# CHAPTER – 11

## SECTION FORMULA

## <u>Exercise – 11.1</u>

**1. Find the co-ordinates of the mid – point of the line segments joining the following pairs of points:**

**(i) (2, – 3), (– 6, 7)**

**(ii) (5, – 11), (4, 3)**

**(iii) $(\boldsymbol{a}+\mathbf{3}, \mathbf{5}\boldsymbol{b}), (\mathbf{2}\boldsymbol{a}-\mathbf{1}, \mathbf{3}\boldsymbol{b}+\mathbf{4})$**

**Solution:**

Co-ordinates of midpoint of line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ = $\left\{\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right\}$

(i) Co-ordinates of midpoint of line joining the points (2, – 3) and (– 6, 7) = $\left\{\frac{(2+(-6))}{2}, \frac{((-3)+7)}{2}\right\}$

= $\left(\frac{-4}{2}, \frac{4}{2}\right)$

= (-2, 2)

Hence the co-ordinates of midpoint of line joining the points (2, – 3) and (– 6, 7) is (-2, 2).

(ii) Co-ordinates of midpoint of line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ = $\left\{\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right\}$

Co-ordinates of midpoint of line joining the points (5, – 11) and (4, 3) = $\left\{\frac{(5+4)}{2}, \frac{((-11)+3)}{2}\right\}$

$= \left(\frac{9}{2}, \frac{-8}{2}\right)$

$= \left(\frac{9}{2}, -4\right)$

Hence the co-ordinates of midpoint of line joining the points (5, – 11) and (4, 3) is $\left(\frac{9}{2}, -4\right)$.

(iii) Co-ordinates of midpoint of line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ = $\left\{\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right\}$

Co-ordinates of midpoint of line joining the points $(a + 3, 5b)$ and $(2a - 1, 3b + 4)$ = $\left\{\frac{(a+3+2a-1)}{2}, \frac{(5b+3b+4)}{2}\right\}$

$= \left\{\frac{(3a+2)}{2}, \frac{(8b+4)}{2}\right\}$

$= \left\{\frac{(3a+2)}{2}, (4b + 2)\right\}$

Hence the co-ordinates of midpoint of line joining the points $(a + 3, 5b)$ and $(2a - 1, 3b + 4)$ is $\left\{\frac{(3a+2)}{2}, (4b + 2)\right\}$.

**2. The co-ordinates of two points A and B are (– 3, 3) and (12, – 7) respectively. P is a point on the line segment AB such that AP: PB = 2: 3. Find the co-ordinates of P.**

**Solution:**

Let the co-ordinates of P (*x, y*) divides AB in the ratio m: n.

A (– 3, 3) and B (12, – 7) are the given points.

Given m: n = 2: 3

$x_1 = -3, y_1 = 3, x_2 = 12, y_2 = -7, m = 2$ and $n = 3$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(2\times12+3\times-3)}{(2+3)}$

$x = \frac{(24-9)}{5}$

$x = \frac{15}{5}$

$x = 3$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(2\times-7+3\times3)}{5}$

$y = \frac{(-14+9)}{5}$

$y = \frac{-5}{5}$

$y = -1$

Hence the co-ordinate of point P are (3, -1).

**3. P divides the distance between A (– 2, 1) and B (1, 4) in the ratio of 2: 1. Calculate the co-ordinates of the point P.**

**Solution:**

Let the co-ordinates of P (*x, y*) divides AB in the ratio m: n.

A (– 2, 1) and B (1, – 4) are the given points.

Given m: n = 2: 1

$x_1 = -2, y_1 = 1, x_2 = 1, y_2 = 4, m = 2$ and $n = 1$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(2\times1+1\times-2)}{(2+1)}$

$x = \frac{(2-2)}{3}$

$x = \frac{0}{3}$

$x = 0$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(2\times4+1\times1)}{(2+1)}$

$y = \frac{(8+1)}{3}$

$y = \frac{9}{3}$

$y = 3$

Hence the co-ordinate of point P are (0, 3).

**4.**

**(i)** **Find the co-ordinates of the points of trisection of the line segment joining the point (3, – 3) and (6, 9).**

**(ii)** **The line segment joining the points (3, – 4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, – 2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q.**

**Solution:**

(i) Let P and Q be the points of trisection of AB i.e. AP = PQ = QB

Given A (3, – 3) and B (6, 9)

$x_1 = 3, y_1 = -3, x_2 = 6, y_2 = 9$

P (*x, y*) divides AB internally in the ratio 1: 2.

$m : n = 1 : 2$

By applying the section formula, the coordination of P are as follow.

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(1\times6+2\times3)}{(1+2)}$

$x = \frac{(6+6)}{3}$

$x = \frac{12}{3}$

$x = 4$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(1\times9+2\times-3)}{(2+1)}$

$y = \frac{(9-6)}{3}$

$y = \frac{3}{3}$

$y = 1$

Hence the co-ordinate of point P are (4, 1).

Now, Q also divides AB internally in the ratio 2: 1.

$m : n = 1 : 2$

By applying the section formula, the coordination of P are as follow.

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(2\times6+1\times3)}{(1+2)}$

$x = \frac{(12+3)}{3}$

$x = \frac{15}{3}$

$x = 5$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(2\times9+1\times-3)}{(2+1)}$

$y = \frac{(18-3)}{3}$

$y = \frac{15}{3}$

$y = 5$

Hence the co-ordinate of point Q are (5, 5).

(ii) Let P (p, – 2) and Q $\left(\frac{5}{3}, q\right)$ be the points of trisection of AB i.e., AP = PQ = QB


Given A (3, – 4) and B (1, 2)

$x_1 = 3, y_1 = -4, x_2 = 1, y_2 = 2$

P (p, – 2) divides AB internally in the ratio 1: 2.

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$p = \frac{(1\times1+2\times3)}{(1+2)}$

$p = \frac{(1+6)}{3}$

$p = \frac{7}{3}$

Now, Q also divides AB internally in the ratio 2: 1.

$m : n = 1 : 2$

Q $\left(\frac{5}{3}, \text{q}\right)$ divides AB internally in the ratio 2: 1.

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$q = \frac{(2\times2+1\times4)}{(2+1)}$

$q = \frac{(4-4)}{3}$

$q = \frac{0}{3}$

$q = 0$

Hence the value of p and q are $\frac{7}{3}$ and 0 respectively.

**5.**

**(i)** **The line segment joining the points A (3, 2) and B (5, 1) is divided at the point P in the ratio 1: 2 and it lies on the line $3x - 18y + k = 0$. Find the value of $k$.**

**(ii)** **A point P divides the line segment joining the points A (3, – 5) and B (– 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$ if P lies on the line $x + y = 0$, then find the value of k.**

**Solution:**

(i) Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A (3, 2) and B (5, 1) are the given points.

Given m: n = 1: 2

$x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 1, m = 1$ and $n = 2$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(1\times5+2\times3)}{(1+2)}$

$x = \frac{(5+6)}{3}$

$x = \frac{11}{3}$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(1\times1+2\times2)}{(1+2)}$

$y = \frac{(1+4)}{3}$

$y = \frac{5}{3}$

Given P lies on the line $3x - 18y + k = 0$

Substitute x and y in above equation

$3 \times \left(\frac{11}{3}\right) - 18 \times \left(\frac{5}{3}\right) + k = 0$

$11 - 30 + k = 0$

$-19 + k = 0$

$k = 19$

Hence the value of k is 19.

(ii) Let the co-ordinates of P (x, y) divides AB in the ratio m: n.

A (3, -5) and B (-4, 8) are the given points.

Given $\frac{AP}{PB} = \frac{k}{1}$

$m : n = 1 : 2$

$x_1 = 3, y_1 = -5, x_2 = -4, y_2 = 8, m = k$ and $n = 1$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(k\times-4+1\times3)}{(k+1)}$

$x = \frac{(-4k+3)}{(k+1)}$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(k\times8+1\times-5)}{(k+1)}$

$y = \frac{(8k+3)}{(k+1)}$

Co-ordinate of P is $\left(\frac{(-4k+3)}{(k+1)}, \frac{(8k+3)}{(k+1)}\right)$

Given P lies on the line $x + y = 0$

Substitute x and y in above equation

$\frac{(-4k+3)}{(k+1)} + \frac{(8k+3)}{(k+1)} = 0$

$(-4k + 3) + (8k - 5) = 0$

$4k - 2 = 0$

$4k = 2$

$k = \frac{2}{4} = \frac{1}{2}$

Hence the value of k is $\frac{1}{2}$.

**6. Find the coordinates of the point which is three-fourth of the way from A (3, 1) to B (-2, 5).**

**Solution:**


Let P be the point which is three-fourth of the way from A (3, 1) to B (-2, 5).

$\frac{AP}{AB} = \frac{3}{4}$

$AB = AP + PB$

$\frac{AP}{AB} = \frac{AP}{AP+PB} = \frac{3}{4}$

$4AP = 3AP + 3PB$

$AP = 3PB$

$\frac{AP}{AB} = \frac{3}{1}$

The ratio m: n = 3: 1

$x_1 = 3, y_1 = 1, x_2 = -2, y_2 = 5$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(3\times5+1\times1)}{(3+1)}$

$x = \frac{(-6+3)}{4}$

$x = \frac{-3}{4}$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(3\times5+1\times1)}{(3+1)}$

$y = \frac{(15+1)}{4}$

$y = \frac{16}{4}$

$y = 4$

Hence the co-ordinates of P are $\left(\frac{-3}{4}, 4\right)$.

**7. Point P (3, – 5) is reflected P' in the x-axis. Also P on reflection in the y-axis is mapped as P''.**

**(i) Find the co-ordinates to P' and P''**

**(ii) Compute the distance P'P''.**

**(iii) Find the middle point of the line segment P'P''.**

**(iv) On which co-ordinate axis does the middle point of the line segment P'P'' lie?**

**Solution:**

**(i) The image of P (3, – 5) when reflected in x-axis will be (3, 5).**

When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).

**Co-ordinates of P' = (3, 5)**

**Image of P (3, – 5) when reflected in y-axis will be (-3, – 5).**

When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into the opposite (its sign is changed).

**Co-ordinates of P’’ = (-3, – 5)**

(ii) Let $P'(x_1, y_1)$ and $P''(x_2, y_2)$ be the given points

By distance formula d (P’P’’) = $\sqrt{[(x_2 - x_1)^2 + (x_2 - x_1)^2]}$

**Co-ordinates of P’ = (3, 5)**

**Co-ordinates of P’’ = (-3, -5)**

Here $x_1 = 3, y_1 = 5, x_2 = -3, y_2 = -5$

D (P’P’’) = $\sqrt{[(-3 - 3)^2 + (-5 - 5)^2]}$

= $\sqrt{[(-6)^2 + (-10)^2]}$

= $\sqrt{36 + 100}$

= $\sqrt{136}$

= $\sqrt{(4 \times 34)}$

= $2\sqrt{34}$

Hence the distance between P’ and P’’ is $2\sqrt{34}$ units.

**(iii) Co-ordinates of P’ = (3, 5)**

**Co-ordinates of P’’ = (-3, -5)**

Here $x_1 = 3, y_1 = 5, x_2 = -3, y_2 = -5$

Let Q (x, y) be the midpoint of P’P’’

By midpoint formula,

$x = \frac{(x_1 + x_2)}{2}$

$y = \frac{(y_1+y_2)}{2}$

$x = \frac{(3+(-3))}{2} = \frac{0}{2} = 0$

$x = \frac{(5+(-5))}{2} = \frac{0}{2} = 0$

Hence the co-ordinate of midpoint of P'P'' is (0, 0).

**(iv) Co-ordinates of P' = (3, -5)**

**Co-ordinates of P'' = (-3, -5)**

Here $x_1 = 3, y_1 = -5, x_2 = -3, y_2 = -5$

Let R (x, y) be the midpoint of P'P''

By midpoint formula,

$x = \frac{(x_1+x_2)}{2}$

$y = \frac{(y_1+y_2)}{2}$

$x = \frac{(3+(-3))}{2} = \frac{0}{2} = 0$

$x = \frac{(-5+(-5))}{2} = \frac{-10}{2} = -5$

So the co-ordinate of midpoint of P'P'' is (0, -5).

Here x co-ordinate is zero.

Hence the point lies on y-axis.

**8. Use graph paper for this question. Take 1 cm = 1 unit on both axes. Plot the points A (3, 0) and B (0, 4).**

**(i)** **Write down the co-ordinates of A1, the reflection of A in the y-axis.**

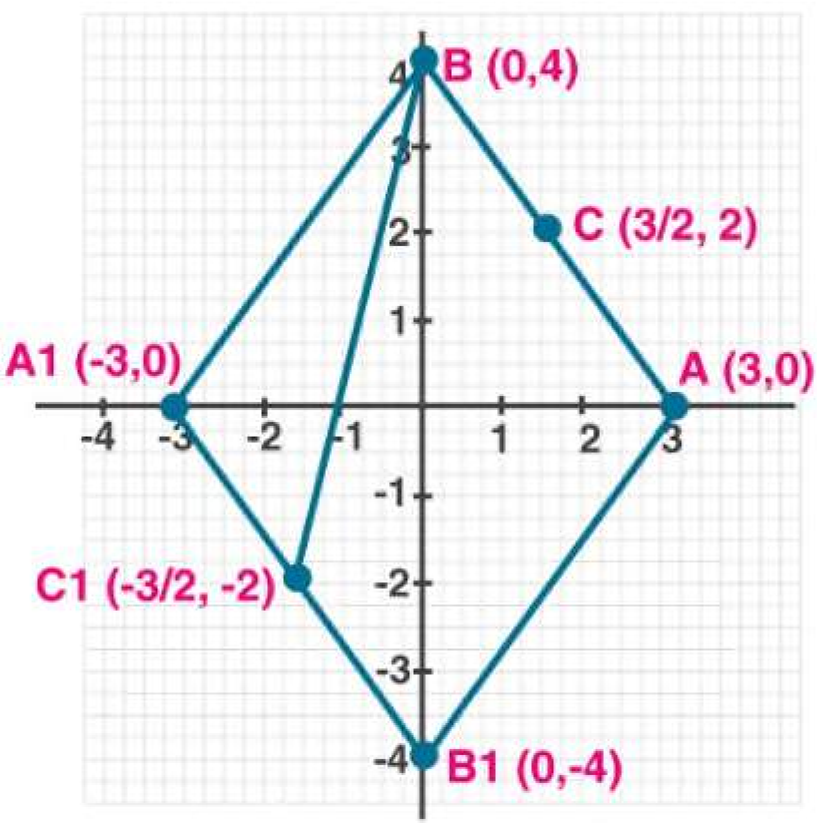
**(ii)** **Write down the co-ordinate of B1, the reflection of B in the x-axis.**

**(iii)** **Assign the special name to the quadrilateral ABA1B1.**

**(iv)** **If C is the midpoint is AB. Write down the co-ordinates of the point C1, the reflection of C in the origin.**

**(v)** **Assign the special name to quadrilateral ABC1B1.**

**Solution:**




(i) Co-ordinate of point A are (3, 0).

When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite (its sign is changed)

Hence the reflection of A in the y-axis is (-3, 0).

(ii) Co-ordinate of point B are (0, 4).

When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed)

Hence the reflection of B in the x-axis is (0, -4).

(iii) The quadrilateral ABA1B1 will be a rhombus.

(iv) Let C be midpoint of AB

Co-ordinate of C = $\left(\frac{(3+0)}{2}, \frac{(0+4)}{2}\right) = \left(\frac{3}{2}, 2\right)$ (midpoint formula)

In a point reflection in the origin, the image of the point (x, y) is the point (-x, -y).

Hence the reflection of C in the origin is $\left(\frac{-3}{2}, -2\right)$

(v) In quadrilateral ABC1B1, ABB1C1

Hence the quadrilateral ABC1B1 is a trapezium.

**9. The line segment joining A (-3, 1) and B (5, -4) is a diameter of a circle whose centre is C. Find the co-ordinates of the point C. (1990)**

**Solution:**

**Given Co-ordinates of A = (-3, 1)**

**Co-ordinates of B = (5, -4)**

Here $x_1 = -3, y_1 = 1, x_2 = 5, y_2 = -4$

Let C (x, y) be the midpoint of AB

By midpoint formula,

$$x = \frac{(x_1+x_2)}{2}$$

$y = \frac{(y_1+y_2)}{2}$

$x = \frac{(-3+5)}{2} = \frac{2}{2} = 1$

$x = \frac{(1+(-4))}{2} = \frac{-3}{2}$

Hence the co-ordinate of midpoint of AB is C $\left(1, \frac{-3}{2}\right)$.

**10. The mid-point of the line segment joining the points (3m, 6) and (-4, 3n) is (1, 2m – 1). Find the values of m and n.**

**Solution:**

Let the midpoint of line joining the points (3m, 6) and (-4, 3n) is (1, 2m – 1).

Here $x_1 = 3m, y_1 = 6, x_2 = -4, y_2 = 3n$

$x = 1, y = 2m - 1$

By midpoint formula,

$x = \frac{(x_1+x_2)}{2}$

$1 = \frac{(3m+4)}{2}$

$3m - 4 = 2$

$3m = 2 + 4$

$m = \frac{6}{3} = 2$

By midpoint formula,

$y = \frac{(y_1+y_2)}{2}$

$2m - 1 = \frac{(6+3n)}{2}$

$4m - 2 = 6 + 3n$

Put $m = 2$ in above equation

$4 \times 2 - 2 = 6 + 3n$

$8 - 2 - 6 = 3n$

$3n = 0$

$n = 0$

Hence the value of m and n are 2 and 0 respectively.

**11. The or-ordinates of the mid-point of the line segment PQ are (1, -2). The co-ordinates of P are (-3, 2). Find the co-ordinates of Q. (1992).**

**Solution:**

Let the co-ordinates of Q be $(x_2, y_2)$

Given co-ordinate of P = $(-3, 2)$

Co-ordinates of midpoint = $(1, -2)$

Here $x_1 = -3, y_1 = 2, x_2 = 1, y_2 = -2$

By midpoint formula,

$x = \frac{(x_1+x_2)}{2}$

$1 = \frac{(-3+x_2)}{2}$

$2 = -3 + x_2$

$x_2 = 2 + 3 = 5$

By midpoint formula,

$$y = \frac{(y_1 + y_2)}{2}$$

$$-2 = \frac{(2 + y_2)}{2}$$

$$-4 = 2 + y_2$$

$$y_2 = -4 - 2$$

$$y_2 = -6$$

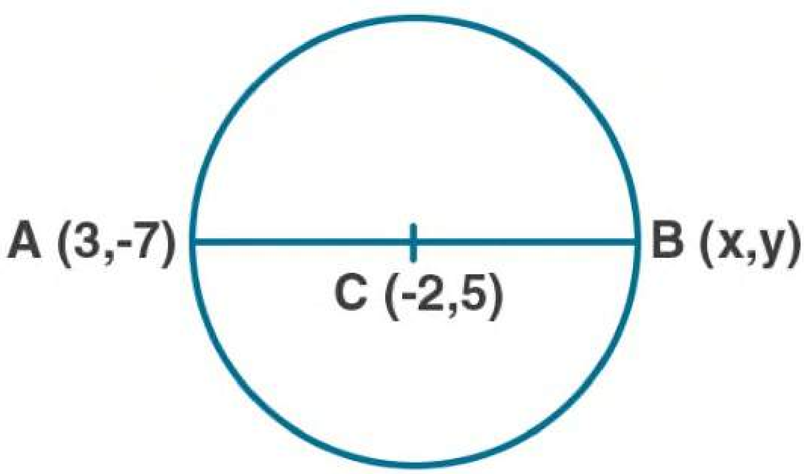
Hence the co-ordinates of Q are (5, -6).

**12. AB is a diameter of a circle with centre C (-2, 5). If point A is (3, -7). Find:**

**(i) The length of radius AC.**

**(ii) The coordinates of B.**

**Solution:**




(i) Length of radius AC = d(A, C)

Co-ordinates of A = (3, -7)

Co-ordinates of C = (-2, 5)

Here $x_1 = 3, y_1 = -7, x_2 = -2, y_2 = 5$

By distance formula d (A, C) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

= $\sqrt{\left[\left((-2) - 3\right)^2 + \left(5 - (-7)\right)^2\right]}$

= $\sqrt{[(-5)^2 + (12)^2]}$

= $\sqrt{[25 + 144]}$

= $\sqrt{169}$

= 13

Hence the radius is 13 units.

(ii) Given AB is the diameter and C is the centre of the circle.

By midpoint formula, $-2 = \frac{(x+3)}{2}$

$-4 = x + 3$

$x = -4 - 3 = -7$

By midpoint formula, $5 = \frac{(-7+y)}{2}$

$10 = -7 + y$

$y = 10 + 7 = 17$

Hence the co-ordinates of B are $(-7, 17)$.

**13. Find the reflection (image) of the point (5, -3) in the point (-1, 3).**

**Solution:**

Let the co-ordinates of the image of the point P (5, -3) be

P1 (x, y) in the point (-1, 3) then the point (-1, 3) will be the midpoint of PP1.

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$-1 = \frac{(5+x_2)}{2}$ $[x = -1, x_1 = 5]$

$-2 = 5 + x_2$

$x_2 = -2 - 5 = -7$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$3 = \frac{(-3+y_2)}{2}$ $[y = 3, y_1 = -3]$

$6 = -3 + y_2$

$y_2 = 6 + 2 = 9$

Hence the co-ordinates of the image of P is (-7, -9).

**14. The line segment joining A $\left(-1, \frac{5}{3}\right)$ the points B (a, 5) is divided in the ratio 1: 3 at P, the point where the line segment AB intersects y-axis. Calculate**

**(i) The value of a**

**(ii) The co-ordinates fo P. (1994)**

**Solution:**

(i) Let P (x, y) divides the line segment joining the points A $\left(-1, \frac{5}{3}\right)$, B (a, 5) in the ratio 1: 3,

Here m: n = 1: 3

$x_1 = -1, y_1 = \frac{5}{3}, x_2 = a, y_2 = 5$

By Section formula $x = \frac{(mx_2+nx_1)}{(m+n)}$

$$x = \frac{(1\times a+3\times -1)}{(1+3)}$$

$$x = \frac{(a-3)}{4} \qquad \ldots \text{(i)}$$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$$y = \frac{\left(\frac{1\times 5+3\times 5}{3}\right)}{(3+1)}$$

$$y = \frac{(5+5)}{4}$$

$$y = \frac{10}{4}$$

$$y = \frac{5}{2} \qquad \ldots \text{(ii)}$$

Given P meets y-axis. So its x co-ordinate will be zero.

i.e. $\frac{(a-3)}{4} = 0$

$$a - 3 = 0$$

$$a = 3$$

(ii) $x = \frac{(a-3)}{4}$ [From (i)]

Substitute a = 3 in above equation.

$$x = \frac{(3-3)}{4} = 0$$

$y = \frac{5}{2}$ [From (ii)]

Hence the co-ordinates of P are $\left(0, \frac{5}{2}\right)$.

**15. The point P (-4, 1) divides the line segment joining the points A (2, -2) and B in the ratio of 3: 5. Find the point B.**

**Solution:**

Let the co-ordinates of B be $(x_2, y_2)$

Given co-ordinates of A = (2, -2)

Co-ordinates of P = (-4, 1)

Ratio m: n = 3: 5

$x_1 = 2, y_1 = -2, x_2 = -4, y_2 = 1$

P divides AB in the ratio 3: 5

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$-4 = \frac{(3\times x_2+5\times 2)}{(3+5)}$

$-4 = \frac{(3x_2+10)}{8}$

$-32 = 3x_2 + 10$

$3x_2 = -32 - 10 = -42$

$x_2 = \frac{-42}{3} = -14$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$1 = \frac{\left(3\times y_2+5\times(-2)\right)}{(3+5)}$

$1 = \frac{(3y_2-10)}{8}$

$8 = 3y_2 - 10$

$3y_2 = 8 + 10 = 18$

$y = \frac{18}{3} = 6$

Hence the co-ordinates of B are (-14, 6).

**16.**

**(i)** **In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6)?**

**(ii)** **In what ratio does the point (-4, b) divide the line segment joining the point P (2, -2), Q (-14, 6)?**

**Solution:**

(i) Let the ratio that the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6) be m: n.

Here $x_1 = 2, y_1 = 1, x_2 = 7, y_2 = 6, x = 5, y = 4$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$5 = \frac{(m\times 7 + n\times 7)}{(m+n)}$

$5 = \frac{(7m+2n)}{(m+n)}$

$5(m + n) = 7m + 2n$

$5m + 5n = 7m + 2n$

$5m - 7m = 2n - 5n$

$-2m = -3n$

$\frac{m}{n} = \frac{-3}{-2} = \frac{3}{2}$

Hence the ratio m: n is 3: 2.

(ii) Let the ratio that the point (-4, b) divide the line segment joining the point (2, -2) and (-14, 6) be m: n.

Here $x_1 = 2, y_1 = -2, x_2 = -14, y_2 = 6, x = -4, y = b$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$-4 = \frac{(m\times -14+n\times 2)}{(m+n)}$

$-4 = \frac{(-14m+2n)}{(m+n)}$

$-4(m+n) = -14m + 2n$

$-4m - 4n = -14m + 2n$

$-4m + 14m = 2n + 4n$

$10m = 6n$

$\frac{m}{n} = \frac{6}{10} = \frac{3}{5}$

Hence the ratio m: n is 3: 5.

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$b = \frac{(3\times 6+5-2)}{(3+5)}$

$b = \frac{(18-10)}{8}$

$b = \frac{8}{8}$

$b = 1$

Hence the value of $b$ is 1 and the ratio m: n is 3: 5.

**17. The line segment joining A (2, 3) and B (6, -5) is intercepted by the x-axis at the point K. Write the ordinate of the point k. Hence,**

**find the ratio in which K divides AB. Also, find the coordinates of the point K.**

**Solution:**

Since the point K is on x-axis, its y co-ordinate is zero.

Let the point K be (x, 0).

Let the point K divides the line segment joining A (2, 3) and B (6, -5) in the ratio m: n.

Here $x_1 = 2, y_1 = 3, x_2 = 6, y_2 = -5, y = 0$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$$0 = \frac{(m\times-5+n\times3)}{(m+n)}$$

$$0 = \frac{(-5m+3n)}{m+n}$$

$$-5m + 3n = 0$$

$$-5m = -3n$$

$$\frac{m}{n} = \frac{-3}{-5} = \frac{3}{5}$$

Hence the point K divides the line segment in the ratio 3: 5.

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$$x = \frac{(3\times6+5\times2)}{(3+5)}$$

$$x = \frac{(18+10)}{8}$$

$$x = \frac{28}{8} = \frac{7}{2}$$

Hence the co-ordinate of K are $\left(\frac{7}{2}, 0\right)$

**18. If A (-4, 3) and B (8, -6),**

**(i) Find the length of AB.**

**(ii) In what ratio is the line joining AB, divided by the x-axis? (2008)**

**Solution:**

(i) Given points are A (-4, 3) and B (8, -6).

Here $x_1 = -4, y_1 = 3$

$x_2 = 8, y_2 = -6$

By distance formula d (AB) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (AB) = $\sqrt{\left[\left(8-(-4)\right)^2 + \left((-6)-3\right)^2\right]}$

D (AB) = $\sqrt{[(12)^2 + (-9)^2]}$

D (AB) = $\sqrt{[144 + 81]}$

D (AB) = $\sqrt{225}$

D (AB) = 15

Hence the length of AB is 15 units.

(ii) Let $m : n$ be the ratio in which the line AB is divided by the x-axis.

Since the line meets x-axis, its y co-ordinate is zero.

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$0 = \frac{(m\times -6+n\times 3)}{(m+n)}$

$0 = \frac{(-6m+3n)}{(m+n)}$

$-6m + 3n = 0$

$-6m = -3n$

$\frac{m}{n} = \frac{-3}{-6} = \frac{3}{6} = \frac{1}{2}$

Hence the ratio is 1: 2.

**19.**

**(i)** **Calculate the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.**

**(ii)** **In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points (3, -1) and (8, 9)? Also, find the coordinates of the point of division.**

**Solution:**

(i) Let $m : n$ be the ratio in which the line segment joining (3, 4) and (-2, 1) is divided by the y-axis.

Since the line meets y-axis, its x co-ordinate is zero.

Here $x_1 = 3, y_1 = 4$

$x_2 = -2, y_2 = 1$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$0 = \frac{(m\times-2+n\times3)}{(m+n)}$

$0 = \frac{(-2m+3n)}{(m+n)}$

$0 = -2m + 3n$

$2m = 3n$

$\frac{m}{n} = \frac{3}{2}$

Hence the ratio m: n is 3: 2.

(ii) Let the line $x - y - 2 = 0$ divide the line segment joining the points (3, -1) and (8, 9) in the ratio m: n at the point P (x, y).

Here $x_1 = 3, y_1 = -1$

$x_2 = 8, y_2 = 9$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(m\times 8+n\times 3)}{(m+n)}$

$x = \frac{(8m+3m)}{(m+n)}$ ... (i)

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(m\times 9+n\times -1)}{(m+n)}$

$y = \frac{(9m-n)}{(m+n)}$ ... (ii)

Since the point P (x, y) lies on the line $x - y - 2 = 0$,

Equation (i) and (ii) will satisfy the equation $x - y - 2 = 0$ ... (iii)

Substitute (i) and (ii) in (iii)

$\left[\frac{(8m+3m)}{(m+n)}\right] - \left[\frac{(9m-n)}{(m+n)}\right] - 2 = 0$

$\left[\frac{(8m+3m)}{(m+n)}\right] - \left[\frac{(9m-n)}{(m+n)}\right] - \left[\frac{2(m+n)}{(m+n)}\right] = 0$

$8m + 3n - (9m - n) - 2(m + n) = 0$

$8m + 3n - 9m + n - 2m - 2n = 0$

$-3m + 2n = 0$

$-3m = -2n$

$\frac{m}{n} = \frac{-2}{-3} = \frac{2}{3}$

Hence the ratio m: n is 2: 3.

Substitute m and n in (i)

$x = \frac{(8m+3m)}{(m+n)}$

$x = \frac{(8\times2+3\times3)}{(2+3)}$

$x = \frac{(16+9)}{5}$

$x = \frac{25}{5} = 5$

Substitute m and n in (ii)

$y = \frac{(9m-n)}{(m+n)}$

$y = \frac{(9\times2-3)}{(2+3)}$

$y = \frac{(18-3)}{5}$

$y = \frac{15}{5} = 3$

Hence the co-ordinates of P are (5, 3).

**20. Given a line segment AB joining the points A (-4, 6) and B (8, -3). Find:**

**(i) The ratio in which AB is divided by the y–axis.**

**(ii) Find the coordinates of the point of intersection.**

**(iii) The length of AB.**

**Solution:**

(i) Let $m : n$ be the ratio in which the line segment joining A (-4, 6) and B (8, -3) is divided by the y-axis.

Since the line meets y-axis, its x co-ordinate is zero.

Here $x_1 = -4, y_1 = 6$

$x_2 = 8, y_2 = -3$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$$0 = \frac{(m\times 8+n\times -4)}{(m+n)}$$

$$0 = \frac{(8m+(-4n))}{(m+n)}$$

$$0 = 8m + (-4n)$$

$$8m = 4n$$

$$\frac{m}{n} = \frac{4}{8} = \frac{1}{2}$$

Hence the ratio m: n is 1: 2.

(ii) By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

Substitute m and n in above equation

$$y = \frac{(1\times(-3)+2\times 6)}{(1+2)}$$

$$y = \frac{(-3+12)}{3}$$

$$y = \frac{9}{3} = 3$$

So the co-ordinates of the point of intersection are (0, 3).

(iii) By distance formula d (AB) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (AB) = $\sqrt{\left[\left(8-(-4)\right)^2+\left((-3)-6\right)^2\right]}$

D (AB) = $\sqrt{[(12)^2+(-9)^2]}$

D (AB) = $\sqrt{[144+81]}$

D (AB) = $\sqrt{225}$

D (AB) = 15

Hence the length of AB is 15 units.

**21.**

**(i)** **Write down the co-ordinates of the point P that divides the line joining A (-4, 1) and B (17, 10) in the ratio 1: 2.**

**(ii)** **Calculate the distance OP where O is the origin.**

**(iii)** **In what ratio does the y-axis divide the line AB?**

**Solution:**

(i) Let P (x, y) divides the line segment joining the points A (-4, 1), B (17, 10) in the ratio 1: 2,

Here $x_1 = -4, y_1 = 1$

$x_2 = 17, y_2 = 10$

$m : n = 1 : 2$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$x = \frac{(1\times17+2\times-4)}{(1+2)}$

$x = \frac{(17+8)}{3}$

$x = \frac{9}{3}$

$x = 3$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(1\times10+2\times1)}{(1+2)}$

$y = \frac{(10+2)}{3}$

$y = \frac{12}{3} = 4$

Hence the co-ordinates of the point P are (3, 4).

(ii) Since O is the origin, the co-ordinates of O are (0, 0).

By distance formula d (OP) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (OP) = $\sqrt{[(0 - 3)^2 + (0 - 4)^2]}$

D (OP) = $\sqrt{[(3)^2 + (4)^2]}$

D (OP) = $\sqrt{(9 + 16)}$

D (OP) = $\sqrt{25}$

D (OP) = 5

Hence the distance OP is 5 units.

(iii) Let $m : n$ be the ratio in which y-axis divide line AB.

Since AB touches y-axis, its x co-ordinate will be zero.

Here $x_1 = -4, y_1 = 1$

$x_2 = 17, y_2 = 10$

By section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$0 = \frac{(m\times17+n\times-4)}{(m+n)}$

$0 = \frac{(17m+4n)}{(m+n)}$

$0 = 17m - 4n$

$17m = 4n$

$\frac{m}{n} = \frac{4}{17}$

$m : n = 4 : 17$

Hence the ratio in which y-axis divide line AB is 4: 17.

**22. Calculate the length of the median through the vertex A of the triangle ABC with vertices A (7, – 3), B (5, 3) and C (3, – 1).**

**Solution:**

Let M (x, y) be the median of ΔABC through A to BC.

M will be the midpoint of BC.

$x_1 = 5, y_1 = 3$

$x_2 = 3, y_2 = -1$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$x = \frac{(5+3)}{2} = \frac{8}{2} = 4$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$y = \frac{(3+(-1))}{2} = \frac{2}{2} = 1$

Hence the co-ordinates of M are (4, 1).

By distance formula d (AM) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$x_1 = 7, y_1 = -3$

$x_2 = 4, y_2 = 1$

D (AM) = $\sqrt{\left[(4-7)^2 + \left(1-(-3)\right)^2\right]}$

D (AM) = $\sqrt{[(-3)^2 + (4)^2]}$

D (AM) = $\sqrt{(9+16)}$

D (AM) = $\sqrt{25}$ = 5

Hence the distance AM is 5 units.

**23. Three consecutive vertices of a parallelogram ABCD are A (1, 2), B (1, 0) and C (4, 0). Find the fourth vertex D.**

**Solution:**

Let M be the midpoint of the diagonals of the parallelogram ABCD.

Co-ordinate of M will be the midpoint of diagonal AC.

Given point are A (1, 2), B (1, 0) and C (4, 0).

Consider the AC.

$x_1 = 1, y_1 = 2$

$x_2 = 4, y_2 = 0$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$x = \frac{(1+4)}{2} = \frac{5}{2}$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$y = \frac{(2+0)}{2} = \frac{2}{2} = 1$

Hence the co-ordinates of M are $\left(\frac{5}{2}, 1\right)$.

M is also the midpoint of diagonal BD.

Consider line BD and M as midpoint.

$x_1 = 1, y_1 = 0$

$x = \frac{5}{2}, y = 1$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$\frac{5}{2} = \frac{(1+x_2)}{2}$

$5 = 1 + x_2$

$x_2 = 5 - 1 = 4$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$1 = \frac{(0+y_2)}{2}$

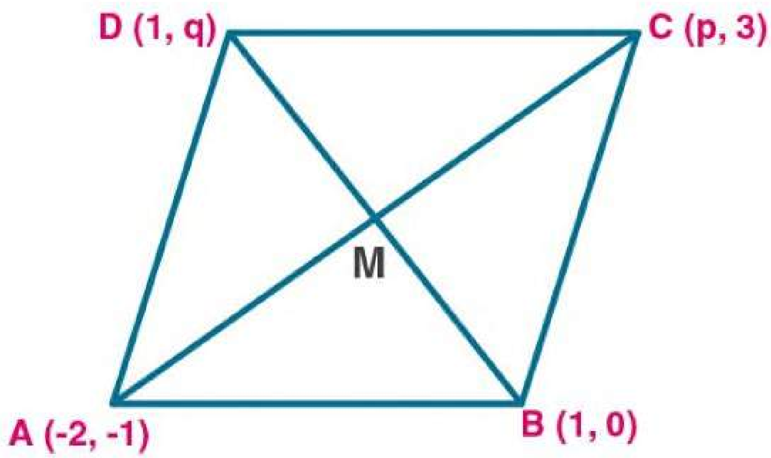
$1 = \frac{y_2}{2}$

$y_2 = 2$

Hence the co-ordinates of D are (4, 2).

**24. If the points A (-2, -1), B (1, 0), C (p, 3) and D (1, q) from, a parallelogram ABCD, find the values of p and q.**

**Solution:**




Given vertices of the parallelogram are A (-2, -1), B (1, 0), C (p, 3) and D (1, q).

Let M (x, y) be the midpoint of the diagonals of the parallelogram ABCD.

Diagonals AC and BD bisect each other at M.

When M is the midpoint of AC

By midpoint formula,

$x = \frac{(-2+p)}{2} = \frac{(p-2)}{2} \quad \ldots \text{(i)}$

$y = \frac{(-1+3)}{2} = \frac{2}{2} = 1 \quad \ldots \text{(ii)}$

When M is the midpoint of BD

By midpoint formula,

$x = \frac{(1+1)}{2} = \frac{2}{2} = 1 \quad \ldots \text{(iii)}$

$y = \frac{(q+0)}{2} = \frac{q}{2} \quad \ldots \text{(iv)}$

Equating (i) and (ii), we get

$\frac{(p-2)}{2} = 1$

$p - 2 = 2$

$p = 2 + 2 = 4$

Equating (iii) and (iv), we get

$\frac{q}{2} = 1$

$q = 2$

Hence the value of p and q are 4 and 2 respectively.

**25. If two vertices of a parallelogram are (3, 2) (-1, 0) and its diagonals meet at (2, -5), find the other two vertices of the parallelogram.**

**Solution:**

Let A (3, 2) and B (-1, 0) be the two vertices of the parallelogram ABCD.

Let M (2, -5) be the point where diagonals meet.

Since the diagonals of the parallelogram bisect each other, M is the midpoint of AC and BD.

Consider A – M – C.

Let Co-ordinate of C be $(x_2,\ y_2)$

$x_1 = 3, y_1 = 2$

$x = 2, y = -5$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$2 = \frac{(3+x_2)}{2}$

$3 + x_2 = 4$

$x_2 = 4 - 3 = 1$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$-5 = \frac{(2+y_2)}{2}$

$-10 = 2 + y_2$

$y_2 = -10 - 2 = -12$

Hence the co-ordinates of the point C are (1, -12).

Consider B – M – D

Let co-ordinate of D be $(x_2,\ y_2)$

$x_1 = -1, y_1 = 0$

$x = 2, y = -5$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$2 = \frac{(-1+x_2)}{2}$

$-1 + x_2 = 4$

$x_2 = 4 + 1 = 5$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$-5 = \frac{(0+y_2)}{2}$

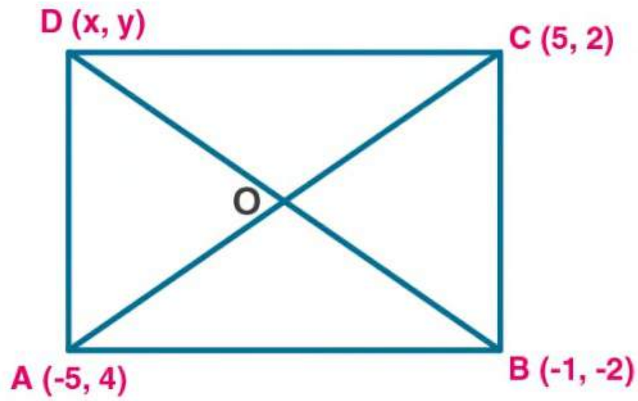
$-10 = 0 + y_2$

$y_2 = -10$

Hence the co-ordinate of the point D are (5, -10).

**26. Prove that the points A (-5, 4), B (-1, -2) and C (5, 2) are the vertices of an isosceles right angled triangle. Find the co-ordinates of D so that ABCD is a square.**

**Solution:**




Given points are A (-5, 4), B (-1, -2) and C (5, 2) are given.

Since these are vertices of an isosceles triangle ABC then AB = BC.

By distance formula d (AB) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = -5, y_1 = 4$

$x_2 = -1, y_2 = -2$

D (AB) = $\sqrt{\left[\left(-1-(-5)\right)^2 + (-2-4)^2\right]}$

D (AB) = $\sqrt{[(4)^2 + (6)^2]}$

D (AB) = $\sqrt{(16 + 36)}$

D (AB) = $\sqrt{52}$ …. (i)

By distance formula d (BC) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = -1, y_1 = -2$

$x_2 = 5, y_2 = 2$

D (BC) = $\sqrt{\left[\left(5-(-1)\right)^2+\left(-2-(-2)\right)^2\right]}$

D (BC) = $\sqrt{[(6)^2+(4)^2]}$

D (BC) = $\sqrt{(36+16)}$

D (BC) = $\sqrt{52}$ …. (ii)

From (i) and (ii) AB = BC

So given points are the vertices of isosceles triangle.

By distance formula d (AC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

Here $x_1=-5, y_1=4$

$x_2=5, y_2=2$

D (AC) = $\sqrt{\left[\left(5-(-5)\right)^2+(2-4)^2\right]}$

D (AC) = $\sqrt{[(10)^2+(-2)^2]}$

D (AC) = $\sqrt{(100+4)}$

D (AC) = $\sqrt{104}$ …. (iii)

Apply Pythagoras theorem to triangle ABC

$AB^2+BC^2=\left(\sqrt{52}\right)^2+\left(\sqrt{52}\right)^2$

= 52 + 52

= 104 …. (iv)

$AC^2=\left(\sqrt{104}\right)^2=104$ …. (v)

From (iv) and (v) we got

$AB^2+BC^2=AC^2$

So Pythagoras theorem is satisfied.

So the triangle is an isosceles right angled triangle.

Hence proved.

If ABCD is a square, let the diagonals meet at O.

Diagonals of a square bisect each other. So, O is the midpoint of AC and BD.

Consider A-O-C

$x_1 = -5, y_1 = 4$

$x_2 = 5, y_2 = 2$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$x = \frac{(-5+5)}{2} = \frac{0}{2} = 0$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$y = \frac{(4+2)}{2} = \frac{6}{2} = 3$

So co-ordinate of O is (0, 3)

Consider B-O-D

Let co-ordinate of D be $(x_2,\ y_2)$

$x_1 = -1, y_1 = -2$

$x = 0, y = 3$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$0 = \frac{(-1+x_2)}{2}$

$-1 + x_2 = 0$

$x_2 = 1$

By midpoint formula, $y = \frac{(x_1+x_2)}{2}$

$3 = \frac{(-2+y_2)}{2}$

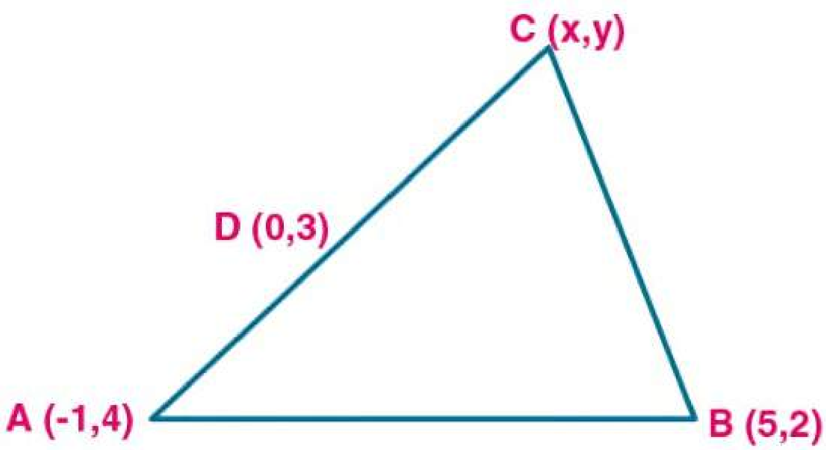
$6 = -2 + y_2$

$y_2 = 6 + 2 = 8$

Hence the co-ordinates of the point D are (1, 8).

**27. Find the third vertex of a triangle if its two vertices are (-1, 4) and (5, 2) and midpoint of one sides is (0, 3).**

**Solution:**




Let A (-1, 4) and B (5, 2) are the vertices of the triangle and let D (0, 3) si the midpoint of side AC.

Let co-ordinate of C be (x, y).

Consider D (0, 3) as midpoint of AC

By midpoint formula,

$\frac{(-1+x)}{2} = 0$

$-1 + x = 0$

$x = 1$

By midpoint formula,

$\frac{(4+y)}{2} = 3$

$4 + y = 6$

$y = 6 - 4 = 2$

So the co-ordinates of C are (1, 2).

Consider D (0, 3) as midpoint of BC

By midpoint formula,

$\frac{(5+x)}{2} = 0$

$5 + x = 0$

$x = -5$

By midpoint formula,

$\frac{(2+y)}{2} = 3$

$2 + y = 6$

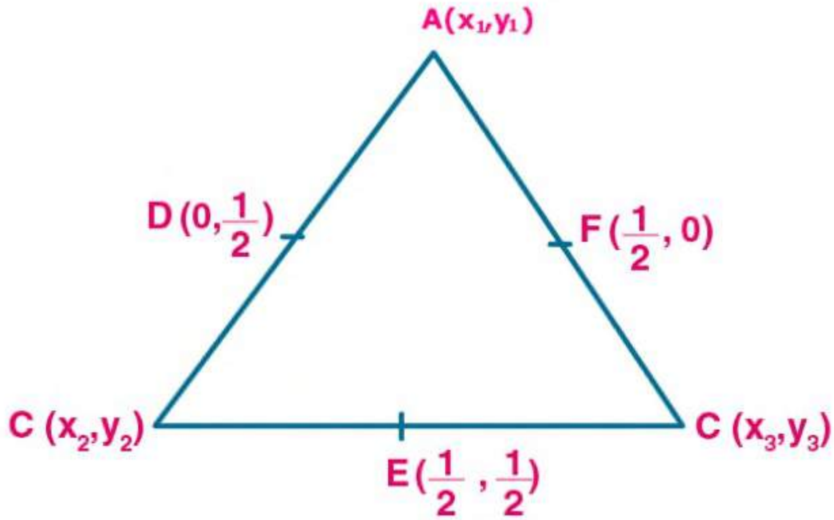
$y = 6 - 2 = 4$

So the co-ordinates of C are $(-5, 4)$.

Hence the co-ordinates of the point C will be (1, 2) or $(-5, 4)$.

**28. Find the coordinates of the vertices of the triangle the middle points of whose sides are $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 0\right)$.**

**Solution:**




Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC.

Consider AB

By midpoint formula, $\frac{(x_1+x_2)}{2} = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2 \qquad \ldots \text{(i)}$

By midpoint formula, $\frac{(y_1+y_2)}{2} = \frac{1}{2}$

$y_1 + y_2 = 0$

$y_1 = -y_2 \qquad \ldots \text{(ii)}$

Consider AC

By midpoint formula, $\frac{(x_1+x_3)}{2} = \frac{1}{2}$

$x_1 + x_3 = 1 \qquad \ldots \text{(iii)}$

By midpoint formula, $\frac{(y_1+y_3)}{2} = 0$

$y_1 + y_3 = 0$

$y_1 = -y_3 \qquad \ldots \text{(iv)}$

Consider BC

By midpoint formula, $\frac{(x_2+x_3)}{2} = \frac{1}{2}$

$x_2 + x_3 = 1$ … (v)

By midpoint formula, $\frac{(y_2+y_3)}{2} = \frac{1}{2}$

$y_2 + y_3 = 1$ … (vi)

Substitute (i) in (iii)

Then (iii) becomes $-x_2 + x_3 = 1$

Equation (v) $x_2 + x_3 = 1$

Adding above two equation, we get

$2x_3 = 2$

$x_3 = \frac{2}{2} = 1$

Substitute $x_3 = 1$ in (iii), we get $x_1 = 0$

$x_2 = 0$ [From (i)]

So $x_1 = 0, x_2 = 0, x_3 = 1$

Substitute (iv) in (ii)

Then (ii) becomes $-y_3 + y_2 = 1$

Equation (vi) $y_2 + y_3 = 1$

Adding above two equation, we get

$2y_2 = 2$

$y_2 = \frac{2}{2} = 1$

Substitute $y_2 = 1$ in (i), we get $y_1 = 0$

$y_3 = 0$

So $y_1 = 0, y_2 = 1, y_3 = 0$

Hence the co-ordinates of vertices are A (0, 0), B (0, 1) and C (1, 0).

**29. Show by section formula that the points (3, -2), (5, 2) and (8, 8) are collinear.**

**Solution:**

Let the point B (5, 2) divides the line joining A (3, -2) and C (8, 8) in the ratio m: n.

Then by section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$5 = \frac{(m\times 8+n\times 3)}{(m+n)}$

$5 = \frac{(8m+3n)}{(m+n)}$

$5m + 5n = 8m + 3n$

$2n = 3m$

$\frac{m}{n} = \frac{2}{3}$ … (i)

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$2 = \frac{(m\times 8+n\times -2)}{(m+n)}$

$2 = \frac{(8m-2n)}{(m+n)}$

$2m + 2n = 8m - 2n$

$6m = 4n$

$\frac{m}{n} = \frac{4}{6} = \frac{2}{3}$ …. (ii)

Here ratios are same.

So the points are collinear.

**30. Find the value of p for which the points (-5, 1), (1, p) and (4, -2) are collinear.**

**Solution:**

Let A (-5, 1) divides the line joining (1, p) and (4, -2) in the m: n.

Then by section formula, $x = \frac{(mx_2+nx_1)}{(m+n)}$

$-5 = \frac{(m\times4+n\times1)}{(m+n)}$

$-5 = \frac{(4m+n)}{(m+n)}$

$-5m + 5n = 4m + n$

$-9m = 6n$

$\frac{m}{n} = \frac{-9}{6} = \frac{-2}{3}$ ... (i)

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$1 = \frac{(m\times8+n\times p)}{(m+n)}$

$2 = \frac{(-2m-pn)}{(m+n)}$

$m + n = -2m + pn$

$3m = (p-1)n$

$\frac{m}{n} = \frac{(p-1)}{3}$ .... (ii)

Equating (i) and (ii)

$\frac{(p-1)}{3} = \frac{-2}{3}$

$p - 1 = -2$

$p = -2 + 1 = -1$

Hence the value of p is – 1.

**31. A (10, 5), B (6, -3) and C (2, 1) are the vertices of triangle ABC. L is the midpoint of AB, M is the mid-point of AC. Write down the co-ordinates of L and M. Show that LM = $\frac{1}{2}$ BC.**

**Solution:**

Given points are A (10, 5), B (6, -3) and C (2, 1)

Let L (x, y) be the midpoint of AB.

Here $x_1 = 10, y_1 = 5$

$x_2 = 6, y_2 = -3$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$x = \frac{(10+6)}{2} = \frac{16}{2} = 8$

By midpoint formula, $y = \frac{(x_1+x_2)}{2}$

$y = \frac{(5-3)}{2} = \frac{2}{2} = 1$

So co-ordinates of L are (8, 1).

Let M (x, y) be the midpoint of AC.

Here $x_1 = 10, y_1 = 5$

$x_2 = 2, y_2 = 1$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$x = \frac{(10+2)}{2} = \frac{12}{2} = 6$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$y = \frac{(5+1)}{2} = \frac{6}{2} = 3$

So co-ordinates of M are (6, 3).

By distance formula d (LM) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Here $x_1 = 8, y_1 = 1$

$x_2 = 6, y_2 = 3$

D (LM) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (LM) = $\sqrt{[(6 - 8)^2 + (3 - 1)^2]}$

D (LM) = $\sqrt{[(-2)^2 + (2)^2]}$

D (LM) = $\sqrt{(4 + 4)}$

D (LM) = $\sqrt{8} = 2\sqrt{2}$ …. (i)

By distance formula d (BC) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are B (6, -3) and C (2, 1)

So $x_1 = 6, y_1 = -3$

$x_2 = 2, y_2 = 1$

D (BC) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (BC) = $\sqrt{\left[(2 - 6)^2 + \left(1 - (-3)\right)^2\right]}$

D (BC) = $\sqrt{[(-4)^2 + (4)^2]}$

D (BC) = $\sqrt{(16 + 16)}$

D (BC) = $\sqrt{32} = 4\sqrt{2}$ …. (ii)

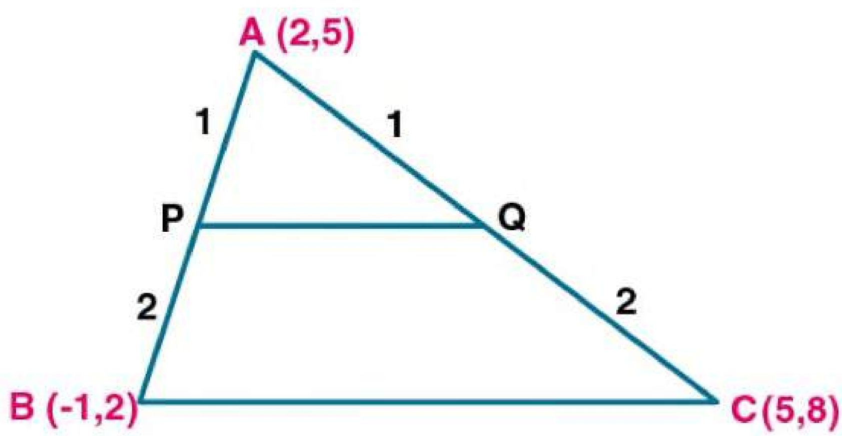
From (i) and (ii), LM = $\frac{1}{2}$ BC.

**32. A (2, 5), B (-1, 2) and C (5, 8) are the vertices of a triangle ABC. P and Q are points on AB and AC respectively such that AP: PB = AQ: QC = 1: 2.**

**(i) Find the co-ordinates of P and Q.**

**(ii) Show that PQ = $\frac{1}{3}$ BC**

**Solution:**




(i) Given vertices of the ABC are A (2, 5), B(-1, 2) and C (5, 8)

P and Q are points on AB and AC respectively such that AP: PB = AQ: QC = 1: 2.

P (x, y) divides AB in the ratio 1: 2.

$x_1 = 2, y_1 = 5$

$x_2 = -1, y_2 = 2$

$m : n = 1 : 2$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$x = \frac{(1\times-1+2\times2)}{(1+2)}$

$x = \frac{(-1+4)}{3}$

$x = \frac{3}{3} = 1$

By section formula $y = \frac{(my_2 + ny_1)}{(m+n)}$

$y = \frac{(1\times2+2\times5)}{(1+2)}$

$y = \frac{(2+10)}{3}$

$y = \frac{12}{3} = 4$

Co-ordinates of P are (1, 4).

Q (x, y) divides AC in the ratio 1: 2.

$x_1 = 2, y_1 = 5$

$x_2 = 5, y_2 = 8$

$m : n = 1 : 2$

By section formula, $x = \frac{(mx_2 + nx_1)}{(m+n)}$

$x = \frac{(1\times5+2\times2)}{(1+2)}$

$x = \frac{(5+4)}{3}$

$x = \frac{9}{3} = 3$

By section formula $y = \frac{(my_2+ny_1)}{(m+n)}$

$y = \frac{(1\times8+2\times5)}{(1+2)}$

$y = \frac{(8+10)}{3}$

$y = \frac{18}{3} = 6$

Co-ordinates of Q are (3, 6).

(ii) By distance formula d (PQ) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are P (1, 4) and Q (3, 6)

So $x_1 = 1, y_1 = 4$

$x_2 = 3, y_2 = 6$

D (PQ) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (PQ) = $\sqrt{[(3 - 1)^2 + (6 - 4)^2]}$

D (PQ) = $\sqrt{[(2)^2 + (2)^2]}$

D (PQ) = $\sqrt{(4 + 4)}$

D (PQ) = $\sqrt{8} = 2\sqrt{2}$ …. (i)

By distance formula d (BC) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

The points are B (-1, 2) and C (5, 8)

Points are B (-1, 2) and C (5, 8).

So $x_1 = -1, y_1 = 2$

$x_2 = 5,\ y_2 = 8$

D (BC) = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

D (BC) = $\sqrt{\left[(5-(-1))^2 + (8-2)^2\right]}$

D (BC) = $\sqrt{[(6)^2 + (6)^2]}$

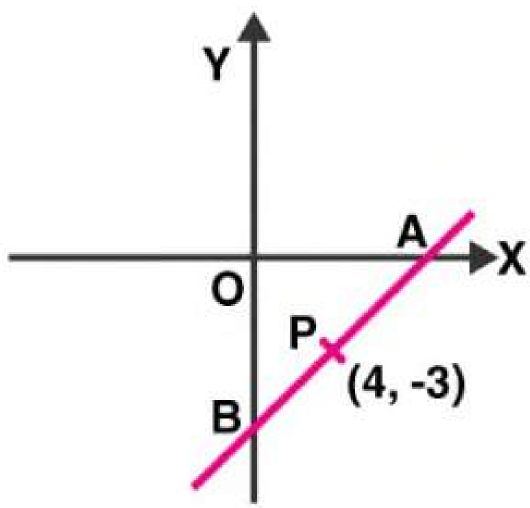
D (BC) = $\sqrt{(36+36)}$

D (BC) = $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$ …. (ii)

$\frac{BC}{3} = \frac{6\sqrt{2}}{3} = \frac{2}{\sqrt{2}} = \text{PQ}$

Hence proved.

**33. The mid-point of the line segment AB shown in the adjoining diagram is (4, -3). Write down the co-ordinates of A and B.**




**Solution:**

Let P (4, -3) be the midpoint of line joining the points A and B.

Since A lies on x-axis, its co-ordinates are $(x_2, 0)$

Since B lies on y-axis, its co-ordinates are $(0, y_1)$

By midpoint formula, $x = \frac{(x_1+x_2)}{2}$

$4 = \frac{(0+x_2)}{2}$

$x_2 = 4 \times 2 = 8$

By midpoint formula, $y = \frac{(y_1+y_2)}{2}$

$-3 = \frac{(y_1+0)}{2}$

$y_1 = -3 \times 2 = -6$

Hence the co-ordinates of A and B are (8, 0) and (0, -6) respectively.

**34. Find the co-ordinates of the centroid of a triangle whose vertices are A (-1, 3), B (1, -1) and C (5, 1). (2006)**

**Solution:**

Given vertices of the triangle are A (-1, 3), B (1, -1) and C (5, 1).

Co-ordinates of the centroid of a triangle, whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$

$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right]$

$(x_1, y_1) = (-1, 3)$

$(x_2, y_2) = (1, -1)$

$(x_3, y_3) = (5, 1)$

$\frac{(x_1+x_2+x_3)}{3} = \frac{(-1+1+5)}{3} = \frac{5}{3}$

$\frac{(y_1+y_2+y_3)}{3} = \frac{(3-1+1)}{3} = \frac{3}{3} = 1$

Hence the co-ordinates of centroid are $\left(\frac{5}{3}, 1\right)$.

**35. Two vertices of a triangle are $(3, -5)$ and $(-7, 4)$. Find the third vertex given that the centroid is (2, -1).**

**Solution:**

Let third vertex be $C(x_3, y_3)$.

Given $(x_1, y_1) = (3, -5)$

$(x_2, y_2) = (-7, 4)$

Co-ordinates of centroid are (2, -1)

Co-ordinates of the centroid of a triangle, whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$

$$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right]$$

$\frac{(x_1+x_2+x_3)}{3} = \frac{(3+7+x_3)}{3} = 2$ [$x$ Co-ordinate of centroid]

$-4 + x_3 = 2 \times 3$

$-4 + x_3 = 6$

$x_3 = 6 + 4$

$x_3 = 10$

$\frac{(y_1+y_2+y_3)}{3} = -1$ [$y$ Co-ordinate of centroid]

$-5 + 4 + y_3 = -1 \times 3$

$-1 + y_3 = -3$

$y_3 = -3 + 1$

$y_3 = -2$

Hence the third vertex is $(10, -2)$.

**36. The vertices of a triangle are A (-5, 3), B (p, -1) and C (6, q). Find the values of p and q if the centroid of the triangle ABC is the point (1, -1).**

**Solution:**

Given vertices of the triangle are A (-5, 3), B (p, -1) and C (6, q).

Co-ordinates of centroid are (1, -1).

Co-ordinates of the centroid of a triangle, whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$

$$\left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right]$$

$$(x_1, y_1) = (-5, 3)$$

$$(x_2, y_2) = (p, -1)$$

$$(x_3, y_3) = (6, q)$$

X co-ordinate of centroid, $\frac{(x_1+x_2+x_3)}{3} = \frac{(-5+p+6)}{3} = 1$

$$p + 1 = 3$$

$$p = 3 - 1$$

$$p = 2$$

Y co-ordinate of centroid, $\frac{(y_1+y_2+y_3)}{3} = \frac{(3-1+q)}{3} = -1$

$$2 + q = 3 \times -1$$

$$2 + q = -3$$

$$q = -3 - 2$$

$$q = -5$$

Hence the value of p and q are 2 and -5 respectively.