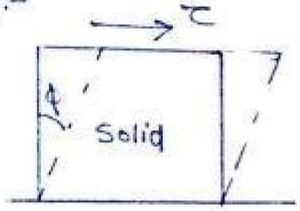
Fluid Properties

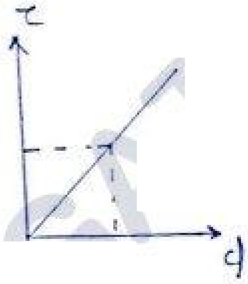
Fluid Mechanics:-
It is the science which deals the fluid (liquid & gas) in rest or in motion.

Fluid :

Solid:-



Hook's law



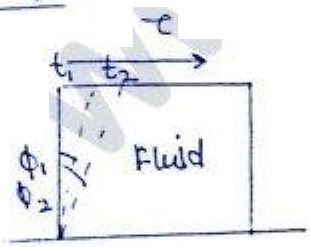
$\tau \propto \phi$ → shear strain

$$\tau = G\phi \Rightarrow G = \frac{\tau}{\phi}$$

$G \propto \frac{1}{\phi}$
↓
rigidity

$\tau \neq 0$
$\phi \to 0 \Rightarrow G \to \infty$

Fluids:-

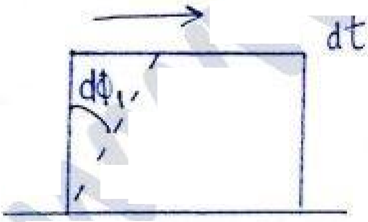


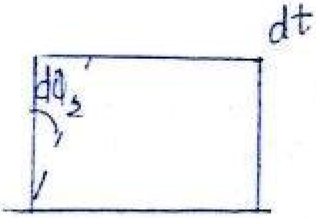
$$\tau \propto \frac{\phi}{t} \qquad \tau \propto \frac{d\phi}{dt} \Rightarrow \text{Rate of shear deformation}$$

$$\tau = \mu \frac{d\phi}{dt}$$

*Viscocity defines only when fluid in motion.

$$\mu = \frac{\tau}{\frac{d\phi}{dt}} \Rightarrow \boxed{\mu \propto \frac{1}{\left(\frac{d\phi}{dt}\right)}}$$





$$\boxed{\text{Fluidity} = \frac{1}{\mu}}$$

$$\left(\frac{d\phi_1}{dt}\right) > \left(\frac{d\phi_2}{dt}\right) \Rightarrow \boxed{\mu_1 < \mu_2}$$

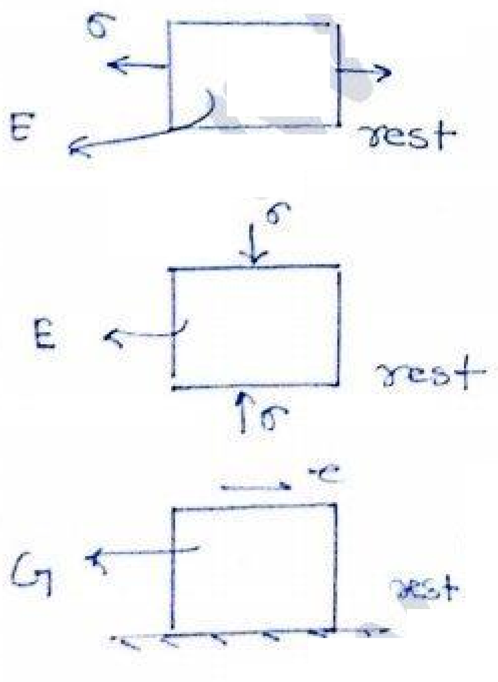
Fluid :-

A Fluid is a substance which is capable of flowing and deforming under the action of shear force, however the small force may be.

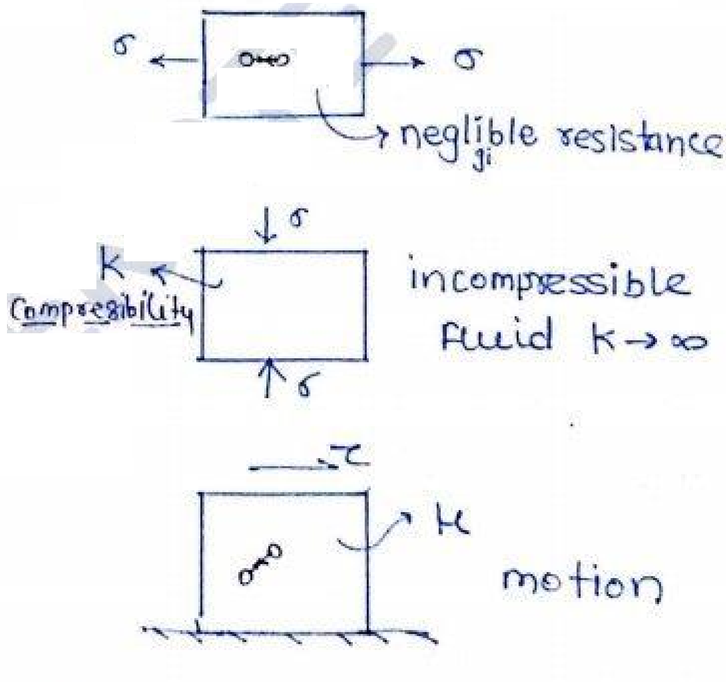
In solids deformation almost Constant with time but in Fluids deformation Changes with time so in Fluids rate of deformation is more important then deformation.

Note :-

Solids



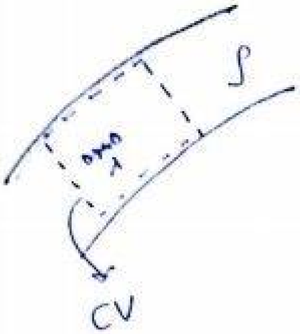
Fluids

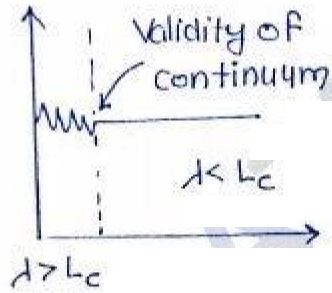


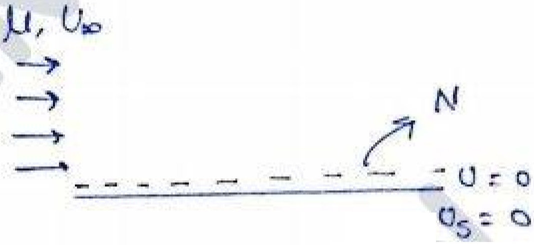
Solids can resist tensile, comp. & shear loads under static condition.

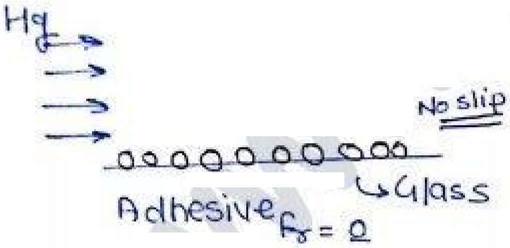
Fluids have negligable resitance for tensile loading. They can resist comp. loads & shear loads under dynamic condn due to viscocity of Fluids so the property known as viscosity observed under dynamic cond^n.

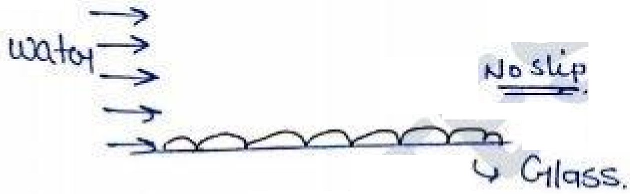
Concept of Continuum :-











when the viscous fluid flows over solid surface bulk of fluid particles aquires the velocity of the surface and this sticking phinomena of viscous fluid is know as ~~concept~~ no slip phinomena.

It is define by Concept of Continuum

Density/ Mass Density :- It is defined as mass per unit volume at a specified temp. and pressure.

$$\boxed{\rho = \frac{m}{V} \left(\frac{kg}{m^3}\right) (SI)}$$

Eg:- <u>water</u>

at 4°c 1 atm $\rho_w = 1000\ kg/m^3$

20°c 1 atm $\rho_w = 998\ kg/m^3$

20°c 100 atm $\rho_w = 1003\ kg/m^3$

<u>Air</u>

1 atm 20°C $\rho_{air} = 1.23\ kg/m^3$

$\underbrace{100 kPa,\ 0°c}_{S.T.P.}$ $\rho_{air} = 1.27\ kg/m^3$

Specific weight/weight density :-

$$\boxed{w = \frac{Weight}{Volume}}\ N/m^3 \qquad w = \rho g$$

$$w = \frac{mg}{V} = \underline{\rho g}\ (N/m^3)$$

<u>Ex</u>: <u>water</u> $w_w = 1000 \times 9.81 \Rightarrow w_w = 9810\ N/m^3$

** $\boxed{1\ kgf = 9.81\ N}$

$w_w = 1000 \times 9.81\ N/m^3$

$w_w = 1000\ kgf/m^3$

Relative density :-

$$R.D. = \frac{\rho_1}{\rho_2}$$ density w.r.t. other fluid density.

Specific Gravity :-

$$* \boxed{(S.G)_f = \frac{\rho_f}{\rho_{standard}}}$$

liquid → $\rho_w = 1000\, kg/m^3$

air → $\rho_{air} = 1.27\, kg/m^3$

<u>liquid</u> $(SG)_f = \frac{\rho_f}{\rho_w} = \frac{\rho_f g}{\rho_w g} = \frac{w_f}{w_w}$

<u>eg</u> $S_{Hg} = 13.6 \Rightarrow \frac{w_{Hg}}{w_w} = 13.6 \Rightarrow w_{Hg} = 13.6\, w_w$

<u>eg</u> $S_{gasoline} = 0.75$

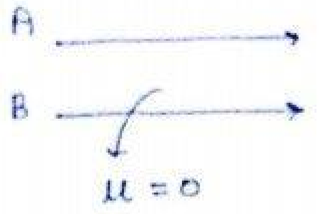
$S_{wood} = 0.6$

$\frac{\rho_{wood}}{\rho_w} = 0.6 \Rightarrow \underline{\rho_{wood}} = 600\, kg/m^3$

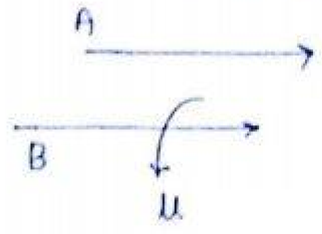
Viscosity :- The property of the fluid which offers internal resistance between two adjucent layer is known as viscosity.

Viscosity is due to (i) Cohesive forces (in liquids)

(ii) molecular momentum transfer in perpendicular dirn b/w two adjucent layer.

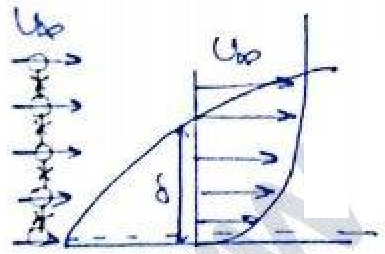


relative motion



Viscosity

Liquid
cohesive force.



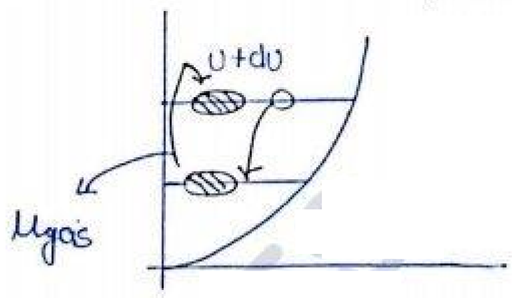
$\mu_{Liquids} \propto \frac{1}{T}$

$\underline{\mu_{liq.} \downarrow}$ as $\underline{T \uparrow}$

$\therefore$ [Cohesive force decrease.]

Gases

molecular momentum transfer



$\mu_{gas} \uparrow \rightarrow$ randomness $\uparrow$

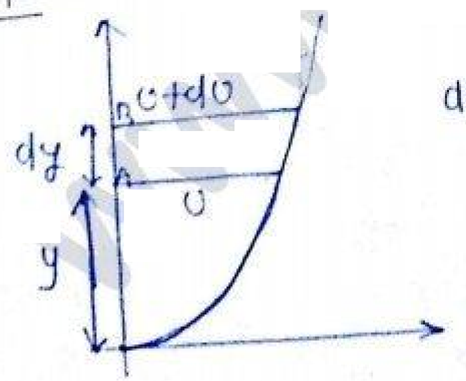
randoness $\propto V_{rms}$ → root mean square.

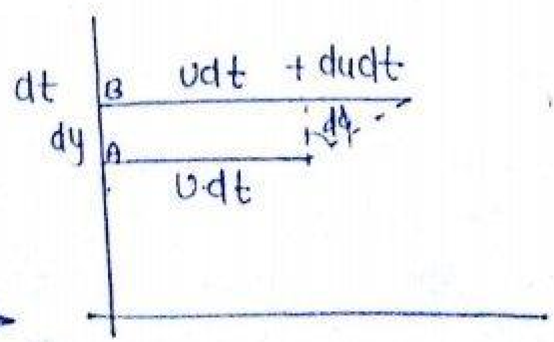
$\mu_{gas} \propto V_{rms}$

$V_{rms} = \sqrt{\frac{3kT}{m}}$

$\boxed{\mu_{gas} \propto \sqrt{T}}$

Note:-





$$\tan d\phi = \frac{du\,dt}{dy}$$

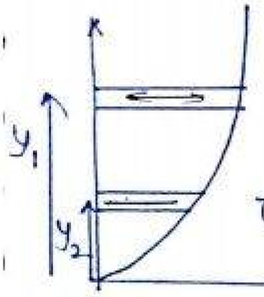
for small angle $\tan d\phi = d\phi$

rate of shear strain $\dfrac{d\phi}{dt} = \dfrac{du}{dy}$

★ Velocity Gradient larger at neares to Surface

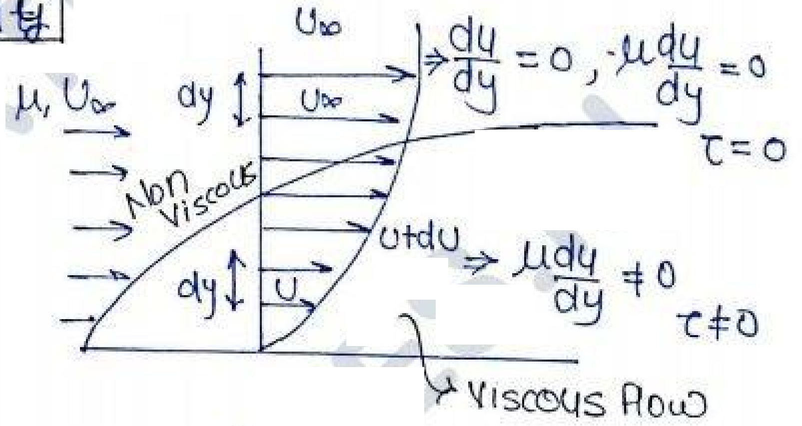
$$\tau = \mu \frac{d\phi}{dt}$$

★ $$\boxed{\tau = \mu \frac{du}{dy}}$$



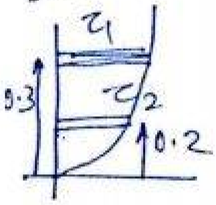
$$\tau_1 = \mu \frac{du}{dy}\Big|_{y=y_1}$$

$$\tau_2 = \mu \frac{du}{dy}\Big|_{y=y_2}$$



Bernoulie's equation is not valid only in Viscous flow region. (or boundary layer region)

Q.31
WB P10



$$u = \frac{3}{4}y - y^2 \qquad \mu = 0.84\ \mathrm{Ns/m^2}$$

$$\tau_1 = \mu \frac{du}{dy} \qquad \frac{du}{dy} = \frac{3}{4} - 2y$$

$$\tau_1 = \mu \frac{du}{dy}\Big|_{y=0.3} = 0.84 \times \left(\frac{3}{4} - 0.3\times 2\right) =$$

$$\tau_2 = \mu \frac{du}{dy}\Big|_{y=0.2} = 0.84 \times \left(\frac{3}{4} - 0.2\times 2\right) =$$

$$\tau_1 = N\tau_2 \Rightarrow N = \frac{\tau_1}{\tau_2} = \underline{0.428}$$

Units of Viscosity :-

Viscosity / Dynamic Viscosity / Absolute Viscosity (μ)

$$\tau = \mu \frac{du}{dy} \Rightarrow$$

$$\Rightarrow \mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \quad \frac{N/m^2}{\left(\frac{m/s}{m}\right)}$$

Force unit $\qquad \mu \Rightarrow \frac{Ns}{m^2}$ (SI)

mass unit $\qquad \mu \Rightarrow \frac{kg}{ms}$ (SI)

$$\boxed{\frac{Ns}{m^2} = \frac{kg}{ms}}$$

$$\Rightarrow \frac{Ns}{m^2} = \frac{(9.81\,N)s}{9.81\ m^2}$$

$$\frac{Ns}{m^2} = \frac{1}{9.81} \frac{kgf\text{-}sec.}{m^2}$$

C.G.S

$$\boxed{\frac{g}{cm\text{-}s} = \text{Poise}}$$

$$\frac{10^{-3}\,kg}{10^{-2}\,m\text{-}s} = \text{Poise}$$

$$0.1 \frac{kg}{m\text{-}s} = 0.1 \frac{NS}{m^2} = \text{Poise}$$

$$N/m^2 = Pa \Rightarrow \boxed{1\,\text{Poise} = 0.1\,Pa\text{-}Sec}$$

$$\mu \begin{cases} \text{SI} \to \dfrac{Ns}{m^2} = \dfrac{kg}{m\text{-sec}} \\ \text{CGS} \to \dfrac{g}{cm\text{-sec}} = \dfrac{dyne\text{-sec}}{cm^2} \end{cases}$$

$$\boxed{1N = 10^5 \text{ dyne}}$$

Ex:

at 20°c $\mu_{water} = 1$ centi Poise

$$\boxed{\mu_w = 10^{-2} \times 0.1 \text{ Pas}}$$

$$* \boxed{\mu_w = 10^{-3} \text{ Pa-s}}$$

$$* \boxed{\mu_{air} = 2 \times 10^{-5} \text{ Pa-s}}$$

$$\frac{\mu_w}{\mu_{air}} = \frac{10^{-3}}{2\times10^{-5}} = 50 \Rightarrow \boxed{\mu_w = 50\,\mu_{air}} *$$

$$\boxed{\mu_w > \mu_{air}}$$

kinematic Viscosity / Momentum diffusivity :- (ν)

S.I. $\boxed{\nu = \frac{\mu}{\rho}}$ $\dfrac{kg/ms}{kg/m^3}$

$$\boxed{\nu \Rightarrow m^2/s} \text{ (S.I.)}$$

C.G.S

$\frac{cm^2}{s}$ = stoke

$10^{-4}\, m^2/s$ = 1 stoke

$$\boxed{m^2/s = 10^4 \text{ stoke}}$$

at 20°c

$\mu_w = 10^{-3}\, kg/ms$	$\mu_{air} = 2\times10^{-5}\, kg/ms$
$\rho_w = 1000\, kg/m^3$	$\rho_{air} = 1.23\, kg/m^3$

$$\nu_w = \frac{10^{-3}}{10^3} = 10^{-6} \qquad \nu_{air} = \frac{2\times10^{-5}}{1.2} = 1.6\times10^{-5}$$

$$\boxed{\nu_w < \nu_{air}}$$

Effect of temprature and pressure on Viscosity

Temp.

$$\begin{array}{ccc} & \mu & \\ \swarrow^{T\uparrow} & & \searrow^{T\uparrow} \\ \mu_{liq} & & \mu_{gas} \end{array}$$

$T\uparrow \rightarrow$ cohesive $\downarrow$

$\mu_{liq}\downarrow$

$\mu_{gas} \propto \sqrt{T}$

$\mu_{gas}\uparrow$

$$\begin{array}{ccc} & \nu & \\ \swarrow^{T\uparrow} & & \searrow^{T\uparrow} \\ \nu_{liq} & & \nu_{gas} \end{array}$$

$\nu_{liq} = \dfrac{\mu_{liq}\downarrow}{\rho_{liq}} \leftarrow$ const almost

$\nu_{liq}\downarrow$

$\nu_{gas} = \dfrac{\mu_{gas}}{\rho_{gas}}$

$P = \rho R T \Rightarrow \rho \propto \dfrac{1}{T}$

$T \rightarrow$ Abs temp

$\nu_{gas} \propto \dfrac{\sqrt{T}}{1/T}$

$\nu_{gas} \propto (T)^{3/2}$

$\nu_{gas}\uparrow$

$T\uparrow$	Liq	Gas
μ	$\downarrow$	$\uparrow$
ν	$\downarrow$	$\uparrow$

Pressure:-

$$\begin{array}{ccc} & \mu & \\ \swarrow^{P\uparrow} & & \searrow^{P\uparrow} \\ \mu_{liq}\times & & \mu_{gas}\ \times \end{array}$$

incompressible.

$\mu_{gas} = \sqrt{\dfrac{3KT}{m}}$

$\mu_{gas} \neq f(P)$

$\mu_{gas} = f(t)$

$$\begin{array}{ccc} & \nu & \\ \swarrow^{P\uparrow} & & \searrow^{P\uparrow} \\ \nu_{liq} & & \nu_{gas} \end{array}$$

$\nu_{liq} = \dfrac{\mu_{liq}\times}{\rho_{liq}\times}$

$\rho \neq f(P)$

$\nu_{gas} = \dfrac{\mu_{gas}}{\rho_{gas}\uparrow}$

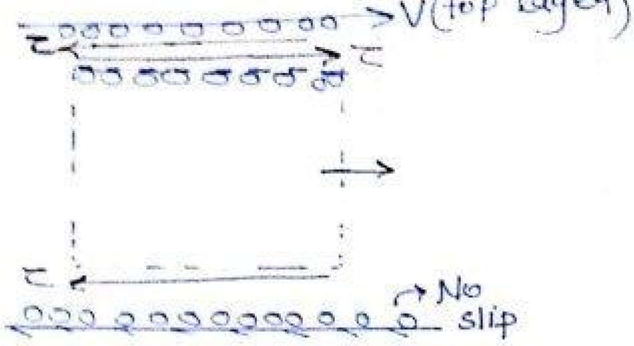
$P = \rho R T$

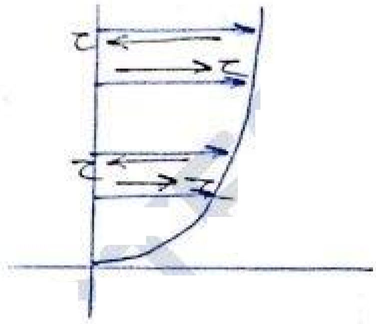
$\rho\uparrow = \dfrac{P\uparrow}{RT}$

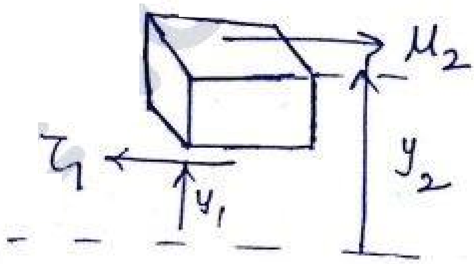
$P\uparrow$	Liq.	Gas.
μ	$\times$	$\times$
ν	$\times$	$\downarrow$

free body diagram:-

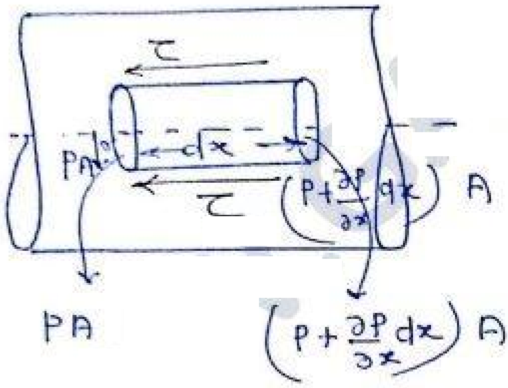
over plate





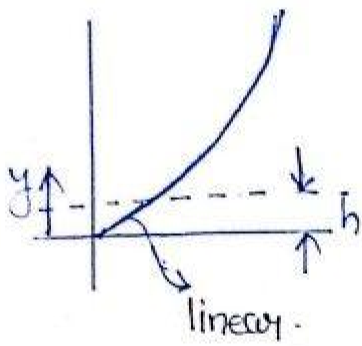


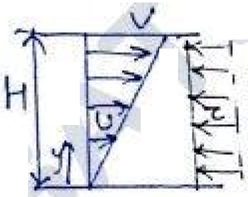
in pipe:-



linear Velocity profile assumption:-

- Assume velocity profile linear for small Gap (shaft + bearing)





$$\frac{U}{y} = \frac{V}{H} \Rightarrow U = \frac{V}{H}y$$

or $U = a + by$

$y = 0,\ U = 0,\ a = 0$

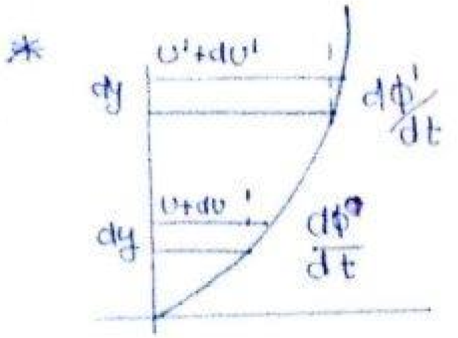
$U = by,\quad y = H,\quad U = V$

$V = bH \Rightarrow \boxed{U = \frac{V}{H}y}$

$$\tau = \mu\frac{dU}{dy}$$

$$\tau = \mu\frac{V}{H} \quad \tau = \text{Const}$$

$$\tau \neq f(y)$$

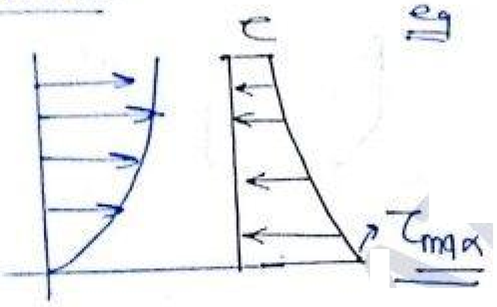


$$\frac{d\phi}{dt} > \frac{d\phi'}{dt}$$

$$\frac{du}{dy} > \frac{du'}{dy} \Rightarrow \mu\frac{du}{dy} > \mu\frac{du'}{dy}$$

$$\underline{\tau > \tau'}$$

in practicle



$u = \frac{3}{4}y - y^2$

eg $\tau = \mu\frac{du}{dy}$ $\quad u = \frac{3}{4}y - y^2$

$$\tau = \mu\left(\frac{3}{4} - 2y\right)$$

$$\tau = \mu\frac{3}{4} - 2\mu y$$

$$y = c - mx$$

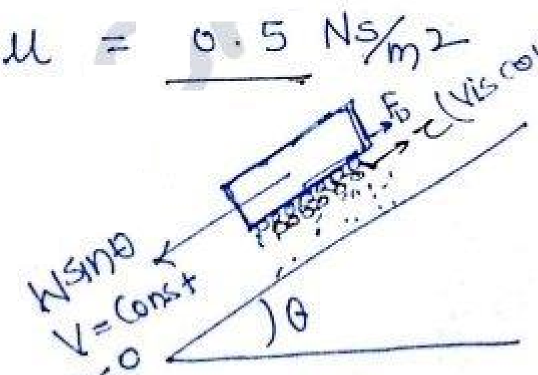
Q.20 $\tau = \mu\frac{du}{dy}$ $\quad \underline{\tau = F \times A}$

$\frac{F \times A}{A} = \mu\frac{V}{y} \Rightarrow$ $\quad \sin\theta = \frac{5}{13}$

$$\frac{130 \times \frac{5}{13}}{1} = \mu \times \frac{0.5 \times 100}{0.5}$$

$$\mu = \underline{0.5\ Ns/m^2}$$

Opposition due to viscous action



$\sin\theta = \frac{5}{13}$

Assume $\tau = \mu\frac{V}{H}$

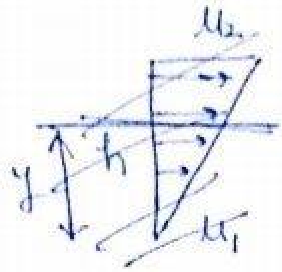
Velocity profile is linear

$\Sigma_{net} = 0$

$$W\sin\theta = F_D = \tau \times A$$

$$130 \times 5/13 = \mu \times \frac{0.5 \times 100}{0.5} \times 1 \Rightarrow \mu = 0.5\ Ns/m^2$$

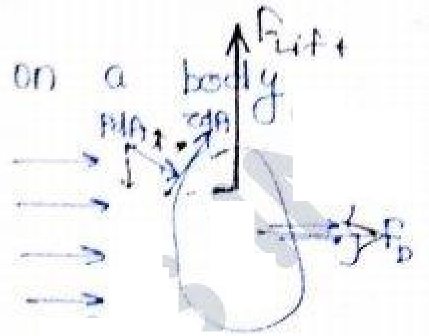
Q. 38



$$\tau_1 = \mu_1 \frac{V_1}{h}$$

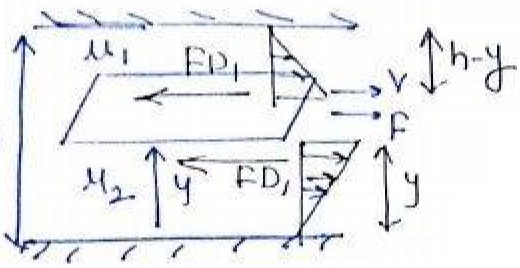
$$\tau_2 = \mu_2 \frac{V_2}{h}$$

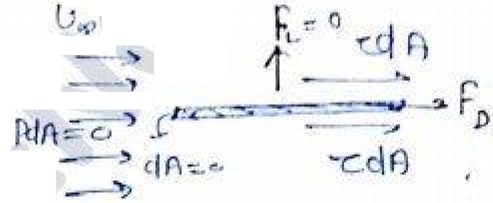
on a body

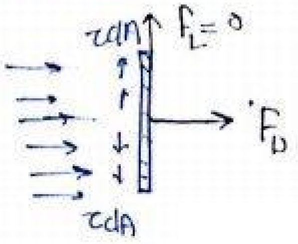


A thin body

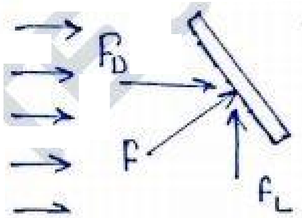
$$\tau = \mu \frac{U}{y}$$







* If there is lift there must be drag. (For lift, drag must)
* If there is drag there may or may not be lift.



Q.38 Sol^n

Assum :- linear Velocity Profile

$$\Sigma_{net} = 0$$

$$F - (F_{D_1} + F_{D_2}) = 0$$

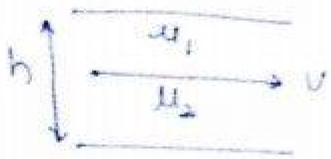
$$F = \tau_1 A + \tau_2 A$$

$$F = \frac{\mu_1 V}{(h-y)} A + \frac{\mu_2 V}{y} A \quad \Rightarrow F = f(y)$$

$$\frac{df}{dy} = 0, \quad 0 = \frac{\mu_1 V}{(h-y)^2} A - \frac{\mu_2 V}{y^2} A \Rightarrow \frac{\mu_1}{(h-y)^2} = \frac{\mu_2}{y^2}$$

$$\boxed{y = \frac{\sqrt{\mu_2}\; h}{\sqrt{\mu_1} + \sqrt{\mu_2}}}$$

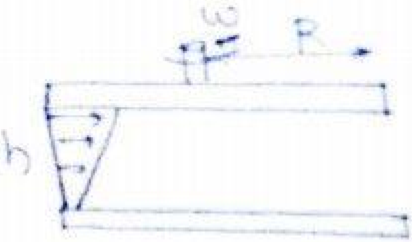
*



if $\mu_1 > \mu_2$

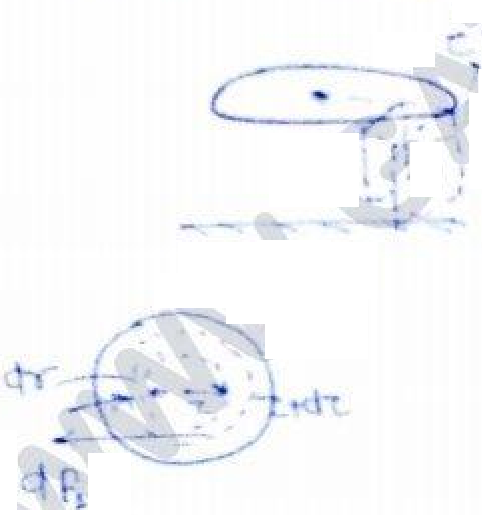
we place the thin plate nearer to less viscous fluid for minimum viscous resistance.

Q.17



$$\tau = \mu \frac{V}{h} = \mu \frac{r\omega}{h}$$

$$\tau \propto r$$



$$dF_t = \tau \times dA$$

$$dT_f = dF_t \times r$$

$$dT_f = dF_f \times r$$

$$dT_f = \tau\, dA \cdot r$$

$$dT_f = \frac{\mu r \omega}{h} \times 2\pi r\, dr \cdot r$$

$$\int_0^{T_f} dT_f = \frac{\mu 2\pi \omega}{h} \int_0^{R} r^3 dr$$

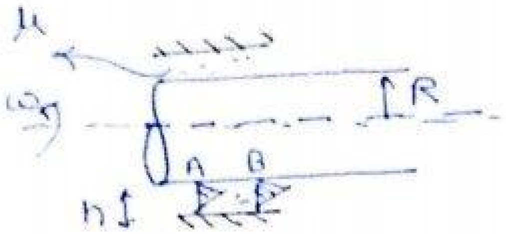
$$T_f = \frac{\pi \mu \omega R^4}{2h} \quad \text{Ans.}$$

or

$$\tau_{avg} = \frac{0 + \frac{\mu R \omega}{h}}{2} = \frac{\mu R \omega}{2h}$$

$$T = \frac{\mu R \omega}{2h} \times \pi R^2 \cdot \frac{R}{2} = \frac{\mu \pi \omega R^4}{2h}$$

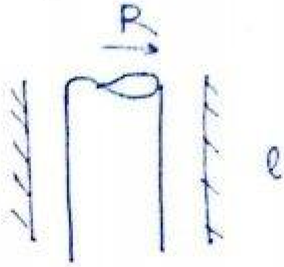
Note :- case of Journal bearing



$$V_A = R\omega = V_B$$

$$\tau = \mu \frac{V}{h} = \text{Constant}$$

$$F_t = \tau \times A$$



$$= \tau \times (2\pi R L)$$

$$= \frac{\mu (R\omega)}{h} \times (2\pi R L)$$

$$T_f = F_f \times R$$

Petro eq^n $$T_f = \frac{\mu R \omega}{h} (2\pi R L) \cdot R$$

Q.3

$$T = \mu W R$$

$$T = 2\pi^2 \left(\frac{Z n}{p}\right) \left(\frac{D}{c}\right) \times W \times \frac{D}{2}$$

$$p = \frac{W}{LD}$$

$$T = 2\pi^2 (Z n)(LD)\left(\frac{D}{C}\right) \times \frac{D}{2}$$

$$\omega = 2\pi n$$

$$T = 2\pi^2 \left(20 \times 10^{-3} \times \frac{20}{2\pi}\right)(0.040 \times 0.040)\left(\frac{0.040}{0.02 \times 10^{-3}}\right) \times \frac{0.040}{2}$$

Newton's law of Viscosity :-

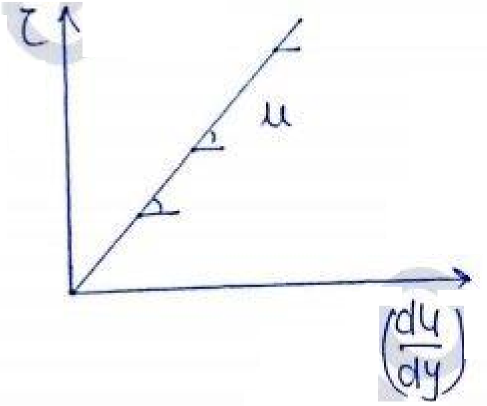
As per Newton's law of Viscosity shear stress between two layers a distance y from the surface is directly proportional to rate of shear deformation.
i.e. Viscosity of the fluid is independent of rate of deformation.

eg:- Water, Air, kerosien, gasoline, diesal most of gases etc. are Newtonion fluid

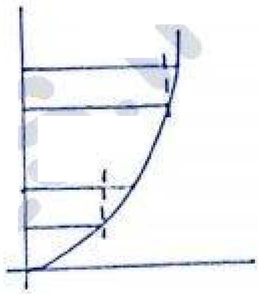
$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

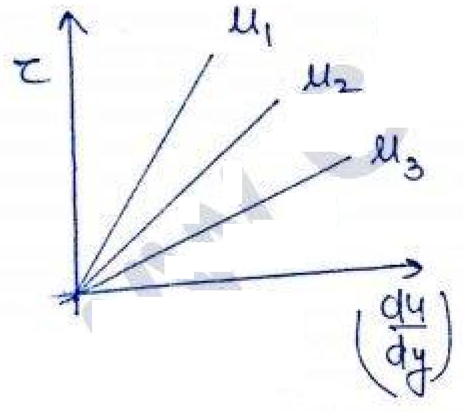
$$\mu \neq f\left(\frac{du}{dy}\right)$$



Newtonian



Non-Newtonion



$$\mu_1 > \mu_2 > \mu_3$$

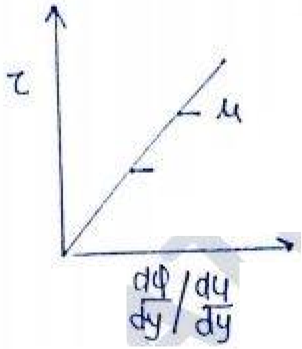
Classification of Fluids :-

Fluids

- Newtonion Fluids (Fluid Mechanics)
- Non-Newtonion Fluids (Rheology)
  - $\mu_{app} = f\left(\frac{du}{dy}\right)$
    - Dilatents (Honey)
    - Pseudoplastic (~~colgate~~)
    - Bingham plastic or ideal plastic
  - $\mu_{app} = f\left(\frac{du}{dy}, \text{time}\right)$
    - Thixotropic (~~ketchup~~)
    - Rheopectic (~~[illegible]~~)

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$



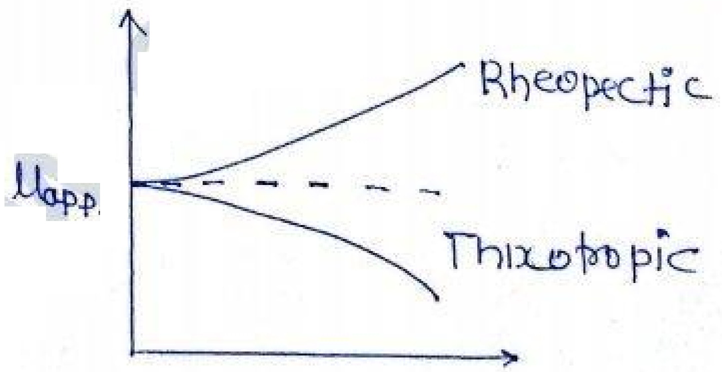
Non-Newtonion fluids.

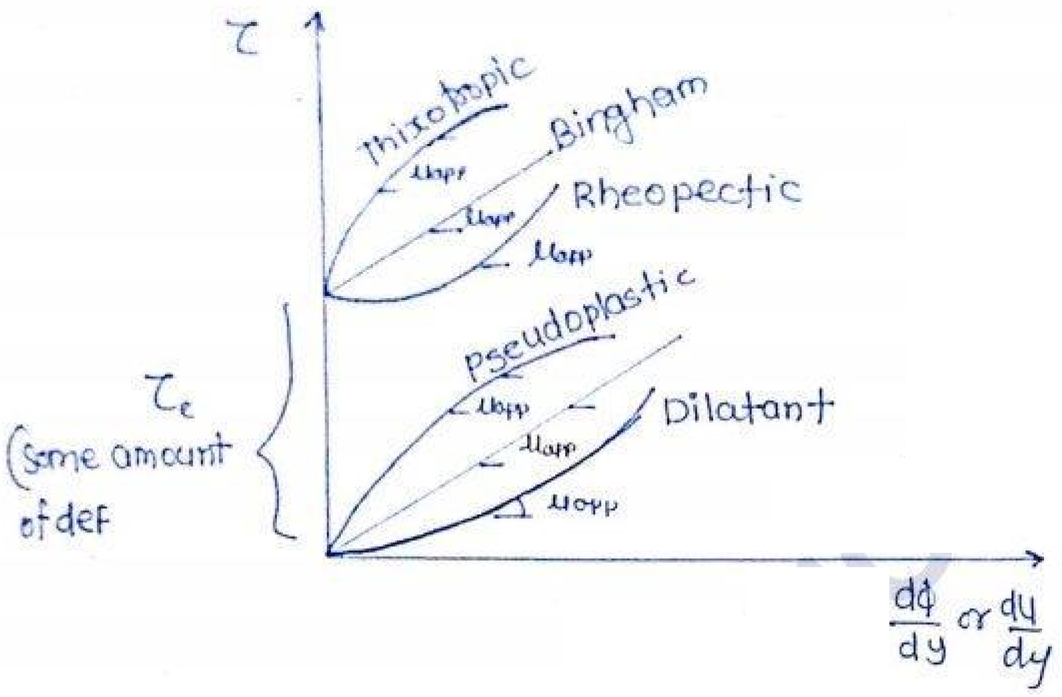
$$\tau = B + k\left(\frac{du}{dy}\right)^n \qquad n = \text{Flow behaviour index}$$

if $B = 0$

$$\tau = k\left(\frac{du}{dy}\right)^n = \underbrace{k\left(\frac{du}{dy}\right)^{n-1}}\left(\frac{du}{dy}\right)$$

$$\tau = \mu_{app}\frac{du}{dy}, \quad \mu_{app} = k\left(\frac{du}{dy}\right)^{n-1}$$

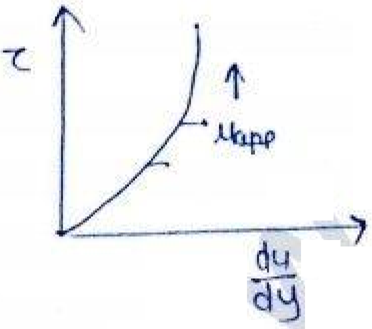




- Fluids which do not follows newton's law of viscosity are defined as non-newtonion fluid but follow power law and defined by apparant viscosity.

~~Dilant fl~~

Dilatant Fluid :-



$$\tau = B + k\left(\frac{du}{dy}\right)^n$$

$$\tau = B + \mu_{app}\left(\frac{du}{dy}\right)$$

$B = 0$ (No initial deformation require)

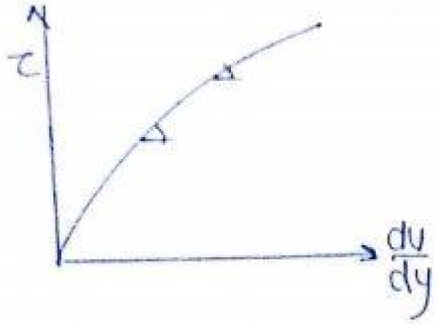
$$\tau = \mu_{app}\left(\frac{du}{dy}\right)$$

$$\mu_{app} = k\left(\frac{du}{dy}\right)^{n-1}$$

$n-1 > 0$ $\quad$ $n > 1$

Eg :- • Butter sol^n
- Honey sol^n
- sugar sol^n of Starch.
- non-Colidal sol^n.
- Quick Sand

Pseudoplastic eg:- Colidal sol^n, Milk, blood, Polymer sol^n



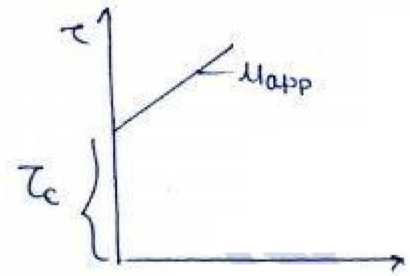
$$\tau = B + \mu_{app}\left(\frac{du}{dy}\right)$$

$$B = 0$$

$$\downarrow \mu_{app} \propto \left(\frac{du}{dy}\right)^{n-1}$$

$$n-1 < 0 \Rightarrow n < 1$$

Bingham Plastic :-
(ideal)



eg: toothpest, drilling mud, gel, creams etc.

$$\tau = B + \mu_{app}\frac{du}{dy}$$

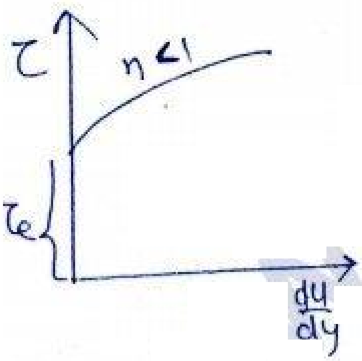
$$B \neq 0, \quad B = \tau_e$$

$$\mu_{app} = k\left(\frac{du}{dy}\right)^{n-1}$$

$$n-1 = 0 \Rightarrow n = 1$$

$$\mu_{app} = K$$

Thixotropic



$$\tau = B + \mu_{app}\left(\frac{du}{dy}\right)$$

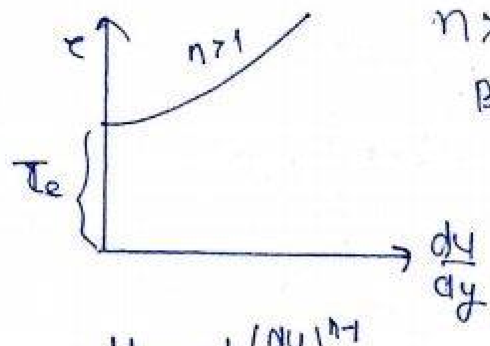
$$n < 1$$

$$B = \tau_e$$

$$\underset{\downarrow}{\mu_{app}} = k\left(\frac{du}{dy}\right)^{n-1}$$

eg: Paints, ketchup
sevage sluge

Rheopectic



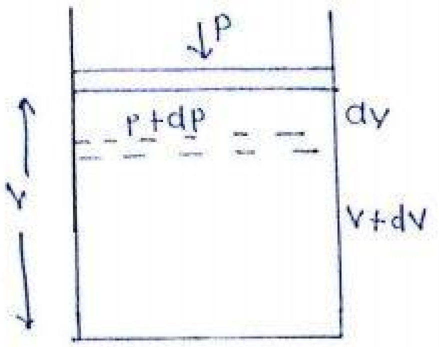
$$n > 1$$

$$B = \tau_e$$

$$\underset{\uparrow}{\mu_{app}} = k\left(\frac{du}{dy}\right)^{n-1}$$

eg Gyspum sol^n,
Printer ink.

Compressibility & Bulk Modulas :-



$P + dp > P \Rightarrow dp > 0$

$v + dv < v \Rightarrow dv < 0$

$(-dv) > 0$

$$dp \propto -\left(\frac{dv}{v}\right)$$

$dp \Downarrow (+ve)$, $\frac{dv}{v} \Downarrow -(-ve)$

$$dp \propto -\left(\frac{dv}{v}\right) \Rightarrow dp = k\left(\frac{-dv}{v}\right)$$

Bulk Modulas

$$\boxed{k = \frac{dp}{\left(\frac{-dv}{v}\right)}} \quad N/m^2$$

$$\boxed{k = \frac{dp}{\left(\frac{d\rho}{\rho}\right)}}$$

Initial

$\rho v = mass$

$\ln \rho + \ln v = \ln m$

$\frac{d\rho}{\rho} + \frac{dv}{v} = 0$

$\frac{d\rho}{\rho} = -\frac{dv}{v}$

$\Rightarrow$ $E = \frac{\sigma}{\varepsilon}$

$E \to \infty$, $\varepsilon \to 0$ rigid

$k \to \infty$ $\left(\frac{-dv}{v}\right) \to 0$

$dp \neq 0$

incompressible fluid

$k \to \infty$, $\beta = 0$

Compressibility

$$\beta = \frac{1}{K}$$

$\beta = 0$ (incompressible fluid)

$$\frac{\frac{d\rho}{\rho}}{dP} = 0$$

$$\frac{1}{\rho}\left(\frac{d\rho}{dP}\right) = 0$$

$\rho \neq 0$ $\quad \frac{d\rho}{dP} = 0$ (incompressible fluid)

eg

$K_w = 2 \times 10^6 \ KN/m^2$ $\quad \frac{K_w}{K_{air}} = 20,000$

$K_{air} = 101 \ KN/m^2$

$$\frac{\beta_{air}}{\beta_w} = 20,000$$

* All fluids are compressible but w.r.t. gases, liquids ~~fluids~~ are incompressible.

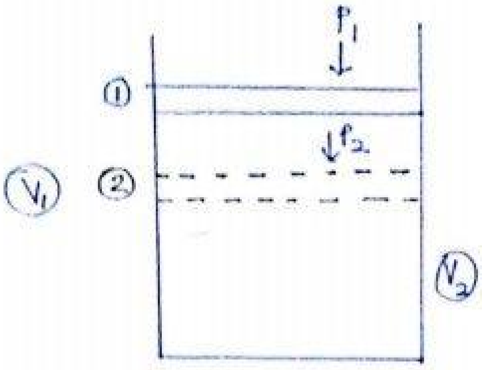
Bulk Modulas :- Bulk Modulas is the measure of modulas of elasticity of fluid it is defined under the compression of fluid.

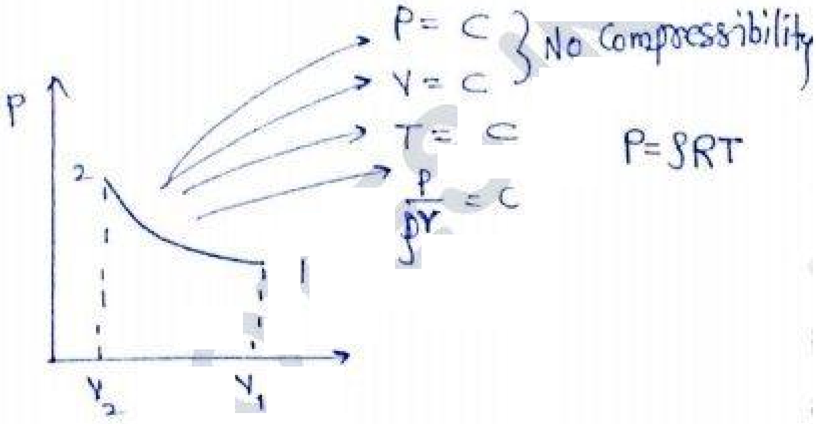
Inverse of bulk Modulas is known as compressibility.

Liquids have large value of bulk Modulas wrt gases.

Liquids can be consider as incompressible fluids.

Bulk Modulas for gases :-





$P = \rho RT$

Isothermal bulk Modulas

$P = \rho RT \qquad T = \text{Cont.}$

$$\frac{dP}{d\rho} = RT$$

$$\rho \frac{dP}{d\rho} = \rho RT$$

$$\boxed{K = P} \rightarrow \text{Absolute}$$

$$\beta = \frac{1}{P}$$

Adiabatic bulk Modulas

$$\frac{P}{\rho^\gamma} = C$$

$$P = C \cdot \rho^\gamma$$

$$\frac{dP}{d\rho} = \gamma \cdot C \cdot \rho^{\gamma - 1}$$

$$\rho \left(\frac{dP}{d\rho}\right) = \gamma \cdot C \rho^\gamma$$

$$\boxed{K = \gamma P}$$

↳ Absolute

Q.4

$$p = \left(3500\,\rho^{1/2} + 2500\right)$$

$$\frac{dp}{d\rho} = 3500 \times \frac{1}{2}(\rho)^{-1/2}$$

$$k = \rho\frac{dp}{d\rho} = 1750(\rho)^{1/2} \quad -(1)$$

$$p = 100\ kPa$$

So $$100\times10^{3} = 3500\ \rho^{1/2} + 2500$$

$$\rho = 776.02$$

$$k = 1750(776.02)^{1/2}$$

$$k = 48750\ N/m^2$$

Q.6 $$\nu_{gas} \propto (T)^{3/2}$$

$$\frac{\nu_{70}}{\nu_{20}} = \left(\frac{70+273}{20+273}\right)^{3/2}$$

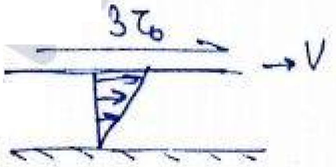
$$\nu_{70} = 2.02\times10^{-6}$$

(14) $$\tau = \tau_0 + K\left(\frac{du}{dy}\right)^{1/2} \Rightarrow 3\tau_0 = \tau_0 + K\left(\frac{du}{dy}\right)^{1/2}$$

$B \neq 0,\ n < 1$

$$\frac{du}{dy} = \frac{4\tau_0^2}{K^2}$$

Velocity profile linear



$$u = 4\left(\frac{\tau}{K}\right)^2 y + c \Rightarrow \begin{matrix} y=0 \\ u=0 \end{matrix} \Big\} \quad c=0$$

$$U_{at\,d} = 4\left(\frac{\tau_0}{K}\right)^2 d$$

(15)

$$u = 0.5y - y^2$$

$$\frac{du}{dy} = 0.5 - 2y$$

$$\tau\Big|_{y=0.20} = \mu \frac{du}{dy}\Big|_{y=0.20}$$

$$= 0.9 \times \left(\frac{0.5}{10} - 2 \times \left(\frac{0.20}{100}\right)\right)$$

$$= 0.9\left(\frac{1}{2} - \frac{2}{5}\right)$$

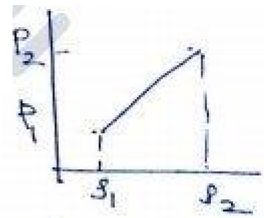
$$= 0.9\left(\frac{5-4}{10}\right)$$

$$\tau = 0.09 \ N/m^2.$$

(35)

$P_1 = 3$ MPa $\qquad \rho_1 = 500 \ kg/m^3$

$P_2 = 3.5$ MPa. $\qquad \rho_2 = 501 \ kg/m^3$



$$K = \rho\left(\frac{dp}{d\rho}\right) = 500 \times \frac{0.5}{1}$$ (Approximate)

$$K = 250 \text{ MPa}$$ Ans linear assumption

Actual

$$K = \frac{dp}{d\rho/\rho} \Rightarrow K\int_1^2 \frac{d\rho}{\rho} = \int_1^2 dp$$

32

$\mu = 10$ Pa-s

$\tau_0 = 10 \times 10^3$ Pa

$h = 10^{-3}$ m

$V = 1$ m/s.

$$\tau = \mu \frac{du}{dy} = 10 \times \frac{1}{10^{-3}}$$

$$\tau = 10 \text{ KPa}$$

Note:-

$$\frac{d\rho}{\rho} < 5\% \quad \text{incompressible flow}$$

20° water

1 atm $\quad \rho_w = 998 \ kg/m^3$ $\quad \frac{d\rho}{\rho} = \frac{5}{998}$

100 atm $\quad \rho_w = 1003 \ kg/m^3$ $\quad \frac{d\rho}{\rho} \times 100 \simeq \frac{5}{1000} \times 100$

$$\frac{d\rho}{\rho}\% = 0.5\%$$

Mach Number:-

$$\boxed{Ma = \frac{V}{C}}$$

C = pressure wave Velocity or sound Velocity

$$\boxed{C = \sqrt{\frac{K}{\rho}}}$$

Solids: $\quad C = \sqrt{\frac{E}{\rho}}$

Liquids:- $\quad C = \sqrt{\frac{K}{\rho}}$

water $\quad C = \sqrt{\frac{2 \times 10^6 \times 10^3}{1000}}$

$$C = 1414 \ m/s$$

Gases:- $P = \rho R T$

$$C = \sqrt{K/\rho}$$

isothermal | adiabatic

$K = P$ | $K = \gamma P$

$C = \sqrt{P/\rho}$ | $C = \sqrt{\frac{\gamma P}{\rho}}$

$C = \sqrt{RT}$ (Newtons) | $C = \sqrt{\gamma R T}$ (laplace)

(Newton's) | laplace eqn $C = 332$ m/s

$Ma = \frac{V}{C}$ → sonic Velocity.

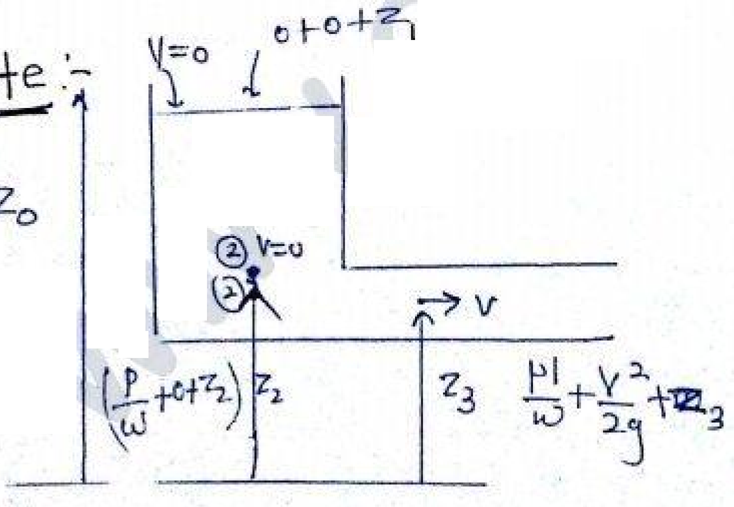
$V < C$, $Ma < 1$ Subsonic speed

$V = C$, $Ma = 1$ Sonic speed

$V > C$, $Ma > 1$ super Sonic Speed

V $Ma > 5$ hyper Sonic Speed.

Note:-



② & ③

$$\frac{P}{w} + 0 + Z_2 = \frac{P1}{w} + \frac{V^2}{2g} + Z_3$$

$$\frac{dP}{\rho g} = \frac{V^2}{2g}$$

$$\frac{1}{\rho}\frac{dp}{\left(\frac{d\rho}{\rho}\right)} = \frac{V^2}{2\left(\frac{d\rho}{\rho}\right)}$$

$$\frac{K}{\rho} = \frac{V^2}{2\left(\frac{d\rho}{\rho}\right)} \qquad \because \sqrt{\frac{K}{\rho}} = c$$

$$\frac{V^2}{c^2} = 2 \times \frac{d\rho}{\rho}$$

$$\frac{V}{c} = \sqrt{2\frac{d\rho}{\rho}} \qquad \frac{d\rho}{\rho} < 0.05$$

incompressible flow.

$$Ma < \sqrt{2 \times 0.05}$$

$$\boxed{Ma < 0.32}$$

Mach no less than this than compressible become incompressible flow

Air $c = 332$

$$\frac{V}{332} < 0.32$$

$V < 106$ m/s. { for $V < 106$ m/s air incomp. }

Note:- Incompressible flow always ~~says~~ shows incompressible flow but compressible fluid flow can be consider incompressible fluid flow when $\frac{d\rho}{\rho} < 0.05$ or $Ma < 0.32$ or total derivative of density wrt. $t = 0$, $\frac{D\rho}{Dt} = 0$

Que 30
WB 10

$S_f = 0.94$ $\quad \frac{\rho_f}{\rho_w} = 0.94$

$\rho_f = 940$

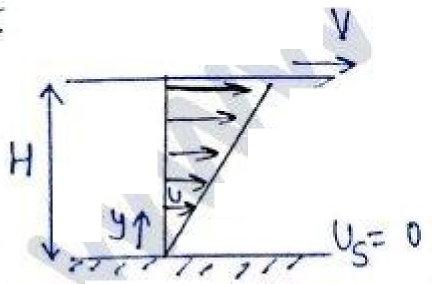
$\frac{d\rho}{\rho} = \frac{-dV}{V} = 0.0022$ $\quad dp = 1400\,kPa$

$c = \sqrt{K/\rho}$ $\quad K = \rho \frac{dp}{d\rho}$

$$c = \sqrt{\frac{dp}{\left(\frac{d\rho}{\rho}\right)\rho}} = \sqrt{\frac{1400}{0.0022 \times 940}}$$

$c = 822.78\ m/s.$

Note

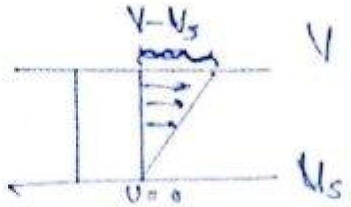


Here $U_s = 0$

$\frac{U}{V} = \frac{y}{H} \Rightarrow U = V\frac{y}{H}$

$$\frac{U - U_s}{V - U_s} = \frac{y - 0}{H - 0}$$

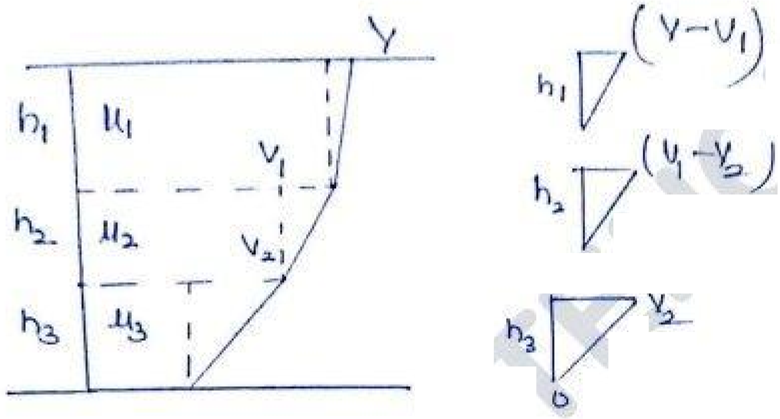
*



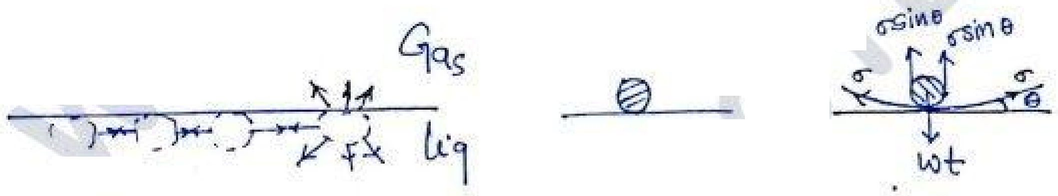
$$\frac{U-U_s}{V-U_s}=\frac{y}{H}$$

$$U=(V-U_s)\frac{y}{H}$$

*



Surface tension:-(σ)



$\sigma_{w-air} = 0.073$ N/m

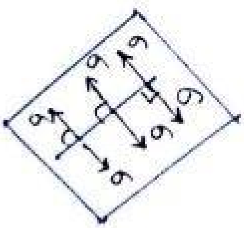
$\sigma_{Hg-air} = 0.497$ N/m

$\sigma_{alchohol-air} = 0.02$ N/m

* $\boxed{\sigma = \frac{\text{surface energy}}{\text{Area}}}$ J/m²

* $\boxed{\sigma = \left(\frac{Fs}{l}\right)}$ N/m

- surface tension is due to only cohesive force.



surface tension normal to line.

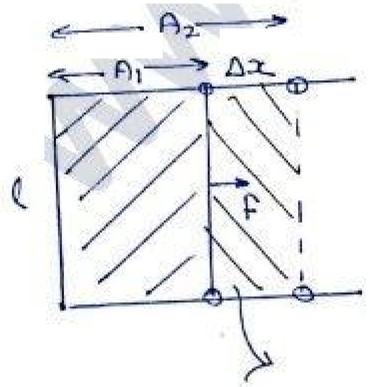
surface ~~ener~~ tension is due to unbalancing Cohesive forces it is the liquid phinomena. it is defined as surface energy per unit area.

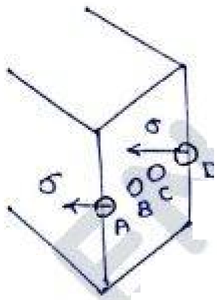
Surface tension is a line force it is define as normal force per unit length.

It is at the liquid surface by increasing temp ~~bo~~ cohesive forces decreases so surface tension increases. By adding salt it increases and by adding ditergent it decreases.

eg:- Floating of insects, leaves on water surface, sap in trees, capliarity etc.

Surface energy Concept:-





$$f_s = (\sigma l + \sigma l)$$

at eqb^m = $f = f_s$

Increase area

$$\Delta A = (l \times \Delta x) + (l \Delta x)$$

$$= 2 l . \Delta x$$

$$\text{Work done} = f . \Delta x$$

$$= f_s . \times \Delta x$$

$$= \sigma \times (2l) \Delta x$$

$$W = \sigma \Delta A$$

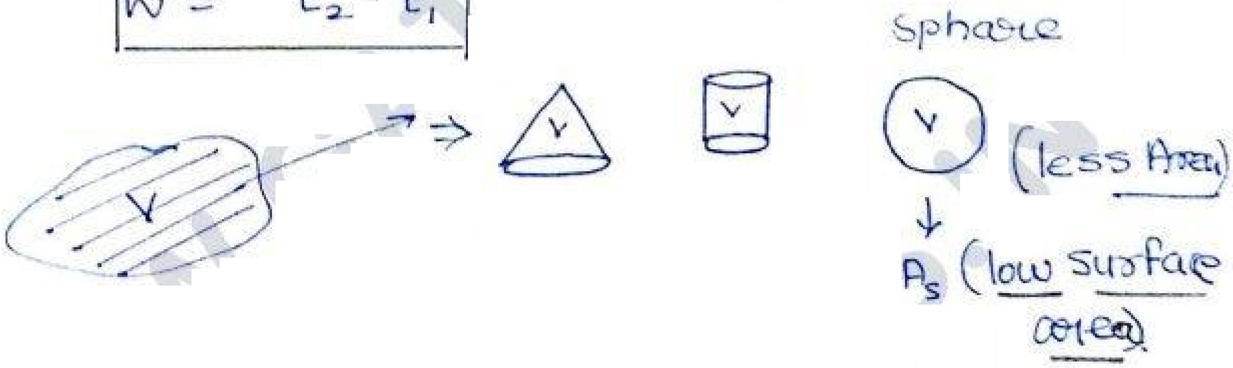
$$\sigma = \frac{W}{\Delta A} \quad J/m^2$$

$$\sigma = \frac{F_s}{l}$$

$$W = \sigma(A_2 - A_1)$$

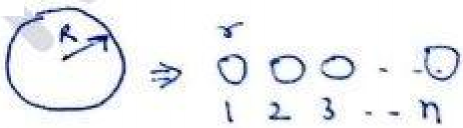
$$\boxed{W = \sigma A_2 - \sigma A_1}$$

$$\boxed{W = E_2 - E_1}$$



Q.19

$$A = 4\pi R^2 \quad V_1 = V_2$$



$$V_1 = V_2 \Rightarrow V = \cancel{\tfrac{4}{3}\pi} R^3 = n\cancel{\tfrac{4}{3}\pi}(r)^3$$

$$r = \frac{R}{(n)^{1/3}} \qquad A_2 = n 4\pi r^2$$

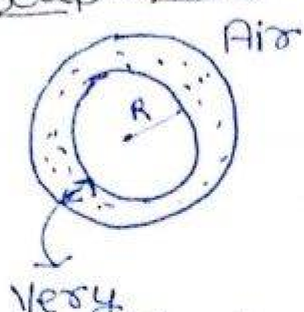
$A_2 =$

Surface energy $E_1 = \sigma A_1$
$$= \sigma(4\pi R^2)$$

$$E_2 = \sigma\left(n 4\pi \frac{R^2}{n^{2/3}}\right) = \sigma n^{1/3} 4\pi R^2$$

$$W = E_2 - E_1 = 4\pi R^2 \sigma(n^{1/3} - 1)$$

* Soap bubble



$$E = \sigma A_S$$

$$= \sigma(4\pi R^2 + 4\pi R^2)$$

(Nelecting thin layer)

Pressure inside the liquid drooplate/ Air bubble :-

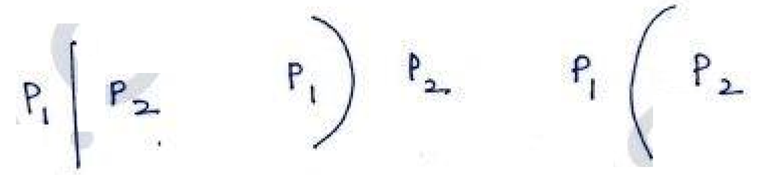
Unbalance cohesion

P_{atm}

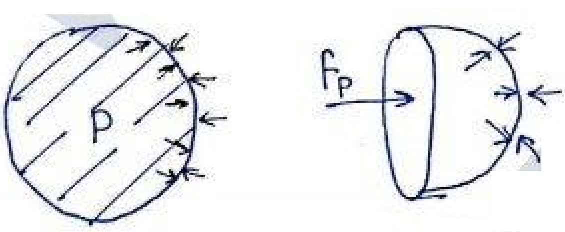
$P > P_{atm}$

$\Delta P = P - P_{atm}$

exess pressure



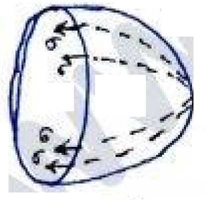
$P_1 = P_2$ $\quad$ $P_1 > P_2$ $\quad$ $P_1 < P_2$



F_P on projected Area $= \frac{\pi d^2}{4}$

(like crushing)
Compression

$$F_P = \Delta P \frac{\pi}{4} d^2$$



$l = \pi d$

Surface tension force (Along length) $\perp^{er}$

$$F_S = \sigma \times l$$

$$F_S = \sigma \times (\pi d)$$

equilibrium

$$F_p = F_s$$

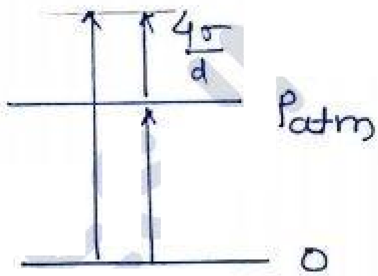
$$\Delta P \frac{\pi}{4} d^2 = \sigma \pi d$$

$$\Delta P = \frac{4\sigma}{d}$$

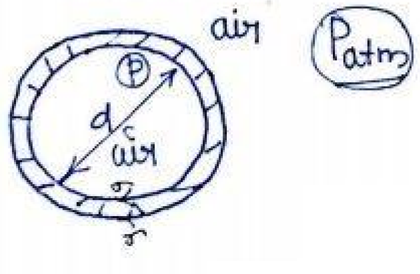
$$P - P_{atm} = \frac{4\sigma}{d}$$

$$P = P_{atm} + \frac{4\sigma}{d} \quad \text{Absolute}$$

Gauge Pressure $\boxed{P = \frac{4\sigma}{d}}$



Pressure inside the Soap bubble:—



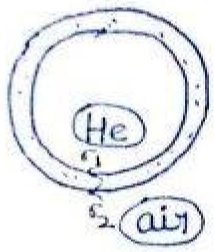
$$\Delta P = \frac{4\sigma}{d_i} + \frac{4\sigma}{d_o}$$

$$d_i \simeq d_o \simeq d$$

$$\Delta P = \frac{8\sigma}{d}$$

Gauge Pressure $\boxed{P = \frac{8\sigma}{d}}$

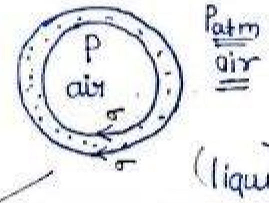
*



$$\Delta P = \frac{4\sigma_1}{d} + \frac{4\sigma_2}{d} = \frac{4(\sigma_1+\sigma_2)}{d}$$

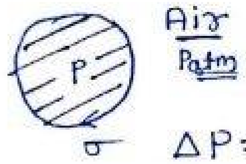
Diff fluids, diff surface tension

*

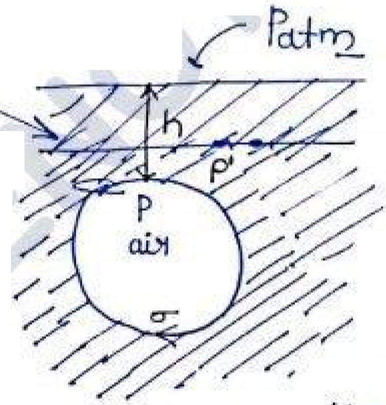


(liquid is filled inside)

(liquid Filled out side)



$$\Delta P = \frac{4\sigma}{d}$$



$$P' = P_{atm} + \rho g h$$

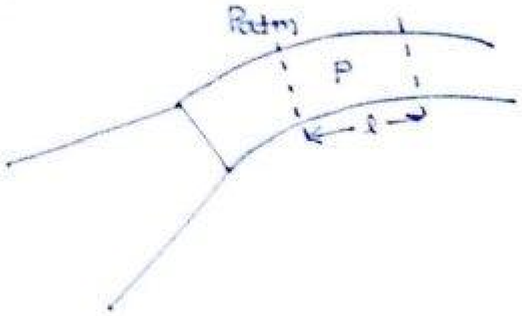
$$\Delta P = P - P' = \frac{4\sigma}{d}$$

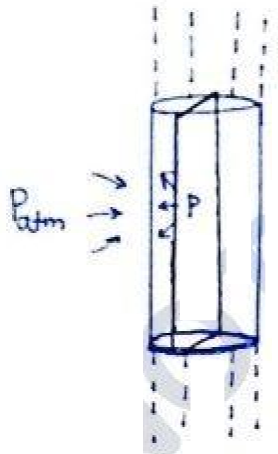
$$P = P_{atm} + \rho g h + \frac{4\sigma}{d}$$

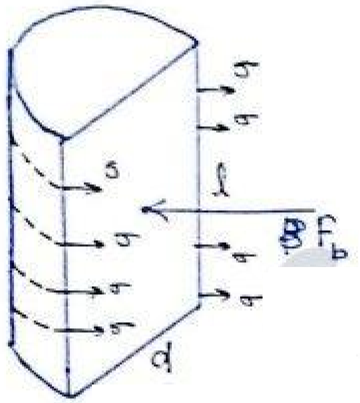
Q.36

$$\Delta P = \frac{4\sigma}{d} = \frac{4 \times 0.072}{0.001} = 288 \ N/m^2$$

Pressure inside the liquid Jet:-







$$F_p = \Delta p \times (l \times d)$$

$$F_s = \sigma \times l + \sigma \times l$$

$$\Delta p (l d) = 2 \sigma l$$

$$\boxed{\Delta p = \frac{2\sigma}{d}}$$

Wetting and non-wetting Phinomena:-

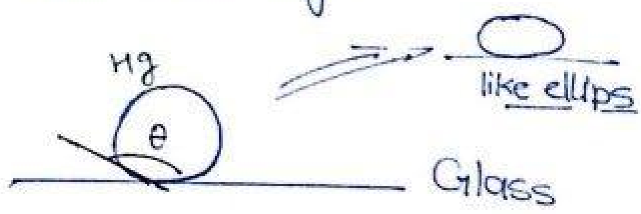
Wetting

Adhesive force > Cohesive force

angle of Contact

$\theta < \pi/2$ acute angle

Non-wetting



Cohesive force > Adhesive Force.

$\theta > \frac{\pi}{2}$

*

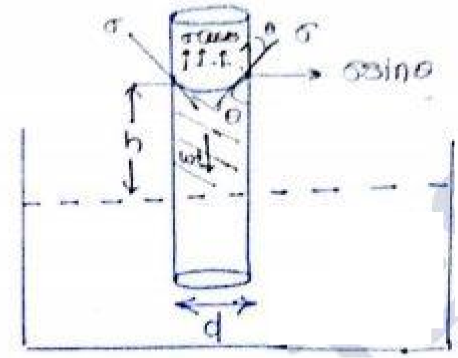
- Pure water = 0° → with Glass
- Normal water - Glass = 29°
- Hg - Glass = 130° - 138°
- kerosene - Glass = 26°

Wetting & Non wetting phinomena are relative phinoman it depands on Cohesion b/w liquid molicule and Cohesion and Adhesion b/w liquid solid surface.

Capillarity: When a small dia tube (less than 10mm) inserted into a liquid there may be rise or fall of liquid in the tube w.r.t. outer surface is known as capiliarity.

Surface tension depands upon Cohesive force i.e. surface tension is due to chesive forces.

capiliarity is due to Cohesive and adhesive forces.



$$F_s = (\sigma \cos\theta) \times l$$

$$F_s = \sigma \cos\theta \, (\pi d)$$

Rise in weight of liquid $\Rightarrow Wt = w \times Vol$

$$= w \times \frac{\pi}{4} d^2 \cdot h$$

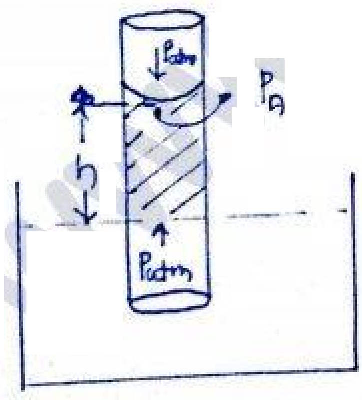
equilibrium (surface tension will balance by wt. rise) (then rising of wt. stop)

$$w \frac{\pi}{4} d^2 \cdot h = \sigma \pi d \cos\theta$$

$$\boxed{h = \frac{4\sigma \cos\theta}{wd} = \frac{4\sigma \cos\theta}{\rho g d}}$$

$$\boxed{h \propto \frac{1}{d}}$$

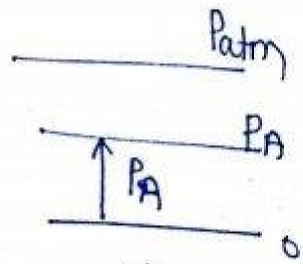
*



$$\boxed{P_A < P_{atm}}$$

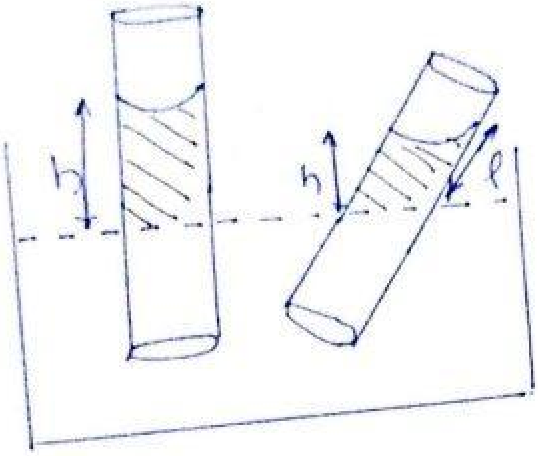
$$P_{atm} - \rho g h = P_A$$

$$P_A = P_{atm} - \rho g h$$

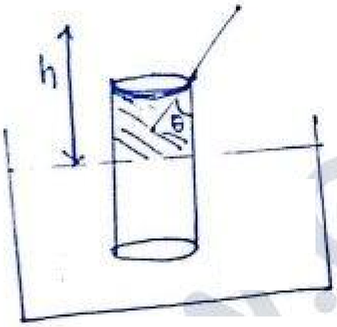


Capiliary Generates Vaccume

*



*

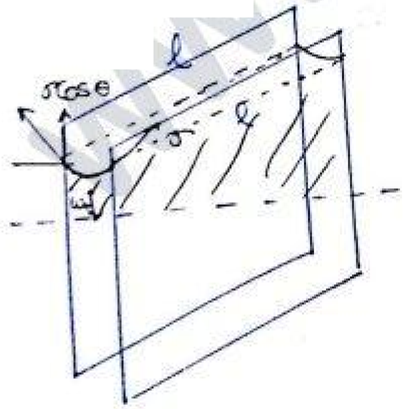


if length of tube is less than $\underline{h}$

in this case angle of contact will increase so $\sigma\cos\theta \downarrow$

(liquid will No Go inside)

*



$$F_s = \sigma\cos\theta \times l + \sigma\cos\theta \times l$$

$$wt = w \times Vol.$$

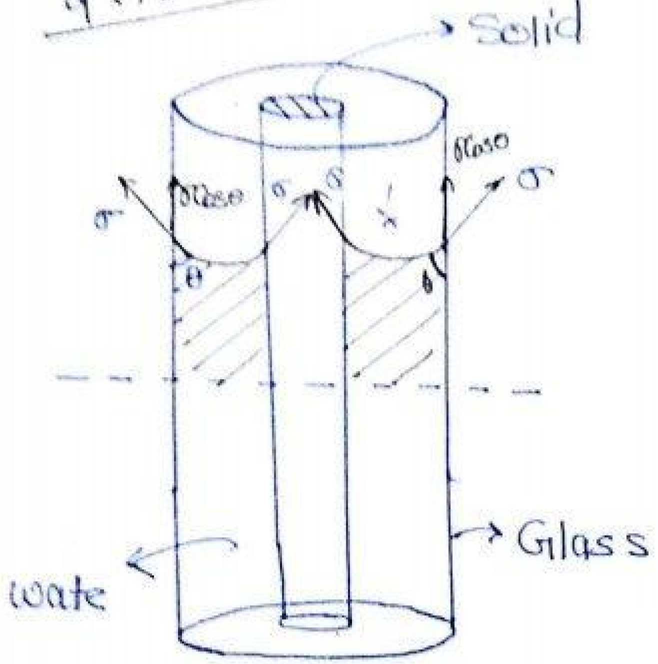
$$= w \times l \times h \times t$$

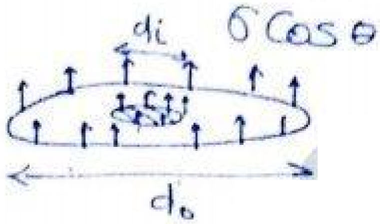
$$wlht = 2\sigma\cos\theta \times l$$

$$h = \frac{2\sigma\cos\theta}{wt}$$

$$\boxed{h = \frac{2\sigma\cos\theta}{\rho g t}}$$

$d < 10\,mm$





$$F_s = (\sigma \cos\theta\, \pi d_o + \sigma \cos\theta\, \pi d_i)$$

$$Wt = w \times Vol$$

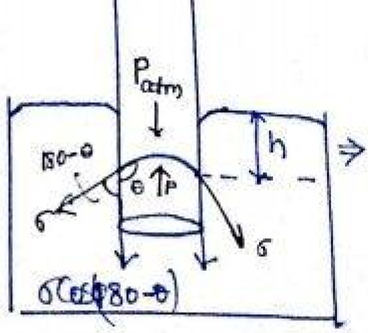
$$= w \times \frac{\pi}{4}(d_o^2 - d_i^2)\, h$$

$$h = \frac{4\sigma \cos\theta}{w(d_o - d_i)} \qquad d_i \to 0 \qquad h = \frac{4\sigma\cos\theta}{w d_o}$$

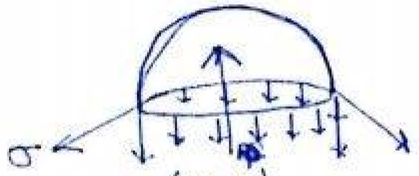
$$* \boxed{h = \frac{4\sigma \cos\theta}{\rho g (d_o - d_i)}}$$

Capillary rise and fall expression are same. -Ve sign in capillary fall indicates the depression of liquid from the outside surface

$$P = P_{atm} + \rho g h$$



$$\Rightarrow P = P_{atm} + \rho g h$$



$\sigma \cos(180-\theta)$

$-\sigma \cos\theta$

$$F_p = \Delta P \frac{\pi d^2}{4}$$

$$P = P_{atm} + \rho g h$$

$$\Delta P = \rho g h$$

$$(-\sigma \cos\theta)\pi d = \Delta P \frac{\pi}{4} d^2$$

$$-\sigma(\cos\theta)\pi = \rho g h \frac{\pi}{4} d$$

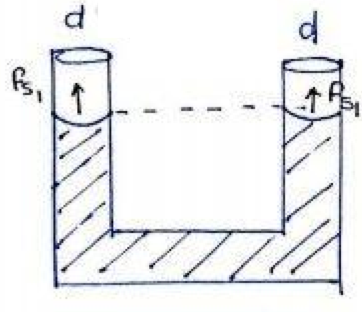
$$h = \frac{-4\sigma\cos\theta}{w d}$$

$\theta > \pi/2$

$\cos\theta = -ve$

$h = +ve$

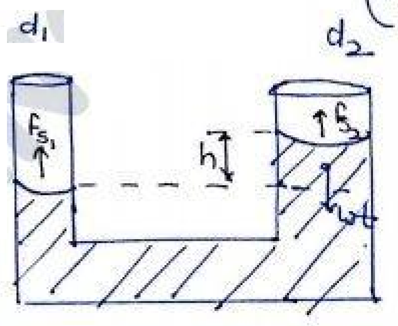
Note:-



$F_{s_1} = (\sigma\cos\theta)\pi d$

$F_{s_2} = (\sigma\cos\theta)\pi d$

(More dia more S.T. force)



$d_2 > d_1$

$F_{s_2} > F_{s_1}$

$F_{s_1} = (\sigma\cos\theta)\pi d_1$

$F_{s_2} = (\sigma\cos\theta)\pi d_2$

$wt = F_{s_2} - F_{s_1}$

$$w \times \frac{\pi}{4} d_2^2 h = (\sigma\cos\theta)\pi(d_2 - d_1)$$

$$\boxed{h_1 = \frac{4\sigma\cos\theta(d_2 - d_1)}{w d_2^2}}$$

$$h = \frac{4\sigma \cos\theta (d_2 - d_1)}{w d_2^2}$$

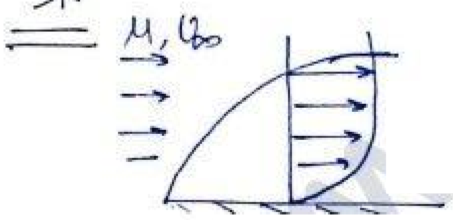
if (i) $d_1 = d_2$ $\quad h = 0$

(ii) $d_1 = 0$ $\quad h = \frac{4\sigma \cos\theta}{w d_2}$

(iii) $d_1 > d_2$ $\quad h < 0$

$$h = \frac{4\sigma \cos\theta (d_1 - d_2)}{w d_1^2}$$

Q.9
*



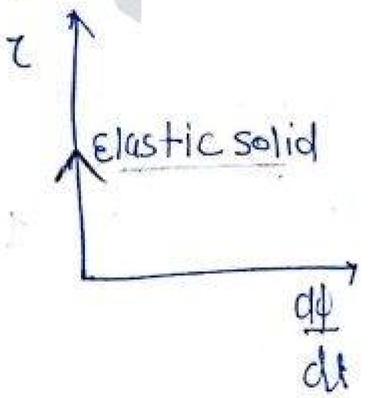
ideal Fluid

$\frac{du}{dy} = 0$

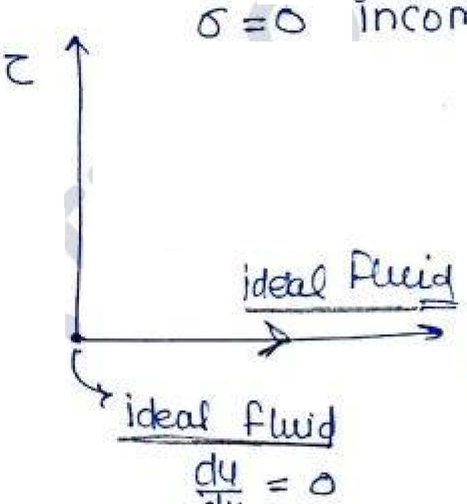
$\mu_0 = 0$, Cohesive force $= 0$

$\sigma = 0$ incompressible

in solid



Fluid



$\frac{du}{dy} = 0$

$\frac{d\phi}{dt} \propto \frac{du}{dy}$

Q.96 $\quad h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.075}{1000 \times 10 \times 1 \times 10^{-3}} = 30\ mm$