

11. Electrostatics

Electric flux

- It is the total number of electric field lines of force crossing the unit surface area in a direction normal to the surface.

$$\phi = \int_S \vec{E} \cdot \vec{ds} = \int_S \vec{E}_n \cdot \vec{ds}$$

- The flux Φ of an electric field E , through a small area element ds is given by

Gauss's law

- It states that the total electric flux through a closed surface enclosing a charge q is equal to $\frac{1}{\epsilon_0}$ times the magnitude of the charge enclosed.

- $\phi = \frac{q}{\epsilon_0}$

- $\phi = \oint_S \vec{E} \cdot \vec{ds}$

Applications of Gauss's Law

- Electric field intensity due to an infinitely long straight wire of linear charge density λ at a point which is at a perpendicular distance r from the wire is given by $\lambda 2\pi\epsilon_0 r n^\wedge$, where n^\wedge is the radial unit vector in the plane normal to the wire passing
- Electric field intensity due to a uniformly charged infinite plane sheet of surface charge density s is given by $\sigma 2\epsilon_0 n^\wedge$, where n^\wedge is a unit vector normal to the plane, acting outward on either side.
- Electric field intensity due to a total charge q distributed along its surface is given by $\frac{q}{4\pi\epsilon r^2}$ ($r \geq R$), 0 ($r < R$)

Here, r is the distance of the point from the centre of the shell and R is the radius of the shell.

- Mechanical force per unit area of a charged conductor, $f = \frac{1}{2}\epsilon E^2$.
- Energy density per unit volume of a charged conductor, $u = \frac{1}{2}\epsilon E^2$.

Electrostatics of conductors

- Inside a conductor, the electric field is zero.
- The interior of a conductor can have no excess charge in static situation.
- The electric field on the surface of a charged conductor is perpendicular to the surface of the conductor at every point.

- Electrostatic potential is constant throughout the volume of the conductor, and has the same value as on its surface.

Non-Polar Dielectrics

- When the Non-polar dielectric is placed in an external electric field the two centres of positive and negative charges in the molecule are separated and the non-polar molecule gets polarised.

Polar Dielectrics

When an external electric field is applied, the individual dipole moments tend to align with the field. A net dipole moment in the direction of the external field is developed. Induced dipole moment P acquired by the molecule may be written as

$$P = \alpha \epsilon_0 E_0$$

When a dielectric is placed in an external electric field \vec{E}_0 due to polarisation there is development of an electric field \vec{E}_p opposite to the \vec{E}_0

\therefore Effective electric field in a polarised dielectric $= E = E_0 - E_p$

Capacitor

- A capacitor is a system of two conductors separated by an insulator.
- Its capacitance, $C = Q/V$, where Q and $-Q$ are the charges on the two conductors and V is the potential difference between them.
- C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. For a parallel plate capacitor (with vacuum between the plates),

$$C = \epsilon_0 A/d$$

Here, A is the area of each plate and d is the separation between them.

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect is called polarisation and it gives rise to a field in the opposite direction.
- The net electric field inside the dielectric, and hence the potential difference between the plates, is thus reduced. Consequently, the capacitance C increases from its value C_0 {when there is no medium (vacuum)} to $C = KC_0$. Here, K is the dielectric constant of the insulating substance.
- Capacitors are of the following types:
 - Parallel plate capacitor
 - Cylindrical capacitor
 - Spherical capacitor
- Capacitance of cylindrical capacitor without dielectric is given as $C = 2\pi\epsilon_0 l \ln(b/a)$
- Capacitance of spherical capacitor without dielectric is given as $C = 4\pi\epsilon_0 ab/(b-a)$

Capacitors in Series

- In series connection the potential difference applied across the combination is the sum of the resulting potential differences across each capacitor.
- In series combination the charge in all of the capacitors is same.
- Total capacitance in a series combination of the capacitors is given as:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel

- In parallel connection the potential difference is same across each capacitor.
- Total capacitance in parallel combination of the capacitors is given as:

$$C_p = C_1 + C_2 + C_3$$

The Energy U stored in a capacitor of capacitance C , with charge Q and voltage V is

- $U = \frac{Q^2}{2C}$
- $U = \frac{1}{2} CV^2$

Van de Graff generator

It works on the principle that charge given to a hollow conductor is transferred to the outer surface and is distributed uniformly on it.

It is a device used for building up high potential differences of the order of a few million volts.

It consists of a large spherical conducting shell. By means of a moving belt and suitable brushes, charge is continuously transferred to the shell and potential difference of the order of several million volts is built up.

It is used as a particle accelerator.