3 CHAPTER

Geometry and Mensuration

LINE

1994

Direction for Question 1 : Data is provided followed by two statements – I and II – both resulting in a value, say I and II. As your answer,

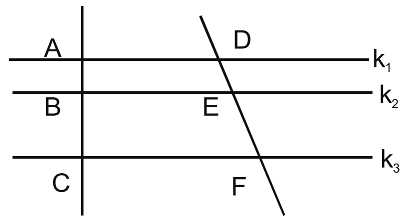
Mark (a) if I > II.

Mark (b) if I < II.

Mark (c) if I = II.

Mark (d) if nothing can be said.

1. k_1, k_2, k_3 are parallel lines. AD = 2 cm, BE = 8 cm and CF = 32 cm.



I. (AB) × (EF)

II. (BC) × (DE)

Direction for Question 2 : The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II, alone,

Mark (c) if both, statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

2. Is segment PQ greater than segment RS?

I. PB $>$ RE, BQ $=$ ES.

II. B is a point on PQ, E is a point on RS.

Direction for Question 3: The question is followed by two statements. As the answer,

Mark:

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both the statements together.

1999

3. A line graph on a graph sheet shows the revenue for each year from 1990 through 1998 by points and joins the successive points by straight-line segments. The point for revenue of 1990 is labelled A, that for 1991 as B, and that for 1992 as C. What is the ratio of growth in revenue between 1991-92 and 1990-91?

I. The angle between AB and X-axis when measured with a protractor is 40°, and the angle between CB and X-axis is 80°.

II. The scale of Y-axis is 1 cm = Rs. 100

TRIANGLE

1991

1. A man starting at a point walks one km east, then two km north, then one km east, then one km north, then one km east and then one km north to arrive at the destination. What is the shortest distance from the starting point to the destination?

(a) $2\sqrt{2}$ km (b) 7 km

(c) $3\sqrt{2}$ km (d) 5 km

1993

Directions for Questions 2 and 3: Use the following information:

ABC forms an equilateral triangle in which B is 2 km from A. A person starts walking from B in a direction parallel to AC and stops when he reaches a point D directly east of C. He, then, reverses direction and walks till he reaches a point E directly south of C.

2. Then D is

(a) 3 km east and 1 km north of A

(b) 3 km east and $\sqrt{3}$ km north of A

(c) $\sqrt{3}$ km east and 1 km south of A

(d) $\sqrt{3}$ km west and 3 km north of A

3. The total distance walked by the person is

(a) 3 km (b) 4 km

(c) $2\sqrt{3}$ km (d) 6 km

1994

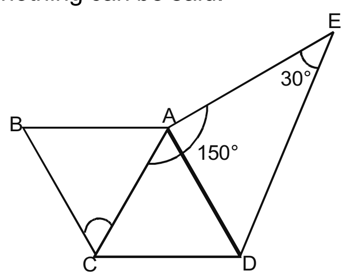
Direction for Question 4 : Data is provided followed by two statements – I and II – both resulting in a value, say I and II. As your answer,

Mark (a) if I > II.

Mark (b) if I < II.

Mark (c) if I = II.

Mark (d) if nothing can be said.



4. In ΔACD, AD = AC and $\angle C = 2\angle E$. The distance between parallel lines AB and CD is h. Then

I. Area of parallelogram ABCD

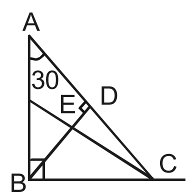
II. Area of ΔADE

1995

5. Which one of the following cannot be the ratio of angles in a right-angled triangle?

(a) 1 : 2 : 3 (b) 1 : 1 : 2
(c) 1 : 3 : 6 (d) None of these

6. AB ⊥ BC, BD ⊥ AC and CE bisects ∠C, ∠A = 30°. Then what is ∠CED?



(a) 30° (b) 60°
(c) 45° (d) 65°

7. The sides of a triangle are 5, 12 and 13 units. A rectangle is constructed, which is equal in area to the triangle, and has a width of 10 units. Then the perimeter of the rectangle is

(a) 30 units (b) 36 units
(c) 13 units (d) None of these

8. The length of a ladder is exactly equal to the height of the wall it is leaning against. If lower end of the ladder is kept on a stool of height 3 m and the stool is kept 9 m away from the wall, the upper end of the ladder coincides with the top of the wall. Then the height of the wall is

(a) 12 m (b) 15 m
(c) 18 m (d) 11 m

Directions for Question 9: The question is followed by two statements, I and II. Mark the answer as:

(a) if the question can be answered with the help of statement I alone.

(b) if the question can be answered with the help of statement II alone.

(c) if both statement I and statement II are needed to answer the question.

(d) if the question cannot be answered even with the help of both the statements.

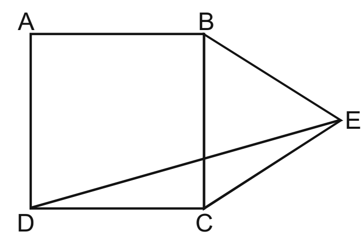
9. What is the area of the triangle?

I. Two sides are 41 cm each.

II. The altitude to the third side is 9 cm long.

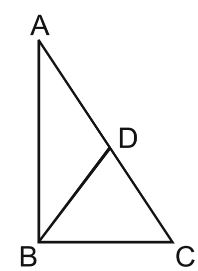
1996

10. If ABCD is a square and BCE is an equilateral triangle, what is the measure of ∠DEC?



(a) 15° (b) 30°
(c) 20° (d) 45°

11. In ΔABC, ∠B is a right angle, AC = 6 cm, and D is the mid-point of AC. The length of BD is



(a) 4 cm (b) $\sqrt{6}$ cm
(c) 3 cm (d) 3.5 cm

1997

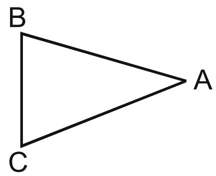
12. In Δ ABC, points P, Q and R are the mid-points of sides AB, BC and CA respectively. If area of Δ ABC is 20 sq. units, find the area of Δ PQR.

(a) 10 sq. units (b) $5\sqrt{3}$ sq. units
(c) 5 sq. units (d) None of these

1998

Directions for Questions 13 and 14: Answer the questions based on the following information.

A cow is tethered at point A by a rope. Neither the rope nor the cow is allowed to enter ΔABC.



$\angle BAC = 30°$

$l(AB) = l(AC) = 10$ m

13. What is the area that can be grazed by the cow if the length of the rope is 8 m?

(a) $134\pi\frac{1}{3}$ sq. m (b) 121π sq. m

(c) 132π sq. m (d) $\frac{176\pi}{3}$ sq. m

14. What is the area that can be grazed by the cow if the length of the rope is 12 m?

(a) $133\pi\frac{1}{6}$ sq. m (b) 121π sq. m

(c) 132π sq. m (d) $\frac{176\pi}{3}$ sq. m

Direction for Question 15: The question is followed by two statements, I and II. Answer the question based on the statements and mark the answer as:

(a) if the question can be answered with the help of any one statement alone but not by the other statement.

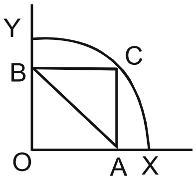
(b) if the question can be answered with the help of either of the statements taken individually.

(c) if the question can be answered with the help of both statements together.

(d) if the question cannot be answered even with the help of both statements together.

15. Find the length of AB if

$\angle YBC = \angle CAX = \angle YOX = 90°$.



I. Radius of the arc is given.

II. OA = 5

2000

16. What is the number of distinct triangles with integral valued sides and perimeter 14?

(a) 6 (b) 5

(c) 4 (d) 3

17. If a, b and c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is

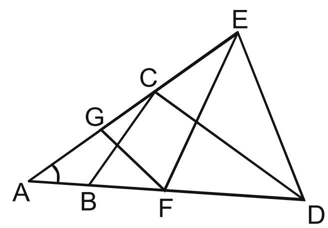
(a) equilateral

(b) isosceles

(c) right-angled

(d) obtuse-angled

18.



In the figure above, AB = BC = CD = DE = EF = FG = GA. Then $\angle$DAE is approximately

(a) 15° (b) 20°

(c) 30° (d) 25°

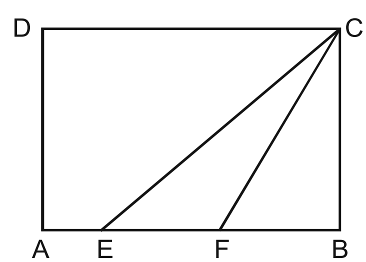
2001

19. A certain city has a circular wall around it, and this wall has four gates pointing north, south, east and west. A house stands outside the city, 3 km north of the north gate, and it can just be seen from a point 9 km east of the south gate. What is the diameter of the wall that surrounds the city?

(a) 6 km (b) 9 km

(c) 12 km (d) None of these

20.



In the above diagram, ABCD is a rectangle with AE = EF = FB. What is the ratio of the areas of ΔCEF and that of the rectangle?

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$

(c) $\frac{1}{9}$ (d) None of these

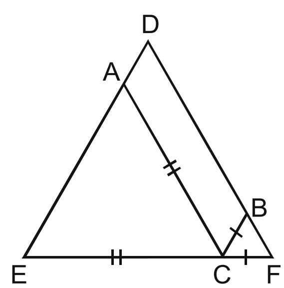
21. A ladder leans against a vertical wall. The top of the ladder is 8 m above the ground. When the bottom of the ladder is moved 2 m farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

(a) 10 m (b) 15 m
(c) 20 m (d) 17 m

22. Euclid has a triangle in mind. Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?

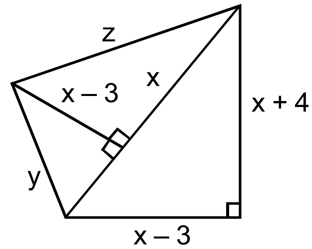
(a) $\sqrt{260}$ (b) $\sqrt{250}$
(c) $\sqrt{240}$ (d) $\sqrt{270}$

23. In ΔDEF shown below, points A, B and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. If $\angle D = 40^0$, then $\angle$ACB =



(a) 140° (b) 70°
(c) 100° (d) None of these

24. Based on the figure below, what is the value of x, if y = 10?



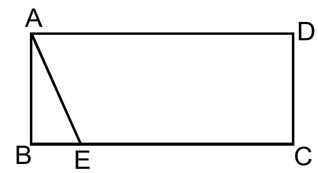
(a) 10 (b) 11
(c) 12 (d) None of these

2002

25. In ΔABC, the internal bisector of $\angle$A meets BC at D. If AB = 4, AC = 3 and $\angle A = 60°$, then the length of AD is

(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$
(c) $\frac{15\sqrt{3}}{8}$ (d) $\frac{6\sqrt{3}}{7}$

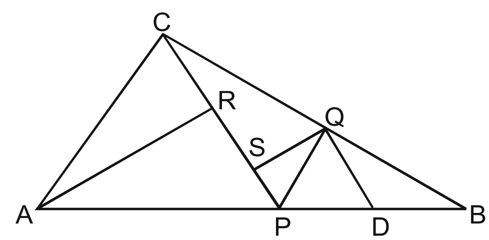
26. In the figure given below, ABCD is a rectangle. The area of the isosceles right triangle ABE = 7 cm^2; EC = 3(BE). The area of ABCD (in cm^2) is



(a) 21 cm^2 (b) 28 cm^2
(c) 42 cm^2 (d) 56 cm^2

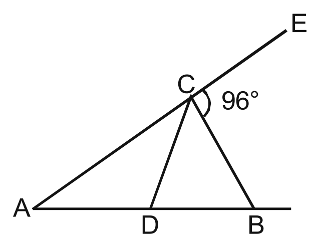
2003(R)

27. In the figure (not drawn to scale) given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, $\angle$ARC = 90°, and in ΔPQS, $\angle$PSQ = 90°. The length of QS is 6 cm. What is the ratio of AP : PD?



(a) 10 : 3 (b) 2 : 1
(c) 7 : 3 (d) 8 : 3

28. In the figure (not drawn to scale) given below, if AD = CD = BC and $\angle BCE = 96°$, how much is the value of $\angle$DBC?



(a) 32°
(b) 84°
(c) 64°
(d) Cannot be determined

2003(L)

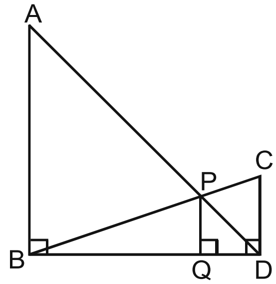
29. In the triangle ABC, AB = 6, BC = 8 and AC = 10. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with center B) is drawn. If the circle cuts AB and BC at P and Q respectively, the AP:QC is equal to

(a) 1 : 1 (b) 3 : 2
(c) 4 : 1 (d) 3 : 8

30. In the diagram given below,

$\angle ABD = \angle CDB = \angle PQD = 90°$.

If AB : CD = 3 : 1, the ratio of CD : PQ is



(a) 1 : 0.69 (b) 1 : 0.75
(c) 1 : 0.72 (d) None of the above

Direction for Question 31: The question is followed by two statements, A and B. Answer the question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements together.

31. D, E, F are the mid points of the sides AB, BC and CA of triangle ABC respectively. What is the area of DEF in square centimeters?

A. AD = 1 cm, DF = 1 cm and perimeter of DEF = 3 cm

B. Perimeter of ABC = 6 cm, AB = 2 cm and AC = 2 cm.

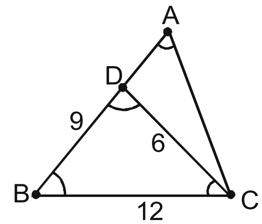
2004

32. A father and his son are waiting at a bus stop in the evening. There is a lamp post behind them. The lamp post, the father and his son stand on the same straight line. The father observes that the shadows of his head and his son's head are incident at the same point on the ground. If the heights of the lamp post, the father and his son are 6 metres, 1.8 metres and 0.9 metres respectively, and the father is standing 2.1 metres away from the post then how far (in metres)is son standing form his father?

(a) 0.9 (b) 0.75
(c) 0.6 (d) 0.45

2005

33. Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$



What is the ratio of the perimeter of ΔADC to that of the ΔBDC?

(a) $\frac{7}{9}$ (b) $\frac{8}{9}$
(c) $\frac{6}{9}$ (d) $\frac{5}{9}$

2008

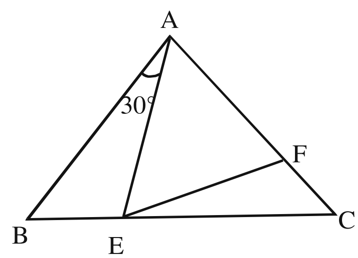
34. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?

(a) 5 (b) 21
(c) 10 (d) 15
(e) 14

MEMORY BASED QUESTIONS

2009

35. In the given figure, ABC is an isosceles triangle with AB = AC. If AE = AF and angle BAE = 30°, then what is the value of angle FEC?



(a) 15° (b) 30°
(c) 60° (d) Cannot be determined

2013

36. E is a point on the side AB of a rectangle ABCD, the adjacent sides of which are in the ratio 2 : 1. If $\angle AED = \angle DEC$, then what is the measure of $\angle AED$?

(a) 15° (b) 45°
(c) 75° (d) Either (a) or (c)

2014

37. The length of the hypotenuse of a right-angled triangle is 240 units. The perimeter of the given triangle is a perfect square. If the perimeter of the given triangle is greater than 550 units, then which of the following can be the length of a side of the given right-angled triangle?

(a) 192 units (b) 168 units
(c) 144 units (d) Both (a) and (c)

38. Let S be an arbitrary point on the side PQ of an acute-angled ΔPQR. Let T be the point of intersection of QR and the straight line PT drawn parallel to SR through P. Let U be the point of intersection of PR and the straight line QU drawn parallel to SR through Q. If PT = a units and QU = b units, then the length of SR is

(a) $\frac{a+b}{ab}$ units (b) $\frac{a-b}{ab}$ units

(c) $\frac{ab}{a+b}$ units (d) $\frac{ab}{a-b}$ units

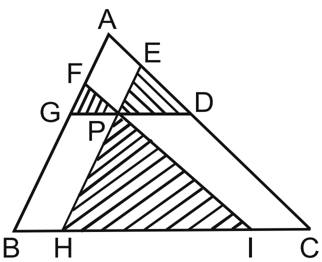
39. The bisector of $\angle BAC$ of ΔABC cuts BC at D and the circumcircle of the triangle at E. If DE = 3 cm, AC = 4 cm and AD = 5 cm, then the length of AB is

(a) 7 cm (b) 8 cm

(c) 9 cm (d) 10 cm

2015

40.



Through point P, lines are drawn parallel to the sides of triangle ABC. The areas of the ΔPED, ΔPFG and ΔPHI are 9, 16 and 49 sq. cm respectively. Find the area (in sq. cm) of triangle ABC.

41. An isosceles right angled triangle with length of its equal sides being 30 cm, is rotated 180° about its centroid to form a new triangle. Find the area of the region common to the original and the new triangles.

(a) 275 sq. cm

(b) 300 sq. cm

(c) 375 sq. cm

(d) 350 sq. cm

42. ΔABC is a right angled triangle, with $\angle B = 90°$, AB = 20 cm and BC = 21cm. A circle with centre O is inscribed in triangle ABC. OD, OE and OF are perpendiculars drawn on the sides AB, BC and CA respectively. Find the ratio of the area of the quadrilateral FOEC to the area of the quadrilateral ADOF.

(a) 15 : 14

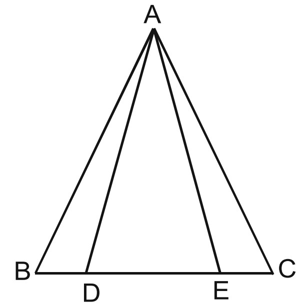
(b) 14 : 15

(c) 12 : 11

(d) 7 : 5

2016

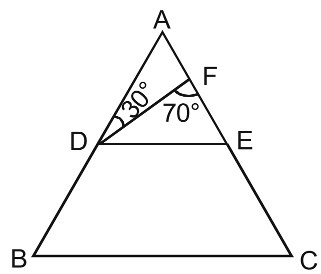
43. In the figure given below, AD = AE = 4 cm and BD = CE = 2 cm. If BC is a straight line and $\angle ADE = 60°$, find the length of AB.



(a) $2\sqrt{7}$ cm (b) 5 cm

(c) $2\sqrt{5}$ cm (d) None of these

44. In an isosceles triangle ABC shown below, AB = AC, DE || BC, $\angle ADF = 30°$ and $\angle DFE = 70°$. If DF = 4 cm, then find the length of DE.



(a) 2 cm (b) 4 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

2018 Slot 1

45. Given an equilateral triangle T1 with side 24 cm, a second triangle T2 is formed by joining the midpoints of the sides of T1. Then a third triangle T3 is formed by joining the midpoints of the sides of T2. If this process of forming triangles is continued, the sum of the areas, in sq cm, of infinitely many such triangles T1, T2, T3,... will be

(a) $248\sqrt{3}$ (b) $192\sqrt{3}$

(c) $188\sqrt{3}$ (d) $164\sqrt{3}$

2018 Slot 2

46. A triangle ABC has area 32 sq units and its side BC, of length 8 units, lies on the line x = 4. Then the shortest possible distance between A and the point (0, 0) is

(a) 8 units (b) 4 units

(c) $2\sqrt{2}$ units (d) $4\sqrt{2}$ units

QUADRILATERALS

1990

1. A square is drawn by joining the midpoints of the sides of a given square. A third square is drawn inside the second square in the same way and this process is continued indefinitely. If side of the first square is 8 cm, the sum of the areas of all the squares such formed (in sq.cm.)is

(a) 128 (b) 120

(c) 96 (d) None of these

1991

Direction for Question 2 : The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II alone,

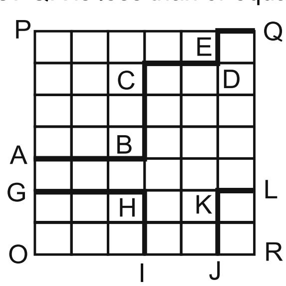
Mark (c) if both the statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

2. What is the area under the line GHI – JKL in the given quadrilateral OPQR, knowing that all the small spaces are squares of the same area?

I. Length ABCDEQ is greater than or equal to 60.

II. Area OPQR is less than or equal to 1512.



3. Let the consecutive vertices of a square S be A, B, C & D. Let E, F & G be the mid-points of the sides AB, BC & AD respectively of the square. Then the ratio of the area of the quadrilateral EFDG to that of the square S is nearest to

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/8

1994

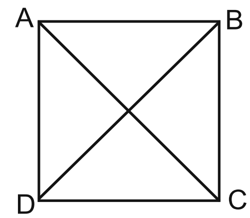
4. Four friends start from four towns, which are at the four corners of an imaginary rectangle. They meet at a point which falls inside the rectangle, after travelling distances of 40, 50 and 60 metres. The maximum distance that the fourth could have traveled is (approximately) ….

(a) 67 metres (b) 52 metres

(c) 22.5 metres (d) Cannot be determined

1995

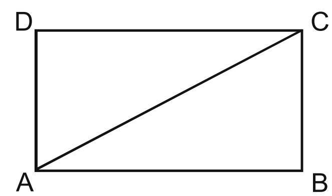
5. ABCD is a square of area 4, which is divided into four non-over lapping triangles as shown in figure. Then the sum of the perimeters of the triangles is



(a) $8(2 + \sqrt{2})$ (b) $8(1+ \sqrt{2})$

(c) $4(1 + \sqrt{2})$ (d) $4(2 + \sqrt{2})$

6. In the adjoining figure, AC+ AB = 5AD and AC – AD = 8. Then the area of the rectangle ABCD is



(a) 36 (b) 50

(c) 60 (d) Cannot be answered

Direction for Question 7: The question is followed by two statements, I and II. Mark the answer as:

(a) if the question can be answered with the help of statement I alone.

(b) if the question can be answered with the help of statement II alone.

(c) if both statement I and statement II are needed to answer the question.

(d) if the question cannot be answered even with the help of both the statements.

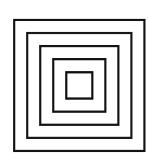
7. What is the length of rectangle ABCD?

I. Area of the rectangle is 48 square units.

II. Length of the diagonal is 10 units.

1997

8. The adjoining figure shows a set of concentric squares. If the diagonal of the innermost square is 2 units, and if the distance between the corresponding corners of any two successive squares is 1 unit, find the difference between the areas of the eighth and the seventh squares, counting from the innermost square.

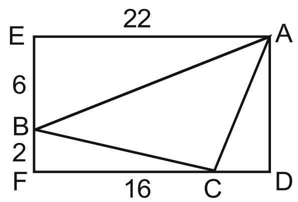


(a) $10\sqrt{2}$ sq. units (b) 30 sq. units

(c) $35\sqrt{2}$ sq. units (d) None of these

9. In a rectangle, the difference between the sum of the adjacent sides and the diagonal is half the length of the longer side. What is the ratio of the shorter to the longer side?
(a) $\sqrt{3} : 2$ (b) $1 : \sqrt{3}$
(c) 2 : 5 (d) 3 : 4

10. In the given figure, EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of EADF and AE = 22, BE = 6, CF = 16 and BF = 2. Find the length of the line joining the mid-points of the sides AB and BC.



(a) $4\sqrt{2}$ (b) 5
(c) 3.5 (d) None of these

1999

Directions for Questions 11 and 12: Answer the questions based on the following information.

A rectangle PRSU, is divided into two smaller rectangles PQTU, and QRST by the line TQ. PQ = 10 cm.

QR = 5 cm and RS = 10 cm. Points A, B, F are within rectangle PQTU, and points C, D, E are within the rectangle QRST. The closest pair of points among the pairs (A, C), (A, D), (A, E), (F, C), (F, D), (F, E), (B, C), (B, D), (B, E)are $10\sqrt{3}$ cm apart.

11. Which of the following statements is necessarily true?
(a) The closest pair of points among the six given points cannot be (F, C)
(b) Distance between A and B is greater than that between F and C.
(c) The closest pair of points among the six given points is (C, D), (D, E), or (C, E).
(d) None of the above

12. AB > AF > BF ; CD > DE >CE ; and BF = $6\sqrt{5}$ cm.
Which is the closest pair of points among all the six given points?
(a) B, F (b) C, D
(c) A, B (d) None of these

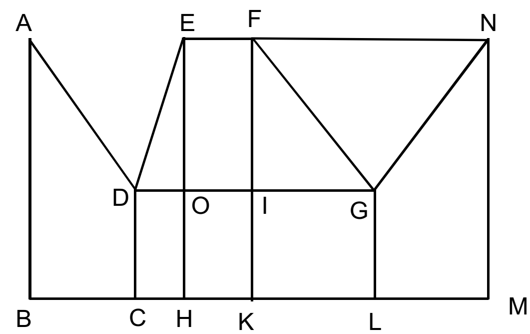
2001

13. A square, whose side is 2 m, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in metres, is
(a) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{2}{\sqrt{2}+1}$
(c) $\frac{2}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}}{\sqrt{2}-1}$

2002

14. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Directions for Questions 15 and 16: Answer the questions based on the following diagram.



In the above diagram, $\angle ABC = 90° = \angle DCH = \angle DOE = \angle EHK = \angle FKL = \angle GLM = \angle LMN$ AB = BC = 2CH = 2CD = EH = FK = 2HK = 4KL = 2LM = MN

15. The magnitude of $\angle FGO$ =
(a) 30° (b) 45°
(c) 60° (d) None of these

16. What is the ratio of the areas of the two quadrilaterals ABCD to DEFG?
(a) 1 : 2 (b) 2 : 1
(c) 12 : 7 (d) None of these

2003(L)

17. A vertical tower OP stands at the center O of a square ABCD. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$ then the relationship between h and b can be expressed as
(a) $2b^2 = h^2$
(b) $2h^2 = b^2$
(c) $3b^2 = 2h^2$
(d) $3h^2 = 2b^2$

2006

18. An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in degrees?

(a) 75 (b) 90
(c) 120 (d) 135
(e) 150

2007

Direction for Question 19: The question is followed by two statements A and B. Indicate your response based on the following directives.

Mark (a) if the questions can be answered using A alone but not using B alone.

Mark (b) if the question can be answered using B alone but not using A alone.

Mark (c) if the question can be answered using A and B together, but not using either A or B alone.

Mark (d) if the question cannot be answered even using A and B together.

19. Rahim plans to draw a square JKLM with point O on the side JK but is not successful. Why is Rahim unable to draw the square?

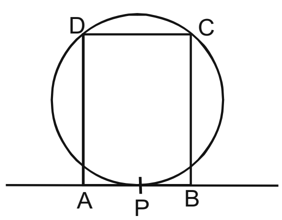
A : The length of OM is twice that of OL.

B : The length of OM is 4 cm.

MEMORY BASED QUESTIONS

2010

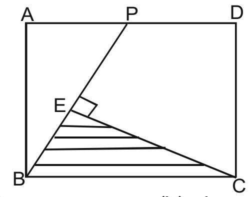
20. In the figure given below, a tangent is drawn at point P on a circle of radius 1 cm. A and B are two points on the tangent and ABCD is a rectangle, where C and D are two points on the circumference of the circle. What is the approximate area (in cm^2) of the rectangle ABCD if 2AB = BC?



(a) 1.77 (b) 1.50
(c) 1.83 (d) 1.60

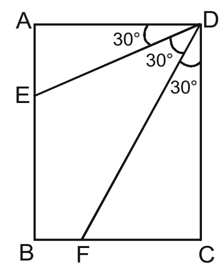
2011

21. In the figure given below ABCD is a square of side 4 cm. P is the midpoint of AD and is joined with vertex B. A perpendicular is drawn from vertex C on BP, which intersects BP at point E. What is the area of ΔBEC?



(a) 6.4 cm^2 (b) 4 cm^2
(c) 3.2 cm^2 (d) None of these

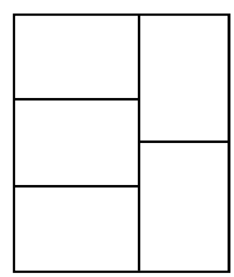
22. In the figure given below ABCD is a rectangle. The ratio of the length of EB to the length of BF is $2:\sqrt{3}$. What is the ratio of the length of BF to the length of FC?



(a) $9:5\sqrt{3}$ (b) $2\sqrt{3}:2$
(c) 2 : 3 (d) None of these

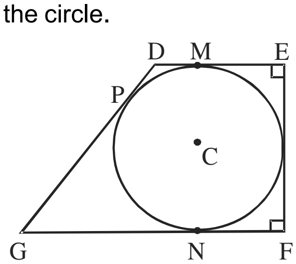
2012

23. A rectangle with perimeter 88 m is partitioned into 5 congruent rectangles, as shown in the diagram given below. The perimeter of each of the congruent rectangles is



(a) 20 m (b) 32 m
(c) 48 m (d) 40 m

24. A trapezium DEFG is circumscribed about a circle that has centre at C. If DM = 1 cm, GN = 4 cm and the measure of $\angle DEF = \angle EFG = 90°$, then find the radius of the circle.



(a) 2 cm (b) 2.5 cm
(c) 2.25 cm (d) 4 cm

2014

25. How many rectangles with integral sides are possible where the area of the rectangle equals the perimeter of the rectangle?

(a) One (b) Three
(c) Two (d) Infinitely many

26. ABCD is a rectangle with points E and F lying on sides AB and CD respectively. If the area of quadrilateral AEFD equals the area of quadrilateral CBEF, then which of the following statements is necessarily false with respect to the rectangle ABCD?

(a) Length of AE is always equal to the length of CF.
(b) If the length of BC is 4 units, then the smallest integral length of EF is 5 units.
(c) Length of AE is equal to the length of DF.
(d) $\angle AEF = \angle EFC$.

2015

27. ABCD is an isosceles trapezium with BC = AD = 10 units, AB = 2 units and CD = 14 units. The mid-points of the sides of the trapezium are joined to form a quadrilateral PQRS. Find the ratio of the area of the circle inscribed in the quadrilateral PQRS to the area of trapezium ABCD.

(a) $\frac{3\pi}{8}$ (b) $\frac{3\pi}{16}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

2017

28. ABCD is a quadrilateral inscribed in a circle with centre O. If $\angle COD$ = 120 degrees and $\angle BAC$ = 30 degrees, then the value of $\angle BCD$ (in degrees) is

2018 Slot 1

29. In a parallelogram ABCD of area 72 sq cm, the sides CD and AD have lengths 9 cm and 16 cm, respectively. Let P be a point on CD such that AP is perpendicular to CD. Then the area, in sq cm, of triangle APD is

(a) $32\sqrt{3}$ (b) $18\sqrt{3}$
(c) $12\sqrt{3}$ (d) $24\sqrt{3}$

30. Points E, F, G, H lie on the sides AB, BC, CD, and DA, respectively, of a square ABCD. If EFGH is also a square whose area is 62.5% of that of ABCD and CG is longer than EB, then the ratio of length of EB to that of CG is

(a) 1 : 3 (b) 3 : 8
(c) 4 : 9 (d) 2 : 5

2018 Slot 2

31. From a rectangle ABCD of area 768 sq cm, a semicircular part with diameter AB and area 72π sq cm is removed. The perimeter of the leftover portion, in cm, is

(a) $80 + 16\pi$ (b) $82 + 24\pi$
(c) $86 + 8\pi$ (d) $88 + 12\pi$

32. A parallelogram ABCD has area 48 sqcm. If the length of CD is 8 cm and that of AD is s cm, then which one of the following is necessarily true?

(a) $5 \le s \le 7$ (b) $s \ne 6$
(c) $s \ge 6$ (d) $s \le 6$

33. The area of a rectangle and the square of its perimeter are in the ratio 1 : 25. Then the lengths of the shorter and longer sides of the rectangle are in the ratio

(a) 2:9 (b) 1:3
(c) 3:8 (d) 1:4

POLYGONS

1993

Direction for Question 1 : The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,
Mark (b) if the question can be answered with the help of statement II, alone,
Mark (c) if both, statement I and statement II are needed to answer the question, and
Mark (d) if the question cannot be answered even with the help of both the statements.

1. What is the area of a regular hexagon?

I. The length of the boundary line of the hexagon is 36 cm.
II. The area of the hexagon is 6 times the area of an equilateral triangle formed on one of the sides.

1999

2. There is a circle of radius 1 cm. Each member of a sequence of regular polygons S1(n), n = 4, 5, 6, …, where n is the number of sides of the polygon, is circumscribing the circle: and each member of the sequence of regular polygons S2(n), n = 4, 5, 6, … where n is the number of sides of the polygon, is inscribed in the circle. Let L1(n) and L2(n) denote the perimeters of the corresponding polygons of S1(n) and S2(n), then $\frac{\{L1(13) + 2\pi\}}{L2(17)}$ is

(a) greater than $\frac{\pi}{4}$ and less than 1

(b) greater than 1 and less than 2

(c) greater than 2

(d) less than $\frac{\pi}{4}$

Direction for Question 3: The question is followed by two statements I and II.

Mark:

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both the statements together.

3. Mr X starts walking northwards along the boundary of a field from point A on the boundary, and after walking for 150 m reaches B, and then walks westwards, again along the boundary, for another 100 m when he reaches C. What is the maximum distance between any pair of points on the boundary of the field?

I. The field is rectangular in shape.

II. The field is a polygon, with C as one of its vertices and A as the mid-point of a side.

2003(R)

4. Let ABCDEF be a regular hexagon. What is the ratio of the area of the Δ ACE to that of the hexagon ABCDEF?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

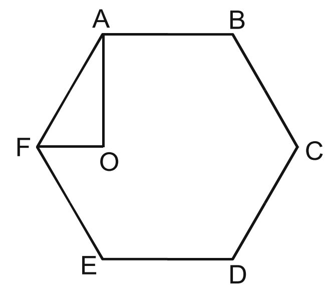
2003(L)

5. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be

(a) 20 (b) 0

(c) 21 (d) 22

6. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{24}$ (d) $\frac{1}{18}$

MEMORY BASED QUESTIONS

2009

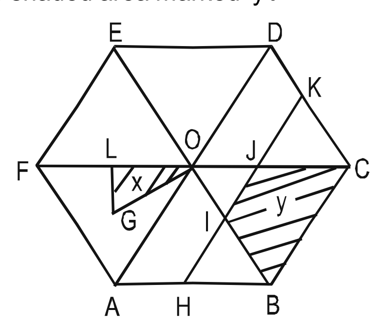
7. ABCDEF is a regular hexagon with AB = 5 cm. Nine line segments are drawn in the hexagon in such a way that AB, DE and these line segments form eleven equidistant parallel lines. If the ends of each of the line segments lie on the perimeter of the hexagon, then what is the sum (in cm) of the lengths of the nine line segments?

(a) 80 (b) 60

(c) 70 (d) 75

2010

8. In the figure given below, ABCDEF is a regular hexagon whose diagonals intersect at point O. G is the centroid of triangle AOF and L is the midpoint of FO. The line segment HK joins the midpoints of AB and CD. Find the ratio of the shaded area marked 'x' to the shaded area marked 'y'.

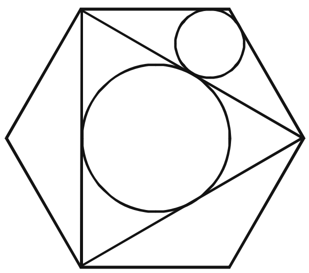


(a) 3 : 7 (b) 2 : 9

(c) 2 : 5 (d) 3 : 11

2012

9. In the regular hexagon shown below, what is the ratio of the area of the smaller circle to that of the bigger circle?



(a) $3:7+2\sqrt{3}$ (b) $3:7+\sqrt{3}$

(c) $3:16+4\sqrt{3}$ (d) $3:7+4\sqrt{3}$

2016

10. In a regular polygon, the number of diagonals is 'k' times the number of sides. If the interior angle of the polygon is θ, then the value of k is

(a) $\frac{3\theta-\pi}{2(\pi-\theta)}$ (b) $\frac{2(3\theta-\pi)}{\pi-\theta}$

(c) $\frac{2(\pi-\theta)}{3\theta-\pi}$ (d) $\frac{(\pi-\theta)}{2(3\theta-\pi)}$

2017

11. Let ABCDEF be a regular hexagon with each side of length 1 cm. The area (in sq cm) of a square with AC as one side is

(a) $3\sqrt{2}$ (c) 3

(c) 4 (d) $\sqrt{3}$

CIRCLE

1991

Direction for Question 1: The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II alone,

Mark (c) if both the statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

1. What is the radius of the circle?

I. Ratio of its area to circumference is > 7.

II. Diameter of the circle is ≤ 32.

2. A circle is inscribed in a given square and another circle is circumscribed about the square. What is the ratio of the area of the inscribed circle to that of the circumscribed circle?

(a) 2 : 3 (b) 3 : 4

(c) 1 : 4 (d) 1 : 2

3. A one rupee coin is placed on a table. The maximum number of similar one rupee coins which can be placed on the table, around it, with each one of them touching it and only two others is

(a) 8 (b) 6

(c) 10 (d) 4

1993

4. Three identical cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is

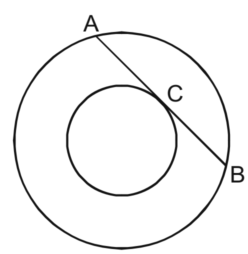
(a) smaller than r

(b) equal to r

(c) larger than r

(d) depends on the height of the cones.

5. The line AB is 6 metres in length and is tangent to the inner one of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer circle is

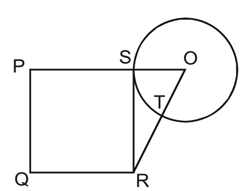


(a) 5 metres (b) 4 metres

(c) 6 metres (d) 3 metres

1995

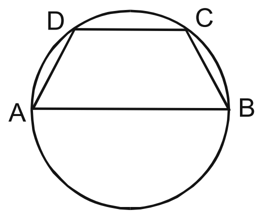
6. PQRS is a square. SR is a tangent (at point S) to the circle with centre O and TR = OS. Then the ratio of area of the circle to the area of the square is



(a) $\frac{\pi}{3}$ (b) $\frac{11}{7}$

(c) $\frac{3}{\pi}$ (d) $\frac{7}{11}$

7. In the given figure, AB is diameter of the circle and points C and D are on the circumference such that $\angle CAD = 30°$ and $\angle CBA = 70°$. What is the measure of $\angle ACD$?



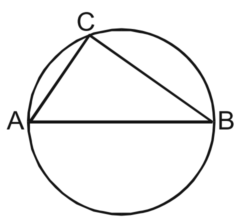
(a) 40° (b) 50°
(c) 30° (d) 90°

1996

8. From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the uncut to the cut portion?

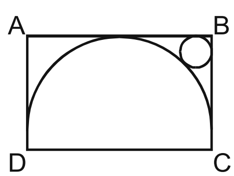
(a) 1 : 3 (b) 4 : 1
(c) 3 : 1 (d) 4 : 3

9. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of $\triangle ABC$.



(a) 60 sq. cm (b) 30 sq. cm
(c) 40 sq. cm (d) 52 sq. cm

10. The figure shows the rectangle ABCD with a semicircle and a circle inscribed inside in it as shown. What is the ratio of the area of the circle to that of the semicircle?



(a) $(\sqrt{2}-1)^2 : 1$ (b) $2(\sqrt{2}-1)^2 : 1$
(c) $(\sqrt{2}-1)^2 : 2$ (d) None of these

Direction for Question 11: The question is followed by two statements, I and II. Mark the answer as:

(a) if the question cannot be answered even with the help of both the statements taken together.

(b) if the question can be answered by any one of the two statements.

(c) if each statement alone is sufficient to answer the question, but not the other one (E.g. statement I alone is required to answer the question, but not statement II and vice versa).

(d) if both statements I and II together are needed to answer the question.

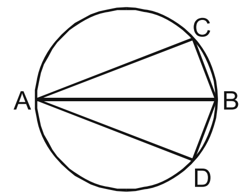
11. A tractor travelled a distance 5 m. What is the radius of the rear wheel?

I. The front wheel rotates 'N' times more than the rear wheel over this distance.

II. The circumference of the rear wheel is 't' times that of the front wheel.

1997

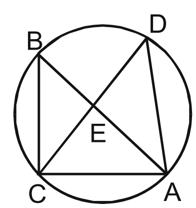
12.



AB is the diameter of the given circle, while points C and D lie on the circumference as shown. If AB is 15 cm, AC is 12 cm and BD is 9 cm, find the area of the quadrilateral ACBD.

(a) 54π sq. cm (b) 216π sq. cm
(c) 162π sq. cm (d) None of these

13. In the adjoining figure, points A, B, C and D lie on the circle. AD = 24 and BC = 12. What is the ratio of the area of $\triangle CBE$ to that of $\triangle ADE$?



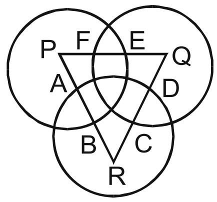
(a) 1 : 4
(b) 1 : 2
(c) 1 : 3
(d) Data insufficient

14. The sum of the areas of two circles, which touch each other externally, is 153π. If the sum of their radii is 15, find the ratio of the larger to the smaller radius.

(a) 4
(b) 2
(c) 3
(d) None of these

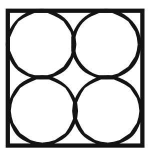
1998

15. Three circles, each of radius 20, have centres at P, Q and R. Further, AB = 5, CD = 10 and EF = 12. What is the perimeter of ΔPQR?



(a) 120 (b) 66
(c) 93 (d) 87

16. Four identical coins are placed in a square. For each coin the ratio of area to circumference is same as the ratio of circumference to area. Then find the area of the square that is not covered by the coins.



(a) $16(\pi - 1)$
(b) $16(8 - \pi)$
(c) $16(4 - \pi)$
(d) $16\left(4 - \frac{\pi}{2}\right)$

Directions for Questions 17 and 18: Each question is followed by two statements, I and II. Answer the questions based on the statements and mark the answer as:

(a) if the question can be answered with the help of any one statement alone but not by the other statement.
(b) if the question can be answered with the help of either of the statements taken individually.
(c) if the question can be answered with the help of both statements together.
(d) if the question cannot be answered even with the help of both statements together.

17. There are two concentric circles C_1 and C_2 with radii r_1 and r_2. The circles are such that C_1 fully encloses C_2. Then what is the radius of C_1?

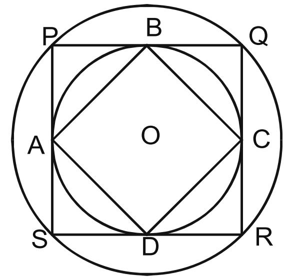
I. The difference of their circumference is k cm.
II. The difference of their areas is m sq. cm.

18. A circle circumscribes a square. What is the area of the square?

I. Radius of the circle is given.
II. Length of the tangent from a point 5 cm away from the centre of the circle is given.

1999

19. The figure below shows two concentric circles with centre O. PQRS is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points B, C, D and A. What is the ratio of the perimeter of the outer circle to that of polygon ABCD?



(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{2}$
(c) $\frac{\pi}{2}$ (d) π

Direction for Questions 20: The question is followed by two statements I and II.

Mark:

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.
(b) if the question can be answered by using either statement alone.
(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
(d) if the question cannot be answered even by using both the statements together.

20. There is a circle with centre C at the origin and radius r cm. Two tangents are drawn from an external point D at a distance d cm from the centre. What are the angles between each tangent and the X-axis?

I. The coordinates of D are given.
II. The X-axis bisects one of the tangents.

2000

21. Consider a circle with unit radius. There are seven adjacent sectors, S_1, S_2, S_3, ... , S_7, in the circle such that their total area is $\frac{1}{8}$ of the area of the circle. Further, the area of the jth sector is twice that of the (j – 1)th sector, for j = 2, ..., 7. What is the angle, in radians, subtended by the arc of S_1 at the centre of the circle?

(a) $\frac{\pi}{508}$ (b) $\frac{\pi}{2040}$
(c) $\frac{\pi}{1016}$ (d) $\frac{\pi}{1524}$

Directions for Questions 22 and 23: Each question is followed by two statements, I and II. Answer each question using the following instructions.

Mark the answer as:

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

22. ΔPQR has $\angle PRQ = 90°$. What is the value of PR + RQ?

I. Diameter of the inscribed circle of the ΔPQR is equal to 10 cm.

II. Diameter of the circumscribed circle of the ΔPQR is equal to 18 cm.

23. 'O' is the centre of two concentric circles, AE is a chord of the outer circle and it intersects the inner circle at points 'B' and 'D'. 'C' is a point on the chord in between 'B' and 'D'.

What is the value of AC/CE?

I. BC/CD = 1

II. A third circle intersects the inner circle at 'B' and 'D' and the point 'C' is on the line joining the centres of the third circle and the inner circle.

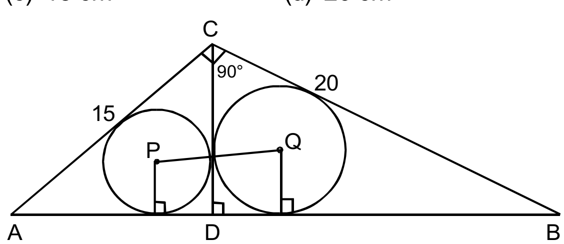
2002

24. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is

(a) 24 cm (b) 25 cm

(c) 15 cm (d) 20 cm

25.



In the above figure, ACB is a right-angled triangle. CD is the altitude. Circles are inscribed within the ΔACD and ΔBCD. P and Q are the centres of the circles. The distance PQ is

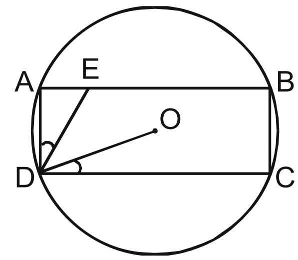
(a) 5 (b) $\sqrt{50}$

(c) 7 (d) 8

2003(R)

26. In the figure below (not drawn to scale), rectangle ABCD is inscribed in the circle with centre at O. The length of side AB is greater than side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : \sqrt{3}$. The line segment DE intersects AB at E such that $\angle ODC = \angle ADE$.

The ratio AE : AD is

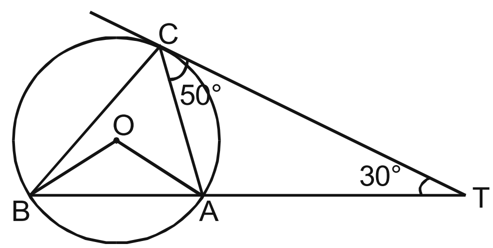


(a) $1:\sqrt{3}$ (b) $1:\sqrt{2}$

(c) $1:2\sqrt{3}$ (d) 1 : 2

27. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C.

If $\angle ATC = 30°$ and $\angle ACT = 50°$, then the angle $\angle BOA$ is



(a) 100°

(b) 150°

(c) 80°

(d) not possible to determine

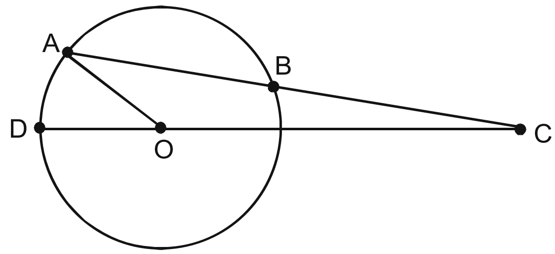
2003(L)

28. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then the area (in square centimeters) of the triangle ABC would be

(a) $\pi\sqrt{12}$ (b) $\frac{9}{\pi}$

(c) $\frac{9\sqrt{3}}{\pi}$ (d) $\frac{6\sqrt{3}}{\pi}$

29. In the figure below, AB is the chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD$ = y degrees and $\angle AOD$ = x degrees such that x = ky, then the value of k is



(a) 3 (b) 2
(c) 1 (d) None of the above

Direction for Question 30: The question is followed by two statements, A and B. Answer the question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.
Choose (b) if the question can be answered by using either statement alone.
Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
Choose (d) if the question cannot be answered even by using both the statements together.

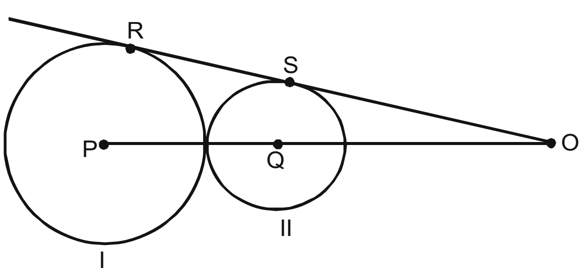
30. AB is a chord of a circle. AB = 5 cm. A tangent parallel to AB touches the minor arc AB at E. What is the radius of the circle?

A. AB is not a diameter of the circle.
B. The distance between AB and the tangent at E is 5 cm.

2004

Directions for Questions 31 to 33: Answer the questions on the basis of the information given below.

In the adjoining figure I and II, are circles with P and Q respectively, The two circles touch each other and have common tangent that touches them at points R and S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4 : 3. It is also known that the length of PO is 28 cm.

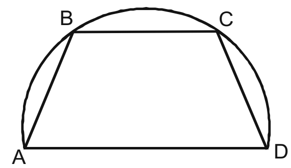


31. What is the ratio of the length of PQ to that of QO?
(a) 1 : 4 (b) 1 : 3
(c) 3 : 8 (d) 3 : 4

32. What is the radius of the circle II?
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

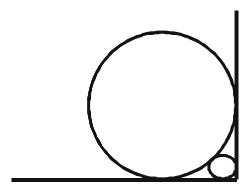
33. The length of SO is
(a) $8\sqrt{3}$ cm (b) $10\sqrt{3}$ cm
(c) $12\sqrt{3}$ cm (d) $14\sqrt{3}$ cm

34. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC?



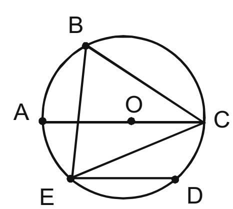
(a) 7.5 (b) 7
(c) 7.75 (d) None of the above

35. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



(a) $3-2\sqrt{2}$ (b) $4-2\sqrt{2}$
(c) $7-4\sqrt{2}$ (d) $6-4\sqrt{2}$

36. In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If $\angle CBE = 65°$, then what is the value of $\angle DEC$?

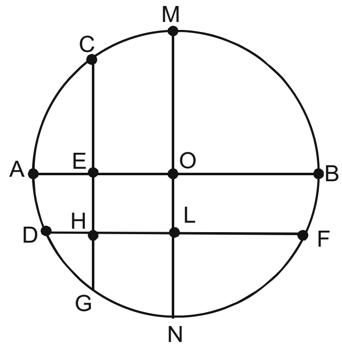


(a) 35° (b) 55°
(c) 45° (d) 25°

2005

37. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?
(a) 1 or 7 (b) 2 or 14
(c) 3 or 21 (d) 4 or 28

38. In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE : EB = 1 : 2, and DF is perpendicular to MN such that NL : LM = 1 : 2. The length of DH in cm is



(a) $2\sqrt{2}-1$ (b) $\frac{(2\sqrt{2}-1)}{2}$

(c) $\frac{(3\sqrt{2}-1)}{2}$ (d) $\frac{(2\sqrt{2}-1)}{3}$

39. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral PQSR?

(a) $2r(1+\sqrt{3})$ (b) $2r(2+\sqrt{3})$

(c) $r(1+\sqrt{5})$ (d) $2r+\sqrt{3}$

2006

40. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semi-circle at D. Given that AC = 2 cm and CD = 6 cm, the area of the semi-circle (in sq. cm)will be:

(a) 32π (b) 50π

(c) 40.5π (d) 81π

(e) undeterminable

2007

41. Two circles with centres P and Q cut each other at two distinct points A and B. The circles have the same radii and neither P nor Q falls within the intersection of the circles. What is the smallest range that includes all possible values of the angle AQP in degrees?

(a) Between 0 and 90

(b) Between 0 and 30

(c) Between 0 and 60

(d) Between 0 and 75

(e) Between 0 and 45

2008

42. In a triangle ABC, the lengths of the sides AB and AC are equal to 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD = 3 cm, then what is the radius (in cm)of the circle circumscribing the triangle ABC?

(a) 17.05 (b) 27.85

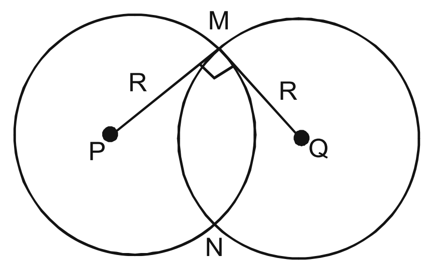
(c) 22.45 (d) 32.25

(e) 26.25

MEMORY BASED QUESTIONS

2009

43. In the given figure, two equal circles (with radius 'R' units) intersect each other at two points M and N such that $\angle PMQ = 90°$. What is the area (in sq units) of the region that is common to both the circles?



(a) $R^2 \times \left(\frac{\pi}{2}-1\right)$

(b) $\frac{1}{2}R^2 \times \left(\frac{\pi}{2}-1\right)$

(c) $\frac{1}{2}R^2 \times \left(1-\frac{\pi}{2}\right)$

(d) $R^2 \times \left(1-\frac{\pi}{2}\right)$

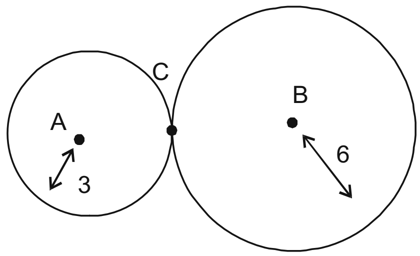
2012

44. A circle of radius 6.5 cm is circumscribed around a right-angled triangle with the sides a, b and c cm where a, b and c are natural numbers. What is the perimeter of the triangle?

(a) 30 cm (b) 26 cm

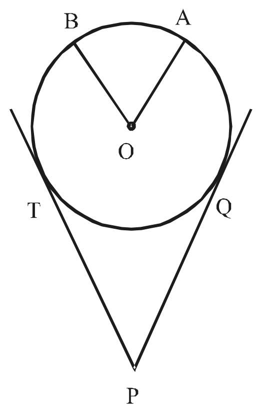
(c) 28 cm (d) 32 cm

45. Two circles with centers A and B touch each other at C. The radii of the two circles are 3 m and 6 m respectively. Ramu and Shamu start simultaneously from C with speeds 6π m/s and 3π m/s and travel along the circles with centers A and B respectively. If Ramu gives Shamu a start of 2 seconds, what time (in seconds)after Ramu's start would they be separated by a distance of 18 m?



(a) 7 (b) 10
(c) 125 (d) Never

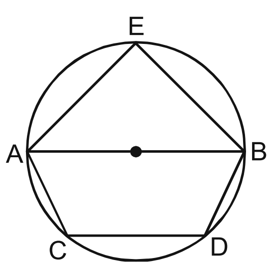
46. From a point P, the tangents PQ and PT are drawn to a circle with centre O and radius 2 units. From the centre O, OA and OB are drawn parallel to PQ and PT respectively. The length of the chord TQ is 2 units. Find the measure of the $\angle AOB$.



(a) 30° (b) 90°
(c) 120° (d) 45°

2013

47. In the figure given below, AB is a diameter of the circle. If AB II CD, AC II BE and ∠BAE = 35°, then the absolute difference between ∠CDB and ∠ABD is



(a) 90° (b) 70°
(c) 55° (d) 125°

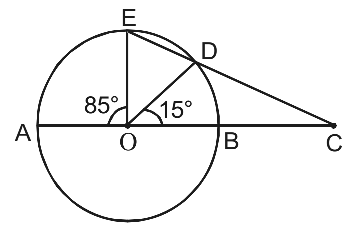
2014

48. AC is a chord of a circle whose centre is at O. If B is any point on the arc AC and ∠OCA = 20°, then the magnitude of ∠ABC is

(a) 110° (b) 70°
(c) 140° (d) Either (a) or (b)

2015

49. In the given figure, AB is the diameter of the circle with centre O. If ∠BOD = 15°, ∠EOA = 85°, then find the measure of ∠ECA.

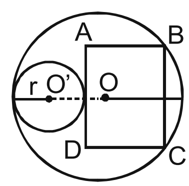


(a) 20° (b) 25°
(c) 35° (d) Cannot be determined

50. The smallest possible circle touching two opposite sides of a rectangle is cut-out from a rectangle of area 60 sq. units. If the area of this circle is $\frac{3}{2}$ times the area left out in the rectangle, find the length of the smaller side of the rectangle.

(a) $\frac{6}{\sqrt{\pi}}$ units (b) $\frac{9}{\sqrt{\pi}}$ units
(c) $\frac{12}{\sqrt{\pi}}$ units (d) $\frac{15}{\sqrt{\pi}}$ units

51.



In the figure, O and O' are the centres of the bigger and smaller circles respectively and small circle touches the square ABCD at the mid point of side AD. The radius of the bigger circle is equal to 15 cm and the side of the square ABCD is 18 cm. Find the radius of the smaller circle.

(a) 4.25 cm (b) 4.5 cm
(c) 4.75 cm (d) 5 cm

2018 Slot 1

52. In a circle, two parallel chords on the same side of a diameter have lengths 4 cm and 6 cm. If the distance between these chords is 1 cm, then the radius of the circle, in cm, is

(a) $\sqrt{11}$ (b) $\sqrt{14}$
(c) $\sqrt{13}$ (d) $\sqrt{12}$

53. Let ABCD be a rectangle inscribed in a circle of radius 13 cm. Which one of the following pairs can represent, in cm, the possible length and breadth of ABCD?

(a) 24, 12 (b) 24, 10
(c) 25, 10 (d) 25, 9

54. In a circle with center O and radius 1 cm, an arc AB makes an angle 60 degrees at O. Let R be the region bounded by the radii OA, OB and the arc AB. If C and D are two points on OA and OB, respectively, such that OC = OD and the area of triangle OCD is half that of R, then the length of OC, in cm, is

(a) $\left(\frac{\pi}{6}\right)^{\frac{1}{2}}$ (b) $\left(\frac{\pi}{4}\right)^{\frac{1}{2}}$

(c) $\left(\frac{\pi}{3\sqrt{3}}\right)^{\frac{1}{2}}$ (d) $\left(\frac{\pi}{4\sqrt{3}}\right)^{\frac{1}{2}}$

2018 Slot 2

55. On a triangle ABC, a circle with diameter BC is drawn, intersecting AB and AC at points P and Q, respectively. If the lengths of AB, AC, and CP are 30 cm, 25 cm, and 20 cm respectively, then the length of BQ, in cm, is

56. A chord of length 5 cm subtends an angle of 60° at the centre of a circle. The length, in cm, of a chord that subtends an angle of 120° at the centre of the same circle is

(a) $6\sqrt{2}$ (b) 8

(c) $5\sqrt{3}$ (d) 2π

MENSURATION

1991

1. A square piece of cardboard of sides ten inches is taken and four equal squares pieces are removed at the corners, such that the side of this square piece is also an integer value. The sides are then turned up to form an open box. Then the maximum volume such a box can have is

(a) 72 cubic inches.

(b) 24.074 cubic inches.

(c) 2000/27 cubic inches

(d) 64 cubic inches.

1993

2. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and resolidified into the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to

(a) 3 (b) 3.5

(c) 4 (d) 4.5

3. The diameter of a hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

(a) 50% (b) less than 50%

(c) more than 50% (d) 100%

1994

4. A right circular cone, a right circular cylinder and a hemisphere, all have the same radius, and the heights of the cone and cylinder equal to their diameters. Then their volumes are proportional, respectively to

(a) 1 : 3 : 1 (b) 2 : 1 : 3

(c) 3 : 2 : 1 (d) 1 : 2 : 3

5. A right circular cone of height h is cut by a plane parallel to the base and at a distance h/3 from the base, then the volumes of the resulting cone and the frustum are in the ratio

(a) 1 : 3 (b) 8 : 19

(c) 1 : 4 (d) 1 : 7

Direction for Question 6 : The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II, alone,

Mark (c) if both, statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

6. What will be the total cost of creating a 1- foot border of tiles along the inside edges of a room?

I. The room is 48 feet in length and 50 feet in breadth.

II. Every tile costs Rs. 10.

1996

7. A wooden box (open at the top) of thickness 0.5 cm, length 21 cm, width 11 cm and height 6 cm is painted on the inside. The expenses of painting are Rs. 70. What is the rate of painting per square centimetres?

(a) Re. 0.7 (b) Re. 0.5

(c) Re. 0.1 (d) Re. 0.2

1997

Direction for Question 8 : The question is followed by two statements. As the answer,

(a) if the question can be answered with the help of one statement alone.

(b) if the question can be answered with the help of any one statement independently.

(c) if the question can be answered with the help of both statements together.

(d) if the question cannot be answered even with the help of both statements together.

8. What is the ratio of the volume of the given right circular cone to the one obtained from it?

I. The smaller cone is obtained by passing a plane parallel to the base and dividing the original height in the ratio 1 : 2.

II. The height and the base of the new cone are one-third those of the original cone.

1999

Direction for Question 9 : The question is followed by two statements. As the answer,

(a) if the question can be answered by any one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both the statements together.

9. A small storage tank is spherical in shape. What is the storage volume of the tank?

I. The wall thickness of the tank is 1 cm.

II. When an empty spherical tank is immersed in a large tank filled with water, 20 l of water overflow from the large tank.

10. There is a square field of side 500 m long each. It has a compound wall along its perimeter. At one of its corners, a triangular area of the field is to be cordoned off by erecting a straight-line fence. The compound wall and the fence will form its borders. If the length of the fence is 100 m, what is the maximum area that can be cordoned off?

(a) 2,500 sq m (b) 10,000 sq m

(c) 5,000 sq m (d) 20,000 sq m

2000

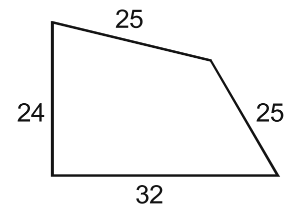
11. A farmer has decided to build a wire fence along one straight side of his property. For this, he planned to place several fence-posts at 6 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 8 m apart. What is the length of the side of his property and how many posts did he buy?

(a) 100 m, 15 (b) 100 m, 16

(c) 120 m, 15 (d) 120 m, 16

2001

12. Two sides of a plot measure 32 m and 24 m and the angle between them is a perfect right angle. The other two sides measure 25 m each and the other three angles are not right angles.



What is the area of the plot?

(a) 768 m^2 (b) 534 m^2

(c) 696.5 m^2 (d) 684 m^2

13. A rectangular pool 20 m wide and 60 m long is surrounded by a walkway of uniform width. If the total area of the walkway is 516 m^2, how wide, in metres, is the walkway?

(a) 43 m (b) 4.3 m

(c) 3 m (d) 3.5 m

Direction for Question 14: The question is followed by two statements, I and II.

Mark

(a) if the question can be answered by one of the statements alone and not by the other.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

14. A square is inscribed in a circle. What is the difference between the area of the circle and that of the square?

I. The diameter of the circle is $25\sqrt{2}$ cm.

II. The side of the square is 25 cm.

2002

15. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area.

(a) 22 m^2 (b) 42 m^2

(c) 84 m^2 (d) 168 m^2

16. Neeraj has agreed to mow a lawn, which is a 20 m × 40 m rectangle. He mows it with 1 m wide strip. If Neeraj starts at one corner and mows around the lawn toward the centre, about how many times would he go round before he has mowed half the lawn?

(a) 2.5

(b) 3.5

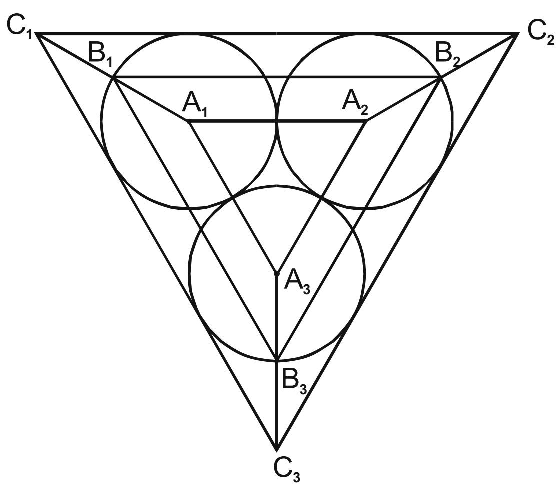
(c) 3.8

(d) 4

2003(R)

Directions for Questions 17 to 19: Answer the questions on the basis of the information given below.

Consider three circular parks of equal size with centres at A_1, A_2 and A_3 respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$ as shown. Three sprinters A, B and C begin running from points A_1, B_1 and C_1 respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.



17. Let the radius of each circular park be r, and the distances to be traversed by the sprinters A, B and C be a, b and c respectively. Which of the following is true?

(a) $b - a = c - b = 3\sqrt{3}\,r$

(b) $b - a = c - b = \sqrt{3}\,r$

(c) $b = \frac{a+c}{2} = 2(1 + \sqrt{3})r$

(d) $c = 2b - a = (2 + \sqrt{3})r$

18. Sprinter A traverses distances A_1A_2, A_2A_3 and A_3A_1 at an average speeds of 20, 30 and 15 respectively. B traverses her entire path at a uniform speed of $(10\sqrt{3} + 20)$. C traverses distances C_1C_2, C_2C_3 and C_3C_1 at an average speeds of $\frac{40}{3}(\sqrt{3}+1)$, $\frac{40}{3}(\sqrt{3}+1)$ and 120 respectively. All speeds are in the same unit. Where would B and C be respectively when A finishes her sprint?

(a) B_1, C_1

(b) B_3, C_3

(c) B_1, C_3

(d) B_1, Somewhere between C_3 and C_1

19. Sprinters A, B and C traverse their respective paths at uniform speeds of u, v and w respectively. It is known that $u^2 : v^2 : w^2$ is equal to Area A : Area B : Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$ respectively. Where would A and C be when B reaches point B_3?

(a) A_2, C_3

(b) A_3, C_3

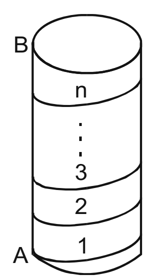
(c) A_3, C_2

(d) Somewhere between A_2 and A_3, Somewhere between C_3 and C_1

Directions for Questions 20 to 22: Answer the questions on the basis of the information given below.

Consider a cylinder of height h cm and radius $r = \frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns).

20. What is the vertical spacing between the two consecutive turns?



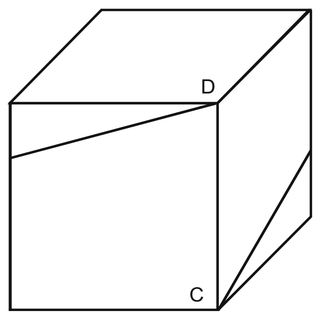
(a) $\frac{h}{n}$ cm

(b) $\frac{h}{\sqrt{n}}$ cm

(c) $\frac{h}{n^2}$ cm

(d) Cannot be determined

21. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string is



(a) $\sqrt{2}$ n cm (b) $\sqrt{17}$ n cm

(c) n cm (d) $\sqrt{13}$ n cm

22. In the set-up of the previous two questions, how is h related to n?

(a) h = $\sqrt{2}$ n (b) h = $\sqrt{17}$ n

(c) h = n (d) h = $\sqrt{13}$ n

23. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side *a* in the following steps: the original square of side *a* is divided into *n* smaller squares, not necessarily of the same size, then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side *a* and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is

(a) 1 : 1 (b) $\sqrt{2}$: 1

(c) $\frac{n(4-\pi)}{4n-\pi}$ (d) $\frac{4n-\pi}{n(4-\pi)}$

24. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s, respectively. Then,

(a) s > t > c

(b) c > t > s

(c) c > s > t

(d) s > c > t

25. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches)of the smaller triangle?

(a) 16.665 (b) 16.565

(c) 15.465 (d) 14.365

26. A square tin sheet of side 12 inches is converted into a box with open top in the following steps. The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?

(a) 3 (b) 4

(c) 1 (d) 2

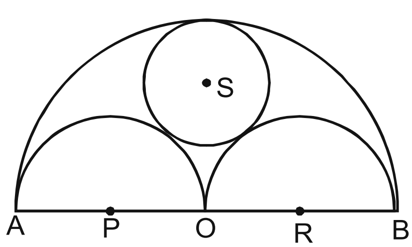
2003(L)

27. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

(a) 85.5 (b) 92.5

(c) 90.5 (d) 87.5

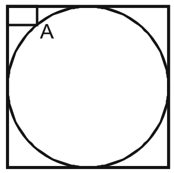
28. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with center at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centers at P and R respectively, and S is the center of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circles and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circle with diameter AB that cannot be grazed by the horses is nearest to



(a) 20 (b) 28

(c) 36 (d) 40

29. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



(a) 10 cm (b) 40 cm
(c) 50 cm (d) None of the above

2004

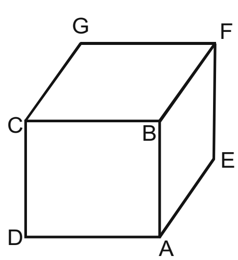
30. A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle?

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{2}$ (d) None of the above

31. Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let $C_1, C_2, C_3, \ldots$ be circles with diameters P_0P_1, P_1P_2, $P_2P_3 \ldots$ respectively. Suppose the circles $C_1, C_2, C_3, \ldots$ are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle is

(a) 8 : 9 (b) 9 : 10
(c) 10 : 11 (d) 11 : 12

32. If the lengths of diagonals DF, AG and CE of the cube shown in the adjoining figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be



(a) equal to the side of cube
(b) $\sqrt{3}$ times the side of the cube
(c) $\frac{1}{\sqrt{3}}$ times the side of the cube
(d) impossible to find from the given information.

2005

33. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in sq. cm of the portion that is common to the two circles is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2} - 1$
(c) $\frac{\pi}{5}$ (d) $\sqrt{2} - 1$

34. Four points A, B, C and D lie on a straight line in the X-Y plane, such that AB = BC = CD, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle is

(a) $3\sqrt{2}$ (b) $1 + \pi$
(c) $\frac{4\pi}{3}$ (d) 5

35. Rectangular tiles each of size 70 cm by 30 cm must be laid horizontally on a rectangular floor of size 110 cm by 130 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is

(a) 4
(b) 5
(c) 6
(d) 7

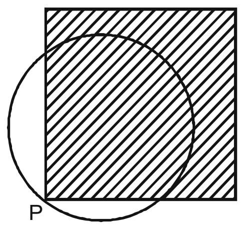
36. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is

(a) 10
(b) 12
(c) 14
(d) 16

2006

Directions for Questions 37 and 38: Answer questions on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



37. The proportion of the sheet area that remains after punching is:

(a) $\frac{(\pi+2)}{8}$ (b) $\frac{(6-\pi)}{8}$

(c) $\frac{(4-\pi)}{4}$ (d) $\frac{(\pi-2)}{4}$

(e) $\frac{(14-3\pi)}{6}$

38. Find the area of the part of the circle (round punch) falling outside the square sheet.

(a) $\frac{\pi}{4}$

(b) $\frac{(\pi-1)}{2}$

(c) $\frac{(\pi-1)}{4}$

(d) $\frac{(\pi-2)}{2}$

(e) $\frac{(\pi-2)}{4}$

2007

Direction for Question 39: The question is followed by two statements A and B. Indicate your response based on the following directives.

Mark (a) if the questions can be answered using A alone but not using B alone.

Mark (b) if the question can be answered using B alone but not using A alone.

Mark (c) if the question can be answered using A and B together, but not using either A or B alone.

Mark (d) if the question cannot be answered even using A and B together.

39. ABC Corporation is required to maintain at least 400 Kilolitres of water at all times in its factory, in order to meet safety and regulatory requirements. ABC is considering the suitability of a spherical tank with uniform wall thickness for the purpose. The outer diameter of the tank is 10 meters. Is the tank capacity adequate to met ABC's requirements?

A : The inner diameter of the tank is at least 8 meters.

B : The tank weights 30,000 kg when empty, and is made of a material with density of 3 gm/cc.

2008

40. Consider a square ABCD with midpoints E, F, G and H of AB, BC, CD and DA respectively. Let L denote the line passing through F and H. Consider points P and Q, on L and inside ABCD, such that the angles APD and BQC both equal 120°. What is the ratio of the area of ABQCDP to the remaining area inside ABCD?

(a) $\frac{4\sqrt{2}}{3}$ (b) $2+\sqrt{3}$

(c) $\frac{10-3\sqrt{3}}{9}$ (d) $1+\frac{1}{\sqrt{3}}$

(e) $2\sqrt{3}-1$

41. Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq. cm.) of the intersecting region?

(a) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ (b) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

(c) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$ (d) $\frac{4\pi}{3}+\frac{\sqrt{3}}{2}$

(e) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

42. Consider a right circular cone of base radius 4 cm and height 10 cm. A cylinder is to be placed inside the cone with one of the flat surfaces resting on the base of the cone. Find the largest possible total surface area (in sq. cm)of the cylinder.

(a) $\frac{100\pi}{3}$ (b) $\frac{80\pi}{3}$

(c) $\frac{120\pi}{7}$ (d) $\frac{130\pi}{9}$

(e) $\frac{110\pi}{7}$

MEMORY BASED QUESTIONS

2009

43. A rectangular floor is fully covered with equal square tiles. All the tiles on the edges of the floor are white and the rest of them are red in colour. The total number of white tiles is half the total number of red tiles on the floor. What is the maximum possible difference between the number of red tiles and the number of white tiles on the floor?

(a) 70 (b) 48
(c) 78 (d) 40

2010

44. A cylindrical pipe of length 75 m, through which water flows at the rate of 54 km/hr, can fill 80% of a cuboidal tank of 118800 m^3 capacity in 14 hours. What is the radius (in cm) of the cross section of the pipe?

(a) 20 (b) 25
(c) 50 (d) Cannot be determined

45. A large cube is formed by bringing together 729 smaller identical cubes. Each face of the larger cube is painted with red colour. How many smaller cubes are there none of whose faces is painted?

(a) 216 (b) 256
(c) 343 (d) None of these

2011

46. A cube of edge 12 cm is cut into 64 equal cubes. All the cubes are now arranged on a table such that one face of each cube touches the table. The resulting figure is a solid cuboid whose length and breadth are in the ratio 4 : 1 respectively. What is the total surface area of the table occupied by the cuboid?

(a) 144 cm^2 (b) 288 cm^2
(c) 576 cm^2 (d) None of these

2013

47. A spherical ball of the maximum possible volume is placed inside a right-circular cone of height 'h' units. If the radius of the base of the cone is equal to $h/\sqrt{3}$ units, then the ratio of the volume of the sphere to that of the cone is

(a) 4 : 9 (b) 5 : 9
(c) $1:\sqrt{3}$ (d) $2:3\sqrt{3}$

2014

48. The top and bottom radii of a frustrum of a solid cone are 3 cm and 6 cm respectively. Its height is 8 cm. There is a conical cavity of height 3 cm and radius 6 cm at the bottom. The amount of material in the solid is

(a) 132 π cm^3 (b) 168 π cm^3
(c) 159 π cm^3 (d) Data Insufficient

2015

49. A field is in the form of a rectangle of dimension 24 m × 56 m. There is 2700 m of fencing that is available. The field has to be divided into many identical smaller square plots, having integral sides (in metres), each of which is to be fenced. Find the side of each of the square plots such that the fencing material that is left out is minimum.

(a) 1 m (b) 2 m
(c) 4 m (d) 8 m

2016

50. An unsharpened cylindrical pencil consists of a layer of wood surrounding a solid cylinder of graphite. The radius of a pencil is 7 mm, the radius of the graphite cylinder is 1 mm and the length of the pencil is 10 cm. Find the cost of the material used in a pencil, if the cost of wood is Rs.0.70/cm^3 and that of graphite is Rs.2.10/cm^3.

(1) Rs.8.76 (2) Rs.10.02
(3) Rs.11.22 (4) Rs.13.74

51. Two circles of radii 'r' units and '2r' units intersect each other in such a way that their common chord is of the maximum possible length. What is the area (in square units) of the region that is common to the two circles?

(1) $\frac{\pi r^2}{2}$ (2) $\frac{7\pi r^2}{6} - \sqrt{3}r^2$
(3) $\frac{11\pi r^2}{6} - \frac{\sqrt{3}r^2}{2}$ (4) $\frac{7\pi r^2}{6} - \frac{\sqrt{3}r^2}{2}$

52. A raindrop consists of 75% water and the rest is dust. However, by the time it reaches the surface of Earth, it is left with 70% water as 2 ml water evaporates on the way. Find the original volume of the raindrop.

(1) 12 ml (2) 15 ml
(3) 40 ml (4) 42 ml

53. A square is inscribed in a circle and the circle is inscribed in a regular octagon. Find the ratio of the area of the square to that of the octagon.

(1) $\left(1+\sqrt{2}\right):4$
(2) $\left(2\sqrt{2}-1\right):4$
(3) $\left(\sqrt{2}-1\right):4$
(4) Cannot be determined

54. From a rectangular sheet of dimensions 30 cm × 20 cm, four squares of equal size are cut from the four corners. Then the resulting four sides are bent upwards to give it the shape of an open box. If the volume of the box is 1056 cm^3, what is the length of the side of the squares cut from the corners?

(1) 2 cm (2) 4 cm
(3) 6 cm (4) 11 cm

2017

55. The base of a vertical pillar with uniform cross section is a trapezium whose parallel sides are of lengths 10 cm and 20 cm while the other two sides are of equal length. The perpendicular distance between the parallel sides of the trapezium is 12 cm. If the height of the pillar is 20 cm, then the total area, in sq cm, of all six surfaces of the pillar is

(1) 1300 (2) 1340
(3) 1480 (4) 1520

56. If three sides of a rectangular park have a total length 400 ft, then the area of the park is maximum when the length (in ft) of its longer side is

57. Let P be an Interior point of a right-angled isosceles triangle ABC with hypotenuse AB. If the perpendicular distance of P from each of AB, BC, and CA is $4(\sqrt{2}-1)$ cm, then the area, in sq cm, of the triangle ABC is

2018 Slot 1

58. A right circular cone, of height 12 ft, stands on its base which has diameter 8 ft. The tip of the cone is cut off with a plane which is parallel to the base and 9 ft from the base. With π = 22/7, the volume, in cubic ft, of the remaining part of the cone is

CO-ORDINATE GEOMETRY

1991

1. What is the distance between the points A(3, 8) and B(–2, –7)?

(a) $5\sqrt{2}$ (b) 5
(c) $5\sqrt{10}$ (d) $10\sqrt{2}$

1991

2. The points of intersection of three lines 2X + 3Y – 5 = 0, 5X – 7Y + 2 = 0 and 9X – 5Y – 4= 0

(a) form a triangle
(b) are on lines perpendicular to each other
(c) are on lines parallel to each other
(d) are coincident

1997

Direction for Question 3: The question is followed by two statements, I and II. Mark the answer

(a) if the question can be answered with the help of one statement alone.
(b) if the question can be answered with the help of any one statement independently.
(c) if the question can be answered with the help of both statements together.
(d) if the question cannot be answered even with the help of both statements together.

3. What is the area bounded by the two lines and the coordinate axes in the first quadrant?

I. The lines intersect at a point which also lies on the lines 3x – 4y = 1 and 7x – 8y = 5.
II. The lines are perpendicular, and one of them intersects the Y-axis at an intercept of 4.

1999

Directions for Questions 4 and 5: Answer the questions based on the following information.

A robot moves on a graph sheet with X and Y-axis. The robot is moved by feeding it with a sequence of instructions. The different instructions that can be used in moving it, and their meanings are:

Instruction	Meaning
GOTO (x, y)	Move to point with coordinates (x, y) no matter where you are currently
WALKX(p)	Move parallel to the X-axis through a distance of p, in the positive direction if p is positive, and in the negative direction if p is negative
WALKY(p)	Move parallel to the Y-axis through a distance of p, in the positive direction if p is positive, and in the negative direction if p is negative.

4. The robot reaches point (6, 6) when a sequence of three instructions is executed, the first of which is a GOTO(x, y)instruction, the second is WALKX(2) and the third is WALKY(4). What are the value of x and y?

(a) 2, 4 (b) 0, 0
(c) 4, 2 (d) 2, 2

5. The robot is initially at (x, y), x > 0 and y < 0. The minimum number of instructions needed to be executed to bring it to the origin (0, 0) if you are prohibited from using the GOTO instruction is

(a) 2 (b) 1
(c) x + y (d) 0

2000

6. ABCD is a rhombus with the diagonals AC and BD intersecting at the origin on the x-y plane. The equation of the straight line AD is x + y = 1. What is the equation of BC?
   (a) x + y = –1
   (b) x – y = –1
   (c) x + y = 1
   (d) None of these

Direction for Question 7: The question is followed by two statements, I and II. Answer the question using the following instructions.

Mark the answer as:

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

7. There are two straight lines in the x-y plane with equations:
   ax + by = c;   dx + ey = f
   Do the two straight lines intersect?
   I. a, b, c, d, e and f are distinct real numbers.
   II. c and f are non-zero.

2002

8. The area of the triangle whose vertices are (a, a), (a + 1, a + 1) and (a + 2, a) is
   (a) a^3 (b) 1
   (c) 2a   (d) $2^{1/2}$

2005

9. Consider a triangle drawn on the X-Y plane with its three vertices of (41, 0), (0, 41) and (0, 0), each vertex being represented by its (X, Y) coordinates. The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is
   (a) 780   (b) 800
   (c) 820   (d) 741

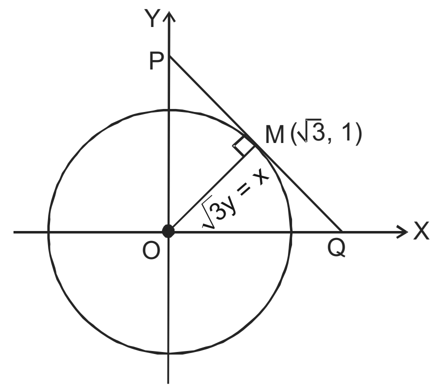
MEMORY BASED QUESTIONS

2011

10. Two lines P and Q intersect at point (3, 2) in the x-y plane. The slope of line P is 45° and line Q is parallel to the x-axis. What is the area (in sq. units) of the triangle formed by P, Q and a line perpendicular to P and passing through point (5, 4)?
    (a) 12   (b) 8
    (c) 6   (d) 4

2015

11. The line $\sqrt{3}y = x$ is the radius of the circle, it meets the circle centred at origin O at point M $(\sqrt{3}, 1)$. If PQ is the tangent to the circle at M as shown, find the length of the PQ.



    (a) $\frac{5}{2}\sqrt{3}$ units   (b) $3\sqrt{3}$ units
    (c) $2\sqrt{3}$ units   (d) $\frac{8}{\sqrt{3}}$ units

12. The coordinates of two diagonally opposite vertices of a rectangle are (4, 3) and (-4,-3). Find the number of such rectangle(s), if the other two vertices also have integral coordinates.
    (a) 1   (b) 4
    (c) 5   (d) 10

2017

13. The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation y = 3x + c, then c is
    (a) –5
    (b) –6
    (c) –7
    (d) –8

ANSWERS

Line

1. (c)	**2.** (c)	**3.** (a)

Triangle

1. (d)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (*)
11. (c)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (a)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (b)
31. (b)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (196)
41. (b)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (b)				

Quadrilaterals

1. (a)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (b)
11. (a)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (e)	**19.** (a)	**20.** (a)
21. (c)	**22.** (c)	**23.** (d)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (90)	**29.** (a)	**30.** (a)
31. (d)	**32.** (c)	**33.** (d)							

Polygons

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)									

Circle

1. (d)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (d)
11. (a)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (b)
21. (c)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (a)
31. (b)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (d)	**36.** (d)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (b)
41. (c)	**42.** (e)	**43.** (a)	**44.** (a)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (d)	**49.** (c)	**50.** (c)
51. (b)	**52.** (c)	**53.** (b)	**54.** (c)	**55.** (24)	**56.** (c)				

Mensuration

1. (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)
11. (d)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (a)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (b)
31. (d)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (e)
41. (e)	**42.** (a)	**43.** (a)	**44.** (a)	**45.** (c)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (c)
51. (b)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (c)	**56.** (200)	**57.** (16)	**58.** (198)		

Coordinate Geometry

1. (c)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (d)
11. (d)	**12.** (c)	**13.** (d)							

EXPLANATIONS

Line

1. c Since the lines are parallel, $\frac{AB}{BC} = \frac{DE}{EF}$, i.e.

AB × EF = BC × DE.

Hence, I = II.

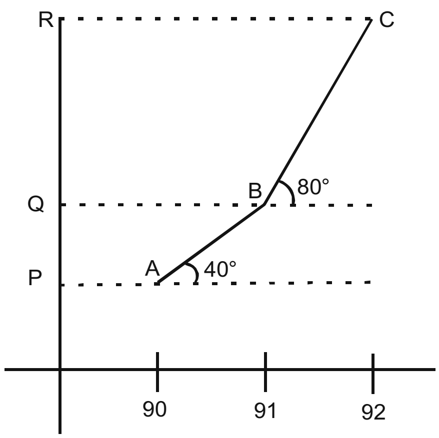
2. c None of the statement alone is sufficient to answer the question.

Using both statements together:

PQ = PB + BQ and RS = RE + ES

If BQ = ES and PB > RE, PQ > PS.

3. a



Ratio of revenues = $\frac{RQ}{QP}$

Since in a line graph, the years are uniformly spaced

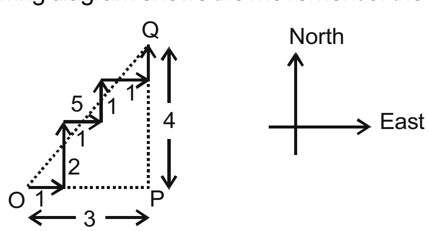
$\Rightarrow \frac{RQ}{QP} = \frac{\tan 80°}{\tan 40°}$

So the ratio can be determined from statement I alone.

Statement II is immaterial because we intend to find the ratio and not absolute figures.

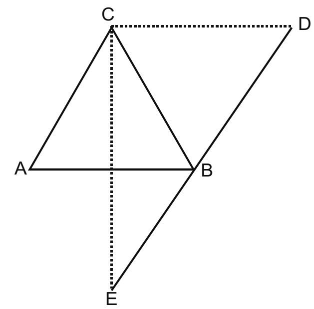
Triangle

1. d Following diagram shows the movement of the man.



$\therefore OQ = \sqrt{3^2 + 4^2} = 5$ km.

For questions 2 and 3:



2. b Since ΔABC is an equilateral triangle with length of the side 2 km, so its altitude will be $\sqrt{3}$ km. As point D is directly east of C, so D is 3 km east and $\sqrt{3}$ km north of A.

3. d ABDC and AEBC, both are rhombus with each side 2 km.

Hence, the total distance walked by the person = BD + DB + BE = 2 + 2 + 2 = 6 km.

4. c Since $\angle C = 2 \angle E$, therefore $\angle BCA = 60°$.

Also since ABCD is a parallelogram,

AB = CD and AD = BC = AC.

Hence, ΔABC and ΔACD are equilateral triangles.

Hence, area of this triangle $= \frac{s^2}{4}\sqrt{3}$, where s is the side of the triangle = AB = AD = DC = BC.

$\therefore$ Area of the parallelogram is twice this area $= \frac{s^2}{2}\sqrt{3}$.

Since $\angle CAD = 60°$, $\angle DAE = 90°$, so ΔEAD is a right triangle with side AD = s. Since it is a 30°-60°-90° triangle, hence side $AE = s\sqrt{3}$.

$\therefore$ Area of this triangle $= \frac{(s \times s\sqrt{3})}{2} = \frac{s^2}{2}\sqrt{3}$.

Hence, the required two areas are equal or I = II.

5. c The largest angle in a right-angled triangle is 90º, which corresponds to the highest part of the ratio.

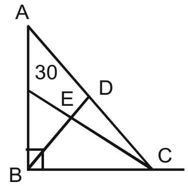
Let us evaluate each option.

In (a), the remaining two angles would be 30º and 60º, which is possible.

In (b), the remaining two angles would be 45º each, which is again possible.

In (c), the remaining two angles would be 15º and 45º, which is not possible as the sum of the angles is not 180º.

6. b



In ΔABC, $\angle ACB = 180° - 90° - 30° = 60°$.

$\therefore$ $\angle DCE = 30°$, since $\angle CDE = 90°$.

In ΔCED, $\angle CED = 180° - 90° - 30° = 60°$.

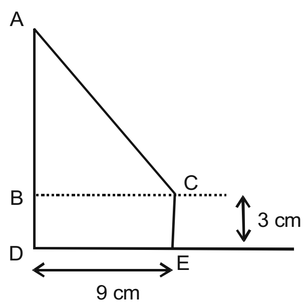
7. d Since 5-12-13 forms a Pythagorean triplet, the triangle under consideration is a right-angled triangle with height 12 and base 5.

So area of the triangle = $\left(\frac{1}{2}\right)(12)(5) = 30$ sq. units.

If area of the rectangle with width 10 units is 30 sq. units, its length = 3 units.

Hence, its perimeter = 2(10 + 3) = 26 units.

8. b



The figure can be drawn as shown above.

Height of the wall = AD = AC = (AB + 3) or AB = (AC – 3).

In right-angled triangle ABC, $AB^2 + BC^2 = AC^2$.

Thus, $(AC - 3)^2 + 81 = AC^2$.

$\therefore$ AC = 15 m.

Hence, height of the wall = 15 m.

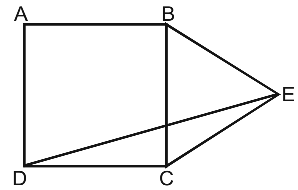
Hint: Please note that the same multiple of all the triplets should also be triplets. E.g. if 3-4-5 is a triplet, then 3(3-4-5)should also be triplet or 9-12-15 is also a triplet.

Note that the base of the triangle is 9, so other two sides should be 12 and 15.

9. c Both statements are required to answer the question. Statement I tells us that the triangle is an isosceles triangle. In an isoceles triangle, the altitude is also the median and bisects the third side.

Hence, if we know the altitude length and the length of the congruent sides in an isoceles triangle, we can find its base. And if we know the base and the height of a triangle, we can find its area.

10. a



Since ΔBCE is an equilateral triangle, CE = BC = BE.

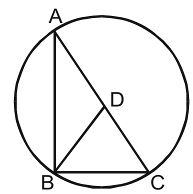
And since ABCD is a square, BC = CD. Hence, CD = CE. So in ΔCDE, we have CD = CE. Hence, $\angle EDC = \angle CED$. Now $\angle BCE = 60°$ (since equilateral triangle)and $\angle BCD = 90°$ (since square).

Hence, $\angle DCE = \angle DCB + \angle BCE = (60° + 90°) = 150°$.

So in ΔDCE, $\angle EDC + \angle CED = 30°$ (since three angles of a triangle add up to 180°).

Hence, we have $\angle DEC = \angle EDC = 15°$.

11. c



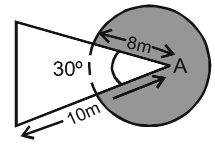
In a right-angled triangle, the length median to the hypotenuse is half the length of the hypotenuse.

Hence, BD = $\frac{1}{2}$ AC = 3 cm. This relationship can be verified by knowing that the diameter of a circle subtends a right angle at the circumference E.g. in the above figure D is the centre of the circle with AC as diameter. Hence, $\angle ABC$ should be 90°. So BD should be the median to the hypotenuse. Thus, we can see that BD = AD = CD = Radius of this circle.

Hence, BD = $\frac{1}{2}$ diameter = $\frac{1}{2}$ AC = $\frac{1}{2} \times$ hypotenuse.

12. c In a triangle, the line joining the mid-points of any two sides is half the length of its third side. Hence, every side of ΔPQR would be half the sides of ΔABC. Hence, area of ΔPQR would be $\frac{1}{4}$ the area of $\Delta ABC = \frac{1}{4} \times 20 = 5$ sq. units.

13. d



It can be seen that if the length of the rope is 8 m, then the cow will be able to graze an area equal to (the area of the circle with radius 8m) – (Area of the sector of the same circle with angle 30°)

$$= \pi(8)^2 - \frac{30}{360}\pi(8)^2$$

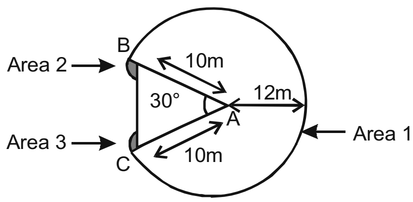
$$= 64\pi - \frac{1}{12}(64\pi)$$

$$= 64\pi\left(\frac{11}{12}\right) = \frac{176\pi}{3} \text{ sq. m}$$

Shortcut:

Area grazed without restriction is 64π m^2 it should be less than 64π sq. m. with restriction. So the correct choice is (d).

14. a

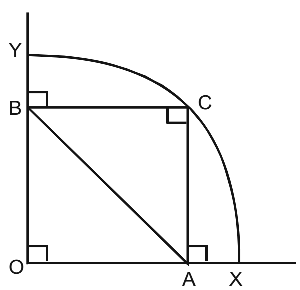


If the length of the rope is 12 m, then the total area that can be grazed by the cow is as depicted in the diagram. Area 1 is (the area of the circle with radius 12)– (Area of the sector of the same circle with angle 30º).

So area 1 = $\pi(12)^2 - \frac{30}{360}\pi(12)^2 = 132\pi$ sq.m.

Since the length of the rope is higher than the sides of the triangle (viz. AB and AC), if the cow reaches point B or C, there would still be a part of the rope (12 – 10)= 2 m in length. With this extra length available the cow can further graze an area equivalent to some part of the circle with radius = 2 m from both points, i.e. B and C. This is depicted as area 2 and area 3 in the diagram. Hence, the actual area grazed will be slightly more than 132π. The only answer choice that supports this is (a).

15. d



Do not make the mistake of assuming O to be the centre of the circle. Since the centre is not known, knowing radius is not of great help. It can be observed that $\angle BCA$ is also $90°$, as in the quadrilateral OBCA, the remaining three angles are $90°$. So the quadrilateral can either be a square or a rectangle. As we do not know even this, we cannot make use of the second statement as well. Hence, both the statements are not sufficient to answer the question.

16. c If the sides of the triangle are a, b and c, then a + b > c. Given a + b + c = 14.

Then the sides can be (4, 4, 6), (5, 5, 4), (6, 5, 3) and (6, 6, 2). Hence, four distinct triangles are possible.

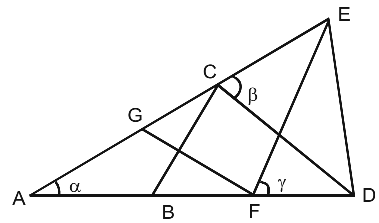
17. a We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 3ab + 3bc + 3ac$

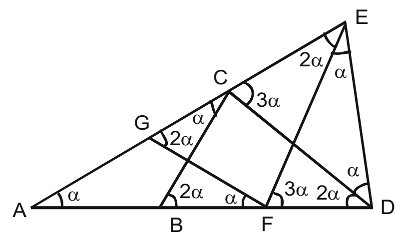
Now assume values of a, b, c and substitute in this equation to check the options.

Short cut: $(a – b)^2 + (b – c)^2 + (c – a)^2 = 0$.

Hence, a = b = c.

18. d





Let $\angle EAD = \alpha$. Then $\angle AFG = \alpha$ and also $\angle ACB = \alpha$.

Therefore, $\angle CBD = 2\alpha$ (exterior angle to ΔABC).

Also $\angle CDB = 2\alpha$ (since CB = CD).

Further, $\angle FGC = 2\alpha$ (exterior angle to ΔAFG).

Since GF = EF, $\angle FEG = 2\alpha$. Now $\angle DCE = \angle DEC = \beta$ (say). Then $\angle DEF = \beta – 2\alpha$.

Note that $\angle DCB = 180° – (\alpha + \beta)$.

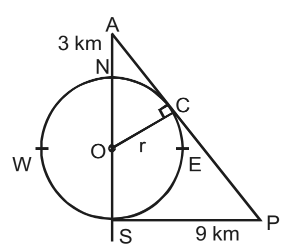
Therefore, in ΔDCB, $180° – (\alpha + \beta) + 2\alpha + 2\alpha = 180°$ or $\beta = 3\alpha$. Further $\angle EFD = \angle EDF = \gamma$ (say).

Then $\angle EDC = \gamma – 2\alpha$. If CD and EF meet at P, then $\angle FPD = 180° – 5\alpha$ (because $\beta = 3\alpha$).

Now in ΔPFD, $180° – 5\alpha + \gamma + 2\alpha = 180°$ or $\gamma = 3\alpha$.

Therefore, in ΔEFD, $\alpha + 2\gamma = 180°$ or $\alpha + 6\alpha = 180°$ or $\alpha = 26°$ or approximately 25°.

19. b



ΔAPS and ΔAOC are similar triangles.

Where OC = r

$$\therefore \frac{r}{r+3} = \frac{9}{\sqrt{81 + (2r + 3)^2}}$$

Now use the options.Hence, the diameter is 9 km.

20. a Let BC = y and AB = x.

Then area of $\Delta CEF = Area(\Delta CEB) - Area(\Delta CFB)$

$$= \frac{1}{2}.\frac{2x}{3}.y - \frac{1}{2}.\frac{x}{3}.y = \frac{xy}{6}$$

Area of ABCD = xy

$\therefore$ Ratio of area of ΔCEF and area of ABCD is

$$\frac{xy}{6} : xy = \frac{1}{6}$$

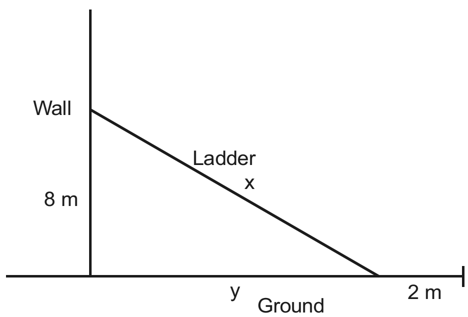
Alternate method:

Join AC, therefore Area of $\Delta ABC = \frac{1}{2}$ Area of ABCD

Also, Area of ΔCAE = Area of ΔCEF = Area of ΔCFB

$\therefore$ Area of $\Delta CEF = \frac{1}{6}$ Area of ABCD

21. d



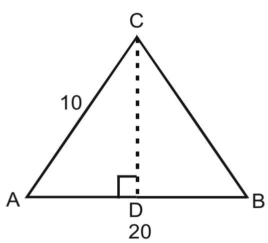
Let the length of the ladder be x meter. We have $8^2 + y^2 = x^2$ and $(y + 2) = x$

Hence, $64 + (x - 2)^2 = x^2$

$\Rightarrow 64 + x^2 - 4x + 4 = x^2$

$\Rightarrow 68 = 4x \quad \Rightarrow \quad x = 17$

22. a



Let's assume AB be the longest side of 20 units and another side AC is 10 units. Here CD $\perp$ AB.

Since area of $\Delta ABC = 80 = \frac{1}{2} AB \times CD$

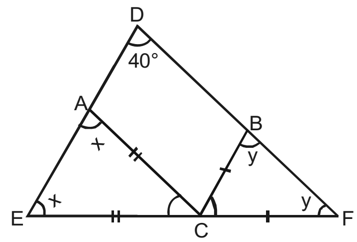
So $CD = \frac{80 \times 2}{20} = 8$.

In ΔACD; $AD = \sqrt{10^2 - 8^2} = 6$

Hence, DB = 20 – 6 = 14.

So $CB = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260}$ units.

23. c



Here $\angle ACE = 180° - 2x$, $\angle BCF = 180° - 2y$

and x + y + 40° = 180° (In ΔDEF)

So x + y = 140°

So $\angle ACB = 180° - \angle ACE - \angle BCF$

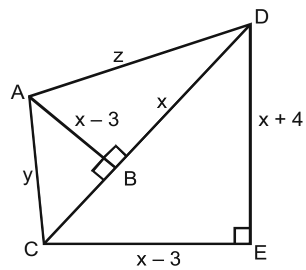
$= 180° - (180° - 2x) - (180° - 2y)$

$= 2(x + y) - 180°$

$= 2 \times 140° - 180°$

$= 100°$

24. b



By Pythagoras Theorem,

$$DC = \sqrt{(x+4)^2 + (x-3)^2}$$

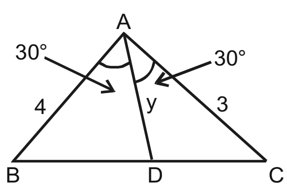
Again by Pythagoras Theorem,

$BC^2 + AB^2 = AC^2$

$$\Rightarrow \left(\sqrt{(x+4)^2 + (x-3)^2} - x\right)^2 + (x-3)^2 = 100$$

We can find the value of x, using the answer choices given in the question. Hence, x = 11.

25. b



Let BC = x and AD = y.

As per Bisector Theorem, $\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3}$

Hence, $BD = \frac{4x}{7}$; $DC = \frac{3x}{7}$

In ΔABD, $\cos 30° = \frac{(4)^2 + y^2 - \frac{16x^2}{49}}{2 \times 4 \times y}$

$$\Rightarrow \quad 2\times 4\times y\times \frac{\sqrt{3}}{2} = 16 + y^2 - \frac{16x^2}{49}$$

$$\Rightarrow \quad 4\sqrt{3}y = 16 + y^2 - \frac{16x^2}{49} \quad \text{... (i)}$$

Similarly, from ΔADC, $\cos 30° = \dfrac{9 + y^2 - \frac{9x^2}{49}}{2\times 3\times y}$

$$\Rightarrow \quad 3\sqrt{3}y = 9 + y^2 - \frac{9x^2}{49} \quad \text{... (ii)}$$

Now (i) × 9 – 16 × (ii), we get

$$36\sqrt{3}y - 48\sqrt{3}y = 9y^2 - 16y^2 \Rightarrow y = \frac{12\sqrt{3}}{7}$$

Alternate solution:

Area of ΔABC = Area of ΔABD + Area of ΔADC

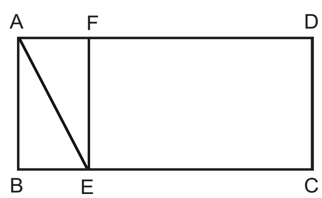
$$\Rightarrow \frac{1}{2}\times 4\times 3 \sin 60° = \frac{1}{2}\times 4\times y$$

$$\sin 30° + \frac{1}{2}\times 3\times y\times \sin 30°$$

$$\Rightarrow \quad 12\sqrt{3} = 4y + 3y$$

$$\Rightarrow \quad y = \frac{12\sqrt{3}}{7}$$

26. d



Area of ΔABE = 7 cm^2

Area of rectangle ABEF = 14 cm^2

$\therefore$ Area of ABCD = 14 × 4 = 56 cm^2

27. c PQ || AC

$$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$$

QD || PC

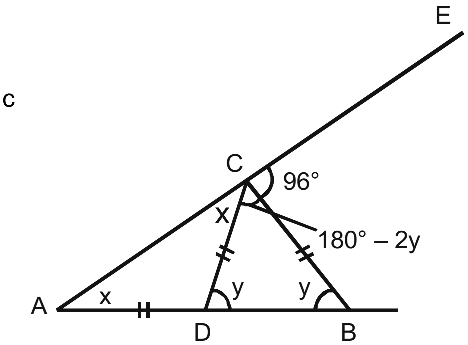
$$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$$

As $\dfrac{PD}{DB} = \dfrac{4}{3}$

$$\therefore PD = \frac{4}{7}PB$$

$$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4}\times\frac{AP}{PB} = \frac{7}{4}\times\frac{4}{3} = 7 : 3$$

28. c



Using exterior angle theorem

$\angle A + \angle B = 96°$

i.e. x + y = 96° ... (i)

Also x + (180° – 2y)+ 96° = 180°

$\therefore$ x – 2y + 96° = 0

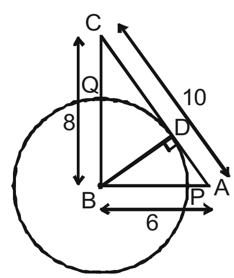
$\therefore$ x – 2y = –96° ... (ii)

Solving (i) and (ii),

y = 64° and x = 32°

$\therefore \angle DBC = y = 64°$

29. d



Triangle ABC is a right angled triangle.

Thus $\frac{1}{2}$ × BC × AB = $\frac{1}{2}$ × BD × AC

Or, 6 × 8 = BD × 10. Thus BD = 4.8.

Therefore, BP = BQ = 4.8.

So, AP = AB – BP = 6 – 4.8 = 1.2 and CQ = BC – BQ = 8 – 4.8 = 3.2.

Thus, AP : CQ = 1.2 : 3.2 = 3 : 8

30. b Using the Basic Proportionality Theorem,

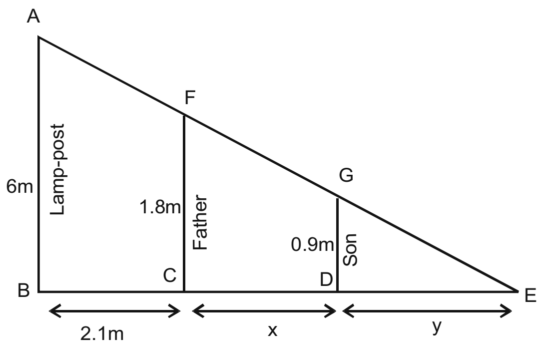
$\dfrac{AB}{PQ} = \dfrac{BD}{QD}$ and $\dfrac{PQ}{CD} = \dfrac{BQ}{BD}$.

Multiplying the two we get, $\dfrac{AB}{CD} = \dfrac{BQ}{QD}$ = 3 : 1.

Thus CD : PQ = BD : BQ = 4 : 3 = 1 : 0.75

31. b The question tells us that the area of triangle DEF will be $\frac{1}{4}$th the area of triangle ABC. Thus by knowing either of the statements, we get the area of the triangle DEF.

32. d



$\Delta ABE \sim \Delta FCE$

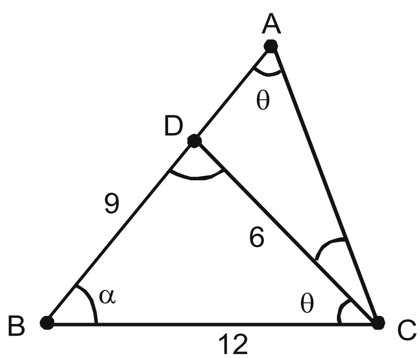
$\therefore \quad \frac{6}{1.8} = \frac{2.1+x+y}{x+y}$ ...(i)

Also $\Delta ABE \sim \Delta GDE$

$\therefore \quad \frac{6}{0.9} = \frac{2.1+x+y}{y}$ ...(ii)

From (i)and (ii), x = 0.45m.

33. a



Here $\angle ACB = \theta + [180° - (2\theta + \alpha)] = 180° - (\theta + \alpha)$

So here we can say that triangle BCD and triangle ABC will be similar. $\Delta BCD \sim \Delta BAC$

Hence, from the property of similar triangles

$$\frac{AB}{12} = \frac{12}{9} \Rightarrow AB = 16$$

$$\frac{AC}{6} = \frac{12}{9} \Rightarrow AC = 8$$

$\therefore \quad AD = 7$

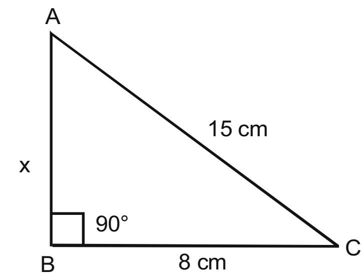
$S_{ADC} = 8 + 7 + 6 = 21$

$S_{BDC} = 27$

Hence, $r = \frac{21}{27} = \frac{7}{9}$

34. c The three sides of the obtuse triangle are 8 cm, 15 cm and x cm. As 15 is greater than 8, hence either x or 15 will be the largest side of this triangle. Consider two cases:

Case I:



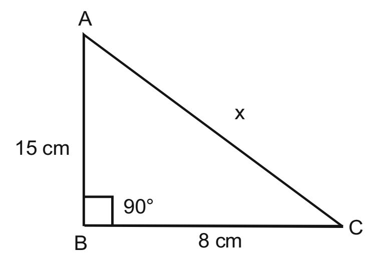
Consider the right ΔABC above,

$$x = \sqrt{15^2 - 8^2} = 12.68 \text{ cm}$$

For all values of x < 12.68, the ΔABC will be obtuse. But as the sum of two sides of triangle must be greater than the third side, hence (x + 8) > 15 or x > 7.

Thus, the permissible values of x are 8, 9, 10, 11 and 12.

Case II:

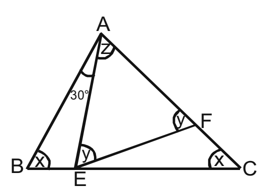


In the right ΔABC above, $x = \sqrt{15^2 + 8^2} = 17$.

For all values of x > 17, ΔABC will be obtuse. But, as the length of third side should be less than the sum of other two sides, hence x < (15 + 8) or x < 23. The permissible values of x are: 18, 19, 20, 21 and 22.

From Case I and II, x can take 10 values.

35. a



Let $\angle ACB = \angle ABC = x$,

$\angle AEF = \angle AFE = y$,

$\angle EAF = z$.

$y = \angle FEC + x$

$\Rightarrow \quad \angle FEC = y - x$

Now $\quad 2y = 180° - z$ ...(i)

Also $\quad 2x + 30° + z = 180°$

$\Rightarrow \quad 2x = 150° - z$ ...(ii)

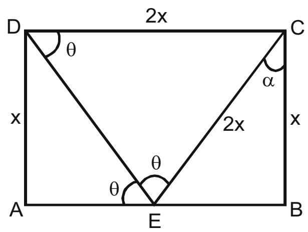
From (i) and (ii): $2(y - x) = 30°$

$\Rightarrow \quad y - x = 15°$

So $\quad \angle FEC = 15°$

36. c Let the two sides of the rectangle be x and 2x and $\angle AED$ be θ.

$\therefore \quad \angle DEC = \theta$



$\therefore \quad \angle CEB = 180 - 2\theta$

Also, $\angle EDC = \angle AED = \theta$ (AB is parallel to CD.)

In ΔDEC,

$\angle DEC = \angle EDC \Rightarrow CD = EC = 2x$

In ΔBCE,

$$\cos\alpha = \frac{x}{2x} \Rightarrow \alpha = 60^o$$

Now, in ΔBCE

$2\theta = 90^o + 60^o$ (Exterior angle is equal to the sum of opposite interior angles.)

$\Rightarrow \quad \theta = 75^o$.

37. d Let the length of the other two sides of the triangle be 'a' and 'b' respectively,

$\Rightarrow \quad a^2 + b^2 = 240^2 = 57600$

Also, a + b + 240 must be a perfect square.

Perimeter of a right angled triangle is maximized when the two sides other than its hypotenuse are equal in length.

Therefore, the maximum perimeter of the right angled triangle will be

$$120(\sqrt{2}) + 120(\sqrt{2}) + 240 = 579\text{(approx)}.$$

Also, the perimeter of the triangle should be greater than twice of the length of the hypotenuse of the triangle.

Therefore, the perimeter of the triangle should be greater than 480 and less than 579.

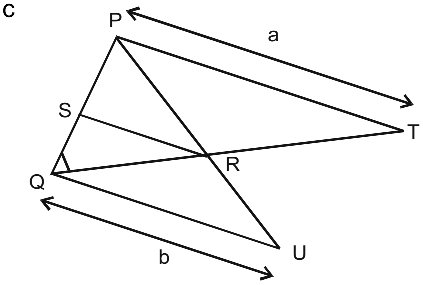
Possible squares in the above range are 484, 529 and 576.

But, as per the information given in the question, the perimeter can only be equal to 576.

The values of 'a' and 'b' will be 192 and 144 units not necessarily in that particular order.

Hence, option (d) is the correct choice.

38. c



Given that PT || SR || QU

PT = a units, QU = b units.

ΔPTQ and ΔSRQ are similar.

$\therefore$ We have

$$\frac{PQ}{PT} = \frac{SQ}{SR}$$

or $$\frac{PQ}{a} = \frac{SQ}{SR} \quad \ldots \text{(i)}$$

ΔUQP and ΔRSP are similar.

$\therefore$ We have $$\frac{PQ}{QU} = \frac{PS}{SR}$$

or $$\frac{PQ}{b} = \frac{PS}{SR} \quad \ldots \text{(ii)}$$

Combining (i) and (ii)

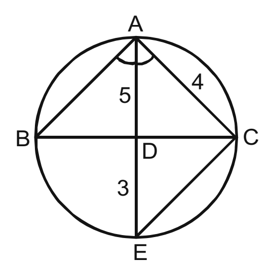
$$\frac{PQ}{a} + \frac{PQ}{b} = \frac{SQ + PS}{SR}$$

or $$PQ\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{PQ}{SR} \text{ [As SQ + PS = PQ]}$$

$$PQ\left[\frac{a+b}{ab}\right] = \frac{PQ}{SR}$$

or $$SR = \frac{ab}{a+b}.$$

39. d



The figure would be as shown above.

Join EC.

Let $\quad \angle BAD = x°$

and $\quad \angle ABD = y°$.

$\Rightarrow \quad \angle DAC = x°$

and $\quad \angle AEC = y°$.

In ΔABD and ΔAEC,

$\angle ABD = \angle AEC$

$$\angle BAD = \angle EAC$$

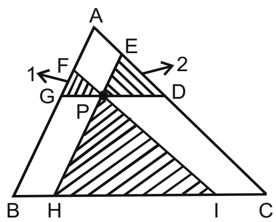
$$\therefore \quad \Delta ABD \sim \Delta AEC$$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{AE}{AC}$$

$$\Rightarrow \quad AB = \frac{AE}{AC} \times AD$$

$$= \frac{8}{4} \times 5 = 10 \text{ cm.}$$

40. 196



ΔPED is similar to ΔGFP

Ratio of area = 9 : 16

Therefore ratio of sides = $\sqrt{\frac{9}{16}}$ = 3 : 4

Hence, P divides GD in the ratio 3 : 4.

$$\Delta AGD = \left(\frac{7}{3}\right)^2 \times 9 = 49 \text{ sq.cm}$$

[ΔAGD similar to ΔFGP]

So area of AEPF = 49 – [16 + 9] = 24 sq. cm

Similarly area of BFI = $\left(\frac{10}{3}\right)^2 \times 9 = 100$ sq. cm

Therefore area of BHPG = 100 – (49 + 9) = 42 sq. cm

Similarly area of PDCI = $\left(\frac{11}{7}\right)^2 \times 49 - 49 - 16$

= 56 sq.cm

Area of triangle ABC = (9 + 16 + 49 + 24 + 42 + 56)

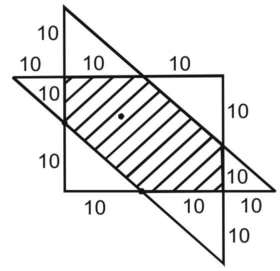
= 196 sq. cm

Alternative method:

Ratio of the corresponding sides is 3:4:7 since the areas are in the ratio 9:16:49 and all the triangles are similar.

Hence GP + PD = BH+IC = HI. So HI is half of BC. Since triangle 3 is similar to Triangle ABC and HI is the corresponding Side to BC , and is half of it the area of triangle 3 must be $\frac{1}{4}$ the area of triangle ABC. So area of triangle ABC = 196 sq cm.

41. b

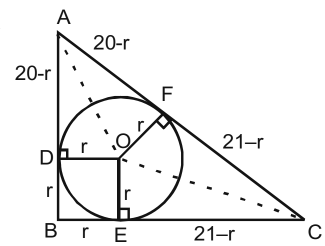


Area of the shaded region

$$= \frac{1}{2} \times 30 \times 30 - 3 \times \left(\frac{1}{2} \times 10 \times 10\right)$$

= 300 sq. cm.

42. a



$AC = \sqrt{AB^2 + BC^2}$ = 29 cm

and (20 – r) + (21 – r) = 29

or, 2r = 41 – 29

$\Rightarrow$ r = 6 cm

$\Delta OEC \cong \Delta OFC$ (RHS)

Area($\square$FOEC) = 2 × Area (ΔOEC)

$$= 2.\frac{1}{2}.15.6 = 90 \text{ cm}^2$$

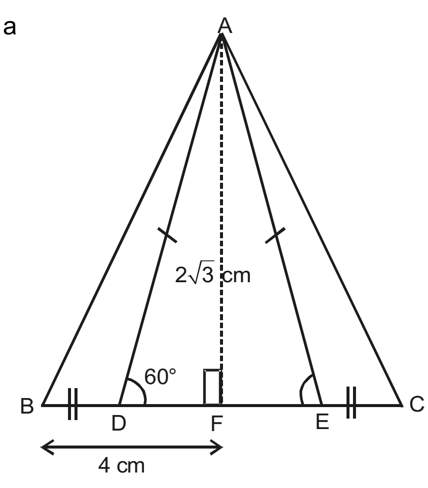
Similarly, $\Delta AOD \cong \Delta AOF$ (RHS)

Area($\square$ADOF) = 2 × Area (ΔAOD)

$$= 2.\frac{1}{2}.14.6 = 84 \text{ cm}^2$$

$$\frac{\text{Area}(\square FOEC)}{\text{Area}(\square ADOF)} = \frac{90}{84} = \frac{15}{14}.$$

43. a



Let's draw AF perpendicular to BC.

In ΔADE, as AD = AE

$\angle$AED = $\angle$ADE = 60°

$\Rightarrow$ $\angle$DAE = 60°

Hence, we can conclude that ΔADE is an equilateral triangle.

$\Rightarrow$ AF $= \frac{\sqrt{3}}{2}\times 4$

$= 2\sqrt{3}$ cm

and DF = FE = 2 cm

In ΔAFB, using Pythagoras' Theorem,

AB $= \sqrt{AF^2 + BF^2}$ =

$\sqrt{(2\sqrt{3})^2 + 4^2} = 2\sqrt{7}$ cm.

44. b In ΔABC, AB = AC

Let $\angle$ABC= $\angle$ACB = x°

and $\angle$BAC = y°

$\therefore$ 2x + y = 180°

In ΔFDE, $\angle$FED = x°

and $\angle$FDE = x – 30° ($\because$ DE || BC)

$\Rightarrow$ x + x – 30 + 70 = 180°

$\therefore$ x = 70°

We see that $\angle$FED = $\angle$DFE

Hence, DFE is an isosceles triangle i.e.

DF = DE = 4 cm.

45. b The triangle formed by joining the mid-points of the sides of an equilateral triangle is itself an equilateral triangle having side length equal to half of that of bigger triangle.

$\therefore$ According to the question,

Sum of the areas of all triangles

$= \frac{\sqrt{3}}{4}\left[(24)^2 + (12)^2 + (6)^2 + (3)^2 + ...\right]$

$= \frac{\sqrt{3}}{4}\times\frac{(24)^2}{1-\frac{1}{4}}$

$\left(\because \text{Sum of an Infinite G.P.} = \frac{a}{1-r}\right)$

$= 192\sqrt{3}$ sq.cm

46. b Point A will be at the shortest possible distance from the (0, 0) when it will lie at a perpendicular distance of 8 units from BC along the X-axis.

Hence, the coordinate of A will be (–8, 0) and the required distance will be 4 units.

Quadrilaterals

1. a The side of every inner square will be $\frac{1}{\sqrt{2}}$ times the side of the immediate outer square.

Hence, the area of every inner square will be half of the area of the immediate outer square.

The area of the outermost square = 64 sq. cm.

So the area of the 2^{nd} square would be 32 sq.cm., the 3^{rd} square would be 16 sq.cm. and so on.

Hence, the sum of all these areas

= 64 + 32 + 16 + 8 + 4 + …

$= \frac{64}{(1 - \frac{1}{2})}$ = 128 sq. cm.

2. c Let length of the smaller square be x.

From statement I, Length of ABCDEQ = 10x $\geq$ 60

$\Rightarrow$ $x \geq 6$

From statement II, Area of rectangle OPQR

$= 42x^2 \leq 1512$ $\Rightarrow$ $x^2 \leq 36$

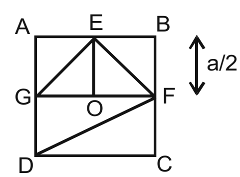
$\therefore$ $x \leq 6$

By combining results from statement I and II

we have, x = 6

Now, area under the line GHI-JKL can be found out using x.

3. a Let side of square be a.



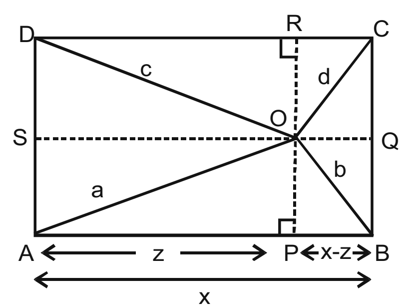
Area quadrilateral EFDG = Area ΔDGF + Area ΔGEF

$= \frac{1}{2}\times\frac{a}{2}\times a + \frac{1}{2}\times a\times\frac{a}{2}$

$= \frac{1}{2}a^2$

$\therefore$ Ar(EFDG) : Ar(S) $= \frac{1}{2}$

4. a



Let x and y be the sides of the rectangle ABCD and z be the length of AP.

Then CR = BP = x – z

By applying Pythagoras Theorem, we have

in ΔAPO, $a^2 = OP^2 + z^2$ ... (i)

in ΔBPO, $b^2 = OP^2 + (x - z)^2$ ... (ii)

in ΔCRO, $d^2 = OR^2 + (x - z)^2$ ... (iii)

in ΔDRO, $c^2 = OR^2 + z^2$ ... (iv)

Solving above equation, we have $a^2 + d^2 = b^2 + c^2$

$\therefore$ For any point inside a rectangle as shown,

$$a^2 + d^2 = b^2 + c^2$$

$\therefore$ Pairing up the distance so that d is to be the maximum, we get $40^2 + d^2 = 50^2 + 60^2$

$\Rightarrow$ d = 67 m.

5. b Since base of each triangle will be counted once,

Sum of perimeters of the triangles = Perimeter of the square + 2 × (Sum of its diagonals).

But each of the other two sides of the triangles is common to two triangles, so it will be counted twice.

Area of the square = 4, therefore length of its side = 2 and perimeter = 8.

Also its diagonal = $2\sqrt{2}$.

So the required perimeter = $(8 + 2 \times 4\sqrt{2}) = 8(1 + \sqrt{2})$.

6. c Since AD = BC (Opposite sides of a rectangle are equal.)

AB + AC = 5BC and AC – BC = 8 or AC = BC + 8

$\therefore$ AB = 4(BC – 2)

By Pythagoras' Theorem, $AB^2 + BC^2 = AC^2$

Expressing AB and AC in terms of BC we get, BC = 5.

$\therefore$ AB = 12 and AC = 13

So area of the rectangle = 5 × 12 = 60.

7. c This can be solved using both the statements together.

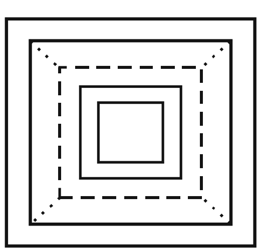
From statement I, we know that L × B = 48 or $B = \frac{L}{48}$.

From statement II, we know $L^2 + B^2 = 100$

Combining statements I and II, $L^2 + \left(\frac{L}{48}\right)^2 = 100$.

L is the only unknown in this equation and can be found out.

8. b



The diagonal of the innermost square is 2 units. The diagonal of every successive square would increase by 2 units (since corners are one unit apart). So the diagonal of the 7th square = 14 units and that of the 8th square = 16 units. Area of the 7th square $= \frac{1}{2}(14)^2$ and that of 8th square $= \frac{1}{2}(16)^2$, i.e. 98 and 128 respectively. Hence, the difference in their areas = 128 – 98 = 30 sq.units.

9. d Let L and B denote the length and the breadth of the rectangle. So the diagonal will be $\sqrt{(L^2 + B^2)}$.

Hence, from the condition given,

$$(L + B) - \sqrt{(L^2 + B^2)} = \frac{1}{2}L$$

$$\Rightarrow \quad \sqrt{(L^2 + B^2)} = \frac{L}{2} + B$$

Squaring both sides, we get

$$(L^2 + B^2) = \left(\frac{L}{2} + B\right)^2$$

$$\Rightarrow \quad L^2 = \frac{L^2}{4} + LB$$

$$\Rightarrow \quad \frac{3L^2}{4} = LB$$

$$\Rightarrow \quad \frac{B}{L} = \frac{3}{4}$$

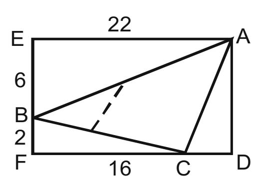
Shortcut:

First write the relation $(L + B) - \sqrt{(L^2 + B^2)} = \frac{1}{2}L$.

or $\frac{L}{2} + B = \sqrt{L^2 + B^2}$

Putting the values of options. Option (d) satisfies. So the answer is (d).

10. b

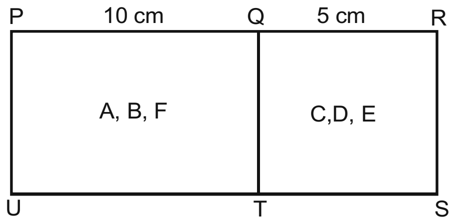


We know that length of the line joining the mid-points of two sides of a triangle is half the length of third side. Hence, the required length is half the length of side AC. Since EADF is rectangle, EF = AD = 8.

CD = (22 – 16) = 6.

So in the right-angled ΔADC, AD = 8 and CD = 6. Therefore, AC = 10. Hence, length of the line joining the mid-points of AB and BC = $\frac{1}{2}(10) = 5$.

For questions 11 and 12:



11. *a The sides of the rectangle PRSU are PR = 15 cm and RS = 10 cm.

$\therefore$ Its diagonal $= \sqrt{15^2 + 10^2} = \sqrt{325} \approx 18$ cm.

Given that the minimum distance between any pair of points formed by taking one from A, B and F and the other from C, D and E is $10\sqrt{3} = \sqrt{300} \approx 17\,\text{cm}$, which is close to the length of the diagonal of PRSU. This implies that each of A, B and F are close to one of the vertices P or U and each of C, D and E are close to one of the vertices S or R.

Note that all the three points in one of the rectangles PQTU and QRST will be close to the same vertex (or else the minimum distance between one of these points and one of the three in the other rectangle will be less than $10\sqrt{3}\,\text{cm}$). Also, the points (A, B, F) and (C, D, E) have to be diagonally opposite as maximum distance between two vertices on same side is 15 cm.

Option (a): The closest two points of the given six points have to be any two out of A, B, F or any two out of C, D, E (since they are closest to the same vertex). Therefore, (F, C)cannot be the closest pair of points as they are diagonally opposite.

Option (b): It is definitely false as A and B are close to the same vertex, while F and C are close to diagonally opposite vertices.

Option (c): It is possible but not necessary that the closest pair of points among the six given points is (C,D), (D, E)or (C, E). The other possibilities are (A, B), (B, F) or (A, F).

12. *d The maximum possible distance between the pairs (C, D), (D, E) or (C, E)is the length of the diagonal of the rectangle QRST, i.e. $\sqrt{10^2 + 5^2} = 5\sqrt{5}$ cm.

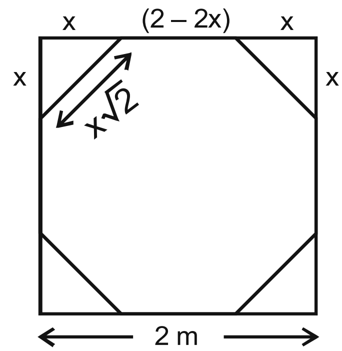
Since AB > AF > BF = $6\sqrt{5}$ cm, the closest pair of points of the given six points will be from the set (C, D, E).

As CD > DE > CE, so (C, E) will be the pair of closest points.

***Note:** There is slight inconsistency regarding the information given in the question. If BF = $6\sqrt{5} \approx 13.4\,\text{cm}$, then A, B and F cannot be close to the same vertex as the length of the diagonal of rectangle PQTU is 14 cm approximately. This in turn will contradict the fact the minimum distance between any point of A, B, F and the other from C, D, E is $10\sqrt{3}$ cm.

A likely possibility is that the information regarding minimum distance between any point of (A, B, F) and the other from (C, D, E) is specific to question no. 11 only.

13. b



Let the length of the edge cut at each corner be x m. Since the resulting figure is a regular octagon,

$\therefore \sqrt{x^2 + x^2} = 2 - 2x \Rightarrow x\sqrt{2} = 2 - 2x$

$\Rightarrow \sqrt{2}\,x\,(1 + \sqrt{2}) = 2 \Rightarrow x = \dfrac{\sqrt{2}}{\sqrt{2}+1}$

$\therefore 2 - 2x = \dfrac{2}{\sqrt{2}+1}$.

14. d Check choices, E.g. $\frac{1}{2} \Rightarrow$ Diagonal = $\sqrt{5}$

Distance saved = $3 - \sqrt{5} \approx 0.75 \neq$ Half the larger side. Hence, incorrect.

$\frac{3}{4} \Rightarrow$ Diagonal = 5

Distance saved = (4 + 3)– 5 = 2 = Half the larger side.

15. d If KL = 1, then IG = 1 and FI = 2

Hence, $\tan\theta = \frac{2}{1} = 2$

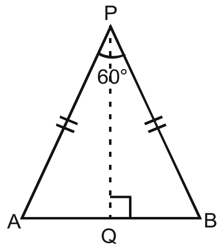
Thus, θ none of 30°, 45° and 60°.

16. c Area of quadrilateral ABCD $= \frac{1}{2}(2x + 4x) \times 4x = 12x$

Area of quadrilateral DEFG $= \frac{1}{2}(5x + 2x) \times 2x = 7x$

Hence, ratio = 12 : 7

17. b



Given $\angle APB = 60°$ and AB = b.

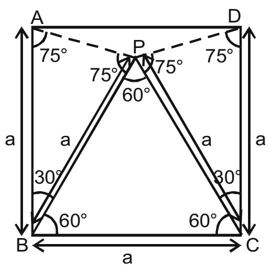
$\therefore \quad PQ = \frac{b}{2} \times \sqrt{3}$

Next, $\frac{b}{2}$, h and PQ form a right angle triangle.

$\therefore \quad \frac{b^2}{4} + h^2 = \frac{3b^2}{4}$

$\Rightarrow \quad 2h^2 = b^2$

18. e



$\angle PBC = \angle CPB = \angle BPC = 60°$

(L's of equilateral triangle)

PC = CD = a

Also, $\angle CPD = \angle PDC = \frac{180° - 30°}{2} = 75°$

Similarly, $\angle BAP = \angle BPA = 75°$

$\angle APD = 360° - 75° - 75° - 60° = 150°$

19. a **Using A:** $\frac{OM}{OL} = \frac{2}{1}$

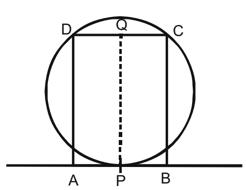
But if O lies on JK, maximum possible value of $\frac{OM}{OL}$ is $\frac{\sqrt{2}}{1}$ (when O lies on K)

So, Rahim is unable to draw such a square

Using B: Nothing specific can be said about the dimensions of the figure.

Hence, option (a)is the correct choice.

20. a



AP must be equal to PB.

Let's assume that the line segment PQ divides the rectangle ABCD into two equal parts (see the figure).

Let AB = 2a; hence, BC = 4a (all lengths in cm).

$$CP = \sqrt{(4a)^2 + a^2} = \sqrt{17}a = DP$$

$$\text{Area of } \Delta CDP = \frac{1}{2} PQ.CD = 4a^2$$

Radius of the circle = Circumradius of ΔCDP

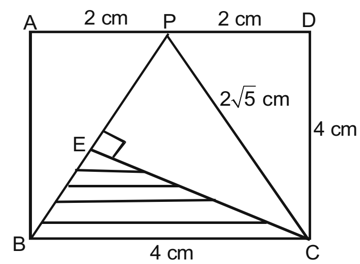
$$= \frac{CD \times CP \times DP}{4(\text{Area of } \Delta CDP)} = \frac{17}{8}a = 1.$$

Hence, $a = \frac{8}{17}$ cm

Area of rectangle ABCD = 2a × 4a = $8a^2$

= 1.77 cm^2 approximately.

21. c



In right triangle PDC, using Pythagoras theorem,

$$PD^2 + DC^2 = PC^2$$

$\Rightarrow \quad PC = 2\sqrt{5}\,cm$ …(i)

$\text{Area } \Delta PBC = \frac{1}{2}$ area sq. ABCD …(ii)

$$= \frac{4 \times 4}{2} = 8\,cm^2$$

Also, area $\Delta PBC = \frac{1}{2} \times CE \times PB$

From equations (i) and (ii), we get

$$\frac{1}{2} \times CE \times 2\sqrt{5} = 8 \quad \text{or} \quad CE = \frac{8}{\sqrt{5}}$$

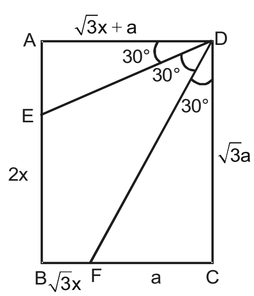
In right triangle CEB, by Pythagoras theorem,

$$BE^2 + CE^2 = BC^2$$

$\Rightarrow \quad BE = \frac{4}{\sqrt{5}}$ cm

Required area $= \frac{1}{2} \times CE \times BE = 3.2\ cm^2$

22. c



Let the length of EB be 2x units.

Therefore, the length of BF = $\sqrt{3}x$ units.

Let the length of FC be a units.

Therefore, the length of DC = $\sqrt{3}a$ units.

(Since triangle DFC is 30°- 60°- 90° triangle.)

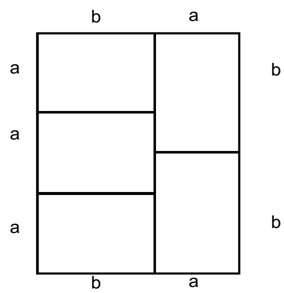
In triangle AED,

$\therefore \quad \sqrt{3}(\sqrt{3}a - 2x) = \sqrt{3}x + a$

$\Rightarrow \quad \frac{\sqrt{3}x}{a} = \frac{2}{3}$

Required ratio = 2 : 3.

23. d Let the lengths and breadths of each of the small rectangles be 'b' m and 'a' m respectively.

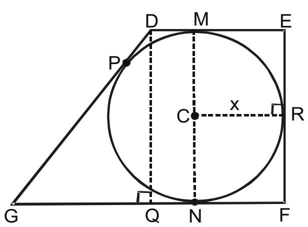


$\therefore$ 5a + 4b = 88 and 3a = 2b

$\Rightarrow$ a = 8 and b = 12

Perimeter of each small rectangle = 2(a + b) = 40

24. a



Let, the radius of circle be 'x' cm.

$\therefore$ CM = CN = CR = x

Given that GN = 4 cm

$\therefore$ GP = 4 cm

Also, DP = DM = QN = 1 cm

$\therefore$ GD = 5 cm

GQ = GM – QN = 4 – 1 = 3 cm

In $\Delta DGQ : DQ = \sqrt{GD^2 - GQ^2} = 4$ cm

$\Rightarrow$ 2x = 4 cm ($\because$ DQ = MN)

$\therefore$ x = 2 cm.

25. c Let the lengths of the sides of the rectangle be a units and b units.

By the problem,

ab = 2a + 2b

a = 2b / (b – 2)

Two cases are possible-

(i) a = 4, b = 4

(ii) a = 3 and b = 6

Thus, two different rectangles are possible.

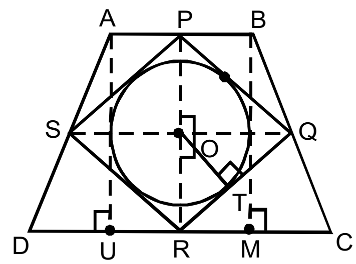
26. b Option (a): This statement is always true.

Option (b): This statement is necessarily false as the smallest possible integral length of EF is 4 units.

Option (c): This statement is true when EF is parallel to BC.

Option (d): This statement is always true.

27. d



PQRS is a rhombus, so the centre of the inscribed circle will be the center of the rhombus PQRS

$SQ = \frac{1}{2}(AB + CD) = \frac{1}{2}(2 + 14) = 8$ units

$OR = \frac{1}{2}PR$

$DU = \frac{1}{2}(CD - MU) = \frac{1}{2}(14 - AB) = 6$ units

$PR = AU = \sqrt{AD^2 - DU^2} = \sqrt{10^2 - 6^2} = 8$ units

PR = SQ = 8

$\Rightarrow$ PQRS is a square.

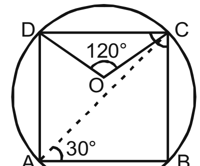
$\Rightarrow OT = 2\sqrt{2}$ units

$\text{Area of trapezium} = \frac{1}{2}(AB + CD) \times PR$

$= 64$ sq. unit

$\text{Required ratio} = \frac{\pi OT^2}{64} = \frac{\pi \times 8}{64} = \frac{\pi}{8}$.

28. 90



$\angle DOC = 120°$

So, $\angle DAC = 60°$

$\Rightarrow \angle DAB = 90°$

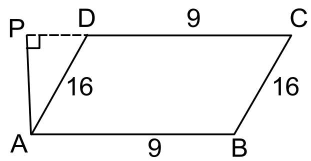
[Angle made at the circumference is half of the angle made at the centre]

As ABCD is a cyclic quadrilateral

So $\angle DAB + \angle BCD = 180°$

$\Rightarrow \angle BCD = 90°$.

29. a



Area of parallelogram ABCD = Base × Height

72 = 9 × h

h = 8 cm = AP

Using Pythagoras formula,

$AD^2 = AP^2 + PD^2$.

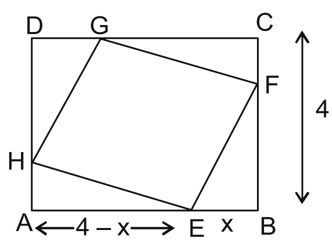
$PD^2 = 256 - 64 = 192$

$\Rightarrow PD = 8\sqrt{3}$

$\therefore$ Area of Δ APD $= \frac{1}{2} \times 8\sqrt{3} \times 8$

$= 32\sqrt{3}$ sq. cm

30. a



Let the side of the square ABCD be 4 units and EB = x

$\Rightarrow$ AE = 4 – x = CG

Area (EFGH) = 62.5% of Area (ABCD)

Area of EFGH = $(EF)^2$

$(EF)^2 = x^2 + (4 - x)^2$

$\therefore \; x^2 + (4 - x)^2 = \frac{5}{8} \times (4)^2$

$x^2 + 16 + x^2 - 8x = \frac{5}{8} \times 16$

$2x^2 - 8x + 6 = 0$

$x^2 - 4x + 3 = 0$

On solving it, we get,

x = 1, x = 3.

x = 3 is not possible as CG = 4 – 3 = 1, which implies CG < EB, which contradicts the given condition.

$\therefore \frac{EG}{CG} = \frac{x}{4 - x} = \frac{1}{3} = 1:3$

31. d Given that : $\frac{\pi r^2}{2} = 72\pi$

$\Rightarrow$ r = 12 cm, AB = 24 cm and BC = 32 cm

Hence, perimeter of the leftover portion

= 32 × 2 + 24 × 1 + π × 12 = 88 + 12π.

32. c Given that AB = CD = 8 cm

$\Rightarrow$ Area of parallelogram = AB × AD sin $\angle$DAB

$\Rightarrow$ 8 × s × sin $\angle$DAB = 48

The maximum value of sin $\angle$DAB will be 1 at 90°

Thus, the minimum value of 's' will be 6 cm

Hence, correct answer will be S $\geq$ 6.

33. d Checking the options we get the required ratio as 1 : 4

Such that, area = 1 × 4 = 4 square units

and $(\text{perimeter})^2 = [2\times(1 + 4)]^2 = 100$

Hence, it gives the original ratio i.e. 4 : 100 = 1 : 25.

Polygons

1. a **From statement I:** We can say that the perimeter of the hexagon is 36 cm, or the length of each side is 6 cm. From this we can find its area. So this statement alone is sufficient to answer the question.

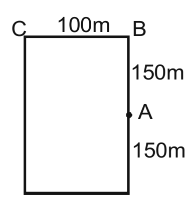
From statement II: It does not provide any other data, but merely states the property of a regular hexagon. So, this statement alone is not sufficient to answer the question.

2. c Following rule should be used in this case: The perimeter of any polygon circumscribed about a circle is always greater than the circumference of the circle and the perimeter of any polygon inscribed in a circle is always less than the circumference of the circle.

Since the circles is of radius 1, its circumference will be 2π. Hence, L1(13)> 2π and L2(17) < 2π. So {L1(13) + 2π } > 4π.

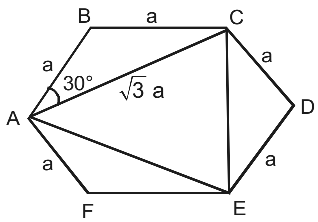
Hence, $\frac{\{L1(13) + 2\pi\}}{L2(17)}$ will be greater than 2.

3. c Statement I by itself does not solve the problem but it does tell us about the shape of the field. However, it fails to give information about the points A, B and C as to whether they be at the end of the field, etc. This data is given by the second statement, from which it is known that



The polygon has the length = 150 × 2 = 300 m and the breadth = 100 m and also that it is a rectangle (from A). Thus, the maximum distance is the diagonal length of the rectangle.

4. b



$\because \Delta ACE$ is equilateral triangle with side $\sqrt{3}$ a .

Area of hexagon = $\frac{\sqrt{3}}{4}\ a^2 \times 6$

Area as $\Delta ACE = \frac{\sqrt{3}}{4}\ (\sqrt{3}a)^2$

Therefore, ratio = $\frac{1}{2}$

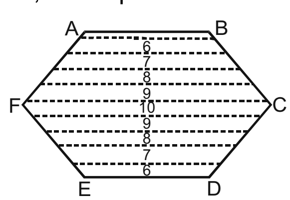
5. c In this kind of polygon, the number of convex angles will always be exactly 4 more than the number of concave angles.

NOTE : The number of vertices have to be even. Hence, the number of concave and convex corners should add up to an even number. This is true only for the answer choice (c).

6. a It is very clear, that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle.

Hence, the required ratio = 1 : 12.

7. c

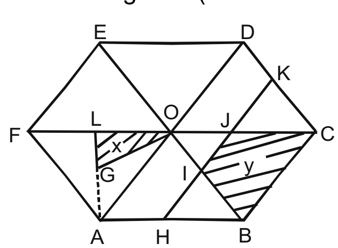


Since AB is 5 cm, CF would be 10 cm.

And the four equidistant lines drawn between them have to be of 6 cm, 7 cm, 8 cm and 9 cm long.

So the total length = 2(6 + 7 + 8 + 9) + 10 = 70 cm

8. b



Let the length of the side of the hexagon be a units.

$\therefore\ \Delta AFO$ and ΔBCO are equilateral triangles with the length of their side equal to a units and area (in sq. units) of each being $\frac{\sqrt{3}}{4}a^2$.

In ΔAFO, as G is the centroid and L is mid point of OF, AL is the median to the side OF.

Also, AL = $\frac{\sqrt{3}}{2}$a units and GL $= \frac{\sqrt{3}}{6}$a units

(Centroid divides medians in the ratio 2:1)

Area of $\Delta GOL = \frac{1}{2}\times LG \times LO$

$= \frac{1}{2}\times\frac{\sqrt{3}}{6}a\times\frac{a}{2}$

$= \frac{\sqrt{3}}{24}a^2$ sq. units ...(i)

As H and K are the mid points of AB and CD respectively, HK is parallel to AD. Thus, HI is parallel to AO and OD is parallel to JK. Hence, I is the mid point of OB and J is the mid point of OC.

$\therefore\ \Delta OIJ \sim \Delta OBC\ \frac{\text{Area}\ \Delta OIJ}{\text{Area}\ \Delta OBC} = \frac{IJ^2}{BC^2} = \frac{1}{4}$

Area of quadrilateral BCJI

$= \frac{3}{4}\text{area}\ \Delta OBC = \frac{\sqrt{3}}{4}a^2 \times \frac{3}{4}$ sq. units. ...(ii)

From (i) and (ii), required ratio = 2:9.

Alternate Method:

ΔAOF and ΔOBC are equilateral triangles with equal area (as length of the sides is the same for the two).

Area $\Delta GOL = \frac{1}{2}$Area ΔFOG

$= \frac{1}{2}\left(\frac{1}{3}\text{Area}\ \Delta AOF\right)$

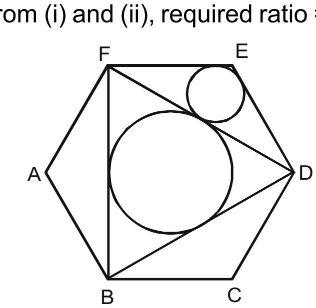
$= \frac{1}{6}$Area ΔAOF ...(i)

Area of quadrilateral

IJCB $= \frac{3}{4}$Area ΔOBC. ...(ii)

From (i) and (ii), required ratio = 2:9.

9. d



Let EF= s unit

In triangle DEF:

$$DF = 2EF\cos 30^0 = \sqrt{3}s$$

Semi perimeter of triangle DEF (in units) = S

$$= \left(1+\frac{\sqrt{3}}{2}\right)s.$$

Area of triangle DEF (in unit^2) = Δ

$$= \frac{1}{2}(EF\sin 30^0)DF = \frac{\sqrt{3}s^2}{4}.$$

Inradius of triangle DEF (in units) = r_1

$$= \frac{\Delta}{S} = \frac{\sqrt{3}s^2}{4\left(1+\frac{\sqrt{3}}{2}\right)s} = \frac{\sqrt{3}s}{4\left(1+\frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{3}s}{2\left(2+\sqrt{3}\right)}$$

Inradius of triangle DEF (in units) = r_2

$$= \frac{1}{3}\left(\frac{\sqrt{3}}{2}DF\right) = \frac{DF}{2\sqrt{3}} = \frac{s}{2}.$$

Ratio of area = $r_1^2 : r_2^2 = 3 : 7+4\sqrt{3}$

10. a Interior angle of an n–sided regular polygon

$$= \frac{(n-2)\pi}{n} = \theta$$

$$\Rightarrow \quad n = \frac{2\pi}{\pi-\theta}$$

The number of diagonals in an n–sided polygon

$$= {}^nC_2 - n = kn$$

$$\Rightarrow \quad \frac{n(n-3)}{2} = kn$$

$$\Rightarrow \quad k = \frac{n-3}{2}$$

Hence, $$k = \frac{\left(\frac{2\pi}{\pi-\theta}\right)-3}{2} = \frac{3\theta-\pi}{2(\pi-\theta)}.$$

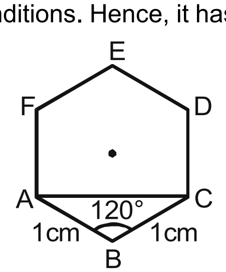
Alternate solution:

If the polygon is an equilateral triangle, then, k = 0 and θ = 60°. Only options (a) and (b) satisfy these conditions.

If the polygon is a square, then, $k = \frac{1}{2}$ and $\theta = 90°$.

Between (a) and (b), only option (a) satisfies these conditions. Hence, it has to be the answer.

11. b



$$\cos 120° = \frac{1^2+1^2-AC^2}{2\times1\times1}$$

$$-\frac{1}{2} = \frac{2-AC^2}{2}$$

$AC^2 = 2 + 1 = 3$

AC = $\sqrt{3}$ cm

Area of square = AC^2 = 3 cm.

Circle

1. d Let the radius of the circle be r.

From statement I, $\frac{\pi r^2}{2\pi r} > 7$

$\Rightarrow r > 14$ ...(i)

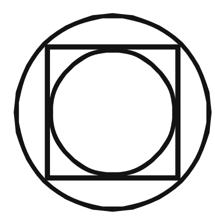
From statement II, $2r \le 32$

$\Rightarrow r \le 16$ ...(ii)

Combining (i)and (ii), $14 < r \le 16$

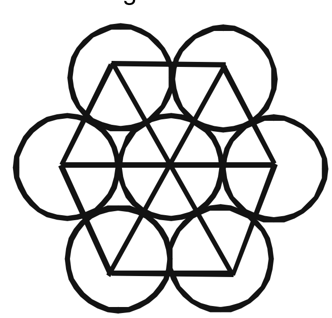
So r can be 15 or 16.

2. d As it is apparent from the following diagram, the diameter of the inscribed circle is equal to the side of the square, while the diameter of the circumscribed square is equal to the diagonal of the square. Since the ratio of any two circles is equal to the ratio of the squares of their diameters, in this case the required ratio is equal to (side)^2 : (diagonal)^2.

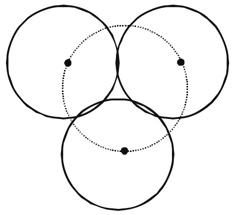


Now, the ratio of the side to the diagonal of a square = $1:\sqrt{2}$, the ratio of their squares will be $1:2$.

3. b It can be seen that if we place 3 coins touching each other, their centers form an equilateral triangle. Hence, the angle made by the centers of the coins around the central coin is 60°. Since the total angle to be covered is 360°, there has to be 6 coins surrounding the central coin.



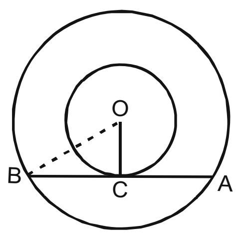
4. c



It can be seen that, if we place the 3 cones in such a way that they touch each other, it will be similar to placing 3 circles touching, with vertices of the cone corresponding to the centers of the circles. The centers of the circle form an equilateral triangle with each side being 2r. A circle that passes through the centers will be the circumcircle to such a triangle. The radius of the circumcircle of an equilateral triangle is $\left(\frac{1}{\sqrt{3}}\right)$ times its side.

Hence, in our case it would be $\left(\frac{2r}{\sqrt{3}}\right)$ and $\left(\frac{2r}{\sqrt{3}}\right) > r$, since $\sqrt{3} = 1.73$ (approx.).

5. a



Let x meters and y meters be the radius of the outer and the inner circles respectively and O be their center.

In right angled Δ OCB,

$CB^2 = OB^2 - OC^2$

$\Rightarrow$ $9 = x^2 - y^2$

$\Rightarrow$ $(x + y)(x - y) = 9 \times 1$

As x and y are integers, therefore, x + y = 9 and x – y = 1.

Thus, x = 5.

Hence, radius of the outer circle is 5 meters.

6. a In the given figure, the area of the circle = πr^2.

To find out the area of the circle, we need to find out the length of the side of the square.

We know, OR = OT + TR = OT + OS = 2r.

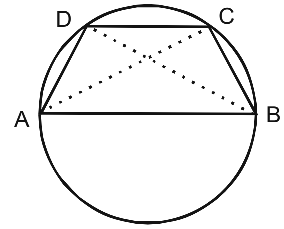
In right-angled triangle ORS, OR = 2r and OS = r.

So $SR^2 = OR^2 - OS^2$.

But SR^2 = Area of the square = $4r^2 - r^2 = 3r^2$.

Hence, the required ratio = $\frac{\pi}{3}$.

7. a



If we draw the imaginary lines AC and BD, we find that $\angle$CAD and $\angle$CBD are subtended by the same chord DC.

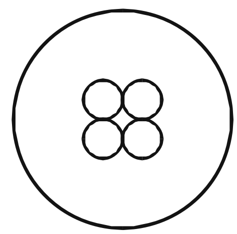
$\therefore$ $\angle CAD = \angle CBD = 30°$.

Thus, $\angle DBA = (70° - 30°) = 40°$.

Also, $\angle$DBA and $\angle$ACD are subtended by the same chord DA.

Hence, $\angle ACD = \angle DBA = 40°$.

8. c



Area of the original paper = $\pi(20)^2 = 400\pi$ cm^2. The total cut portion area = $4(\pi)(5)^2 = 100\pi$ cm^2. Therefore, area of the uncut (shaded) portion = $(400 - 100) = 300\pi$ cm^2.

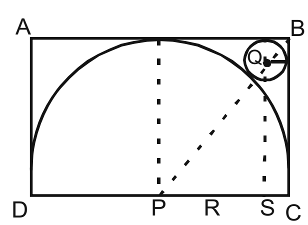
Hence, the required ratio = $300\pi : 100\pi = 3 : 1$.

9. b The radius of the circle is 6.5 cm. Therefore, its diameter = 13 cm and AB = 13 cm. Since the diameter of a circle subtends 90° at the circumference, $\angle ACB = 90°$. So ΔACB is a right-angled triangle with AC = 5 cm, AB = 13 cm.

Therefore, CB should be equal to 12 cm (as 5-12-13 form a Pythagorean triplet).

Hence, the area of the triangle $= \frac{1}{2} \times AC \times CB = \frac{1}{2} \times 5 \times 12 = 30$ sq.cm.

10. d



Let radius of the semicircle be R and radius of the circle be r.

Let P be the centre of semicircle and Q be the centre of the circle.

Draw QS parallel to BC.

Now, $\Delta PQS \sim \Delta PBC$

$\therefore \quad \frac{PQ}{PB} = \frac{QS}{BC}$

$\Rightarrow \quad \frac{R+r}{\sqrt{2}R} = \frac{R-r}{R}$

$\Rightarrow \quad R + r = \sqrt{2}R - \sqrt{2}r$

$\Rightarrow \quad r(1+\sqrt{2}) = R(\sqrt{2}-1)$

$\Rightarrow \quad r = R\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$

$\Rightarrow \quad r = R(\sqrt{2}-1)^2$

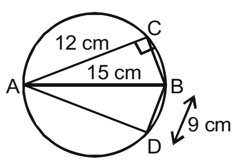
Required ratio = $\frac{\pi r^2}{\pi R^2} \times 2$

$= \frac{\pi R^2(\sqrt{2}-1)^4 \times 2}{\pi R^2}$

$= 2(\sqrt{2}-1)^4 : 1$

11. a None of the statements is useful in finding the radius of the rear wheel. In the question, distance travelled is given. But the number of rotations taken by it is not given.

12. d



Since AB is the diameter of the circle, $\angle ACB$ would be right angle. In this triangle, we know AB = 15 and AC = 12. So we can find BC. Hence, BC = 9. Since BC = BD, AD = AC (similar triangles).

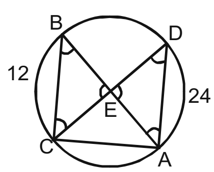
Hence, area of ΔACB = Area of Δ ABD

$= \frac{1}{2}$ AC × CB

$= \frac{1}{2}$ × 12 × 9 = 54 cm^2

So the area of quadrilateral ACBD = 2 × 54 = 108 sq. cm.

13. a



In ΔBEC and ΔAED

$\angle CBE = \angle CDE$ ($\because$ Angles in the same segment of a circle are equal)

Similarly, $\angle BCE = \angle EAD$ ($\because$ Angles in the same segment of a circle are equal)

$\angle BEC = \angle AED$ (Vertical angles are equal)

$\therefore$ By AAA similarity

$\Delta CEB \sim \Delta AED$

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides

$\therefore \frac{Area(\Delta BEC)}{Area(\Delta AED)} = \left(\frac{BC}{DA}\right)^2 = \left(\frac{12}{24}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

14. a If the radii of two circles are r_1 and r_2, then the two equations can be written $\pi r_1^2 + \pi r_2^2 = 153\pi$

or $(r_1^2 + r_2^2) = 153$ and $r_1 + r_2 = 15$.

Now $r_1^2 + r_2^2 = (r_1 + r_2)^2 - 2r_1r_2$

Therefore, $153 = (15)^2 - 2r_1r_2$ or $r_1r_2 = 36$.

If 36 is to be expressed as the product of two integers, it could be (36 × 1), (18 × 2), (12 × 3), (9 × 4), (6 × 6).

The only two factors that add up to 15 are 12 and 3. Hence, $r_1 = 12$, $r_2 = 3$. Therefore, the ratio of larger radius to the smaller one is 12 : 3 = 4.

15. c PQ = PE + FQ – FE

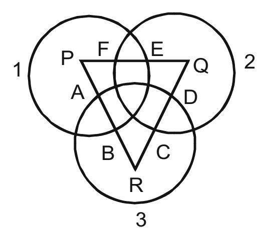
= Radius of circle 1 + Radius of circle 2 – FE

= 20 + 20 – 12 = 28

Similarly, QR = 20 + 20 – CD = 40 – 10 = 30

and PR = 20 + 20 – AB = 40 – 5 = 35

So perimeter of ΔPQR = 28 + 30 + 35 = 93



16. c Let R be the radius of each circle. Then $\frac{\pi R^2}{2\pi R} = \frac{2\pi R}{\pi R^2}$ which implies that $\frac{R}{2} = \frac{2}{R}$, i.e. $R^2 = 4$, i.e. R = 2.

Then the length of the square is 8. Thus, the area of the square is 64, while the area covered by each coin is $\pi \times 2^2 = 4\pi$. Since there are four coins, the area covered by coins is $4(4\pi) = 16\pi$.

Hence, the area not covered by the coins is $64 - 16\pi = 16(4 - \pi)$.

17. c The information given in the question implies that $r_1 > r_2$.

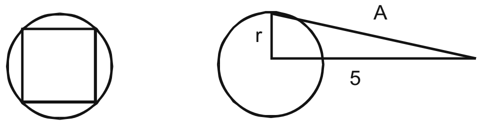
Statement I suggests that $(r_1 - r_2) = \frac{k}{2\pi}$.

Hence, this statement alone does not give the value of r_1.

Statement II implies that $(r_1^2 - r_2^2) = \frac{m}{\pi}$.

Hence, again this statement alone is not sufficient to answer the question. But in the second equation, we simplify $(r_1^2 - r_2^2)$ as $(r_1 + r_2)(r_1 - r_2)$ and then substitute the value of $(r_1 - r_2)$ from the first equation, we will get the value of $(r_1 + r_2)$. Now we have two equations in r_1 and r_2, which can be solved simultaneously to get the value of r_1. Hence, both the statements when together taken can answer the question.

18. b



Statement I itself is sufficient to answer the question. As, if we know the radius of the circle we can find out the length of the diagonal of the square (which will be the diameter) and if we know the diagonal of a square we can find the length of its sides and hence the area.

Again, the second statement in itself can answer the question. As, from the data that is given we can find the radius of the circle and hence the area of the square (as given before). This can be explained from the diagram given. Since the tangent makes a right angle with the radius at the circumference, the triangle is a right-angled triangle. Hence, $A^2 = 5^2 + r^2$. Hence, knowing the value of A, we can find out r. Hence, both statements in itself can answer the question. Therefore, the answer is (b).

19. c Let the radius of the outer circle be OQ = x.

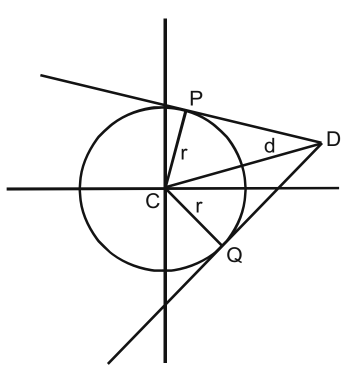
Hence, perimeter of the circle $= 2\pi x$

But OQ = BC = x (diagonals of the square BQCO)

Perimeter of ABCD = 4x

Hence, ratio = $\frac{2\pi x}{4x} = \frac{\pi}{2}$.

20. b

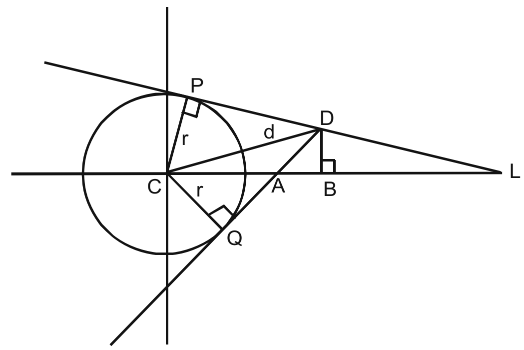


r and d are given

From **statement I,** when co-ordinates of D are given, only one pair of tangents can be drawn onto the given circle from D. So angle made by x-axis for each can be found out.

Hence, statement I alone is sufficient.

Consider **statement II.** Let the x-axis bisect the tangent QD, i.e. QA = AD.



Here $QA = \frac{1}{2}QD = \frac{1}{2}\sqrt{d^2 - r^2}$. So using trigonometric ratios in right ΔCQA, we can determine $\angle CAQ$. Therefore, $\angle DAB$ is equal to $\angle CAQ$ (vertically opposite angles).

Consider the other tangent DP. Let it intersect x-axis at point L.

$\angle CDQ$ can be determined using trigonometric ratios (as two of the sides are given in right ΔCDQ). Also, $\angle CDQ$ is equal to $\angle CDP$ (since the two right ΔCQD and ΔCPD are congruent). Drop a perpendicular DB on x-axis. In right ΔDBL, we can find $\angle BDL = 180^o - (\angle ADB + \angle CDQ + \angle CDP)$ $= 180^o - 2\angle CDQ - \angle ADB$. Applying angle sum property of triangle, we can determine $\angle DLB$.

Hence, statement II alone is sufficient.

21. c Let the area of sector S_1 be x units. Then the area of the corresponding sectors shall be 2x, 4x, 8x, 16x, 32x and 64x. Since every successive sector has an angle that is twice the previous one, the total area then shall be 127x units. This is $\frac{1}{8}$ of the total area of the circle.

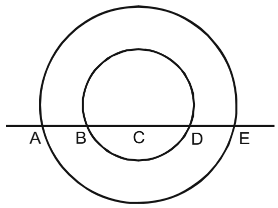
Hence, the total area of the circle will be 127x × 8

= 1016x units. Hence, angle of sector S_1 is $\frac{\pi}{1016}$.

22. c For a given inradius and circumradius, there is only one possible value of (PR + RQ).

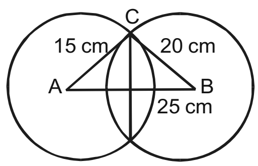
Hence, both the statements are required to answer the question.

23. b You can see from the following diagram that both statements individually



imply towards C being the mid-point of BD. The ratio of AC/CE will be one by using any statement.

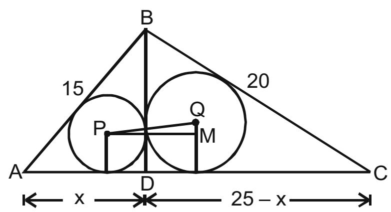
24. a



Let the length of the chord be x cm.

$\therefore \frac{1}{2}(15 \times 20) = \frac{1}{2} \times 25 \times \frac{x}{2} \Rightarrow x = 24$ cm

25. b



$(15)^2 - x^2 = (20)^2 - (25 - x)^2$

$\Rightarrow$ x = 9

$\Rightarrow$ BD = 12

Area of $\Delta ABD = \frac{1}{2} \times 12 \times 9 = 54$

$s = \frac{1}{2}(15 + 12 + 9) = 18$

$r_1 = \frac{\text{Area}}{s} \Rightarrow r_1 = 3$

Area of $\Delta BCD = \frac{1}{2} \times 16 \times 12 = 96$

$s = \frac{1}{2}(16 + 20 + 12) = 24$

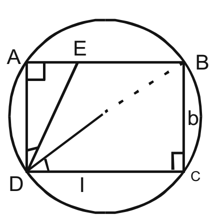
$r_2 = \frac{\text{Area}}{s} \Rightarrow r_2 = 4$

In ΔPQM, PM = $r_1 + r_2$ = 7 cm

QM = $r_2 - r_1$ = 1 cm

Hence, PQ = $\sqrt{50}$ cm

26. a



BD = 2r

$$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi r^2}{lb} = \frac{\pi}{\sqrt{3}}$$

$$\frac{r^2}{lb} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\frac{d^2}{4}}{lb} = \frac{1}{\sqrt{3}} \Rightarrow \frac{d^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2 + b^2}{4lb} = \frac{1}{\sqrt{3}} \Rightarrow \frac{l^2 + b^2}{lb} = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{l}{b} + \frac{b}{l} = \frac{4}{\sqrt{3}} \quad \text{... (i)}$$

Now $\Delta AED \sim \Delta CBD$

$$\therefore \frac{AE}{CB} = \frac{AD}{DC} \Rightarrow \frac{AE}{AD} = \frac{BC}{DC} \Rightarrow \frac{AE}{AD} = \frac{b}{l}$$

$\therefore$ We have to find $\frac{AE}{AD}$, i.e. $\frac{b}{l}$.

Let $\frac{b}{l} = x$

Therefore, from (i), we get

$$\frac{1}{x} + x = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{1 + x^2}{x} = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} + \sqrt{3}x^2 = 4x$$

$$\Rightarrow \sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\therefore x = \frac{-(-4) \pm \sqrt{16 - 4\left(\sqrt{3}\right)\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2\sqrt{3}} = \frac{4 \pm 2}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} \text{ OR } \frac{2}{2\sqrt{3}}$$

$$= \frac{\sqrt{3}}{1} \text{ OR } \frac{1}{\sqrt{3}}$$

From options, the answer is $\frac{1}{\sqrt{3}}$, i.e. $1 : \sqrt{3}$.

27. a $\angle BAC = \angle ACT + \angle ATC = 50^\circ + 30^\circ = 80^\circ$

And $\angle ACT = \angle ABC$ (Angle in alternate segment)

So $\angle ABC = 50^\circ$

$\angle BCA = 180^\circ - (\angle ABC + \angle BAC)$

$= 180^\circ - (50^\circ + 80^\circ) = 50^\circ$

Since $\angle BOA = 2\angle BCA = 2 \times 50^\circ = 100^\circ$

Alternative Method:

Join OC

$\angle OCT = 90^\circ$ (TC is tangent to OC)

$\angle OCA = 90^\circ - 50^\circ = 40^\circ$

$\angle OAC = 40^\circ$ (OA = OC being the radius)

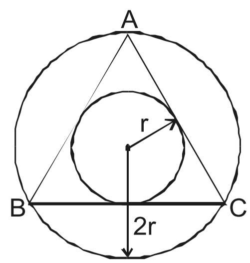
$\angle BAC = 50^\circ + 30^\circ = 80^\circ$

$\angle OAB = 80^\circ - 40^\circ = 40^\circ = \angle OBA$

(OA = OB being the radius)

$\angle BOA = 180^\circ - (\angle OBA + \angle OAB) = 100^\circ$

28. c



Since the area of the outer circle is 4 times the area of the inner circle, the radius of the outer circle should be 2 times that of the inner circle.

Since AB and AC are the tangents to the inner circle, they should be equal. Also, BC should be a tangent to inner circle. In other words, triangle ABC should be equilateral.

The area of the outer circle is 12.

Hence, the area of inner circle is 3 or the radius is $\sqrt{\frac{3}{\pi}}$.

The area of equilateral triangle = $3\sqrt{3}$ r^2, where r is the inradius.

Hence, the answer is $\frac{9\sqrt{3}}{\pi}$.

29. a If y = 10°,

$\angle BOC$ = 10° (opposite equal sides)

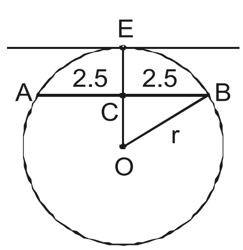
$\angle OBA$ = 20° (external angle of ΔBOC)

$\angle OAB$ = 20 (opposite equal sides)

$\angle AOD$ = 30° (external angle of ΔAOC)

Thus k = 3

30. a



We can get the answer using the second statement only. Let the radius be r.

AC = CB = 2.5 and using statement B, CE = 5, thus OC = (r – 5).

Using Pythagoras theorem, $(r - 5)^2 + (2.5)^2 = r^2$

We get r = 3.125 cm.

NOTE: You will realize that such a circle is not possible (if r = 3.125 how can CE be 5). However we need to check data sufficiency and not data consistency. Since we are able to find the value of r uniquely using second statement the answer is (a).

31. b $\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

OP = 28

OQ = 21

PQ = OP – OQ = 7

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

32. b PR + QS = PQ = 7 $= \frac{PR}{QS} = \frac{4}{3}$

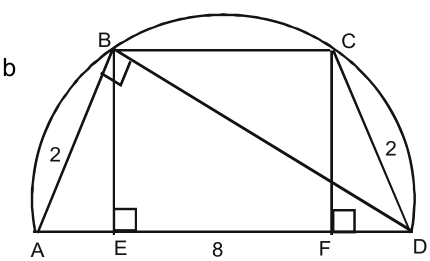
$\Rightarrow QS = 3$ cm.

33. c $SO = \sqrt{OQ^2 - QS^2}$

$= \sqrt{21^2 - 3^2}$

$= \sqrt{24 \times 18} = 12\sqrt{3}$ cm.

34. b



$$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$$

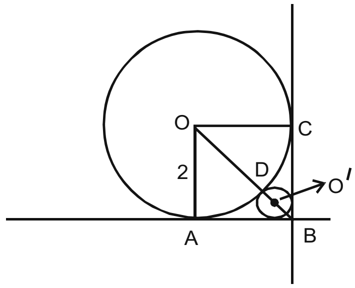
$$\Rightarrow 2\sqrt{8^2 - 2^2} = 8 \times BE$$

$$\Rightarrow BE = \frac{\sqrt{60}}{4} = \frac{\sqrt{15}}{2}$$

$$AE = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$$

$$BC = EF = 8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$$

35. d



Let the radius of smaller circle be ‘r’.

$\therefore \quad O'B = r\sqrt{2}$

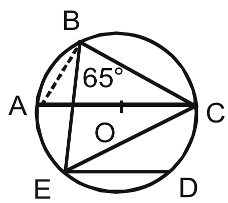
$\therefore \quad OB = O'B + O'D + OD = r\sqrt{2} + r + 2$

Also OB = $2\sqrt{2}$

$\Rightarrow \quad r\sqrt{2} + r + 2 = 2\sqrt{2}$

$\Rightarrow \quad r = 6 - 4\sqrt{2}$

36. d



In ΔABC, $\angle B = 90°$ (Angles in semicircle)

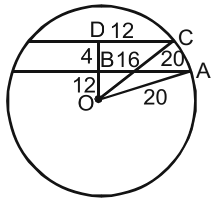
Therefore, $\angle ABE = 90 - 65 = 25°$

Also, $\angle ABE = \angle ACE$ (Angle subtended by same arc AE)

Also, $\angle ACE = \angle CED$ [$AC \parallel ED$]

Therefore, $\angle CED = 25°$

37. d **Case I:** Chords on same side of the centre.



$OB^2 = OA^2 - AB^2 = 20^2 - 16^2 = 144$

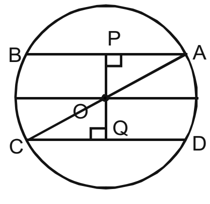
$OB = 12$

$OD^2 = 20^2 - 12^2 = 400 - 144 = 256$

$OD = 16$

$BD = 4$ cm

Case II: Chords on opposite side of the centre.



AB = 32 cm, CD = 24 cm

$OP = \sqrt{AO^2 - AP^2} = \sqrt{(20)^2 - (16)^2}$

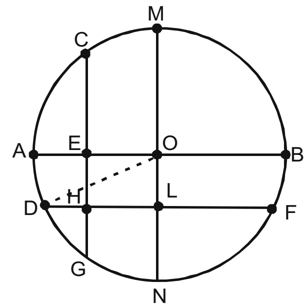
OP = 12 cm

& $OQ = \sqrt{(OC)^2 - (CQ)^2} = \sqrt{(20)^2 - (12)^2}$

OQ = 16 cm

Distance = PQ = 12 + 16 = 28 cm.

38. b



AE = 1 cm, BE = 2 cm & NL = 1 cm, ML = 2 cm

$HL = OE = \frac{1}{2}$

DL = DH + HL

$\Rightarrow \quad DL = DH + \frac{1}{2}$

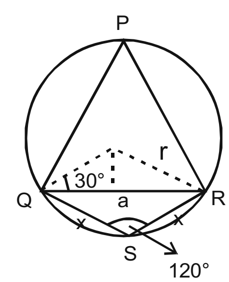
OB = AO = radius = 1.5

$DO^2 = OL^2 + DL^2$

$\Rightarrow \quad \left(\frac{3}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(DH + \frac{1}{2}\right)^2$

$\Rightarrow \quad \left(DH + \frac{1}{2}\right)^2 = 2 \quad \Rightarrow \quad DH = \sqrt{2} - \frac{1}{2}$.

39. a



Here $\cos 30° = \frac{a}{2r}$

$\Rightarrow \quad a = r\sqrt{3}$

Here the side of equilateral triangle is $r\sqrt{3}$

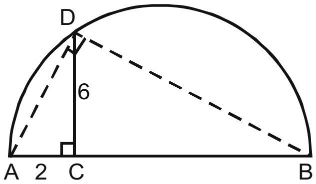
From the diagram $\cos 120° = \frac{x^2 + x^2 - a^2}{2x^2}$

$\Rightarrow \quad a^2 = 3x^2$

$x = r$

Hence, the circumference will be $2r\left(1 + \sqrt{3}\right)$.

40. b



$\angle ADB = 90°$ (Angle in semicircle)

$CD^2 = AC \times CB$

$\Rightarrow (6)^2 = 2 \times CB$

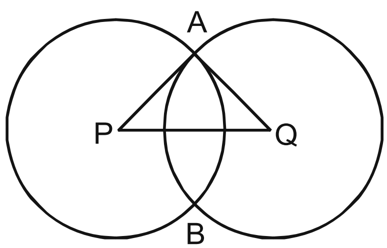
$\Rightarrow 36 = 2 \times CB$

$\Rightarrow CB = 18$ cm

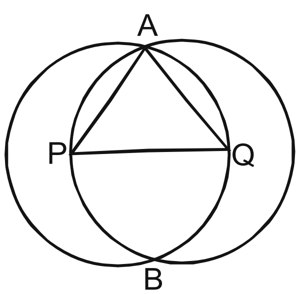
$\therefore AB = AC + CB = 20$ cm

Hence, area of semicircle $= \frac{1}{2}\pi(10)^2 = 50\pi$ sq. cm.

41. c



If P and Q lie on the intersections of the circles as shown in the figure given below.



In this case triangle APQ is equilateral. So the maximum possible measure of the angle AQP is 60°. The answer is between 0 and 60.

42. e We can use the formula for the circum radius of a triangle:

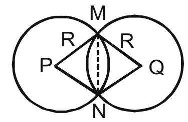
$$R = \frac{a \times b \times c}{4 \times (\text{Area of the triangle})}$$

or $$R = \frac{a \times b \times c}{4 \times \left(\frac{1}{2} \times b \times AD\right)}$$

$$= \frac{a \times c}{2 \times AD}$$

$$= \frac{17.5 \times 9}{2 \times 3} = 26.25 \text{ cm}.$$

43. a



Area of $\Delta MNQ = \frac{1}{2} \times R \times R = \frac{1}{2}R^2$

Area of sector $MNQ = \frac{1}{4} \times \pi R^2$

So the required area $= 2\left[\frac{1}{4}\pi R^2 - \frac{1}{2}R^2\right]$

$$= 2 \times \frac{1}{2}R^2\left[\frac{\pi}{2} - 1\right] = R^2\left[\frac{\pi}{2} - 1\right] \text{ sq units}$$

44. a Centre of the circle will be mid-point of the hypotenuse of the triangle. So, hypotenuse = 2 × 6.5 = 13 cm.

Since all the sides are natural numbers, obviously other sides are 5 cm and 12 cms and the perimeter is 5 + 12 + 13 = 30 cm.

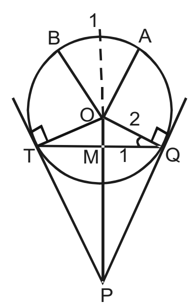
45. d If Ramu and Shamu are at the diametrically opposite ends (in which case distance = 18 m)

Then

1.5 + 3 x N = 4 M where M, N are natural numbers

This is never possible.

46. c



Since, OQ = TQ = 2 units, therefore ΔOTQ is an equilateral.

$\therefore \angle TOQ = 60°$

Since, PQ is a tangent to the circle, therefore $\angle OQP = 90°$.

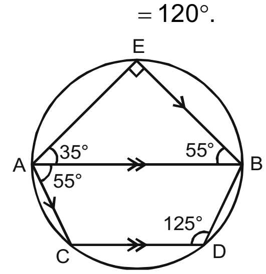
Since, PQ is a parallel to OA therefore $\angle AOQ = 90°$.

For the same reason $\angle BOT = 90°$.

$\therefore \angle AOB = 360° - (\angle TOQ + \angle AOQ + \angle BOT)$

$= 120°$.

47. b



$\angle AEB = 90°$ (Angle in a semicircle)

$\therefore \quad \angle ABE = 180 - (90 + 35) = 55°$

$\therefore \quad \angle BAC = 55°$ (Alternate interior angles)

Now, $\angle CDB = 180 - 55 = 125°$ (Sum of opposite angles of a cyclic quadrilateral is 180°.)

$\therefore \quad \angle ABD = 180 - 125 = 55°$

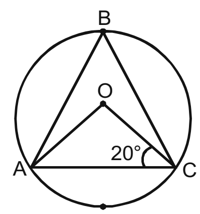
(Sum of interior opposite angles is 180°.).

Hence $\angle CDB - \angle ABD = 125 - 55 = 70°$.

48. d There would be 2 cases.

Case I: When B lies in greater arc AC.

The figure would be as below.



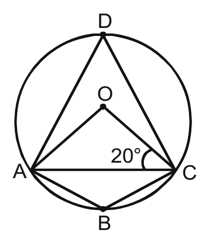
$\angle OAC = \angle OCA = 20°$

$\Rightarrow \quad \angle AOC = 140°$

$\therefore \quad \angle ABC = \frac{1}{2}\angle AOC = 70°$

Case II: When B lies in smaller arc AC.

The figure would be as given below.



Let D be a point O is the greater arc AC.

$\angle OAC = \angle OCA = 20°$

$\Rightarrow \quad \angle OAC = 140°$

$\therefore \quad \angle ADC = \angle AOC = 70°$

ABCD is a cyclic quadrilateral

$\therefore \quad \angle ABC = 180° - 70° = 110°$.

49. c $\angle EOA = 85°$, $\angle BOD = 15°$

$\therefore \angle EOD = 180° - (85° + 15°) = 80°$

$\therefore$ In ΔOED, OE = OD (radius)

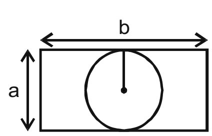
$\therefore \angle OED = \angle ODE = 50°$

In ΔEOC,

$\angle EOC = 80° + 15° = 95°$ and $\angle OEC = 50°$

$\therefore \angle ECA = 180° - (95° + 50°) = 35°$

50. c



Let 'a' be the length of smaller side

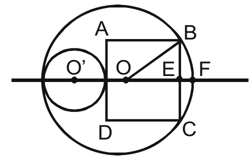
$\therefore$ Radius $= \frac{a}{2}$

$\Rightarrow \pi\left(\frac{a}{2}\right)^2 = \frac{3}{2}\left[60 - \pi\left(\frac{a}{2}\right)^2\right]$

$\Rightarrow 5\pi\left(\frac{a}{2}\right)^2 = 180 \Rightarrow \frac{\pi a^2}{4} = 36$

$a = \sqrt{\frac{36 \times 4}{\pi}} = \frac{12}{\sqrt{\pi}}$ units .

51. b



Let the radius of the bigger circle be R and that of the smaller circle be r and the side of the square is 2a.

$\therefore \quad OE = R - EF$

$= R - [2R - (2r + 2a)]$

$OE^2 + EB^2 = OB^2$

i.e $[2a + 2r - R]^2 + a^2 = R^2$

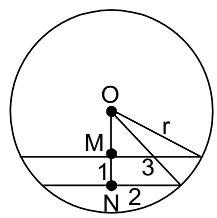
$a = 9$ ($\because$ 2a = 18); R = 15

$\therefore \quad (18 + 2r - 15)^2 + 9^2 = 15^2$

$\therefore \quad 2r + 3 = 12$

$\therefore \quad r = \frac{9}{2} = 4.5$ cm

52. c



Using Pythagoras formula,

$r^2 - 9 = OM^2$.

$r^2 = 4 + (OM + 1)^2$.

$r^2 = 4 + OM^2 + 1 + 2\,OM$ ....(i)

Now put $OM = \sqrt{r^2 - 9}$ in eq. (i)

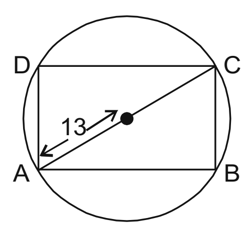
$r^2 = 4 + r^2 - 9 + 1 + 2\sqrt{r^2 - 9}$

$4 = 2\sqrt{r^2 - 9}$

$r^2 = 13$

$r = \sqrt{13}$ cm

53. b



AC = 26 cm

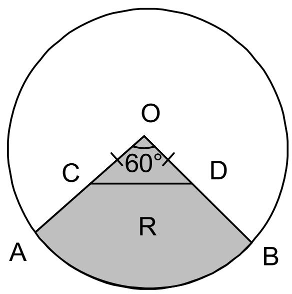
So, $AB^2 + BC^2 = (26)^2 = 576$

Among the options, length = 24 and breadth = 10cm, will satisfy the above equation

$(24)^2 + (10)^2 = 576$

Hence, (24, 10) can be the possible pair.

54. c



Area of $\Delta OCD = \frac{1}{2}$ Area of region R.

$\frac{1}{2} \times \pi (1)^2 \times \frac{60}{360} = \frac{\pi}{12}$ sq. cm

As OC = CD and $\angle COD = 60°$

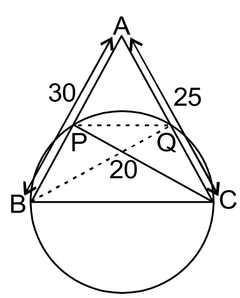
$\therefore \angle OCD = \angle ODC = 60°$

Hence, ΔOCD is an equilateral Δ.

$\therefore \frac{\sqrt{3}}{4} OC^2 = \frac{\pi}{12}$

$OC = \left(\frac{\pi}{3\sqrt{3}}\right)^{\frac{1}{2}}$ cm.

55. 24 Triangles BPC and BQC will be right angled triangles as they are made inside semicircle.



Since, area of triangle ABC

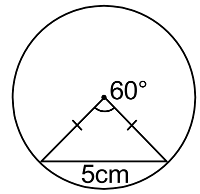
$= \frac{1}{2} \times AB \times CP = \frac{1}{2} \times AC \times BQ$

$\Rightarrow AB \times CP = AC \times BQ$

$\Rightarrow 30 : 20 = 25 \times BQ$

Hence, BQ = 24 cm.

56. c The chord extends an angle of 60° at the center so an equilateral triangle will be formed with the other two sides being the radius of the circle.



Thus, radius = 5 cm

Let the length of the chord that extends 120° be 'a' cm.

$\Rightarrow a^2 = 5^2 + 5^2 - 2 \times 5 \times 5\cos 120°$

....(using cosine rule)

Hence, a = $5\sqrt{3}$ cm.

Mensuration

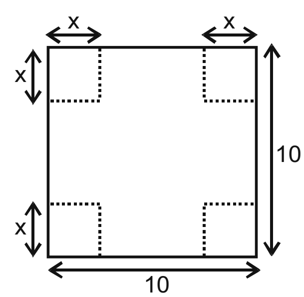
1. a If x = 1, then for the rectangular box : l = 8, b = 8 and h = 1, so volume = 64.

If x = 2, l = 6, b = 6 and h = 2 and volume = 72.

If x = 3, l = 4, b = 4 and h = 3 and volume = 48.

Therefore, we can see that for a value of x between 2 and 3, the volume of box decreases and will go on decreasing further as x increases.

Hence, the maximum volume that the box can have is 72 cubic inches.



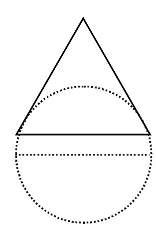
2. b Let l be the length of the rod, then

Volume of slab = Volume of rod

$\Rightarrow 8 \times 11 \times 2 = \pi \left(\frac{8}{2}\right)^2 \times l\pi$

$\Rightarrow l = 3.5$ inches.

3. c It can be seen that if a spherical ball is placed inside a hollow cone of same diameter, the ball won't go up to the diameter. In other words, because of the slanting edges of the cone, only less than 50% of the ball would enter the cone. i.e, more than 50% of the ball would be outside the cone.



4. a If the diameters and the heights of a cone and a cylinder are same, then the volume of cone is always 1/3rd the volume of the cylinder.

So the ratio of the volume of cone to the volume of cylinder = 1 : 3.

The only answer choice that supports this is (a).

5. b Let the radius and height of original cone be 'r' and 'h' respectively.

$\therefore$ The volume of the original cone $(V) = \frac{\pi r^2 h}{3}$.

The height and radius of the smaller cone are $\frac{2h}{3}$ and $\frac{2r}{3}$ respectively.

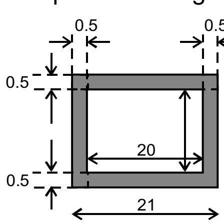
So its volume $= \frac{\pi}{3} \times \left(\frac{2r}{3}\right)^2 \times \frac{2h}{3} = \frac{8V}{27}$.

$\therefore$ Volume of frustum = $\left(V - \frac{8V}{27}\right) = \frac{19V}{27}$.

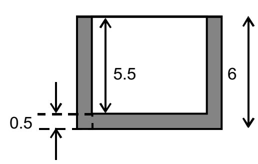
$\therefore$ Ratio of the volumes = 8 : 19.

6. d From the statement I, we can find out the area that needs to be bordered. And from the statement II, we can find out the cost of each tile. But to find the total cost, we require the total number of tiles and to find this we require the dimension of each tile. Since this is not known, we cannot answer the question using either statements.

7. c



When seen from top



When seen from front

As it can be seen from the diagram, because of the thickness of the wall, the dimensions of the inside of the box is as follows: length = (21 – 0.5 – 0.5) = 20 cm, width = (11 – 0.5 – 0.5) = 10 cm and height = (6 – 0.5)= 5.5 cm.

Total number of faces to be painted = 4 walls + one base (as it is open from the top).

The dimensions of two of the walls = (10 × 5.5), that of the remaining two walls = (20 × 5.5) and that of the base = (20 × 10).

So the total area to be painted = 2 × (10 × 5.5) + 2 × (20 × 5.5) + (20 × 10) = 530 cm^2.

Since the total expense of painting this area is Rs. 70, the rate of painting = $\frac{70}{530}$ = 0.13 = Re. 0.1 per sq.cm. (approximately).

8. b Let V_1 be the original volume and r_1 and h_1 be the radius and height of the cone respectively.

$V_1 = \left(\frac{1}{3}\right) \times \pi \times (r_1)^2 \times h_1$. Consider statement I.

If the cone is cut parallel to base and dividing the height in the ratio 1 : 2, then $r_2 = (\frac{1}{2}) \times r_1$ and $h_2 = (\frac{1}{2}) \times h_1$, where r_2 and h_2 are the radius and height of the new cone respectively. If V_2 is the volume of new cone, then $V_2 = (\frac{1}{3}) \times \pi (r_2)^2 \times h_2$

$= (\frac{1}{3}) \times \pi \times (\frac{1}{2} \times r_1)^2 \times ((\frac{1}{2})h_1) = (\frac{1}{8}) \times V_1$

Hence, statement I alone is sufficient to answer the question (as we get the ratio as 1 : 8).

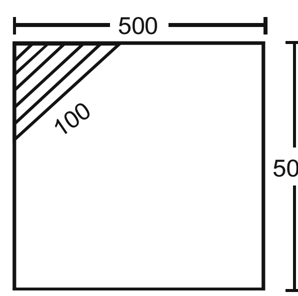
Similarly, based on statement II alone, we can find the ratio (which will be 1 : 27).

9. c Statement I gives the thickness of the wall which is of no use to find the volume of the tank since we do not know the radius of the sphere.

Statement II gives us the answer as the volume of water displaced is equal to the volume of the immersed tank (from Archimedes' principle).

So to find the exact storage volume of the tank both the statements are needed.

10. a



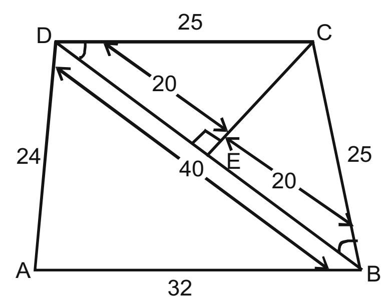
Area of shaded region $= \frac{1}{2} \times \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}} = 2{,}500$ sq m

Area of a Δ is maximum when it is an isosceles Δ.

So perpendicular sides should be of length $\frac{100}{\sqrt{2}}$ m.

11. d Work with options. Length of wire must be a multiple of 6 and 8. Number of poles should be one more than the multiple.

12. d



$CE = \sqrt{25^2 - 20^2} = 15$

(Since DBC is isosceles triangle.)

ABCD is a quadrilateral where AB = 32 m, AD = 24 m, DC = 25 m, CB = 25 m and $\angle DAB$ is right angle.

By Pythagoras Theorem: DB = 40 m

So area of Δ ADB $= \frac{1}{2} \times 32 \times 24 = 384$ sq. m

Now in isosceles ΔBCD, perpendicular CE from C to BD bisects BD.

$BE = DE = \frac{40}{2} = 20$ m.

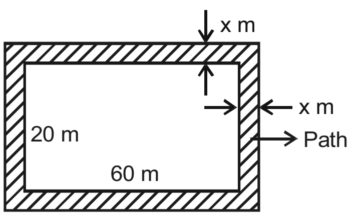
Now by Pythagoras, Theorem:

$CE = \sqrt{25^2 - 20^2} = 15$ m.

So area of $\Delta BCD = 2 \times \frac{1}{2} \times 15 \times 20 = 300$ sq. m

Hence, area of ABCD = 384 + 300 = 684 sq. m

13. c



Let width of the path be x metres.

Then area of the path = 516 sq. m

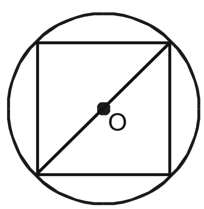
$\Rightarrow$ (60 + 2x)(20 + 2x)– 60 × 20 = 516

$\Rightarrow$ 1200 + 120x + 40x + $4x^2$ – 1200 = 516

$\Rightarrow$ $4x^2 + 160x - 516 = 0 \Rightarrow x^2 + 40x - 129 = 0$

Using the answer choices, we get x = 3 m.

14. b



We know that the diameter of circle will be the diagonal of the square.

Thus, from any of the two statements, we can find out the areas of the circle and square.

15. a Total area = 14 × 14 = 196 m^2

Grazed area $= \left(\frac{\pi \times r^2}{4}\right) \times 4 = \pi r^2 = 22 \times 7$ (r = 7 m) = 154 m^2

Ungrazed area is less than (196 – 154) = 42 m^2, for which there is only one option i.e. 22 m^2.

16. c Area = 40 × 20 = 800 m^2.

If 3 rounds are done, area = 34 × 14 = 476 m^2

$\Rightarrow$ Area > 3 rounds

If 4 rounds $\Rightarrow$ Area left = 32 × 12 = 347 m^2

Hence, area should be slightly less than 4 rounds.

For questions 17 to 19:

$A_1A_2 = 2r$, $B_1B_2 = 2r + r\sqrt{3}$, $C_1C_2 = 2r + 2r\sqrt{3}$

Hence, a = 3 × 2r;b = 3 × (2r + $r\sqrt{3}$); c = 3 × $\left(2r + 2r\sqrt{3}\right)$

17. a Difference between (a)and (b) is $3\sqrt{3}r$ and that between (b) and (c)is $3\sqrt{3}r$. Hence, (a) is the correct choice.

18. c Time taken by $A = \frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \left(\frac{2r \times 9}{60}\right) = \frac{3}{10}r$

Therefore, B and C will also travel for time $\frac{3}{10}r$.

Now speed of B = $\left(10\sqrt{3} + 20\right)$

Therefore, the distance covered

$= \left(10\sqrt{3} + 20\right) \times \frac{3}{10}r = \left(\sqrt{3} + 2\right) \times 10 \times \frac{3}{10}r$

$= \left(2r + \sqrt{3}r\right) \times 3 = B_1B_2 + B_2B_3 + B_3B_1$

$\therefore$ B will be at B_1.

Now time taken by for each distance are

$\frac{C_1C_2}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_2C_3}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_3C_1}{120}$

$\frac{3}{40} \times \frac{\left(2 + 2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{3}{40} \times \frac{\left(2 + 2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{\left(2 + 2\sqrt{3}\right)r}{120}$

i.e. $\frac{3}{40} \times 2r, \frac{3}{40} \times 2r, \frac{\left(1 + \sqrt{3}\right)}{60}r$

i.e. $\frac{3}{20}r, \frac{3}{20}r, \frac{\left(1 + \sqrt{3}\right)}{60}r$

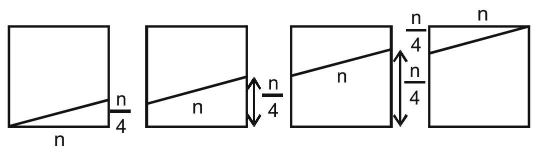
We can observe that time taken for C_1C_2 and C_2C_3 combined is $\frac{3}{20}r + \frac{3}{20}r = \frac{3}{10}r$, which is same as time taken by A.Therefore, C will be at C_3.

19. b In similar triangles, ratio of Area = Ratio of squares of corresponding sides.

Hence, A and C reach A_3 and C_3 respectively.

20. a The whole height h will be divided into n equal parts. Therefore, spacing between two consecutive turns = $\frac{h}{n}$.

21. b The four faces through which string is passing can be shown as:



Therefore, length of string in each face

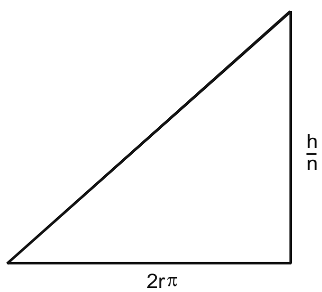
$$= \sqrt{n^2 + \left(\frac{n}{4}\right)^2} = \sqrt{n^2 + \frac{n^2}{16}} = \frac{\sqrt{17}n}{4}$$

Therefore, length of string through four faces

$$= \frac{\sqrt{17}n}{4} \times 4 = \sqrt{17}n$$

22. c Length of string is same.

Let us open 1 turn of string around cylinder.



So length of string

$$= n \times \sqrt{(2\pi r)^2 + \frac{h^2}{n^2}} = n\sqrt{4^2 + \frac{h^2}{n^2}}$$

From previous question length of string $= \sqrt{17}n$.

$$\Rightarrow \sqrt{17}n = n\sqrt{16 + \frac{h^2}{n^2}} \Rightarrow h = n$$

23. a Consider a square of side x.

Therefore, its area = x^2

Therefore, area of the largest circle = $\pi\left(\frac{x}{2}\right)^2$,

which can be cut from square = $\frac{\pi x^2}{4}$.

Therefore, area scrapped = $x^2 - \frac{\pi}{4}x^2 = x^2\left(1 - \frac{\pi}{4}\right)$

$$\therefore \frac{\text{Area scrapped}}{\text{Area of square}} = \frac{x^2\left(1-\frac{\pi}{4}\right)}{x^2} = 1 - \frac{\pi}{4} = \text{Contant}$$

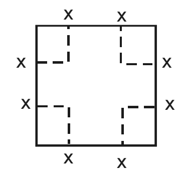
As this ratio is constant whether we cut a circle from small square or larger square, scrapped area will be a fixed percentage of square. Therefore, in our problem as two squares are of the same size, the ratio will be 1 : 1.

24. c It's standard property among circle, square and triangle, for a given parameter, area of circle is the highest and area of the triangle is least whereas area of the square is in-between, i.e. c > s > t.

25. d The required answer is 34 × 0.65 × 0.65 = 14.365

Because we get two similar triangles and area is proportional to square of its side.

26. d



Volume = l × b × h

$V = (12 - 2x)(12 - 2x) \times x$

$V' = (12 - 2x)\ (12 - 2x)\ 4x$

Where V' = 4 V

Now sum = 12 – 2x + 12 – 2x + 4x = 24 [Constant]

As sum is constant for maximum product 12 – 2x = 12 – 2x = 4x

Therefore, x = 2

27. d The surface area of a sphere is proportional to the square of the radius.

Thus, $\frac{S_B}{S_A} = \frac{4}{1}$ (Surface area of B is 300% higher than A)

$\therefore \frac{r_B}{r_A} = \frac{2}{1}$

The volume of a sphere is proportional to the cube of the radius.

Thus, $\frac{V_B}{V_A} = \frac{8}{1}$

Or, V_A is $\frac{7}{8}$th less than B i.e. $\frac{7}{8} \times 100 = 87.5\%$.

28. b If the radius of the field is r, then the total area of the field = $\frac{\pi r^2}{2}$.

The radius of the semi-circles with centre's P and $R = \frac{r}{2}$.

Hence, their total area = $\frac{\pi r^2}{4}$

Let the radius if the circle with centre S be x.

Thus, OS = (r – x), OR = $\frac{r}{2}$ and $RS = \left(\frac{r}{2} + x\right)$.

Applying Pythagoras Theorem, we get

$$(r - x)^2 + \left(\frac{r}{2}\right)^2 = \left(\frac{r}{2} + x\right)^2$$

Solving this, we get $x = \frac{r}{3}$.

Thus the area of the circle with centre $S = \frac{\pi r^2}{9}$.

The total area that can be grazed

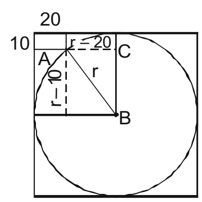
$$= \pi r^2\left(\frac{1}{4}+\frac{1}{9}\right) = \frac{13\pi r^2}{36}$$

Thus the fraction of the field that can be grazed

$$= \frac{26}{36}\left(\frac{\text{Area that can be grazed}}{\text{Area of the field}}\right)$$

The fraction that cannot be grazed = $\frac{10}{36}$ = 28% (approx.)

29. c

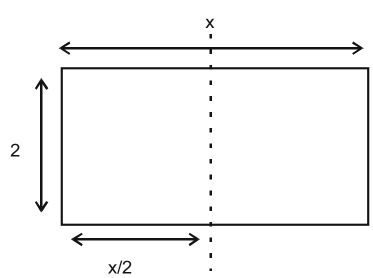


Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have $(r-10)^2 + (r-20)^2 = r^2$

i.e. $r^2 - 60r + 500 = 0$. Thus r = 10 or 50.

It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

30. b



In original rectangle ratio = $\frac{x}{2}$

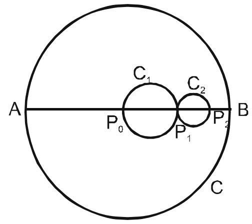
In smaller rectangle ratio = $\frac{2}{\left(\frac{x}{2}\right)}$

Given $\frac{x}{2} = \frac{2}{\frac{x}{2}} \Rightarrow x = 2\sqrt{2}$

Area of smaller rectangle

$$= \frac{x}{2}\times 2 = x = 2\sqrt{2}\text{ sq. units}.$$

31. d



Circle	Radius
C	r
C_1	$\frac{r}{4}$
C_2	$\frac{r}{8}$
C_3	$\frac{r}{16}$
⋮	⋮

$$\Rightarrow \frac{\text{Area of unshaded portion of C}}{\text{Area of C}}$$

$$= 1 - \frac{\text{Area of shaded portion}}{\text{Area of C}}$$

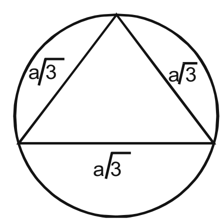
$$= 1 - \frac{\pi\left(\left(\frac{r}{4}\right)^2 + \left(\frac{r}{8}\right)^2 + \ldots\right)}{\pi r^2}$$

$$= 1 - \left(\frac{1}{4^2} + \frac{1}{8^2} + \ldots\right) = 1 - \frac{\frac{1}{16}}{1-\frac{1}{4}} = \frac{11}{12}$$

32. a DF, AG and CE are body diagonals of cube.

Let the side of cube be a.

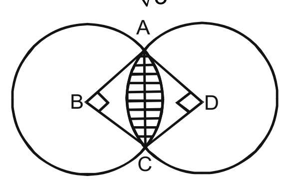
Therefore, body diagonal is $a\sqrt{3}$



Circum radius for equilateral triangle = $\frac{\text{Side}}{\sqrt{3}}$

Therefore, $\frac{a\sqrt{3}}{\sqrt{3}} = a$

33. b

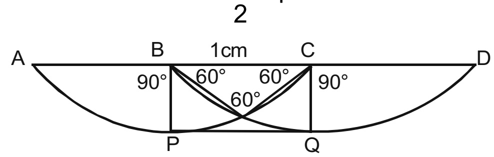


Shaded area = 2 × (area of sector ADC – area of ΔADC)

$$= 2\times\left(\frac{\pi}{4}\times 1^2 - \frac{1}{2}\times 1\times 1\right)$$

$$= \frac{\pi}{2} - 1$$

34. b



Drawn figure since it have not to be within distance of 1 m so it will go along APQD, which is the path of minimum distance.

$$AP = \frac{90}{360} \times 2\pi \times 1 = \frac{\pi}{2}$$

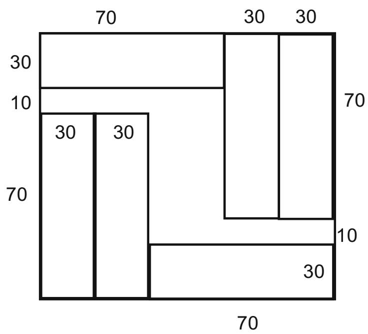
Also, $AP = QD = \frac{\pi}{2}$

So the minimum distance

$= AP + PQ + QD$

$= \frac{\pi}{2} + 1 + \frac{\pi}{2} = 1 + \pi$

35. c



36. b Let the rectangle has m and n tiles along its length and breadth respectively.

The number of white tiles

$W = 2m + 2(n - 2) = 2(m + n - 2)$

And the number of Red tiles $= R = mn - 2(m + n - 2)$

Given W = R

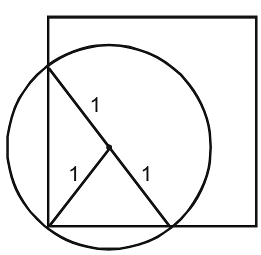
$\Rightarrow$ $4(m + n - 2) = mn$

$\Rightarrow$ $mn - 4m - 4n = -8$

$\Rightarrow$ $(m - 4)(n - 4) = 8$

As m & n are integers so $(m - 4)$ & $(n - 4)$ are both integers. The possibilities are $(m - 4, n - 4) \equiv (1, 8)$ or $(2, 4)$ giving, (m, n) as $(5, 12)$ or $(6, 8)$ so the edges can have 5, 12, 6 or 8 tiles.

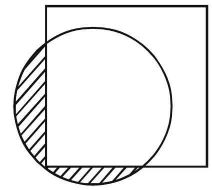
37. b



Remaining area $= 4 - \left(\frac{\pi}{2} + \frac{1}{2} \times 1 \times 2\right) = \frac{6 - \pi}{2}$

Remaining proportion $= \frac{6 - \pi}{8}$

38. d



Area $= \pi(1)^2 - \left(\frac{\pi}{2} + 1\right) = \pi - \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

39. b **Using A:** Inner radius of the tank is atleast 4 m.

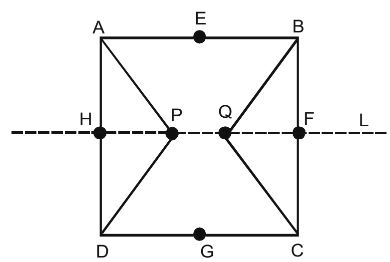
So volume $= \frac{4}{3}\pi r^3$ where $4 < r < 10$

This volume can be greater as well as smaller than 400 for different r.

Using B: The given data gives the volume of the material of tank, which can be expressed as $\frac{4}{3}\pi(10^3 - r^3)$, which will give the value of r which is unique and sufficient to judge if the capacity is adequate.

Hence, option (b)is correct choice.

40. e



Let the length of AH be 'x' cm.

By symmetry of the figure given above, we can conclude that ΔAPD and ΔBQC will have the same area.

$\because \angle APD$ is 120° and line 'L' divides the square ABCD in 2 equal halves, therefore $\angle APH = \angle HPD = 60°$

In $\Delta AHP : \frac{AH}{HP} = \tan 60° = \sqrt{3} \Rightarrow HP = \frac{x}{\sqrt{3}}$ cm

Area of $\Delta APD = 2 \times \text{area}(\Delta AHP)$

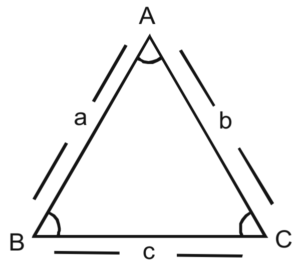
$$= 2 \times \frac{1}{2} \times x \times \frac{x}{\sqrt{3}} = \frac{x^2}{\sqrt{3}} \text{ cm}$$

Area of ABQCDP = area (ABCD) – 2 area (ΔAPD)

$$= 4x^2 - \frac{2x^2}{\sqrt{3}} = \frac{2x^2\left(2\sqrt{3} - 1\right)}{\sqrt{3}}$$

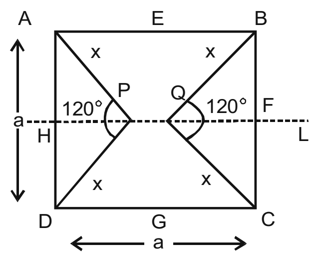
$$\text{Required ratio} = \frac{\frac{2x^2\left(2\sqrt{3} - 1\right)}{\sqrt{3}}}{\frac{2x^2}{\sqrt{3}}} = 2\sqrt{3} - 1$$

Alternate method: Concepts used:



$\Rightarrow \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Also, area of Δ ABC $= \dfrac{1}{2}$ ab sin C $= \dfrac{1}{2}$ bc sin A $= \dfrac{1}{2}$ ac sin B. In the given figure



For ΔAPD, Let AP = PD = x cm

$\Rightarrow \dfrac{a}{\sin 120°} = \dfrac{x}{\sin 30°} = \dfrac{x}{\sin 30°}$

$\Rightarrow$ sin 120° = sin (90° + 30°)

$= \cos 30° = \dfrac{\sqrt{3}}{2}$, $\sin 30° = \dfrac{1}{2}$

$\Rightarrow \dfrac{a}{\frac{\sqrt{3}}{2}} = \dfrac{x}{\frac{1}{2}}$

$\Rightarrow x = \dfrac{a}{\sqrt{3}}$ cm

Thus, area of ΔAPD is $\dfrac{1}{2} \times AP \times PD \times \sin 120°$

$= \dfrac{1}{2} \times \dfrac{a}{\sqrt{3}} \times \dfrac{a}{\sqrt{3}} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{a^2}{4\sqrt{3}} cm^2$

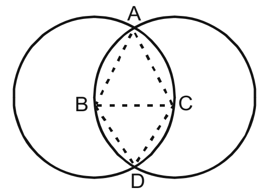
by symmetry, Area of ΔAPD = Area of ΔBQC

Thus, ratio of $\dfrac{\text{Area of ABQCDP}}{\text{[Removing area inside square ABCD}}$

$= \dfrac{\text{Area of square ABCD} - 2 \times (\text{Area of } \Delta APD)}{2 \times (\text{Area of } \Delta \text{ APD})}$

$= 2\sqrt{3} - 1$

41. e



It is given that AB = BC = AC = BD = DC = 1 cm.

Therefore, ΔABC is an equilateral triangle.

Hence, $\angle ACB = 60°$

Now area of sector $\overset{\frown}{AB} = \dfrac{60}{360} \times \pi(1)^2 = \dfrac{\pi}{6}$

Area of equilateral triangle $\Delta ABC = \dfrac{\sqrt{3}}{4}(1)^2 = \dfrac{\sqrt{3}}{4}$

Area of remaining portion in the common region

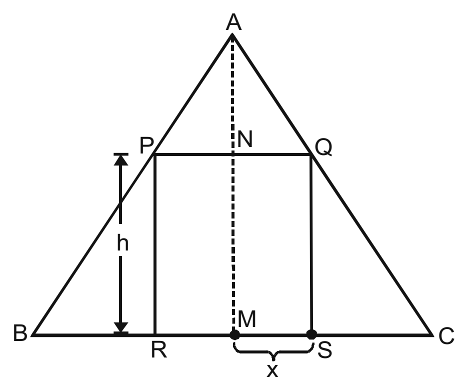
$\overset{\frown}{ABC}$ excluding ΔABC $= 2 \times \left(\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}\right)$

Hence, the total area of the intersecting region

$= 2 \times \dfrac{\sqrt{3}}{4} \times (1)^2 + 4 \times \left(\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}\right)$

$= \dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}$ sq. cm.

42. a



Let the height of the cylinder be 'h' cm and radius be 'x' cm.

ΔANQ is similar to ΔQSC

$\Rightarrow \dfrac{AN}{NQ} = \dfrac{QS}{SC} \Rightarrow \dfrac{10-h}{x} = \dfrac{h}{4-x}$

$\Rightarrow \dfrac{10}{h} - 1 = \dfrac{x}{4-x} \Rightarrow \dfrac{10}{h} = \dfrac{4}{4-x}$

$\therefore\ h = \dfrac{5}{2}(4-x)$

Surface area of the cylinder PQSR

$= 2\pi\left[x^2 + hx\right] = 2\pi\left[x^2 + \dfrac{5x}{2}(4-x)\right]$

$= 2\pi\left[x^2 - \frac{5}{2}x^2 + 10x\right] = 2\pi\left[10x - \frac{3}{2}x^2\right]$

$= 2\pi\left[-\frac{3}{2}\left(x - \frac{10}{3}\right)^2 + \frac{50}{3}\right]$

Maximum value of surface area of the cylinder will be at $x = \frac{10}{3}$.

43. a Let's assume that the rectangle has m and n tiles along its length and breadth respectively.

The number of white tiles

$W = 2m + 2(n - 2)$
$= 2(m + n - 2)$

The number of red tiles

$R = (m - 2)(n - 2)$
$= mn - 2m - 2n + 4$

Also, $2W = R$ (given)

$\Rightarrow$ $4m + 4n - 8 = mn - 2m - 2n + 4$

$\Rightarrow$ $mn - 6m - 6n + 12 = 0$

$\Rightarrow$ $(m - 6)(n - 6) - 36 + 12 = 0$

$\Rightarrow$ $(m - 6)(n - 6) = 24$

As m and n are integers, both (m – 6) and (n – 6) are integers as well. The possible sets of values where m, n are positive integers:

(m – 6, n – 6) = (24,1), (12, 2), (8, 3), (6, 4)

So (m, n) = (30, 7), (18, 8), (14, 9), (12, 10)

The maximum possible difference

= R – W = 2W – W = W

= 2(m + n – 2)

= 2(30 + 7 – 2)

= 70

44. a Let the radius of the cross section of the pipe be r.

Speed (v) at which water flows

= 54 km/hr = 54000 m/hr

Rate of water flow

= (Cross-sectional area of the pipe) × v

$\therefore \quad \pi r^2 \times 54 \times 10^3 \times 14 = \frac{80}{100} \times 118800$

$\Rightarrow \quad r = \frac{2}{10} m = 20$ cm.

45. c There are 8 smaller cubes (on the corners) which have exactly three sides painted.

There are 7 × 12 i.e. 84 smaller cubes (on the edges) which have exactly two sides painted.

There are 7 × 7 × 6 i.e. 294 smaller cubes (on the faces) which have exactly one side painted.

The total number of smaller cubes with at least one side painted = 8 + 84 + 294 = 386

So the total number of smaller cubes with none of the sides painted = 729 – 386 = 343.

Alternate Method:

Each edge of the larger cube is made of 9 smaller cubes. It can be observed that there is another cube whose edge is made of 7 smaller cubes which lies inside this larger cube, such that none of the cubes in it makes to the surface of the larger cube (and didn't get painted as a result).

The total number of smaller cubes in this cube = $7^3 = 343$.

46. c Volume of each smaller cube

$= \frac{12 \times 12 \times 12}{64} = 27 \text{ cm}^3$

Edge of each smaller cube = 3 cm

Let the number of cubes along the length and the breadth of the cuboid be 4x and x respectively.

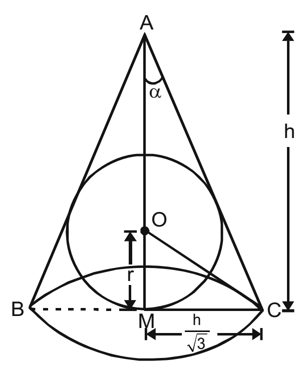
$4x \times x = 64$ or $x = 4$

Length of the cuboid = 4x × 3 = 48 cm

Breadth of the cuboid = x × 3 = 12 cm

Required surface area = 48 × 12 = 576 cm^2

47. a



Let the semi vertical angle of the cone be α.

$\tan\alpha = \frac{h/\sqrt{3}}{h} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^\circ$

The vertical angle of the cone = 60°

Therefore, the triangle ABC is an equilateral triangle.

The given case i.e. placing of a sphere of the maximum volume inside a cone will be identical to the case of inscribing a circle inside a triangle. Therefore, the center (O) of the circle will coincide with the centroid of the triangle ABC.

In triangle ACM, OC is angle bisector of $\angle$ACM.

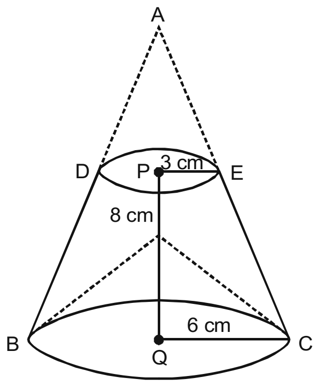
$\therefore \quad \angle OCM = 30^\circ$

In ΔOCM,

$\tan 30^\circ = \dfrac{r}{h/\sqrt{3}} \Rightarrow r = \dfrac{h}{3}$

Hence, the ratio of the volume of the sphere to that of the cone $= \dfrac{\frac{4}{3}\pi\left(\frac{h}{3}\right)^3}{\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 \times h} = 4:9.$

48. a



In the figure given above, ABC represents the actual cone from which the frustum was made. DEBC represents the frustum and BRC represents the conical cavity.

Now, the volume of the material in the solid can be calculated by subtracting the volume of the cone ADE and the conical cavity BRC from the cone ABC.

ΔAPE and ΔAQC are similar,

$\Rightarrow \quad \dfrac{AP}{AQ} = \dfrac{PE}{QC}$

$\Rightarrow \quad AP = 8$ cm.

Volume of cone ABC $= \dfrac{1}{3}\pi(6)^2 \times 16$

Volume of cone APE $= \dfrac{1}{3}\pi(3)^2 \times 8$

Volume of conical cavity BRC = $\dfrac{1}{3}\pi(6)^2 \times 3$

Required volume of solid

$= \dfrac{1}{3}\pi(6)^2 \times 16 - \dfrac{1}{3}\pi(3)^2 \times 8 - \dfrac{1}{3}\pi(6)^2 \times 3$

$= 132\pi$ cm^3.

49. b Since the HCF of 24 and 56 is 8, the side of the identical square plots must be one of the factors of 8.

The factors of 8 are 1, 2, 4 and 8

If side of the square plot is 1 m, the length of fencing material required is (25 × 56 + 57 × 24) = 2768 m

But 2768 m > 2700 m.

If side of the square plot is 2 m, the length of fencing material required is

(29 × 24 +13 × 56) = 1464 m < 2600 m

$\therefore$ For minimum fencing material to be left the side of identical square plot = 2m.

50. Volume of a cylinder = $\pi r^2 h$, where

r = radius of the cylinder

h = height of the cylinder

Volume of the graphite cylinder

$$= \pi\left(\frac{1}{10}\right)^2 \times 10 = \frac{\pi}{10}\text{ cm}^3$$

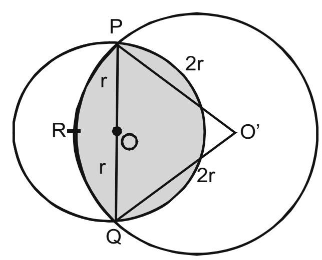
Volume of the layer of wood

$$= 10\pi\left[\left(\frac{7}{10}\right)^2 - \left(\frac{1}{10}\right)^2\right] = \frac{48\pi}{10}\text{ cm}^3$$

Cost of the material in a pencil

$$= \frac{\pi}{10} \times 2.10 + \frac{48\pi}{10} \times 0.70 = \text{Rs.}11.22$$

51.



The common chord will be of the maximum length if it is the diameter of the smaller circle.

In ΔPO'Q, PQ = 2r and PO' = QO' = 2r, therefore, ΔPO'Q is an equilateral triangle.

Required area (in sq. units) of the shaded region

$= \dfrac{1}{2}$ × Area of smaller circle + Area of segment PRQ

$= \dfrac{1}{2}$ × Area of smaller circle + (Area of sector O'PQ – Area of triangle PO'Q)

$$= \frac{\pi r^2}{2} + \left\{\frac{\pi(2r)^2}{6} - \frac{\sqrt{3}(2r)^2}{4}\right\} = \frac{7\pi r^2}{6} - \sqrt{3}r^2.$$

52. Let the original volume of the raindrop be 'x' ml.

Therefore, initial volume of water in the raindrop will be 0.75x ml.

Remaining volume of water after evaporation

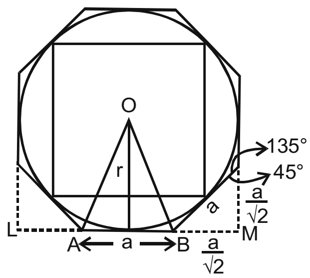
= (0.75x – 2) ml

Remaining volume of water is 70% of (x – 2) ml.

$\therefore \quad 0.75x - 2 = (x - 2) \times 0.7$

$\Rightarrow \quad x = 12.$

53. Let the length of the side of the octagon and the radius of the circle be 'a' units and 'r' units respectively. Therefore, the length of the side of the square will be $\left(\frac{2r}{\sqrt{2}}\right)$ units.



LM = LA + AB + BM

$$\Rightarrow \quad 2r = \frac{a}{\sqrt{2}} + a + \frac{a}{\sqrt{2}}$$

$$\Rightarrow \quad r = \frac{a(1+\sqrt{2})}{2}$$

Hence, the ratio $= \dfrac{\left(\frac{2r}{\sqrt{2}}\right)^2}{8 \times \frac{1}{2} \times r \times a}$

$$= \frac{r}{2a}$$

$$= \frac{a\left(\frac{1+\sqrt{2}}{2}\right)}{2a} = \left(1+\sqrt{2}\right) : 4.$$

54. Let the side of the squares be 'a' cm.

So the dimensions of the open box will be:

Length = (30 – 2a) cm

Breadth = (20 – 2a) cm

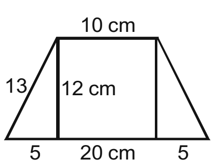
Height = a cm

$\therefore$ (30 – 2a) × (20 – 2a) × a = 1056

$\Rightarrow$ (15 – a) × (10 – a) × a = 264 = 11 × 6 × 4

Hence, a = 4.

55.



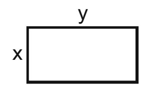
Area of a base = $\frac{1}{2}(10+20) \times 12 = 180$ cm

Total area of all six surfaces of the pillar is

= 10 × 20 + 20 × 20 + 2 × 180 + 2 × (13 × 20)

= 200 + 400 + 360 + 520 = 1480 cm^2.

56. 00



So x + 2y = 400

x = 400 – 2y

Area = xy

= (400 – 2y)y

= 400 y – 2y^2

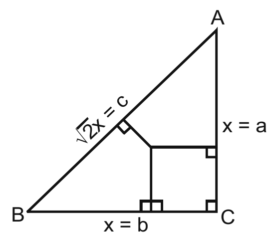
Maximum value of quadratic function is $\frac{-b}{2a}$ (when a is –ve)

So, value of y = $\frac{-400}{-2 \times 2} = 100,$

So if y = 100

x = 200.

57. 6



In-radius of a right $\angle\Delta$ is r = $\frac{a+b-c}{2}$

$$\Rightarrow 4(\sqrt{2}-1) = \frac{x+x-\sqrt{2}x}{2}$$

$$\Rightarrow 4(\sqrt{2}-1) = \frac{2x-\sqrt{2}x}{2}$$

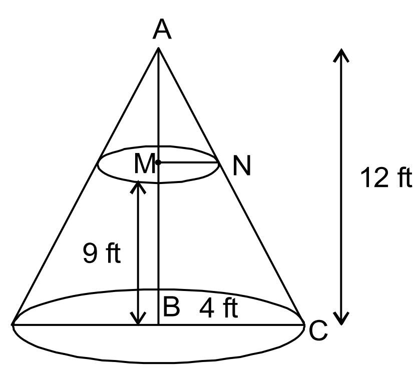
$$\Rightarrow 4(\sqrt{2}-1) = \frac{\sqrt{2}x(\sqrt{2}-1)}{2}$$

x = $4\sqrt{2}$ cm.

$\therefore$ Area $= \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2}$

= 16 cm^2

58.



AB = 12 ft, MB = 9 ft

$\therefore$ AM = 3ft

As $\Delta AMN \sim \Delta ABC$

$\therefore \frac{12}{4} = \frac{3}{MN}$

$\Rightarrow MN = 1$ ft

$\therefore$ Volume of the remaining part of the cone

$$= \frac{1}{3}\pi(4)^2 \times 12 - \frac{1}{3} \times \pi(1)^2 \times 3.$$

$$= \frac{1}{3}\pi \times 3[64 - 1]$$

$$= \frac{22}{7} \times 63 = 198 \text{ cubic ft.}$$

Co-ordinate Geometry

1. c The distance between two points (x_1, y_1) and (x_2, y_2) is given as $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Hence, required distance

$= \sqrt{(-2-3)^2 + (-7-8)^2}$

$= 5\sqrt{10}$.

2. d The three lines can be expressed as

$$Y = \frac{5}{3} - \frac{2X}{3},\ Y = \frac{5X}{7} + \frac{2}{7} \text{ and } Y = \frac{9X}{5} - \frac{4}{5}.$$

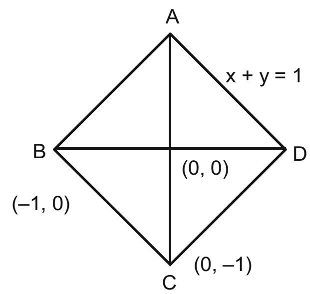
Therefore, the slopes of the three lines are $\frac{-2}{3}, \frac{5}{7}$ and $\frac{9}{5}$ respectively and their Y intercepts are $\frac{5}{3}, \frac{2}{7}$ and $\frac{4}{5}$ respectively. For any two lines to be perpendicular to each other, the product of their slopes = –1. We find that the product of none of the slopes is –1. For any two be parallel, their slopes should be the same. This is again not the case. And finally for the two lines to be intersecting at the same point, there should be one set of values of (X, Y)that should satisfy the equations of 3 lines. Solving the first two equations, we get X = 1 and Y = 1. If we substitute this in the third equation, we find that it also satisfies that equation. So the solution set (1, 1)satisfies all three equations, suggesting that the three lines intersect at the same point, viz. (1, 1). Hence, they are coincident.

3. c If we solve the two given equations, we get the point of intersection as (3, 2). Let A = (3, 2). The lines of our interest (let it be L_1 and L_2)also pass through A. One of the lines passes through (0, 4). Let L_1 passes through (0, 4), but it also passes through (3, 2). Hence, we can find the slope of L_2 (which is equal to $-\frac{2}{3}$). Hence, slope of L_2 will be $\frac{3}{2}$ since L_1 and L_2 are perpendicular. Hence, equations of L_1 and L_2 can be obtained by using slope point form. (Students! we need not really find out the equations.) After getting both the equations, we can find the area bounded by L_1 and L_2 and coordinate axes.

4. c The final point is (6, 6). The previous point is (6, 2) and the one before is (4, 2).

5. a Two instructions are needed, one parallel to the X-axis and the other parallel to the Y-axis. i.e. WALKX(–x)and WALKY (–y)

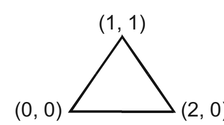
6. a The gradient of the line AD is –1. Coordinates of B are (– 1, 0).



Equation of line BC is x + y = –1.

7. d Both the statements combined also do not tell us if they are intersecting or not. The two lines can be parallel also depending on the values of a, b, d, e.

8. b

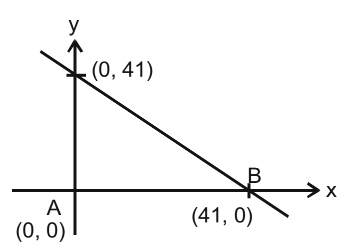


Let a = 0

Hence, area = $\frac{1}{2}(2)\ (1) = 1$

Note: Answer should be independent of a and area of the triangle does not have square root.

9. a



equation of line $\equiv x + y = 41$. If the (x, y) co-ordinates of the points are integer, their sum shall also be integers so that x + y = k (k, a variable) as we have to exclude points lying on the boundary of triangle; k can take all values from 1 to 40 only. k = 0 is also rejected as at k = 0 will give the point

A; which can't be taken.

Now, x + y = k, (k = 1, 2, 3, ...,40)

with k = 40; x + y = 40; taking integral solutions.

We get points (1, 39), (2, 38); (3, 37); ..., (39, 1)

i.e. 39 points

x + y = 40 will be satisfied by 39 points.

Similarly, x + y = 39 is satisfied by 38 points.

x + y = 38 by 37 points.

x + y = 3 by 2 points.

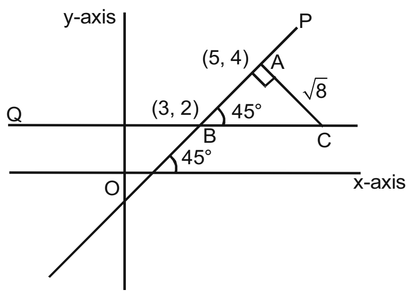
x + y = 2 is satisfied by 1 point.

x + y = 1 is not satisfied by any point.

So, the total no. of all such points is:

$$39+38+37+36+...+3+2+1=\frac{39\times 40}{2}=780 \text{ points.}$$

10. d



As slope of line P is 45°, $\angle ABC = 45°$.

(Corresponding angles)

In triangle ABC, length of AB

$$=\sqrt{(5-3)^2+(4-2)^2}=2\sqrt{2} \text{ units}$$

Therefore, length of AC = $2\sqrt{2}$ units

Required area $=\frac{1}{2}\times 2\sqrt{2}\times 2\sqrt{2}=4$ sq. units

11. d PQ is perpendicular to line $y=\frac{x}{\sqrt{3}}$

$\therefore$ Slop of PQ $=\frac{-1}{\frac{1}{\sqrt{3}}}=-\sqrt{3}$

$\therefore$ Let equation of line PQ be $y=-\sqrt{3}x + c$

At point M, when x = $\sqrt{3}$, y = 1.

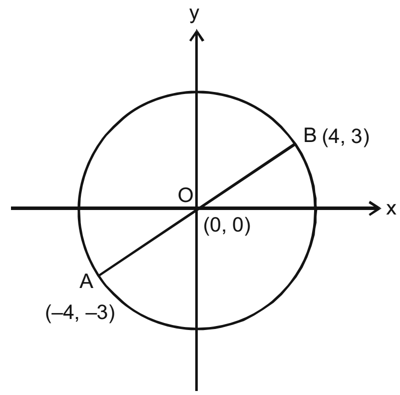
$\therefore$ c = 4

$\Rightarrow$ $y=-\sqrt{3}x+4$

$\therefore$ Co-ordinates of point Q = $\left(\frac{4}{\sqrt{3}}, 0\right)$ and Co-ordinates of point P = (0, 4).

Hence, PQ = $\sqrt{\left(\frac{4}{\sqrt{3}}\right)^2+4^2}=4\sqrt{\frac{1}{3}+1}=\frac{8}{\sqrt{3}}$ units

12. c

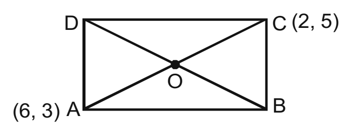


Other two vertices will make two right angled triangles with AB as the common hypotenuse. So they must lie on the circle with AB as the diameter and O as the centre. Radius of that circle will be 5 units.

There will be 5 such pairs in which both the coordinates are integers.

[(5, 0), (–5, 0)], [(–4, 3), (4, – 3)],
[(–3, 4), (3, –4)] [(–3, –4), (3, 4)] and [(0, 5), (0, –5)]

13. d



Since diagonals of a rectangle bisect each other so O is the mid point of AC.

Co-ordinates of O = $\left(\frac{6+2}{2}, \frac{5+3}{2}\right)$ = (4, 4)

Point O will satisfy the equation of the other diagonal too. So,

y = 3x + c

4 = 3 × 4 + c

c = – 8.