

Therefore, (x - 2 + √3)(x - 2 - √3) should be a factor of given equation.
Also, (x - 2 + √3)(x - 2 - √3) = x^2 - 2x - √3x -2x + 4 + 2√3 + √3x - 2√3 – 3

= $x^2 - 4x + 1$

To find other zeroes, we divide given equation by $x^2 - 4x + 1$

$$x^2 - 2x - 35$$

$$x^2 - 4x + 1 \,\big)\, x^4 - 6x^3 - 26x^2 + 138x - 35$$

$$x^4 - 4x^3 + x^2$$

$$- \quad + \quad -$$

$$-2x^3 - 27x^2 + 138x - 35$$

$$-2x^3 + 8x^2 - 2x$$

$$+ \quad - \quad +$$

$$-35x^2 + 140x - 35$$

$$-35x^2 + 140x - 35$$

$$+ \quad - \quad +$$

$$0$$

We get ,

$x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now factorizing $x^2 - 2x - 35$ we get,

$x^2 - 2x – 35$ is also a factor of given polynomial and $x^2 - 2x – 35 = (x-7)(x + 5)$

The value of polynomial is also zero when,

$x - 7 = 0$

or $x = 7$

And, $x + 5 = 0$

Or $x = -5$

Hence, 7 and -5 are also zeroes of this polynomial.

Q. 5 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2k + k$ the remainder comes out to be $x + a$, find k and a.

Answer:

To solve this question divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ by long division method

Let us divide, by $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$

$$x^2 - 4x + (8 - k)$$

$$x^2 - 2x + k \sqrt{x^4 - 6x^3 + 16x^2 - 25x + 10}$$

$$x^4 - 2x^3 + kx^2$$

$$- \quad + \quad -$$

$$-4x^3 + (16 - k)x^2 - 25x + 10$$

$$-4x^3 + 8x^2 \quad -4kx$$

$$+ \quad - \quad +$$

$$(8 - k)x^2 + (4k - 25)x + 10$$

$$(8 - k)x^2 + (16 - 2k)x + (8 - k)x$$

$$- \quad + \quad -$$

$$(2k - 9)x + (10 - 8k + k^2)$$

So, remainder $= (2k - 9)x + (10 - 8k + k^2)$

But given remainder $= x + a \Rightarrow (2k - 9)x + (10 - 8k + k^2) = x + a$

Comparing coefficient of x, we have $2k - 9 = 1 \Rightarrow 2k = 10 \Rightarrow k = 5$ and Comparing constant term, $10 - 8k + k^2 = a$

$\Rightarrow a = 10 - 8(5) + 5^2$

$\Rightarrow a = 10 - 40 + 25 \Rightarrow a = -5$. So, the value of k is 5 and a is -5.