Expectation

Ex-1 X ~~denote~~ is the discrete random variable, which denote the number of heads that might come up while fliping two coin.

→

	TT	HT,TH	HH
X	0	1	2
P(X)	1/4	1/2	1/4

SS = {HH, HT, TH, TT}
1/4 1/4 1/4 1/4

Exceptation of X,

$$\boxed{E(X) = \sum_{i:p(i)>0} i\, p(X=i)} = 0\cdot 1/4 + 1\cdot 1/2 + 2\cdot 1/4$$

$$= 1$$

E-2 X is the discrete random variable, which denot the number when roll a die.

X	~~0~~	1	2	3	4	5	6
P(X)		1/6	1/6	1/6	1/6	1/6	1/6

~~E~~ Exeptation of X,

$$\boxed{E(X) = \sum_{i=1}^{n} i\, p(X=i)}$$

$$= \frac{1+2+3+4+5+6}{6}$$

$$= \frac{7}{2} \Rightarrow 3.5$$

→ In gate question they can be asked "what is the Exceptation of it". E(X)

** [Ex-3] Expectation of a bernoulli Random variable with parametere 'p'.

P → probability of success.

$(X=0) \rightarrow$ failure.

$(X=1) \rightarrow$ success

→

X	0	1
P(X)	(1-P)	P

Exceptation of X,

$$E(X) = 0 \times (1-P) + 1 \times P$$

$$\boxed{E(X) = P}$$

The excepted number of successes that can happed In ~~trial~~ trail is 'p'.

~~On~~ '1' trial – P success.

∴ $\boxed{1/p \text{ trials} - 1 \text{ success.}}$

** [Ex-4] Expectation of Binomial random variable : (n, P)
↓
Prob

$$\boxed{P(i) = P(X=i) = {}^{n}C_{i}\, P^{i} (1-P)^{n-i}}$$

Expectation of a RV is,

$$E[X] = \sum_{i=\cancel{0}1}^{n} i\, \underline{P(X=i)}$$

$$= \sum_{i=\cancel{0}1}^{n} i\, {}^{n}C_{i}\, P^{i} (1-P)^{n-i}$$

$$= \sum_{i=\cancel{0}1}^{n} i \frac{n!}{(n-i)!\, i!} P^{i} (1-P)^{n-i}$$

$$= \sum_{i=\cancel{0}1}^{n} \frac{n!}{(n-i)!\,(i-1)!} P^{i} (1-P)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n(n-1)!}{(n-i)!\,(i-1)!} p \cdot p^{i-1} (1-p)^{n-i}$$

let $(i-1) = K$ | $i = 1$ to n

$K = i-1$

$= 0, (n-1)$

$$= np \sum_{K=0}^{n-1} \frac{(n-1)!}{(n-1-K)!\,K!} p^K (1-p)^{n-1-K}$$

$$= np \sum_{K=0}^{n-1} {}^{n-1}C_K \, p^K (1-p)^{n-1-K}$$

$$= np \, [p + (1-p)]^{n-1}$$

* $\boxed{E[x] = np}$

$\boxed{Ex-5}$ Calculate $E[x]$ if x is a poisson random variable with parameter 'λ'. $P(x=i) = \sum_{i=0}^{n} \left(e^{-\lambda} \frac{\lambda^i}{i!}\right)$

$$E[x] = \sum_{i=0}^{\infty} i \, P(x=i)$$

$$\Rightarrow \sum_{i=1}^{\infty} i \left(\frac{e^{-\lambda} \lambda^i}{i!}\right)$$

$$\Rightarrow \sum_{i=1}^{\infty} \frac{e^{-\lambda} \lambda^i}{(i-1)}$$

$$\Rightarrow \lambda e^{-\lambda} \sum_{i=1}^{\infty} \frac{\lambda^{(i-1)}}{(i-1)!}$$

$\boxed{\text{Let, } K = i-1}$ $K = 0$ to ∞

$i = 1$ to ∞

$$\Rightarrow \lambda e^{-\lambda} \sum_{K=0}^{\infty} \frac{\lambda^K}{K!}$$

$$\left(e^{\lambda} = \frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots\right)$$

$$\Rightarrow \lambda e^{-\lambda} e^{\lambda}$$

* $\boxed{E[x] \Rightarrow \lambda}$ — Just remember it.

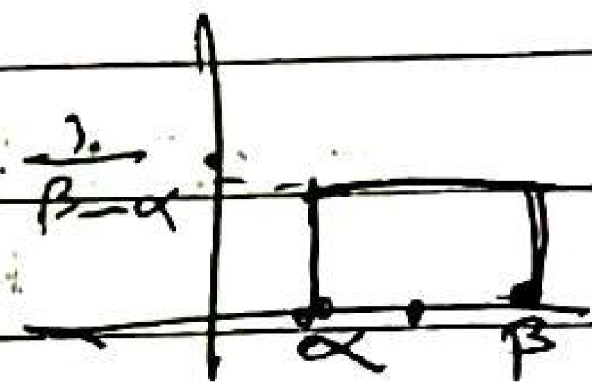
- **The continuous case:**

→ If x is a continuous RV having a propability function $f(x)$, then the expected value of x is defined by,

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx.$$

- Expectation of uniform RV distributed over (α, β)

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{other wise} \end{cases}$$



$$\boxed{E[x] = \frac{\beta + \alpha}{2}}$$

- Expectation of exponentital Random variable -

Let x be exponentially distributed with parameter λ

$$\boxed{E[x] = \frac{1}{\lambda}}$$ λ - will give.

- Expectation of normal RV: Let x be a normally distributed RV with parameters μ and σ^2.

$$\boxed{E[x] = \mu}$$ — Just remember the final answer.